Torque Behaviour of 1-Phase Permanent Magnet AC Motor

Mircea Popescu[1)], T. J.E. Miller[1)] Malcolm I. McGilp[1)],

Giovanni Strappazzon[2)], Nicola Trivillin[2)], Roberto Santarossa[2)]

1) SPEED Laboratory, University of Glasgow, G12 8LT, United Kingdom
2) Electrolux Compressors, Comina 33170, Pordenone, Italy

Abstract–**This paper presents a detailed comparative study of two starting and running methods for a single-phase permanent magnet synchronous motor, equipped with a squirrel-cage rotor. The analysis of the motor performance is realized for a pulse width modulated (PWM) inverter fed motor and for a capacitor-start, capacitor-run motor. The developed approach may be extended to any 1-phase AC motor – induction, synchronous reluctance or synchronous permanent magnet.**

Index terms–**permanent magnet motor, capacitor motor, torque**

NOMENCLATURE

v_m – main supply voltage instantaneous value
$v_{d,q}$ – d-q axis voltage components instantaneous value
$i_{d,q}$ – d-q axis current components instantaneous value
$\underline{V}_m$ – complex main supply voltage
$\underline{\psi}_{d,q}$, $\underline{I}_{d,q}$ – complex d-q axis flux-linkage/current components
R_s – stator winding resistance
R_m – stator main winding resistance
R_a – stator auxiliary winding resistance
X_{ls} – stator leakage reactance
X_{lm} – stator main leakage reactance:
X_{la}, – stator auxiliary leakage reactance
β – turns ratio (main/aux)
ζ – shift electrical angle between stator windings
θ – rotor angle in electrical degrees
$\phi_{m,a}$ – wire diameter of the main/auxiliary winding
R_{rd}, R_{rq} – rotor resistance for d-q axis
X_{lrd}, X_{lrq} – rotor leakage reactance for d-q axis
X_{md}, X_{mq} - magnetization reactance for d-q axis
X_d, X_q - synchronous reactance for d-q axis
C – capacitance value
m, P – phases and pole numbers
p – time derivative operator
ω – synchronous angular velocity [elec. rad/sec]
ω_r – rotor angular velocity [elec. rad/sec]
ω_k – k^{th} harmonic angular velocity [elec. rad/sec]
E_0 – open-circuit induced voltage [V rms]
T_e – electromagnetic torque
T_m – magnet braking torque
T_{rel} – reluctance torque
T_{cage} – cage rotor torque
$T_{(avg)\pm}$ – symmetrical components cage rotor torque

I. INTRODUCTION

Permanent magnet AC motors, equipped with a cage rotor, may represent a higher-efficiency alternative to induction motors. These motors run synchronously and in this way the cage rotor losses are minimized at nominal load. The single-phase version of the permanent magnet AC motor is mainly employed in home appliances, such as refrigerator compressors.

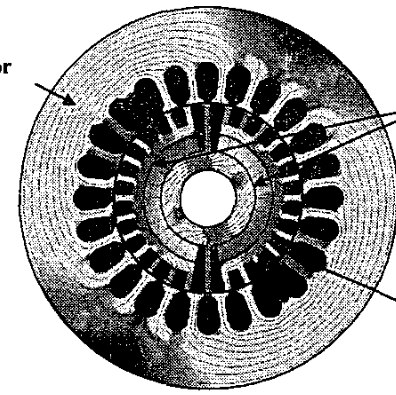

Fig. 1 Cross-section of a 1-phase permanent magnet AC motor

The excitation realised with permanent magnets cannot be switched off. Hence, it will considerably affect the starting capabilities of such motors. The torque oscillations, during the starting transient, are much higher than for an induction motor or a wound field synchronous motor equipped with a cage rotor.

This paper investigates two starting configurations for a single-phase permanent magnet AC motor: a capacitor start, capacitor run motor and a PWM inverter supplied motor. Fig.1 illustrates the flux paths in a cross-section of a 1-phase permanent magnet motor under load.

The subject of the analysis for the single-phase starting performance is a single-phase permanent AC motor, 50 Hz, two-pole motor with concentric windings. The rotor consists of an aluminium rotor cage, with interior ferrite magnets.

II. MATHEMATICAL model

A. General d-q axis model

Analysis of an m-phase balanced permanent magnet AC motor with salient-pole rotor is carried out using the voltage equations in the rotor reference frame under the following assumptions:

- No saturation effect, i.e. motor inductance is not affected by current level;

- Negligible spatial m.m.f. harmonics, i.e. the stator windings and rotor magnets are arranged to produce sinusoidal m.m.f. airgap distribution;
- The effects of stator slots may be neglected;
- There is no fringing of the magnet field;
- The magnetic field intensity is constant and radially directed across the air-gap, such that the permanent magnet may be equivalent to a super conducting winding (e), where the equivalent current is always constant with $i_e = E_0/X_{md}$;
- Eddy current and hysteresis effects are negligible.

The rotor cage is substituted with two equivalent sinusoidally distributed windings (*rd* and *rq*), with magnetic axes shifted by 90 electrical degrees. The voltage equations are expressed through the following matrix equation:

$$\begin{bmatrix} v_d \\ v_q \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s+\frac{p}{\omega}X_d & \frac{\omega_r}{\omega}X_q & 0 & \frac{p}{\omega}X_{md} & -\frac{\omega_r}{\omega}X_{md} \\ \frac{\omega_r}{\omega}X_d & R_s+\frac{p}{\omega}X_q & \frac{\omega_r}{\omega}X_{md} & \frac{\omega_r}{\omega}X_{mq} & \frac{p}{\omega}X_{mq} \\ 0 & 0 & 0 & 0 & 0 \\ \frac{p}{\omega}X_{md} & 0 & 0 & R_{rd}+\frac{p}{\omega}X_{lrd} & 0 \\ 0 & \frac{p}{\omega}X_{mq} & 0 & 0 & R_{rq}+\frac{p}{\omega}X_{lrq} \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \\ i_e \\ i_{rd} \\ i_{rq} \end{bmatrix} \quad (1)$$

The following expressions compute the instantaneous electromagnetic torque and its components (reluctance, cage and alignment i.e. excitation):

$$T_e = \frac{mP}{2\,\omega}\cdot\left[\left(X_{md}-X_{mq}\right)i_d\,i_q + \left(X_{md}i_{dr}i_q - X_{mq}i_{qr}i_d\right) + X_{md}i_e i_q\right]$$
$$= T_{rel} + T_{cage} + T_m \quad (2)$$

$$T_{rel} = \frac{mP}{2\omega}\left(X_{md}-X_{mq}\right)i_d\,i_q = \frac{mP}{2\omega_s}\left(X_d - X_q\right)i_d\,i_q \quad (3)$$

$$T_{cage} = \frac{mP}{2\omega}\left(X_{md}\,i_{dr}\,i_q - X_{mq}\,i_{qr}\,i_d\right) \quad (4)$$

$$T_m = \frac{mP}{2\omega}X_{md}\,i_e\,i_q = \frac{mP}{2\omega}E_0\,i_q \quad (5)$$

At synchronous speed (steady-state) the time derivative terms become zero. Note that for the case of $E_0 = 0$ we can obtain the equation system for a synchronous reluctance motor. For zero excitation voltage plus the additional condition $X_d = X_q$ (no reluctance effect), the induction motor case can be obtained.

B. Capacitor-start, capacitor-run motor

The single-phase permanent magnet AC motor may exhibit asymmetries both on the stator (unbalanced windings and supply voltages) and the rotor (saliency). This makes the classical *d-q* axes analysis on a certain reference frame impossible, without the assumption that the stator windings have the same copper weight and distribution, i.e. $R_s = R_m = \beta^2 R_a$, $X_{ls} = X_{lm} = \beta^2 X_{la}$, $\phi_a = \beta^{1/2}\phi_m$. Torque behaviour may be determined using: quasi steady-state analysis [1], steady-state analysis [2], [4] or a dynamic analysis [2].

Quasi steady-state performance may be described with the relations [1]:

$$T_{(avg)+} = \frac{m}{2}\cdot\frac{P}{2}\cdot\mathrm{Re}\left\{\left(\underline{\psi}_{q+}\right)^{*}\underline{I}_{d+} - \left(\underline{\psi}_{d+}\right)^{*}\underline{I}_{q+}\right\} \quad (6)$$

$$T_{(avg)-} = \frac{m}{2}\cdot\frac{P}{2}\cdot\mathrm{Re}\left\{\left(\underline{\psi}_{q-}\right)^{*}\underline{I}_{d-} - \left(\underline{\psi}_{d-}\right)^{*}\underline{I}_{q-}\right\} \quad (7)$$

$$T_m = \frac{P}{2}\cdot\sin(\zeta)\left[\beta\cdot\psi_{dm}I_{qm} - \frac{1}{\beta}\cdot\psi_{qm}I_{dm}\right] \quad (8)$$

$$T_e = T_{(avg)+} + T_{(avg)-} + T_m \quad (9)$$

For the case of dynamic analysis, the transformed d-q axis voltages and currents are linked to the real values through the relations [2]:

$$v_d = v_s\left[\left(1+\cos(\zeta)\right)\cos(\theta) + \frac{\sin(\zeta)}{\beta}\sin(\theta)\right] - v_c\cos(\theta) \quad (10)$$

$$v_q = v_s\left[-\left(1+\cos(\zeta)\right)\sin(\theta) + \frac{\sin(\zeta)}{\beta}\cos(\theta)\right] + v_c\sin(\theta) \quad (11)$$

$$i_d = i_m\left[\beta\sin(\theta)\sin(\zeta) + \cos(\theta)\cos(\zeta)\right] + i_a\cos(\theta) \quad (12)$$

$$i_q = i_m\left[\beta\cos(\theta)\sin(\zeta) - \sin(\theta)\cos(\zeta)\right] - i_a\sin(\theta) \quad (13)$$

The capacitor voltage is a state variable and must be incorporated in the terminal constraints of the permanent magnet AC motor, which are:

$$v_c = \frac{1}{C}\int i_a\,dt; \qquad v_m = v_s; \qquad v_a = v_s - v_c; \qquad i_s = i_m + i_a \quad (14)\text{-}(17)$$

In steady-state operation the motor performance can be predicted using the symmetrical components theory [4]. Thus the positive sequence voltage is characterized by the following electromagnetic torque components:

$$T_{e+} = T_{rel} + T_m \quad (18)$$

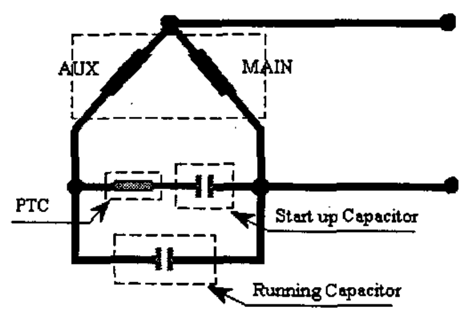

Fig. 2. Schematic structure of a capacitor start, capacitor run single-phase permanent magnet AC motor

For a 1-phase permanent magnet motor, the positive and negative sequence voltages V_+, V_- depends on the equivalent positive and negative impedances $\underline{Z}_+$, $\underline{Z}_-$.

If the positive sequence impedance $\underline{Z}_+$ is variable with the load angle δ, and determined through an iterative process, the negative sequence impedance may be computed as the average between *d* and *q* axis impedance [1].

The complete analysis of the steady-state performance for a 1-phase permanent magnet motor includes the negative sequence voltage contribution. The corresponding torque is computed as a ratio between the electromagnetic power and the synchronous angular velocity:

$$T_{e-} = \frac{P}{2\omega}\cdot\left[\mathrm{Re}\left(\underline{V}_-\cdot(\underline{I}_-)^*\right) - m\cdot R_s\cdot|\underline{I}_-|^2\right] \quad (19)$$

where the negative sequence current is computed as:

$$\underline{I}_- = \frac{\underline{V}_-}{\underline{Z}_-} \quad (20)$$

The total electromagnetic torque at steady-state operation will be:

$$T_e = T_{e+} + T_{e-} \quad (21)$$

B. PWM inverter fed motor

Quasi steady-state performance may be described with the following relations, considering a balanced supply voltage system [1]:

$$T_e = T_{(avg)+} + T_m \quad (22)$$

For the case of the dynamic analysis, the transformed *d-q* axis voltages and currents are linked to the real values through the relations [2]:

$$v_d = v_m\left[\cos(\zeta)\cdot\cos(\theta) + \frac{\sin(\zeta)}{\beta}\cdot\sin(\theta)\right] + v_a\cos(\theta) \quad (23)$$

$$v_q = v_m\left[-\cos(\zeta)\cdot\sin(\theta) + \frac{\sin(\zeta)}{\beta}\cdot\cos(\theta)\right] - v_a\sin(\theta) \quad (24)$$

$$i_d = i_m\left[\beta\sin(\theta)\sin(\zeta) + \cos(\theta)\cos(\zeta)\right] + i_a\cos(\theta) \quad (25)$$

$$i_q = i_m\left[\beta\cos(\theta)\sin(\zeta) - \sin(\theta)\cos(\zeta)\right] - i_a\sin(\theta) \quad (26)$$

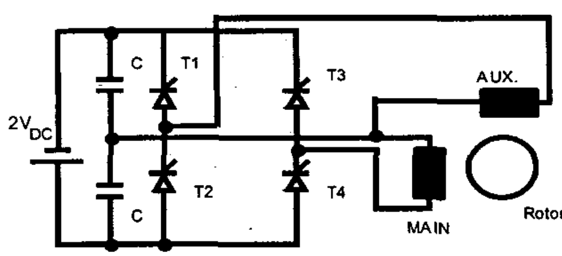

Fig. 3. Schematic structure of a PWM controlled inverter-fed single-phase permanent magnet AC motor

The supply voltages v_m and v_a are described by the same time variation pattern, a six-step pulse width modulated voltage inverter with k_r rectifier coefficient, displaced in phase with 90^0, and their amplitude ratio is equal to the turns ratio β [8]:

$$|v_{main}| = \frac{\sqrt{2}}{\pi}\cdot V_{DC} = \frac{\sqrt{2}}{\pi}\cdot k_r\cdot|v_m| \quad (27)$$

$$|v_{aux}| = \frac{1}{\beta}\cdot\frac{\sqrt{2}}{\pi}\cdot V_{DC} = \frac{1}{\beta}\cdot\frac{\sqrt{2}}{\pi}\cdot k_r\cdot|v_m| \quad (28)$$

The steady-state performance may be computed with the previous expressions (22) – (28), but considering the harmonic effects and substituting for each k^{th} harmonic:

$$\cos(\theta_k) = \frac{\omega_k}{\omega} \quad (29)$$

$$\sin(\theta_k) = j\frac{\omega_k}{\omega} \quad (30)$$

III. Experimental and Simulation Results

The experiments were performed on a motor equipped with identical stator windings. As the purpose of this work was to study the torque behaviour for different driving methods, the prototype motor was not optimised.

First, the starting method was implemented using two selected capacitor values for the capacitor motor connection: 23 μF (C2) at low speed (starting) and 3 μF (C4) at high speed (speed > 80-90% of synchronous speed). The selection criteria were maximum starting torque/current and balanced operation and/or equal current densities in the stator windings [9] at nominal load and synchronous speed. The starting capacitor is connected in series with a positive temperature coefficient resistor (PTC). For a complete comparison tests were performed with:

- starting capacitor value equal to 10 μF (C1), 23 μF (C2) and 40 μF (C3);
- run capacitor value equal to 3 μF (C4).

The use of a run capacitor for the synchronous operation of the 1-phase permanent magnet motor achieves maximum efficiency for only one load point. Thus, if the load varies (as it does in of refrigerator compressors) a suggested solution would be to use a permanently balanced supply voltage system. This justifies the second analysed starting method that can be implemented using a two-phase (e.g. Fig. 3) or a three-phase PWM inverter [7]. Experimental data were obtained for the three-phase solution that comprises six MOSFET transistors. The rectifier–inverter assembly used for the second starting method was directly connected to the AC mains (220 V, 50 Hz). Thus, the input sinusoidal modulated voltage waveform had a 50 Hz frequency and 200 Vrms. The switching frequency for the triangular modulation signal was 5.5 kHz. The motor was driven in an open loop at constant frequency. At synchronous speed and variable load the measured output fundamental voltage value was between 150 V rms and 132 V rms.

Fig. 4 shows the settings for the test stand and Figs. 5, 7 illustrate the experimental quasi-steady state torque vs. speed and current vs. speed respectively. An hysteresis brake (coupled with an acquisition and control system) was used to test the motor. The automatic procedure starts running the motor without load, then gradually decreases the speed measuring at each step the torque and power. The experiments were intended to study the torque behaviour during starting, synchronization and synchronous operation, for a wide range of capacitance values. The tested motors exhibited large torque oscillations at low speed. This phenomenon makes any measurement for locked rotor or low speed conditions very difficult.

During starting, the accelerating torque of the permanent magnet AC motor is the average cage torque minus the magnet braking torque and the load torque. The average cage torque is developed by "induction motor action", except that the saliency and the unbalanced stator voltages complicate the analysis and may compromise performance. The magnet braking torque is produced by the fact that the magnet flux generates currents in the stator windings, and is associated with the loss in the stator circuit resistance. The variation of this torque with speed follows a pattern similar to that in the induction motor, but the per-unit speed takes the place of the slip. The magnet braking torque should not be confused with the synchronous "alignment" torque that arises at synchronous speed, even though the magnet braking torque is still present at synchronous speed and therefore diminishes the output and the efficiency. The magnet alignment torque has a non-zero average value (i.e., averaged over one revolution or electrical cycle) only at synchronous speed. At all other speeds it contributes an oscillatory component of torque. The same is true of the reluctance torque. As the rotor approaches synchronous speed, the screening effect of the cage becomes less, and as the slip is very small, the oscillatory synchronous torques (alignment and reluctance) cause large variations in speed that may impair the ability to synchronize large-inertia loads.

Starting simulation results are presented for the quasi steady-state torque (Fig. 6) and current (Fig.8), transient speed (Fig. 9), currents (Figs. 10, 11) and torque (Figs. 12, 13). The dynamic computation for the capacitor motor was implemented using a capacitor-start value of 23 μF and a capacitor-run value of 3 μF. The starting torque may reach values almost 3 times higher for the capacitor motor when compared to the inverter fed motor, for the same main supply voltage amplitude. The PWM inverter fed motor exhibits a lower starting current level (3 times lower in the analysed case) and a much lower harmonic torque content. This is explained by the elimination of the negative sequence voltage and consequently of the unbalanced stator cage torques and pulsating torque components [1]. However, the torque/current ratio exhibits similar values for the capacitor and inverter fed motor. From the starting torque capabilities point of view, the capacitor motor performance easily overcomes the open loop inverter fed motor performance. Test data and computed data demonstrate that for high load starting conditions the capacitor motor represents the optimum solution. For low load starting conditions, the PWM inverter fed motor represents a reasonable alternative solution. Another drawback for the inverter fed motor solution is the longer period required for synchronization: 1.3 seconds compared to 0.45 seconds for the capacitor motor with 23 μF starting capacitance (C2). Fig. 14 illustrates the test and computed data at synchronous operation (current vs. torque) for the capacitor motor (run capacitor = 3 μF) and for the case of the PWM inverter fed motor. Note the difference between test and computed data especially at higher load torque level. This is explained by the fact that the computation considered only the fundamental harmonic and has neglected the friction losses. The balanced supply voltage (i.e. no negative sequence voltage presence) is responsible for the better torque performance in the inverter fed motor case. The torque behaviour at synchronous operation demonstrates a much higher stability for load variations when the motor is supplied through an inverter. The experiments showed an increased efficiency with up to 9 – 10% for the case of the PWM inverter supplied motor. The capacitor motor efficiency peaks at 75%, while the PWM inverter fed motor runs with a maximum efficiency of 85%. The cost of the components was slightly more favourable for the capacitor motor case.

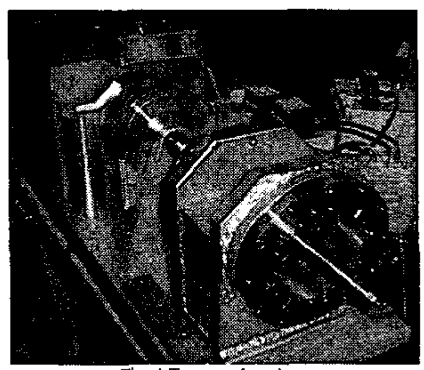

Fig. 4 Test stand settings

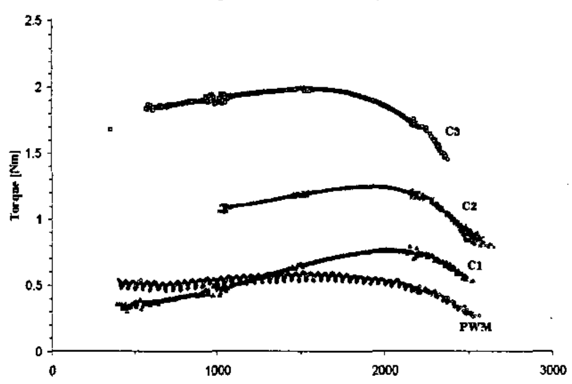

Fig.5 Experimental torque variation vs. speed during starting (capacitor motor – C1, C2, C3 and inverter fed motor – PWM)

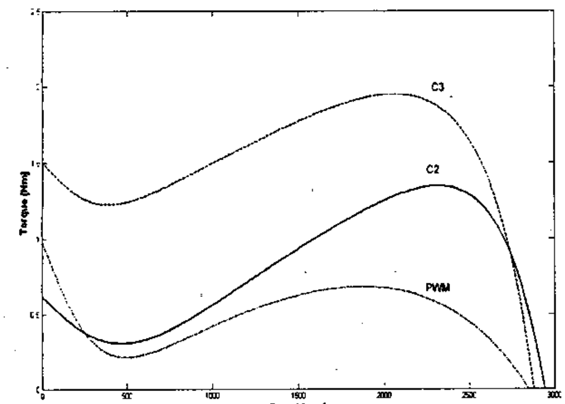

Fig.6 Computed torque variation vs. speed during starting (capacitor motor – C1, C2, C3 and inverter fed motor – PWM)

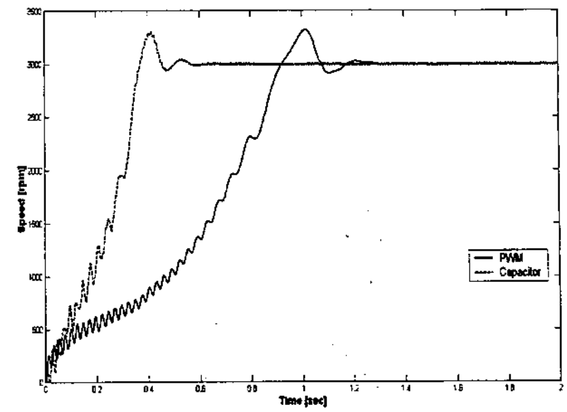

Fig. 9 Speed vs. time variation during starting (capacitor motor and inverter fed motor – PWM)

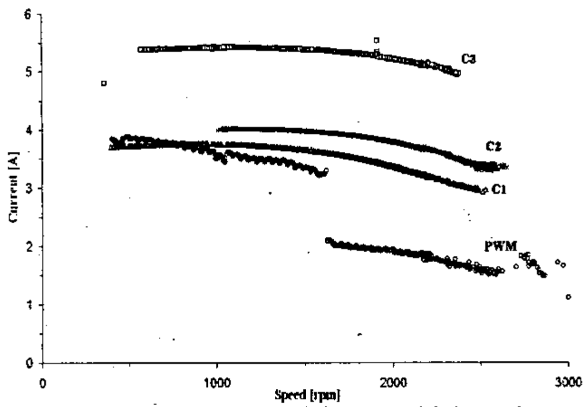

Fig.7 Experimental current variation vs. speed during starting (capacitor motor – C1, C2, C3 and inverter fed motor – PWM)

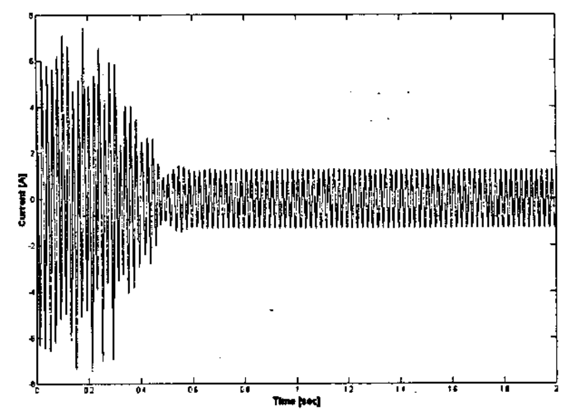

Fig. 10 Current vs. time variation for capacitor motor during starting

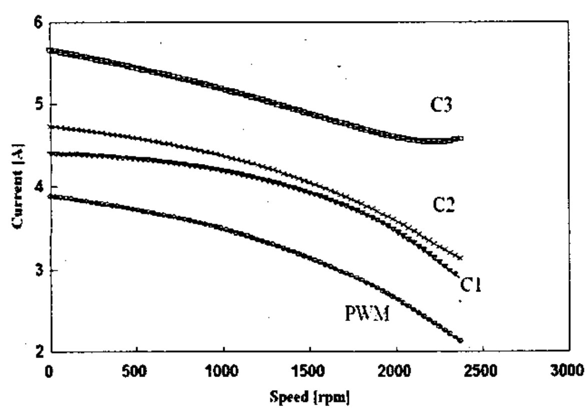

Fig.8 Computed current variation vs. speed during starting (capacitor motor – C1, C2, C3 and inverter fed motor – PWM)

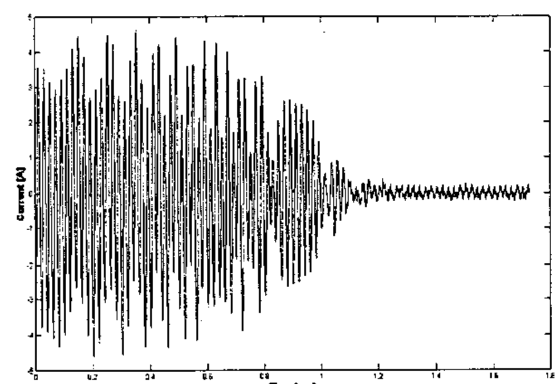

Fig. 11 Current vs. time variation for PWM inverter fed motor during starting

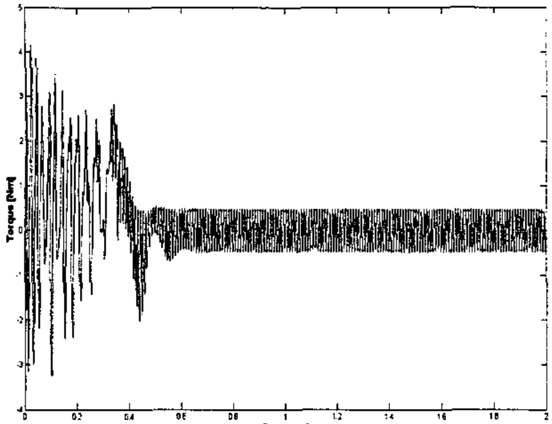

Fig. 12 Torque vs. time variation for capacitor motor during starting

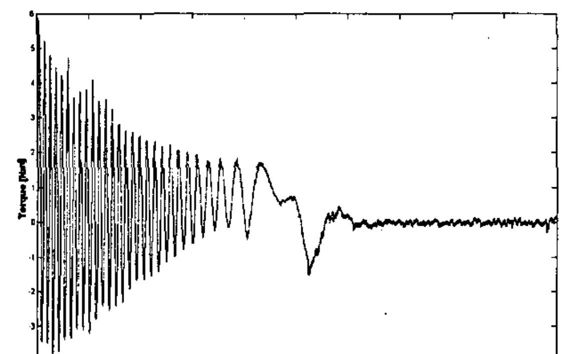

Fig. 13 Torque vs. time variation for PWM inverter fed motor during starting

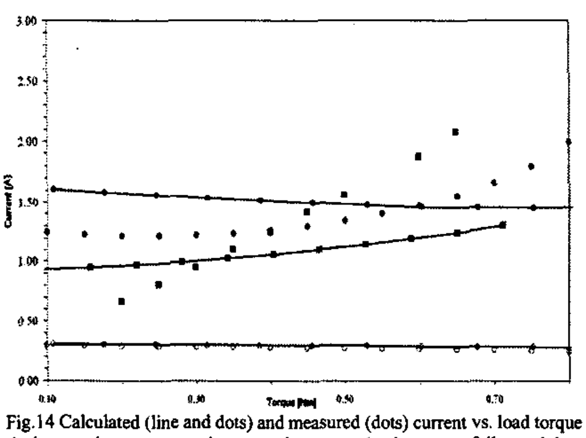

Fig.14 Calculated (line and dots) and measured (dots) current vs. load torque during synchronous operation: capacitor motor (main current full round dots and auxiliary current empty round dots) and inverter fed motor (square dots)

All simulations have been implemented neglecting saturation and core losses. However, the motor modelling equations may be modified to include these non-linear effects. The equivalent circuit parameters defined in the nomenclature have been either measured or computed with the use of a specially developed program and finite element analysis.

IV. CONCLUSIONS

Two starting and running methods for a single-phase permanent magnet AC motor are compared: capacitor start/run and PWM inverter fed. The torque behaviour prediction for a single-phase permanent magnet AC motor can be made for asynchronous operation using the quasi steady-state plus the dynamic analysis and for synchronous operation using the steady-state analysis.

Measurements and simulations demonstrate that both starting methods have advantages and drawbacks.

The electromagnetic torque vs. current ratio during asynchronous operation exhibit similar values in both analysed cases. However, for a high load torque application the capacitor motor solution is preferable while for a low load torque application the PWM inverter fed solution seems to be more advantageous.

At synchronous speed operation the PWM inverter fed solution leads to a torque vs. current ratio value that is not achievable with a capacitor motor.

Torque oscillations are minimised for the PWM inverter fed motor solution as the machine always runs at synchronous speed as a 2-phase balanced motor.

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