Deformation of a Paper Roll Loaded Against a Nip Roller

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Summary An elastoplastic material model accounting for the interlayer slippage of paper layers is used to study the deformation of a paper roll compressed against a rigid nip roller. Calculated load–deformation paths are compared to experimental results. It is found that the interlayer slippage in the nip area plays a key part in the deformation mechanism of the roll.

Introduction

Material constants for paper have been measured and reported by many authors [1], [2], [3]. However, the elastic material parameters obtained from the tests are not directly applicable to a wound roll. A purely elastic continuum model cannot correctly describe the layered structure of a paper roll and account for the slippage of the paper layers during the deformation. This can be a real drawback, since the interlayer slippage can play a key role in the mechanism of deformation [4], [5]. This is especially true in the case of nip[1] loads when high local stress concentrations in the roll can occur.

In this paper, an orthotropic elastoplastic material model, with the interlayer slippage described by plastic shear [6], is used to study the load–deformation paths, permanent deformations and layer–to–layer slippage in a paper roll loaded against a nip roller. It is demonstrated that compression tests of paper rolls against solid nip rollers can be used in determining the material parameters of paper by fitting the corresponding computed and measured results. Using this approach, values for the radial and shear moduli of paper are determined. The model can correctly predict the permanent deformations and hysteresis found in experimental tests.

Constitutive model for layered structure

The detailed mathematical formulation of the model used in this work is given in [6]. The main idea of the model is that the layered structure of the roll is embedded in the constitutive equation defining the material response. Larger elements of this jointed material, each spanning over several hundreds of layers, can then be used in the finite element model of the roll. The description of slippage between the layers in the material is based on the theory of plasticity. Since the layers in a roll are closely spaced compared to the characteristic dimensions of the domain of the model, they can be smeared into a continuum with slip surfaces. The outcome was an elastoplastic jointed material model for orthotropic materials with shear limits based on Coulomb friction. Finally, the model was implemented in ABAQUS/Standard finite element structural analysis software to perform the calculations.

Compression test and FE-model

In the compression test a roll of catalog paper was compressed against a solid nip roller as shown in Fig. 1. Since the initial radial pressure distribution, generated in the roll during winding, can have a significant effect on the slippage of the layers, it had to be taken into account. This pressure distribution was estimated using pull tab measurements. The indentation δ, *i.e.*, the relative

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[1] the contact region between the paper roll and nip roller

displacement of the centres of the roll and roller, as well as the nip load P and paper to paper coefficient of friction μ_{pp} were also measured.

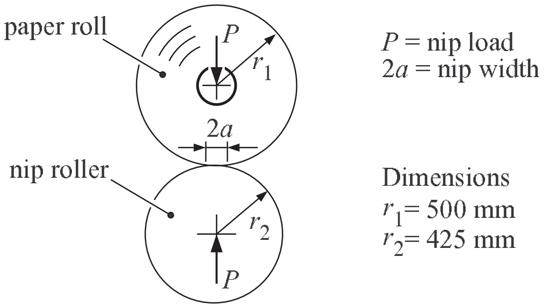

Figure 1: Paper roll compressed against a nip roller. The radii of the roll and roller are r_1 and r_2, respectively.

To model the compression test, a finite element simulation of a hollow cylinder of orthotropic jointed material around a rigid core, representing the paper roll, compressed against a rigid roller was performed. During one load cycle the roller was first compressed against the roll and then withdrawn. A total of two load cycles were simulated. A two dimensional model under plane strain conditions was used. Only one half of the roll had to be modelled when using the appropriate symmetry conditions. Numerical studies indicated that the results were

- practically independent of the Poisson's ratios $\nu_{r\theta}$, ν_{rz}, $\nu_{\theta z}$ and the coefficient of friction between the paper roll and nip roller μ_{pr}. For the Poisson's ratios typical values from [7] were used.
- only slightly dependent on the elastic moduli E_θ and E_z. The modulus E_θ was measured. A value typical for catalog paper was used for E_z.

The values for E_r and $G_{r\theta}$ were determined by fitting the calculated and measured results. It should be noted that values for the shear modulus $G_{r\theta}$ are rarely reported in existing literature, and yet it is an important parameter in modern winding models including the nip action. A summary of the parameter values used in the ultimate calculations are given in Table 1.

Table 1: Values of the parameters used in the calculation. The indices r, θ and z refer to the radial, circumferential and axial directions of the roll, respectively.

Elastic moduli	Poisson's ratios	Shear modulus	Coefficients of friction
$E_r = 10.0\,\mathrm{MPa}$	$\nu_{r\theta} = -0.0055$	$G_{r\theta} = 42.0\,\mathrm{MPa}$	$\mu_{pp} = 0.275$
$E_\theta = 5100\,\mathrm{MPa}$	$\nu_{rz} = -0.0035$		$\mu_{pr} = 0.4$
$E_z = 2600\,\mathrm{MPa}$	$\nu_{\theta z} = 0.37$		

Results

The indentation δ as a function of the nip load P is shown in Fig. 2. A preload of $1\,\mathrm{kN/m}$ was applied to remove the entrained air below the top layers of the paper roll and to set the zero point for the displacement. The calculated results (solid line) are in relatively good agreement with the experimental results (dotted line). Note that the loading and unloading do not happen along the same path. Instead, a hysteresis cycle can be seen and a permanent indentation of approximately $0.3\,\mathrm{mm}$ remains after the first load cycle. For comparison, the calculation was repeated using a purely elastic, orthotropic model for the paper roll. The result is shown in Fig. 2 (dashed line). Obviously the elastic model cannot produce the hysteresis cycle and, thus, the loading and unloading occur along the same path. The elastic model is also more than 30% stiffer than the elastoplastic model. This can be understood, since in the plastic model layer–to–layer slippage softens the behaviour.

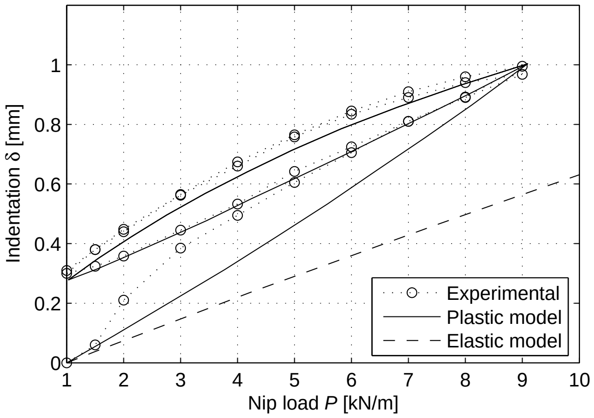

Figure 2: Indentation δ as a function of the nip load P. Experimental results (circles and dotted line) and calculated results obtained using the elastoplastic model (solid line) and elastic model (dashed line).

Regions of interlayer slippage

Since the interlayer slippage contributes significantly to the indentation–load behaviour, it is interesting to see where the slippage occurs. The permanent plastic engineering shearing strain (or permanent interlayer slippage) after two load cycles is shown in Fig. 3.

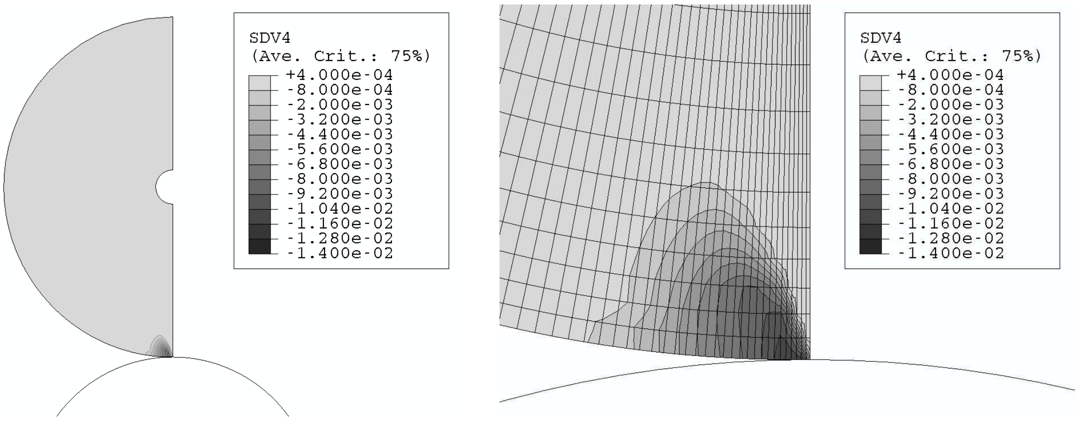

Figure 3: Plastic engineering shearing strain (or interlayer slippage) in the paper roll after two completed load cycles. A close up of the nip region and the finite element mesh are shown on the right. Only the left half of the roll is shown.

The area of high slippage is found on the side of the nip region close to the roll surface, penetrating about $40\,\mathrm{mm}$ into the roll. This can be expected, since this is the region of high shear stresses of cylindrical contact problems where shear stresses overcome the frictional forces and slippage occurs. Note also that in this region the radial compressive stresses are not as high as right below the nip area.

Concluding remarks

An orthotropic elastoplastic jointed material model accounting for the slippage and separation of the paper layers has been used to study the deformation of a paper roll compressed against a nip roller. The calculated results were compared to experimental data and a good correspondence was found. If other parameter values are known, the present model can also be utilized in an indirect method for determining the shear modulus $G_{r\theta}$ of paper from roll compression tests. A direct measurement of $G_{r\theta}$ is difficult due to the thinness of paper. It was also shown that a purely elastic model highly exaggerates the stiffness of the roll. In conclusion, the interlayer slippage plays a key role in the deformation mechanism of a paper roll, and it has to be taken into account in the calculations if accurate results are to be achieved.

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