
This is an electronic reprint of the original article.
This reprint may differ from the original in pagination and typographic detail.

Partanen, Mikko; Tulkki, Jukka

Momentum and rest mass of the covariant state of light in a medium

Published in:
Physics and Simulation of Optoelectronic Devices XXV

DOI:
[10.1117/12.2250877](https://doi.org/10.1117/12.2250877)

Published: 01/01/2017

Document Version
Publisher's PDF, also known as Version of record

Please cite the original version:
Partanen, M., & Tulkki, J. (2017). Momentum and rest mass of the covariant state of light in a medium. In Physics and Simulation of Optoelectronic Devices XXV (pp. 1-7). [100980P] (Proceedings of SPIE : the International Society for Optical Engineering; Vol. 10098). DOI: 10.1117/12.2250877

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.

PROCEEDINGS OF SPIE

[SPIDigitalLibrary.org/conference-proceedings-of-spie](https://spiedigitallibrary.org/conference-proceedings-of-spie)

Momentum and rest mass of the covariant state of light in a medium

Mikko Partanen, Jukka Tulkki

Mikko Partanen, Jukka Tulkki, "Momentum and rest mass of the covariant state of light in a medium," Proc. SPIE 10098, Physics and Simulation of Optoelectronic Devices XXV, 100980P (22 February 2017); doi: 10.1117/12.2250877

SPIE.

Event: SPIE OPTO, 2017, San Francisco, California, United States

Momentum and rest mass of the covariant state of light in a medium

Mikko Partanen and Jukka Tulkki

Engineered Nanosystems group, School of Science
Aalto University, P.O. Box 12200, 00076 Aalto, Finland

ABSTRACT

Conventionally, theories of electromagnetic waves in a medium assume that only the energy of the field propagates in a transparent medium and the medium is left undisturbed. Consequently, the transport of mass density and the related kinetic and elastic energies of atoms is neglected. We have recently presented foundations of a covariant theory of light propagation in a medium by considering a light wave simultaneously with the dynamics of the medium atoms driven by optoelastic forces between the induced dipoles and the electromagnetic field. In the previously discussed mass-polariton (MP) quasiparticle approach, we considered the light pulse as an isolated coupled state between the photon and matter and showed that the momentum and the transferred mass of MP follow unambiguously from the Lorentz invariance and the fundamental conservation laws of nature. In the present work, we combine the electrodynamics of continuous media and elasticity theory to account for the space- and time dependent dynamics of the light pulse and the associated mass and momentum distributions of the mass density wave (MDW). In this optoelastic continuum dynamics (OCD) approach, we obtain a numerically accurate solution of the Newtonian continuum dynamics of the medium when the light pulse is propagating in it. For an incoming Gaussian light pulse having total energy E_0 in vacuum, the OCD simulations of the light pulse propagating in a crystal having refractive index n give the same momentum $p = nE_0/c$ and the transferred mass $\delta m = (n^2 - 1)E_0/c^2$ as the MP quasiparticle approach. Since the elastic forces are included in our theory on equal footing with the optical forces, our theory also predicts how the mass and thermal equilibria are re-established by elastic waves.

Keywords: mass-polariton, photon momentum, optical forces, electrodynamics, optomechanics

1. INTRODUCTION

It is well known in the electrodynamics of continuous media that, when a light pulse propagates in a medium, the atoms are a subject of field-dipole forces.¹ However, the coupled dynamics of the field and the medium driven by the field-dipole forces has been a subject of very few detailed studies. In this work, we elaborate how these driving forces give rise to a mass density wave (MDW) in the medium when a light pulse is propagating in it. We have previously studied MDW in the mass-polariton (MP) quasiparticle picture.² In this picture, the coupled state of the field and matter is considered isolated from the rest of the medium and thus a subject of the covariance principle and the general conservation laws of nature. The MP quasiparticle picture also gives a transparent resolution to the centenary Abraham-Minkowski controversy of the momentum of light in a medium,³⁻⁸ which has also gained much experimental interest.⁹⁻¹⁵

The problem of light propagation in a medium is depicted in Fig. 1, which presents the state of the photon before (left) inside the medium (middle) and after leaving the medium (right). In this work we apply the electrodynamics of continuous media and continuum mechanics to compute the dynamics of the medium when a light pulse is propagating in it. In this optoelastic continuum dynamics (OCD) approach, the total force on a medium element consists of the field-dipole force and the elastic force resulting from the density variations in the medium. In brief, we will show that accounting for MDW resulting from the coupled dynamics of the field and matter allows formulating a fully consistent covariant theory of light propagation in a medium. OCD theory enables numerical calculation of the mass and momentum distribution of the atoms moving with the light pulse. We will show that OCD theory provides an independent but complementary view of how the covariance principle governs the field-matter coupling in the case of a light pulse propagating in a medium.

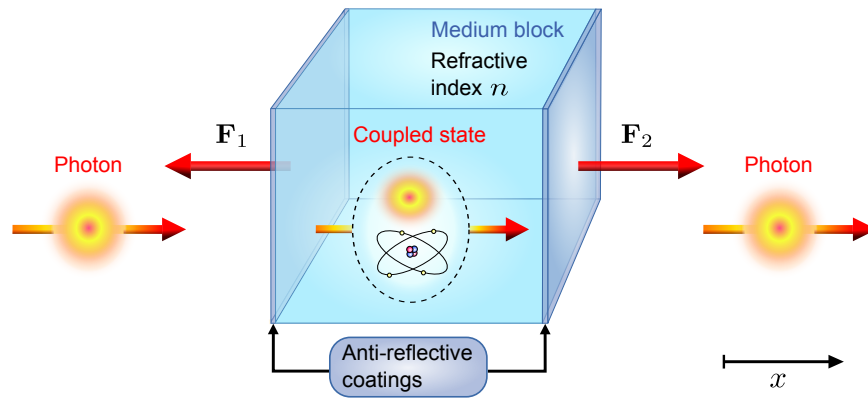


Figure 1. (Color online) Schematic illustration of a photon propagating through a transparent medium block with a refractive index n . On the left, the photon is incident from vacuum. Inside the medium block, the photon couples to atoms forming a quasiparticle, which continues to propagate inside the block. On the right, the photon continues to propagate in vacuum after the transmission. At the photon entrance and exit, the medium block experiences recoil forces \mathbf{F}_1 and \mathbf{F}_2 that depend on the total momentum of light in a medium. The anti-reflective coatings at the interfaces are included only to simplify the conceptual understanding of the problem.

Recently Leonhardt⁸ has applied fluid dynamics to study the momentum transport of light in fluids. In his work, Lendhardt calculates the force of the field-dipole interaction on fluid elements. However, Leonhardt does not study the dynamics of the fluid in the time scale of the oscillating harmonic components of light as he concentrates on the deformation of the fluid surface due to a stationary light beam. In the present work, we show that to calculate the dynamics of the medium one has to account for the exact time dependence of the field induced force to obtain correct total momentum and transferred mass for the light wave. In addition, Leonhardt assumes that the fluid is incompressible. This makes the light induced force field to propagate at infinite speed in the fluid ruining the relativistic invariance and leading to a fundamentally unsound theory regarding the momentum of light in a medium. The OCD theory presented in this work can be straightforwardly generalized to fluids but, in the present simulations, we consider only elastic non-dispersive solids.

2. OPTOELASTIC CONTINUUM DYNAMICS

2.1 Newton's equation of motion and optoelastic forces

We will use Newtonian formulation of the continuum mechanics to show that the optical force gives rise to MDW. Together with the associated recoil effect, the MDW effect perturbs the mass density of the medium from its equilibrium value ρ_0 . In this OCD model, the perturbed atomic mass density of the medium becomes $\rho_a(\mathbf{r}, t) = \rho_0 + \rho_{\text{rec}}(\mathbf{r}, t) + \rho_{\text{MDW}}(\mathbf{r}, t)$, where $\rho_{\text{rec}}(\mathbf{r}, t)$ is the mass density perturbation due to the recoil effect and $\rho_{\text{MDW}}(\mathbf{r}, t)$ is the mass density of MDW. The mass density perturbances related to the recoil and MDW effects become spatially separated after the light pulse has penetrated in the medium. The recoil effect will be shown to lead to the mass density perturbation at the interface while the MDW follows the light pulse inside the medium. As the atomic velocities are nonrelativistic in the reference frame where the medium is initially at rest, the Newtonian mechanics can be applied for the description of the movement of atoms. Newton's second law for the mass density of the medium is written as

$$\rho_a(\mathbf{r}, t) \frac{d^2 \mathbf{r}_a(\mathbf{r}, t)}{dt^2} = \mathbf{f}_{\text{opt}}(\mathbf{r}, t) + \mathbf{f}_{\text{el}}(\mathbf{r}, t), \quad (1)$$

where $\mathbf{r}_a(\mathbf{r}, t)$ is the atomic displacement field of the medium, $\mathbf{f}_{\text{opt}}(\mathbf{r}, t)$ is the optical force density, and $\mathbf{f}_{\text{el}}(\mathbf{r}, t)$ is the elastic force density.

The optical force density originates from the interaction between the induced dipoles and the electromagnetic field and it effectively also accounts for the interaction between the induced dipoles. Using Maxwell's equations

and the Lorentz force law, it can be shown that the optical force density experienced by the induced dipoles in the medium is given in terms of the electric field $\mathbf{E}(\mathbf{r}, t)$ and the Poynting vector $\mathbf{S}(\mathbf{r}, t)$ as¹⁶

$$\mathbf{f}_{\text{opt}}(\mathbf{r}, t) = -\frac{\varepsilon_0}{2}E(\mathbf{r}, t)^2\nabla n^2 + \frac{n^2 - 1}{c^2}\frac{\partial}{\partial t}\mathbf{S}(\mathbf{r}, t). \quad (2)$$

As the atoms are displaced from their equilibrium positions due to the optical force, they are also affected by the elastic force density $\mathbf{f}_{\text{el}}(\mathbf{r}, t)$ following from Hooke's law. In the case of a homogeneous isotropic elastic medium, the stiffness tensor in Hooke's law has only two independent entries. Typically, these entries are described by using the Lamé parameters or any two independent elastic moduli, such as the bulk modulus B and the shear modulus G .¹⁷ The elastic force density of a homogeneous isotropic medium is then given in terms of the material displacement field $\mathbf{r}_a(\mathbf{r}, t)$ as¹⁸

$$\mathbf{f}_{\text{el}}(\mathbf{r}, t) = (B + \frac{4}{3}G)\nabla[\nabla \cdot \mathbf{r}_a(\mathbf{r}, t)] - G\nabla \times [\nabla \times \mathbf{r}_a(\mathbf{r}, t)]. \quad (3)$$

The factor $B + \frac{4}{3}G$ in the first term of Eq. (3) is also known as the P-wave modulus.¹⁷ In the special case of non-viscous fluids, we could set the shear modulus G to zero, in which case the second term of Eq. (3) becomes zero and the bulk modulus B is the only elastic modulus that remains.

2.2 Transferred mass and momentum of the mass density wave

The optical and elastic force densities in Eqs. (2) and (3) and Newton's equation of motion in Eq. (1) can be used to simulate the motion of atoms in the medium as a function of space and time. The total displacement of atoms at position \mathbf{r} is solved from Eq. (1) by integration as

$$\mathbf{r}_a(\mathbf{r}, t) = \int_{-\infty}^t \int_{-\infty}^{t''} \frac{d^2\mathbf{r}_a(\mathbf{r}, t')}{dt'^2} dt' dt'' = \int_{-\infty}^t \int_{-\infty}^{t''} \frac{\mathbf{f}_{\text{opt}}(\mathbf{r}, t') + \mathbf{f}_{\text{el}}(\mathbf{r}, t')}{\rho_a(\mathbf{r}, t')} dt' dt''. \quad (4)$$

As the total mass of atoms inside the light pulse is very large when compared to mass equivalent of the field energy, the perturbed mass density of the medium $\rho_a(\mathbf{r}, t)$ is extremely close to the equilibrium mass density ρ_0 . Therefore, when using Eq. (4), it is well justified to approximate the mass density in the denominator of the integrand with the equilibrium mass density ρ_0 .

When the light pulse has passed through the medium, the displacement of atoms at the position \mathbf{r} is given by $\mathbf{r}_a(\mathbf{r}, \infty)$. We then obtain the displaced volume as $\delta V = \int \mathbf{r}_a(\mathbf{r}, \infty) \cdot d\mathbf{A}$, where the integration is performed over the transverse plane with a surface element vector $d\mathbf{A}$. Using the solution of the atomic displacements in Eq. (4), one obtains an equation for the total transferred mass $\delta m = \rho_0 \delta V$, given by

$$\delta m = \int \int_{-\infty}^{\infty} \int_{-\infty}^t [\mathbf{f}_{\text{opt}}(\mathbf{r}, t') + \mathbf{f}_{\text{el}}(\mathbf{r}, t')] dt' dt \cdot d\mathbf{A}. \quad (5)$$

The total transferred mass δm is given in terms of the mass density of MDW as $\delta m = \int \rho_{\text{MDW}}(\mathbf{r}, t) dV$. Using Eq. (5) and the relation $cdt = ndx$, we then obtain the mass density of MDW, given by

$$\rho_{\text{MDW}}(\mathbf{r}, t) = \frac{n}{c} \int_{-\infty}^t [\mathbf{f}_{\text{opt}}(\mathbf{r}, t') + \mathbf{f}_{\text{el}}(\mathbf{r}, t')] \cdot \hat{\mathbf{x}} dt', \quad (6)$$

where $\hat{\mathbf{x}}$ is the unit vector in the direction of light propagation. With the expressions for the optical and elastic forces in Eqs. (2) and (3), one can use Eq. (6) for numerical simulations of the propagation of light and the associated MDW in the medium.

The velocity distribution of the medium is given by $\mathbf{v}_a(\mathbf{r}, t) = d\mathbf{r}_a(\mathbf{r}, t)/dt$. Therefore, the momentum of MDW is directly given by integration of the classical momentum density $\rho_0 \mathbf{v}_a(\mathbf{r}, t)$ as

$$\mathbf{p}_{\text{MDW}} = \int \rho_0 \mathbf{v}_a(\mathbf{r}, t) d^3r = \int \rho_{\text{MDW}}(\mathbf{r}, t) \mathbf{v} d^3r, \quad (7)$$

where \mathbf{v} is the velocity vector of MP with length $v = c/n$. In numerical simulations described in Sec. 3, it is verified that both forms in Eq. (7) give an equal result.

2.3 Comparison of the OCD and MP quasiparticle approaches

For the light pulse of energy E_0 , the total mass transferred by MDW, given in Eq.(5), can be also written as

$$\delta m = \int \rho_{\text{MDW}}(\mathbf{r}, t) d^3 r = (n^2 - 1)E_0/c^2. \quad (8)$$

The right hand side equals the result obtained from the MP quasiparticle model.² The total momentum of the coupled state of the field and matter is a sum of the momenta of the field and MDW, given by

$$\mathbf{p}_{\text{MP}} = \int \rho_0 \mathbf{v}_a(\mathbf{r}, t) d^3 r + \int \frac{\mathbf{S}(\mathbf{r}, t)}{c^2} d^3 r = \frac{nE_0}{c} \hat{\mathbf{x}}. \quad (9)$$

The first term on the left is the MDW momentum in Eq. (7) and the second term is the momentum density of the electromagnetic field. On the right, we have the momentum density of the coupled state obtained from the MP quasiparticle model.² In the simulations described in Sec. 3, it is found that the OCD and MP quasiparticle model results agree within the numerical accuracy of the simulations.

3. SIMULATIONS OF THE MASS TRANSFER

We simulate the propagation of a light pulse in the geometry of a cubic diamond crystal block with anti-reflective coatings illustrated in Fig. 2. In the x direction, the first and second interfaces of the crystal are located at positions $x = 0$ and $x = 100$ mm. In the y and z directions, the geometry is centered so that the trajectory of the light pulse center follows the line $y = z = 0$. For the diamond crystal, we use a refractive index $n = 2.4$,¹⁹ mass density $\rho_0 = 3500$ kg/m³,²⁰ bulk modulus $B = 443$ GPa,²¹ and shear modulus $G = 478$ GPa.²²

We assume a titanium-sapphire laser pulse with a wavelength $\lambda_0 = 800$ nm in vacuum and total energy $E_0 = 5$ mJ. The wavelength in the diamond crystal is then $\lambda = \lambda_0/n = 333$ nm. The Gaussian form of the laser pulse is assumed to propagate in the direction of the positive x -axis. The energy density of the laser pulse averaged over the harmonic cycle is then given by

$$u(\mathbf{r}, t) = E_0 \frac{n\Delta k_x \Delta k_y \Delta k_z}{\pi^{3/2}} e^{-(n\Delta k_x)^2 (x-ct/n)^2} e^{-(\Delta k_y)^2 y^2} e^{-(\Delta k_z)^2 z^2}, \quad (10)$$

where Δk_x , Δk_y , and Δk_z are the standard deviations of the wave vector components in vacuum, defining the pulse width in the x , y , and z directions. In the simulations, we use the relative spectral width of the pulse given

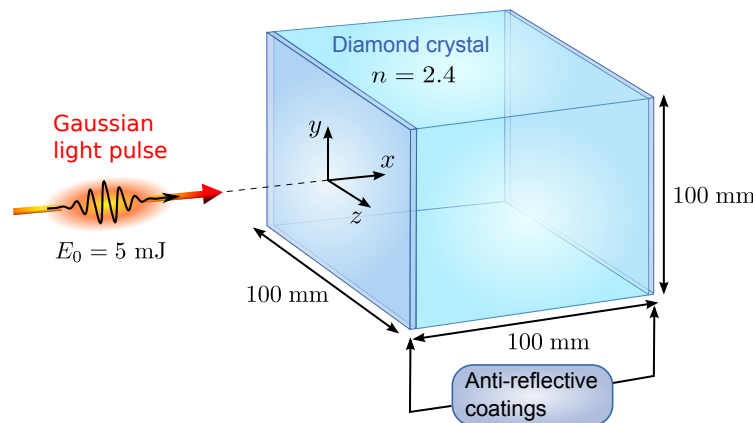


Figure 2. (Color online) Illustration of the simulation geometry consisting of a cubic diamond crystal block coated with anti-reflective coatings. The refractive index of the crystal is $n = 2.4$. A Gaussian light pulse of energy $E_0 = 5$ mJ propagates in the direction of the positive x -axis and enters the crystal from the left. The geometry is centered so that the center of the light pulse enters the crystal at $x = y = z = 0$. The first interface of the crystal is located at $x = 0$ and the second interface at $x = 100$ mm.

by $\Delta\omega/\omega = \Delta k_x/k_0 = 10^{-5}$. The corresponding standard deviation of position is $\Delta x = 1/(\sqrt{2}\Delta k_x) \approx 7$ mm, and the standard deviation of the pulse width in time is $\Delta t = n\Delta x/c \approx 60$ ps. In the transverse direction, we use $\Delta k_y = \Delta k_z = 10^{-4}k_0$ corresponding to the standard deviation of position given by $\Delta y = \Delta z \approx 0.7$ mm.

3.1 Simulation in one dimension

In the one-dimensional simulation, the simulation geometry corresponds to a plate that has thickness $L = 100$ mm in the x direction and is infinite in the y and z directions. The one-dimensional light pulse is Gaussian only in the x direction, which is the direction of propagation. The one-dimensional pulse is obtained from the three-dimensional pulse in Eq. (10) by dropping out the y and z dependence and renormalizing the light pulse so that its integral over x gives $E_0/(\lambda/2)^2$. This corresponds to very high power per unit area, which was chosen so that we obtain an order of magnitude estimate of how large atomic displacements we obtain if the whole vacuum energy $E_0 = 5$ mJ of the laser pulse can be coupled to a single mode fiber having a cross section $(\lambda/2)^2$. The discretization length in the one-dimensional simulation is $h_x = 250$ μm , which is small compared to the pulse width.

Figure 3(a) shows the MDW as a function of position when the light pulse is propagating in the middle of the crystal. MDW mass density $\rho_{\text{MDW}}(\mathbf{r}, t)$ is calculated by using Eq. (6). MDW is driven by the optoelastic forces due to the Gaussian light pulse. The mass density perturbation $\rho_{\text{rec}}(\mathbf{r}, t)$ at the first interface due to the interface force is not shown in the figure. One can see that the form of MDW clearly follows the Gaussian form of the light pulse as expected. When the MDW mass density in Fig. 3(a) is integrated over the light pulse, we obtain the total transferred mass of 2.6×10^{-19} kg. Dividing this by the photon number of the light pulse $N_0 = E_0/\hbar\omega = 2.0 \times 10^{16}$, we then obtain the value of the transferred mass per photon, given by $\delta m = (n^2 - 1)\hbar\omega/c^2 = 7.4$ eV/ c^2 , which corresponds to the value obtained in the MP quasiparticle approach.²

Figure 3(b) presents the corresponding atomic displacements as a function of position. On the left from the first interface at $x = 0$, the atomic displacement is zero as there are no atoms in vacuum. As a result of the optoelastic recoil effect described by the first term of Eq. (2), a thin material layer at the interface recoils to the left. In the simulation, the width of the material layer that takes the recoil energy is the discretization length of $h_x = 250$ μm . Therefore, we do not account for the atomic scale effects on the refractive index at the interfaces and the negative atomic displacement at the interface is only an approximate. Between the positions $x = 0$ and $x = 40$ mm, the atomic displacement has a constant value of of 2.7 nm. This results from the optical force in the second term of Eq. (2). To the right of the position $x = 60$ mm, the atomic displacement is zero as the leading edge of the light pulse has not yet reached these positions. In Fig. 3(b), MDW is manifested by the fact that atoms are more densely spaced at the position of the light pulse since the atoms on the left of the pulse have been displaced forward and the atoms on the right of the pulse are still at their initial positions.

Figure 3(c) shows the atomic displacements just after the light pulse has left the medium. One can see that all atoms inside the crystal have been displaced forward from their initial positions. At the both interfaces, the

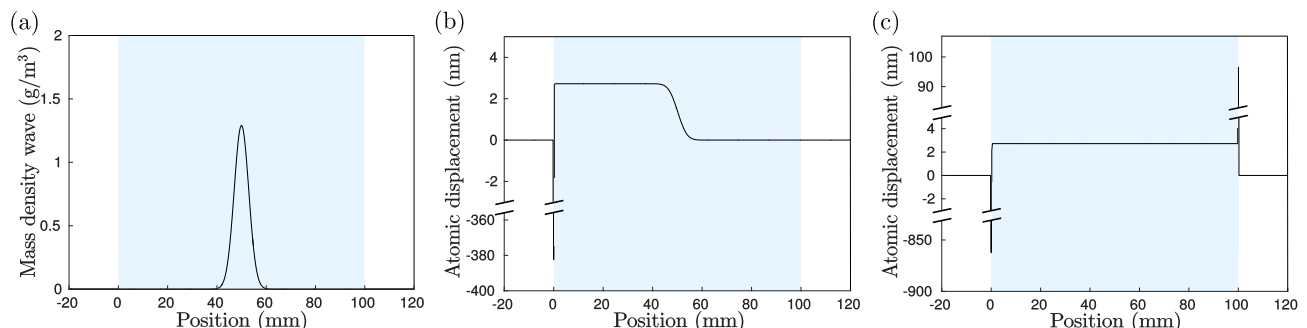


Figure 3. (Color online) Simulation of the mass transfer in a single-mode diamond waveguide due to a Gaussian light beam. (a) The calculated mass density of MDW as a function of position when the light pulse is in the middle of the crystal. The light blue background represents the region of the crystal between positions $x = 0$ and $x = 100$ mm. (b) The calculated atomic displacements when the light pulse is in the middle of the crystal. (c) The calculated atomic displacements when the light pulse has just left the crystal. Note the breaks in the scales of the figures.

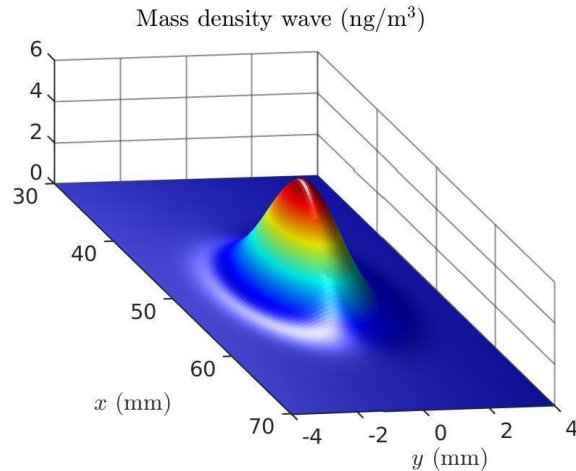


Figure 4. (Color online) Simulation of MDW driven by optoelastic forces. The MDW mass density $\rho_{\text{MDW}}(\mathbf{r}, t)$ is plotted as a function of position in the plane $z = 0$. The light pulse propagates in the direction of the positive x -axis. The front end of the crystal is located at $x = 0$ and the back end at $x = 100$ mm, not shown in this figure.

surface atoms have been displaced outwards from the medium due to the optoelastic recoil effect. After the transmission of the light pulse, the elastic forces start to restore the mass density equilibrium in the crystal. Therefore, due to the elastic forces, the magnitudes of the atomic displacements at the interfaces are changing as a function of time. When the mass equilibrium has been re-established, the elastic energy that was left in the crystal, after the transmission of the light pulse, is converted to lattice heat.

3.2 Simulation in three dimensions

Figure 4 presents the mass density of MDW simulated in the full three-dimensional geometry in Fig. 2. The MDW mass density calculated by using Eq. (6) is shown as a function of position in the plane $z = 0$ when the light pulse is propagating in the middle of the crystal. Again, one can see that the form of MDW clearly follows the Gaussian form of the light pulse. If the MDW mass density in Fig. 4(a) is integrated over the light pulse, we obtain the same total transferred mass of 2.6×10^{-19} kg as in the case of the one-dimensional simulation. Therefore, the simulation results are fully consistent with the results obtained in the MP quasiparticle approach.² In the three-dimensional simulations, the maximum atomic displacement after the light pulse is found to be 1.5×10^{-17} m, which is significantly smaller than the value obtained in the one-dimensional simulation, where the field is spatially restricted into a smaller volume.

4. CONCLUSIONS

In conclusion, we have presented a covariant OCD approach for the study of light propagation in a nondispersive medium, and compared the OCD approach to the previously discussed MP quasiparticle picture. Our analysis shows that, when a photon enters the crystal, its energy and momentum are shared by the medium atoms and the propagating electromagnetic field. Both the MP quasiparticle picture and the OCD approach lead to transfer of mass with the light wave. The transfer of mass with the light wave, in turn, gives rise to nonequilibrium of the mass density in the medium. When the mass equilibrium is re-established by relaxation, a small amount of initial photon energy is converted to lattice heat. These discoveries fundamentally change our understanding of light-matter interaction. We have calculated the mass transfer numerically for one- and three-dimensional Gaussian wave packets and the diamond crystal with realistic material parameters. The mass transfer and dissipation are real world phenomena that can also be studied experimentally. Thus, our work is of great interest to scientists experimenting with light.

ACKNOWLEDGMENTS

This work has in part been funded by the Academy of Finland and the Aalto Energy Efficiency Research Programme.

REFERENCES

- [1] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Electrodynamics of continuous media*, Pergamon, Oxford, 4 ed., 1984.
- [2] M. Partanen, T. Häyrynen, J. Tulkki, and J. Oksanen, “Photon mass drag and the momentum of light in a medium,” arXiv:1603.07224v2.
- [3] A. Cho, “Century-long debate over momentum of light resolved?,” *Science* **327**, p. 1067, 2010.
- [4] U. Leonhardt, “Momentum in an uncertain light,” *Nature* **444**, pp. 823–824, 2006.
- [5] R. N. C. Pfeifer, T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztein-Dunlop, “Colloquium: Momentum of an electromagnetic wave in dielectric media,” *Rev. Mod. Phys.* **79**, pp. 1197–1216, Oct 2007.
- [6] S. M. Barnett, “Resolution of the Abraham-Minkowski dilemma,” *Phys. Rev. Lett.* **104**, p. 070401, Feb 2010.
- [7] S. M. Barnett and R. Loudon, “The enigma of optical momentum in a medium,” *Phil. Trans. R. Soc. A* **368**(1914), pp. 927–939, 2010.
- [8] U. Leonhardt, “Abraham and Minkowski momenta in the optically induced motion of fluids,” *Phys. Rev. A* **90**, p. 033801, Sep 2014.
- [9] G. K. Campbell, A. E. Leanhardt, J. Mun, M. Boyd, E. W. Streed, W. Ketterle, and D. E. Pritchard, “Photon recoil momentum in dispersive media,” *Phys. Rev. Lett.* **94**, p. 170403, May 2005.
- [10] R. E. Sapiro, R. Zhang, and G. Raithel, “Atom interferometry using Kapitza-Dirac scattering in a magnetic trap,” *Phys. Rev. A* **79**, p. 043630, Apr 2009.
- [11] R. V. Jones and J. C. S. Richards, “The pressure of radiation in a refracting medium,” *Proc. R. Soc. Lond. A* **221**(1147), pp. 480–498, 1954.
- [12] R. V. Jones and B. Leslie, “The measurement of optical radiation pressure in dispersive media,” *Proc. R. Soc. Lond. A* **360**(1702), pp. 347–363, 1978.
- [13] G. B. Walker and D. G. Lahoz, “Experimental observation of Abraham force in a dielectric,” *Nature* **253**, pp. 339–340, 1975.
- [14] W. She, J. Yu, and R. Feng, “Observation of a push force on the end face of a nanometer silica filament exerted by outgoing light,” *Phys. Rev. Lett.* **101**, p. 243601, Dec 2008.
- [15] L. Zhang, W. She, N. Peng, and U. Leonhardt, “Experimental evidence for Abraham pressure of light,” *New J. Phys.* **17**(5), p. 053035, 2015.
- [16] P. W. Milonni and R. W. Boyd, “Momentum of light in a dielectric medium,” *Adv. Opt. Photon.* **2**, pp. 519–553, Dec 2010.
- [17] G. Mavko, T. Mukerji, and J. Dvorkin, *The Rock Physics Handbook*, Cambridge University Press, Cambridge, 2003.
- [18] A. Bedford and D. S. Drumheller, *Introduction to Elastic Wave Propagation*, Wiley, Chichester, 1994.
- [19] H. R. Phillip and E. A. Taft, “Kramers-Kronig analysis of reflectance data for diamond,” *Phys. Rev.* **136**, pp. A1445–A1448, Nov 1964.
- [20] D. R. Lide, ed., *CRC Handbook of Chemistry and Physics*, CRC Press, Boca Raton, FL, 85 ed., 2004.
- [21] C. Kittel, *Introduction to Solid State Physics*, Wiley, Hoboken, NJ, 8 ed., 2005.
- [22] H. J. McSkimin and P. Andreatch, “Elastic moduli of diamond as a function of pressure and temperature,” *J. Appl. Phys.* **43**, pp. 2944–2948, 1972.