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Characterization of polarization fluctuations in random electromagnetic beams

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Abstract. Two stationary, partially polarized electromagnetic beams with equal degrees of polarization may exhibit completely different time evolutions of the instantaneous polarization state. In this work, we derive a statistical quantity that describes the rate at which the field intensity in the beam, on average, is redistributed between the beam's polarization state at any time and the state orthogonal to it. This method allows one to treat the dynamical properties of the polarization fluctuations both theoretically and experimentally. We demonstrate the method by applying it to important special cases, such as fields obeying Gaussian statistics, black-body radiation pencils and depolarized laser beams. We also prove that a geometric approach introduced earlier is closely connected with the present model.

Radiation from all sources, whether natural or artificial, exhibits fluctuations of the electric-field strength and polarization state, due to either the inherent properties of the source or the randomness of the medium in which the field propagates. Thus, by measuring and characterizing the nature of these fluctuations, one can extract useful information about the source or the transmitting medium. The importance of the polarization fluctuations has been addressed in many theoretical and experimental studies, such as those on polarization mode dispersion in optical fibers [1, 2], particle shape determination [3], polarimetric radar imaging [4], physics of vertical-cavity surface-emitting lasers [5], atom–field interaction [6], supercontinuum

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generation [7], and polarimetry of cosmic radiation, including the microwave background radiation from the early universe [8]–[10]. Even the most fundamental notions of the interaction between electromagnetic radiation and matter, such as the quantum-mechanical selection rules [11], explicitly depend on the polarization state of the field. Recently, the statistical properties of the scattered or fluorescent light have been used to probe the local environments in complex systems, such as biological or disordered nanoscopic media [12]. However, the dynamical properties of the polarization fluctuations, which can distinguish the fields from each other even if their degrees of polarization are equal, have not been directly studied.

In this work, we characterize the temporal fluctuations of polarization in electromagnetic beam fields by using a statistical quantity, $\gamma_{\mathrm{p,\,Jones}}(\tau)$, constructed using the Jones vectors. This quantity is obtained by evaluating the time-averaged fraction of the electric-field intensity that at time $t+\tau$ remains in the same polarization state in which the field instantaneously was at time t. It has an unambiguous physical meaning in terms of energy exchange between orthogonal polarization modes. On introducing $\gamma_{\mathrm{p,\,Jones}}(\tau)$ and assessing its main properties, we apply this quantity to illustrative examples and elucidate its relation to an analogous quantity that describes the time-evolution of the Poincaré vector [13].

Let us consider the temporal fluctuations of the electric field in an electromagnetic beam at some given point in space. The complex-valued electric field (a two-element Jones vector) and the field intensity at a time instant t are represented by $\mathbf{E}(t)$ and $I(t) = \mathbf{E}^*(t)\cdot\mathbf{E}(t)$, respectively. The asterisk denotes complex conjugation. The normalized Jones vector, which describes the polarization state in the complex-vector domain, is given by [14]

$$\mathbf{e}(t) = \mathbf{E}(t)/\sqrt{I(t)}. \tag{1}$$

Since the field $\mathbf{E}(t)$ is a random process, at time $t+\tau$ the polarization state may differ from that at time t. However, at $t+\tau$, the vector field may still have a certain fraction of its intensity in the polarization state $\mathbf{e}(t)$. This fraction,

$$\gamma_{\mathbf{e}}(t, t+\tau) \equiv \frac{I_{\mathrm{still\ in}\,\mathbf{e}(t)}(t+\tau)}{I(t+\tau)} \tag{2}$$

is given by

$$\gamma_{\mathbf{e}}(t, t+\tau) = \frac{|\mathbf{e}^*(t)\cdot\mathbf{E}(t+\tau)|^2}{I(t+\tau)} = |\mathbf{e}^*(t)\cdot\mathbf{e}(t+\tau)|^2, \tag{3}$$

where the dot stands for scalar product. If $\mathbf{e}(t)$ and $\mathbf{e}(t+\tau)$ are the same, $\gamma_{\mathbf{e}}(t, t+\tau)$ assumes its maximum value of 1, and if they are orthogonal, $\gamma_{\mathbf{e}}(t, t+\tau)$ takes on the minimum value of 0.

The quantity $\gamma_{\mathbf{e}}(t, t+\tau)$ accounts only for the changes in the polarization state, regardless of the intensity fluctuations. If one were to take a time average of $\gamma_{\mathbf{e}}(t, t+\tau)$ in a stationary beam field, the instants of time at which the intensities $I(t)$ and $I(t+\tau)$ are low would lead to equal contributions to the average as compared to those instants when $I(t)$ and $I(t+\tau)$ are high. Therefore, we weight $\gamma_{\mathbf{e}}(t, t+\tau)$ by the intensity product $I(t)I(t+\tau)$, and obtain

$$\begin{aligned}\gamma_{\mathbf{E}}(t, t+\tau) &= I(t)I(t+\tau)|\mathbf{e}^*(t)\cdot\mathbf{e}(t+\tau)|^2 \\ &= |\mathbf{E}^*(t)\cdot\mathbf{E}(t+\tau)|^2.\end{aligned} \tag{4}$$

In terms of quantum optics, the product $I(t)I(t+\tau)$ describes the probability of coexistence of photons at t and $t+\tau$, while $|\mathbf{e}^*(t)\cdot\mathbf{e}(t+\tau)|^2$ accounts for the probability that these photons have the same state of polarization.

The maximum value of the time average $\langle \gamma_{\mathbf{E}}(t, t+\tau)\rangle$ obviously is $\langle I(t)I(t+\tau)\rangle$, corresponding to a situation in which the polarization states $\mathbf{e}(t)$ and $\mathbf{e}(t+\tau)$ are the same. For a statistically stationary beam $\langle \gamma_{\mathbf{E}}(t, t+\tau)\rangle$ and $\langle I(t)I(t+\tau)\rangle$ depend only on the time difference τ. We make use of $\langle \gamma_{\mathbf{E}}(t, t+\tau)\rangle$ and define, as is customary in optical coherence theory, a normalized function $\gamma_{\mathrm{p,\,Jones}}(\tau)$ as

$$\gamma_{\mathrm{p,Jones}}(\tau) = \frac{\langle |\mathbf{E}^{*}(t)\cdot \mathbf{E}(t+\tau)|^2\rangle}{\langle I(t)I(t+\tau)\rangle}. \tag{5}$$

If $I(t)$ and $\mathbf{e}(t)$ are statistically independent, equation (5) reduces simply to $\gamma_{\mathrm{p,\,Jones}}(\tau) = \langle \gamma_{\mathbf{e}}(t, t+\tau)\rangle = \langle |\mathbf{e}^{*}(t)\cdot \mathbf{e}(t+\tau)|^2\rangle$. Such an independence ensues, for instance, if either $I(t)$ or $\mathbf{e}(t)$ is constant, as is the case in intensity-stabilized lasers or in polarized random fields.

The function $\gamma_{\mathrm{p,\,Jones}}(\tau)$ has the following properties. By definition, $\gamma_{\mathrm{p,\,Jones}}(0) = 1$. If the polarization state does not change, $\gamma_{\mathrm{p,\,Jones}}(\tau)$ assumes for all τ its largest allowed value equal to 1. If there exists a time interval $\tau_{\perp}$ during which the field intensity is fully transferred to an orthogonal polarization state, then $\gamma_{\mathrm{p,\,Jones}}(\tau_{\perp}) = 0$, which also is the smallest possible value. Starting from $\tau = 0$, the function $\gamma_{\mathrm{p,\,Jones}}(\tau)$ decreases if the polarization state of the field evolves in time, and $\gamma_{\mathrm{p,\,Jones}}(\tau)$ remains constant if the field polarization does not vary; in general, $0 \leqslant \gamma_{\mathrm{p,\,Jones}}(\tau) \leqslant 1$.

We proceed now to calculate $\gamma_{\mathrm{p,\,Jones}}(\tau)$ for random electromagnetic beam fields obeying Gaussian statistics. Let the electric-field vector oscillate in the xy plane of a Cartesian coordinate system, while the field propagates in the z-direction. Expressing the fourth-order correlation functions in terms of the second-order ones by means of the Gaussian moment theorem [15], we find

$$\langle I(t)I(t+\tau)\rangle = (I_x + I_y)^2 + I_x^2|\gamma_{xx}(\tau)|^2 + I_y^2|\gamma_{yy}(\tau)|^2 + I_x I_y[|\gamma_{xy}(\tau)|^2 + |\gamma_{yx}(\tau)|^2], \tag{6}$$

where I_x and I_y are the time-averaged intensities of the $E_x(t)$ and $E_y(t)$ components, respectively, and $\gamma_{ij}(\tau)$ is the intensity-normalized mutual correlation function of the i and j vector components, i.e.

$$\gamma_{ij}(\tau) = \frac{1}{\sqrt{I_i I_j}}\langle E_i^{*}(t)E_j(t+\tau)\rangle. \tag{7}$$

Similarly, the numerator in equation (5) can be expressed in terms of $\gamma_{ij}(\tau)$ as

$$\langle |\mathbf{E}^{*}(t)\cdot \mathbf{E}(t+\tau)|^2\rangle = I_x^2 + I_y^2 + 2I_x I_y|\gamma_{xy}(0)|^2 + |I_x\gamma_{xx}(\tau) + I_y\gamma_{yy}(\tau)|^2. \tag{8}$$

On substituting equations (6) and (8) into equation (5), we obtain in a straightforward way a lengthy expression for $\gamma_{\mathrm{p,\,Jones}}(\tau)$ that can be rewritten in a compact and more insightful way as

$$\gamma_{\mathrm{p,\,Jones}}(\tau) = \frac{1 + P^2 + 2|\gamma_{\mathrm{W}}(\tau)|^2}{2[1 + \gamma_{\mathrm{EM}}^2(\tau)]}, \tag{9}$$

where P is the degree of polarization of the beam field [15], $\gamma_{\mathrm{W}}(\tau)$ is a complex electric-field correlation function [16, 17], and $\gamma_{\mathrm{EM}}(\tau)$ is the electromagnetic degree of coherence [18, 19], respectively. Explicitly, these quantities are given by the expressions [13]

$$P^2 = 2\frac{\mathrm{tr}[\boldsymbol{\mathcal{E}}^2(0)]}{\mathrm{tr}^2[\boldsymbol{\mathcal{E}}(0)]} - 1, \tag{10}$$

$$\gamma_{\mathrm{W}}(\tau) = \frac{\mathrm{tr}[\boldsymbol{\mathcal{E}}(\tau)]}{\mathrm{tr}[\boldsymbol{\mathcal{E}}(0)]}, \tag{11}$$

$$\gamma_{\mathrm{EM}}^2(\tau) = \frac{\mathrm{tr}[\mathcal{E}(\tau)\mathcal{E}(-\tau)]}{\mathrm{tr}^2[\mathcal{E}(0)]}, \tag{12}$$

where tr denotes the trace and $\mathcal{E}(\tau)$ is the coherence matrix, whose elements are $\mathcal{E}_{ij}(\tau) = \sqrt{I_i I_j}\gamma_{ij}(\tau)$. Since the intensities and correlation functions appearing in equations (6) and (8) can be measured by conventional methods, the quantity $\gamma_{\mathrm{p,\,Jones}}(\tau)$ is experimentally measurable as well. Equation (9) indicates that two beams with the same degree of polarization can have different polarization dynamics.

For $\tau = 0$, $\gamma_{\mathrm{W}}(0) = 1$ and $\gamma_{\mathrm{EM}}^2(0) = (P^2+1)/2$, so that equation (9) gives $\gamma_{\mathrm{p,\,Jones}}(0) = 1$, as expected. In the limit as $\tau \to \infty$, both $\gamma_{\mathrm{W}}(\tau)$ and $\gamma_{\mathrm{EM}}(\tau)$ tend to 0 and the function $\gamma_{\mathrm{p,\,Jones}}(\tau)$ approaches $(P^2+1)/2$. For an unpolarized beam $\gamma_{\mathrm{p,\,Jones}}(\tau)$ approaches $1/2$, because on average, when $\tau \to \infty$, the field intensity is equally distributed between any two orthogonal polarization states.

Let us apply equation (9) to calculate $\gamma_{\mathrm{p,\,Jones}}(\tau)$ for a black-body field [20]. Since a pencil of black-body radiation is unpolarized [21], $P = 0$. The functions $\gamma_{\mathrm{W}}(\tau)$ and $\gamma_{\mathrm{EM}}(\tau)$ are given by [13]

$$\gamma_{\mathrm{W}}(\tau) = \frac{90}{\pi^4}\zeta\left(4,\, 1 + \mathrm{i}\frac{k_{\mathrm{B}}T}{\hbar}\tau\right), \tag{13}$$

$$\gamma_{\mathrm{EM}}(\tau) = \frac{90}{\sqrt{2}\pi^4}\left|\zeta\left(4,\, 1 + \mathrm{i}\frac{k_{\mathrm{B}}T}{\hbar}\tau\right)\right|, \tag{14}$$

where k_{B} is Boltzmann's constant, T the temperature, $\hbar$ Planck's constant divided by 2π and $\zeta(s, a)$ the generalized Riemann–Hurwitz zeta function [14]. Substituting these equations into equation (9), we obtain

$$\gamma_{\mathrm{p,\,Jones}}(\tau) = \frac{1 + 2\left|\frac{90}{\pi^4}\zeta\left(4,\, 1 + \mathrm{i}\frac{k_{\mathrm{B}}T}{\hbar}\tau\right)\right|^2}{2 + \left|\frac{90}{\pi^4}\zeta\left(4,\, 1 + \mathrm{i}\frac{k_{\mathrm{B}}T}{\hbar}\tau\right)\right|^2}. \tag{15}$$

In figure 1, this quantity is plotted as a function of the propagation distance $c\tau$ of the field for three different values of T (c is the speed of light). Starting from $\gamma_{\mathrm{p,\,Jones}}(0) = 1$, the function $\gamma_{\mathrm{p,\,Jones}}(\tau)$ smoothly decreases toward $1/2$ with a rate that depends on the temperature. The polarization time (τ_{p}) and polarization length ($l_{\mathrm{p}} = c\tau_{\mathrm{p}}$) can be defined by using $\gamma_{\mathrm{p,\,Jones}}(\tau)$, e.g. by requiring that $\gamma_{\mathrm{p,\,Jones}}(\tau_{\mathrm{p}}) = 3/4$. It can be seen from figure 1 that at room temperature the polarization state of a black-body beam remains essentially unchanged over a propagation distance of several micrometers; at 10 K the polarization length is already of the order of 100 μm.

As another demonstrative example we consider an optical system that is used to depolarize laser light. In this system, a linearly polarized laser beam is split into two beams with equal powers and orthogonal polarizations (say, x and y states) by using a polarizing beam splitter. The beams propagate different distances, after which they are recombined into a single beam with another polarizing beam splitter. If the time delay, τ_{d}, of one beam with respect to the other is much longer than the coherence time τ_{c} of the field, the resulting beam can be considered unpolarized [22]. For the beam at the system's output, we write

$$\gamma_{\mathrm{p,\,Jones}}(\tau) = \frac{2 + 2|\gamma_{xy}(0)|^2 + 4|\gamma_{xx}(\tau)|^2}{4 + 2|\gamma_{xx}(\tau)|^2 + |\gamma_{xy}(\tau)|^2 + |\gamma_{yx}(\tau)|^2}. \tag{16}$$

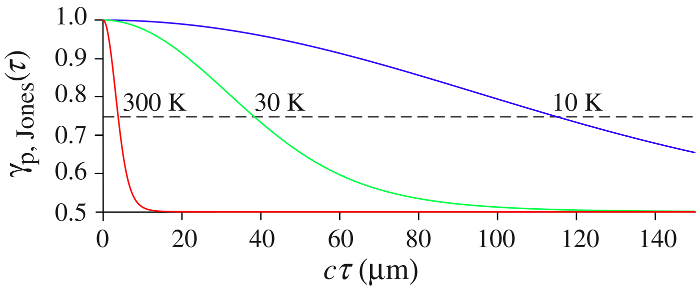

Figure 1. Behavior of $\gamma_{\mathrm{p,\,Jones}}(\tau)$ for black-body radiation; $c\tau$ is the propagation distance. The red, green and blue curves correspond to black-body temperatures of 300, 30 and 10 K, respectively. At room temperature, the polarization state of the beam is effectively unchanged over a propagation distance of several micrometers. The horizontal dashed line marks the criterion, $\gamma_{\mathrm{p,\,Jones}}(\tau_{\mathrm{p}}) = 3/4$, for the polarization length $c\tau_{\mathrm{p}}$.

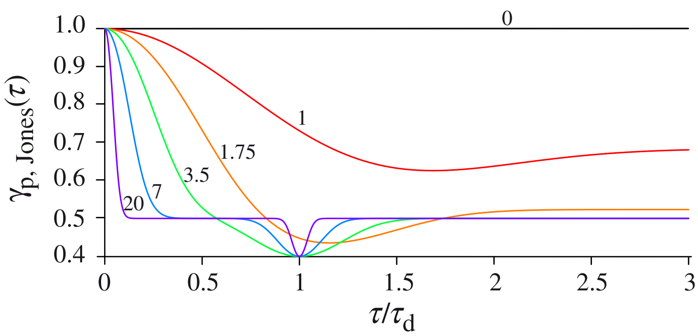

Figure 2. Behavior of $\gamma_{\mathrm{p,\,Jones}}(\tau)$ for a depolarized laser beam. An originally fully polarized field is divided into two orthogonally polarized beams, which are then recombined with a certain time delay τ_{d} with respect to each other. The quantity $\gamma_{\mathrm{p,\,Jones}}(\tau)$ of the emerging beam is plotted as a function of τ/τ_{d} for several values of $\tau_{\mathrm{d}}/\tau_{\mathrm{c}}$, where τ_{c} is the coherence time of the incident beam. The parameters $\tau_{\mathrm{d}}/\tau_{\mathrm{c}}$ are 0 (black curve), 1 (red), 1.75 (orange), 3.5 (green), 7 (blue) and 20 (violet).

This expression is obtained by using equations (5), (6) and (8), and the fact that $I_x = I_y$ and $\gamma_{xx}(\tau) = \gamma_{yy}(\tau)$. Since the x- and y-polarized beams are essentially time-delayed versions of the original beam, we may rewrite equation (16) in terms of the absolute value of the normalized input-beam coherence function, taken to be Gaussian $|\gamma_{\mathrm{o}}(\tau)| = \exp(-\tau^2/2\tau_{\mathrm{c}}^2)$, as

$$\gamma_{\mathrm{p,\,Jones}}(\tau) = \frac{2 + 2e^{-(\tau_{\mathrm{d}}^2/\tau_{\mathrm{c}}^2)} + 4e^{-(\tau^2/\tau_{\mathrm{c}}^2)}}{4 + 2e^{-(\tau^2/\tau_{\mathrm{c}}^2)} + e^{-[(\tau-\tau_{\mathrm{d}})^2/\tau_{\mathrm{c}}^2]} + e^{-[(\tau+\tau_{\mathrm{d}})^2/\tau_{\mathrm{c}}^2]}}, \tag{17}$$

here we have made use of the equalities $|\gamma_{xy}(0)|^2 = |\gamma_{\mathrm{o}}(\tau_{\mathrm{d}})|^2$, $|\gamma_{xx}(\tau)|^2 = |\gamma_{\mathrm{o}}(\tau)|^2$, $|\gamma_{xy}(\tau)|^2 = |\gamma_{\mathrm{o}}(\tau - \tau_{\mathrm{d}})|^2$, and $|\gamma_{yx}(\tau)|^2 = |\gamma_{\mathrm{o}}(\tau + \tau_{\mathrm{d}})|^2$. In figure 2, the quantity $\gamma_{\mathrm{p,\,Jones}}(\tau)$ is shown as a function of τ/τ_{d} for several values of $\tau_{\mathrm{d}}/\tau_{\mathrm{c}}$, ranging from 0 to 20. With increasing τ, all these curves tend to the value of $(\mathrm{e}^{-\tau_{\mathrm{d}}^2/\tau_{\mathrm{c}}^2} + 1)/2$. Note that the degree of polarization in this case is

$P = e^{-\tau_d^2/2\tau_c^2}$ so that with a sufficiently long τ_d the beam, indeed, is close to unpolarized and the value of $\gamma_{p,\,Jones}(\tau)$ approaches $1/2$.

For $\tau_d/\tau_c = 0$ (black curve in figure 2), the beam is fully polarized and $\gamma_{p,\,Jones}(\tau) = 1$ at all τ. The polarization time τ_p is infinite in this case. It is infinite also for all values of τ_d/τ_c that are smaller than about 0.75. Above this level, when τ_d/τ_c becomes larger, the polarization time of the field decreases, i.e. the instantaneous polarization state evolves faster in time and deviates more from the average. Note, however, that at large values of τ_d/τ_c the time τ_p is close to the coherence time τ_c of the field, which can also be verified by setting $\gamma_{p,\,Jones}(\tau)$ in equation (17) equal to $3/4$ and evaluating τ.

For $\tau_d/\tau_c = 1$ (red curve in figure 2), the emerging beam is partially polarized ($P = 0.6$) and $\gamma_{p,\,Jones}(\tau)$ saturates to 0.68 at large τ. However, at $\tau/\tau_d \approx 1.6$, the function $\gamma_{p,\,Jones}(\tau)$ has a local minimum with the value of about 0.63. When τ_d/τ_c is increased, the local minimum becomes more pronounced and $\gamma_{p,\,Jones}(\tau)$ starts to have values less than $1/2$. This means that, on average, no matter what the polarization state of the light is at time t, at time $t+\tau$ the orthogonal polarization state will have a higher intensity than the original state of polarization. The time interval within which the values of $\gamma_{p,\,Jones}(\tau)$ are lower than $1/2$ can be as long as several coherence times τ_c.

When the delay time τ_d increases, the local minimum of $\gamma_{p,\,Jones}(\tau)$ shifts toward $\tau/\tau_d = 1$. It can be seen from equation (17) that the quantity $\gamma_{p,\,Jones}(\tau_p)$ approaches the value of $2/5$, when τ_d considerably exceeds τ_c, (which is exactly the condition for the beam to be depolarized). Thus, on average, a fraction of $3/5$ of the light intensity will after a time interval τ_d belong to the polarization state that is orthogonal to the original one. This detail in the polarization dynamics of such depolarized laser beams is an example of the polarization fluctuation effects that can be predicted by using our model.

We finally discuss the connection between $\gamma_{p,\,Jones}(\tau)$ and a function introduced previously by us to describe the dynamics of polarization fluctuations in random beam-like fields [13]. That function is written in terms of two instantaneous Poincaré vectors, $\mathbf{S}(t)$ and $\mathbf{S}(t+\tau)$, as

$$\gamma_{p,\,Poincaré}(\tau) = \frac{\langle \mathbf{S}(t)\cdot\mathbf{S}(t+\tau)\rangle}{\langle S_0(t)S_0(t+\tau)\rangle}, \tag{18}$$

where $S_0(t)$ is the Stokes parameter equal to the instantaneous intensity of the electric field at time t. The construction of this function was based on the fact that the dot product of the two Poincaré vectors decreases when the polarization states corresponding to these vectors separate as a function of τ. Searching for the connection between $\gamma_{p,\,Jones}(\tau)$ and $\gamma_{p,\,Poincaré}(\tau)$, we find that the normalized (unit-length) Poincaré vectors $\mathbf{s}(t)$ and $\mathbf{s}(t+\tau)$ and the normalized Jones vectors $\mathbf{e}(t)$ and $\mathbf{e}(t+\tau)$ satisfy the following general relation:

$$\mathbf{s}(t)\cdot\mathbf{s}(t+\tau) = 2|\mathbf{e}^*(t)\cdot\mathbf{e}(t+\tau)|^2 - 1. \tag{19}$$

Multiplying both sides by $I(t)I(t+\tau)$ and performing time averaging we then obtain

$$\langle \mathbf{S}(t)\cdot\mathbf{S}(t+\tau)\rangle = 2\langle|\mathbf{E}^*(t)\cdot\mathbf{E}(t+\tau)|^2\rangle - \langle I(t)I(t+\tau)\rangle, \tag{20}$$

from which it follows that

$$\gamma_{p,\,Poincaré}(\tau) = 2\gamma_{p,\,Jones}(\tau) - 1. \tag{21}$$

Thus, the function $\gamma_{p,\,Poincaré}(\tau)$ is a scaled, intensity-weighted time average of the quantity $I_{\text{still in }\mathbf{e}(t)}(t+\tau)/I(t+\tau)$. The scaling ensures that $\gamma_{p,\,Poincaré}(\tau)$ approaches the squared degree of polarization, P^2, when $\tau \to \infty$.

It is obvious that both $\gamma_{\mathrm{p,\,Jones}}(\tau)$ and $\gamma_{\mathrm{p,\,Poincaré}}(\tau)$ can be used equally well to describe the dynamics of the fluctuating polarization state. In particular, they characterize how fast, on average, the instantaneous polarization state can change as a function of time. Furthermore, they provide different additional information on the polarization dynamics. The quantity $\gamma_{\mathrm{p,\,Poincaré}}(\tau)$ describes how the tip of the instantaneous Poincaré vector moves on the Poincaré sphere, and it shows that the effective deviation of the tip from its average position is determined by the degree of polarization. On the other hand, the quantity $\gamma_{\mathrm{p,\,Jones}}(\tau)$ directly describes the energy exchange between the field's polarization states. We note that the expressions in equations (5) and (18) are quite general, since as long as the instantaneous polarization state and intensity can be defined, $\gamma_{\mathrm{p,\,Jones}}(\tau)$ and $\gamma_{\mathrm{p,\,Poincaré}}(\tau)$ can be evaluated.

In summary, we have introduced a function $\gamma_{\mathrm{p,\,Jones}}(\tau)$ that describes the dynamical properties of polarization fluctuations in stationary, beam-like (two-dimensional) electromagnetic fields. This function characterizes the ability of the field to preserve its intensity in a certain polarization state within a fixed time interval. We have derived a compact, insightful expression for this quantity for the case of electromagnetic beams obeying Gaussian statistics. This expression allows one to evaluate or measure the function $\gamma_{\mathrm{p,\,Jones}}(\tau)$. We have presented useful examples of the application of our model, by calculating $\gamma_{\mathrm{p,\,Jones}}(\tau)$ for black-body radiation and depolarized laser beam. Finally, we have shown that the functions $\gamma_{\mathrm{p,\,Jones}}(\tau)$ and $\gamma_{\mathrm{p,\,Poincaré}}(\tau)$ have a one-to-one correspondence, which adds physical insight to the geometry-based concept of $\gamma_{\mathrm{p,\,Poincaré}}(\tau)$ introduced by us previously [13]. Both functions can be used to introduce the notions of 'polarization time' and 'polarization length', over which the beam's polarization state remains essentially unchanged.

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