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Impact of the shape of demand distribution in decision models for operations management

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ABSTRACT

Decision support tools are increasingly used in operations where key decision inputs such as demand, quality, or costs are uncertain. Often such uncertainties are modeled with probability distributions, but very little attention is given to the shape of the distributions. For example, state-of-the-art planning systems have weak, if any, capabilities to account for the distribution shape. We consider demand uncertainties of different shapes and show that the shape can considerably change the optimal decision recommendations of decision models. Inspired by discussions with a leading consumer electronics manufacturer, we analyze how four plausible demand distributions affect three representative decision models that can be employed in support of inventory management, supply contract selection and capacity planning decisions. It is found, for example, that in supply contracts flexibility is much more appreciated if demand is negatively skewed, i.e., has downside potential, compared to positively skewed demand. We then analyze the value of distributional information in the light of these models to find out how the scope of improvement actions that aim to decrease demand uncertainty vary depending on the decision to be made. Based on the results, we present guidelines for effective utilization of probability distributions in decision models for operations management.

1. Introduction

Organizations use mathematical models to support decision making in uncertain environments. Such models often account for many uncertain variables: in manufacturing, product demand varies from period to another; cost parameters change over time due to fluctuating raw-material prices or rising wages; and suppliers may not supply as promised due to constrained capacity or quality problems. Uncertain demand, in particular, is a key variable for operations management and supply chain planning: for example, push type supply chains are typically coordinated using a forecast of demand for a given planning period. This demand forecast is built on expert knowledge and/or mathematical forecast and serves as a basis for other supply chain planning activities from operational to strategic level decisions [41]. In principle, accurate forecasts would allow cost-efficient coordination, but forecasting is difficult in turbulent environments. As a result, increasing attention has been paid to the question of how demand and supply uncertainties should be accounted for in supply chain modeling [10]. These uncertainties impact all levels of operations management: strategic (e.g., [14,26]), tactical (e.g., [18]) and operational (e.g., [36]).

Considerable efforts have been made to develop both stochastic (distribution based) and robust (distribution free) models to support decision making under uncertainty. Such models can be used to derive insights under very general assumptions; for example, they can be used to study how a given supply contract shares demand risk in a supply chain, or how lead time variability reduction can systematically lower inventory levels while keeping the shortage risk constant. But at a more concrete level (such as when implementing decision support systems) detailed assumptions about the uncertainties are required for setting numerical contract parameters or calculating target inventory levels, for example. Thus, the estimation of uncertainties is critical for model implementation.

In this paper, we study how the shape of demand distribution can impact the results of decision making models in operations management, and discuss the value of distributional knowledge in these models. In particular, we focus on demand uncertainty and show how seemingly similar but qualitatively different uncertainties impact three widely employed models. We use different

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demand distributions, which exhibit at least one of the following statistical properties: (i) symmetry, (ii) positive skewness, (iii) negative skewness, and (iv) bimodality. Arguably, these properties can be used to describe the qualitative attributes of a large share of realistic demand types. We assume demands which have identical expected value and variance, but which differ in shape with respect to (i)–(iv). By drawing upon these examples, we also analyze how much value the knowledge about a distribution can offer, compared to a case where the distribution is not known. Similar analysis has been done before for individual decision models, but not extensively for multiple models as we do here. Based on the resulting insights we provide high-level guidelines for managers who seek to address uncertainties in all levels of decision making in operations.

Our study is motivated by a large consumer electronics company which is in the process of designing a new sourcing strategy. The company sought better understanding on how demand uncertainty can be managed when there are different types of demand, depending on the product category and market segment. Such challenges are not unique: because common planning systems make only use of point demand forecast, and deviation at best, the impact of the shape of demand distribution is largely neglected. According to Van Nieuwenhuyse et al. [32], companies lack capability to analyze demand uncertainty and use the results as decision support. For example, they note that SAP's Advanced Planning and Optimization module "disregards uncertainty". However, the same authors have developed an advanced software module that accounts for stochastic demand and they report promising results from two cases in different manufacturing industries.

Other promising applications have been reported in this area: Talluri et al. [42] present a simple enhancement for lead time demand estimation that could lead to large saving in inventory costs at a pharmaceutical company. Nagali et al. [31] describe the Procurement Risk Management approach at Hewlett-Packard, where instead of a point forecast, a scenario-based approach to demand forecasting has been applied successfully with improved component availability and significant cost savings. Sodhi [40] presents exploratory work on managing Sales and Operations Planning process (S&OP) at a consumer electronics manufacturer. He demonstrates how the value of flexibility and risk of shortages or excess inventories can be analyzed with a stochastic demand model. Finally, according to survey of 180 executives by Jain et al. [20], "Non-normal demand distributions that make traditional forecast modeling difficult" was pointed out as one of the biggest challenges of demand management across companies. Jain et al. concluded that companies with best-in-class demand management capabilities reap multiple benefits in form of, e.g., improved invetory turns and higher order fulfillment rates.

The rest of the paper is structured as follows: Section 2 relates our work to earlier approaches in uncertainty modeling and operations management. Section 3 covers the example demand distributions and Section 4 describes the models and corresponding numerical results. Subsequently, Section 5 elaborates the value of distributional information in these models. Finally, Section 6 discusses the implications for managers and Section 7 presents conclusions.

2. Approaches to uncertainty modeling in operations management

Extensive reviews of quantitative research in operations management include Kouvelis et al. [22], Tang [43] and Peidro et al. [34], who also refer to many applications that consider uncertainties. We discuss mostly optimization based approaches in connection with uncertainty modeling. In this respect, two complementary approaches can be identified: *stochastic optimization* and *robust optimization*. Gupta and Maranas [18] further divide stochastic optimization to *scenario approach* where uncertainties are presented by a set of discrete scenarios, and *distribution approach* where probability distributions are used; this latter approach is the focus in this paper, but our aim is to study the sensitivity of models with respect to demand distribution. Robust optimization, on the other hand, has recently received attention as a distribution-free optimization approach (e.g., [16,4]). There are also studies that fall into both categories: e.g., Andersson et al. [1] select a distribution based on entropy maximization principle, which makes the approach both distribution based and robust at the same time.

We use safety stock calculation for inventory management as an example of an operational model. Various studies have discussed the impact of demand distribution in this context: Naddor [30] compares inventory costs under Poisson, beta, negative binomial, uniform, and 2-point (extreme) distributions. He finds that false assumptions about a distribution can become costly in extreme cases, but with realistic distributions, inventory management is insensitive to distribution choice and only first two moments of the distribution are essential. Fortuin [15] studies a similar inventory policy with Gaussian, logistic, gamma, log-normal, and Weibull distributions. He finds that these yield similar policies and thus recommends using logistic distribution, because it results in simple ordering formula. Lau and Zaki [25] note that mean and variance are not sufficient for safety stock calculation, but also skewness and kurtosis should be accounted for. Eppen and Martin [12] study safety stock calculation when demand distribution is estimated from data; they also present a bimodal demand distribution in their motivational example, which is one of the rare cases that bimodality is explicitly considered at least in some level. In robust approaches, in his seminal paper Scarf [37] considers ordering when only the minimum and maximum demands are known and develops a (conservative) ordering policy which is currently known as the *Scarf's rule*. More recently, Gallego and Moon [16], Yue et al. [44], Perakis and Roels [35] and Andersson et al. [1] have extended the literature of robust inventory management; the latter found that maximum entropy approach works well under both risk-neutral and risk-averse objective in a newsvendor setting. We note that all aforementioned references treat the specific topic of inventory management in more detail than we do. In this respect, our contribution relates to ability to compare insights from this case with two other models' outcomes, which yields a more complete picture over all planning horizons of a company.

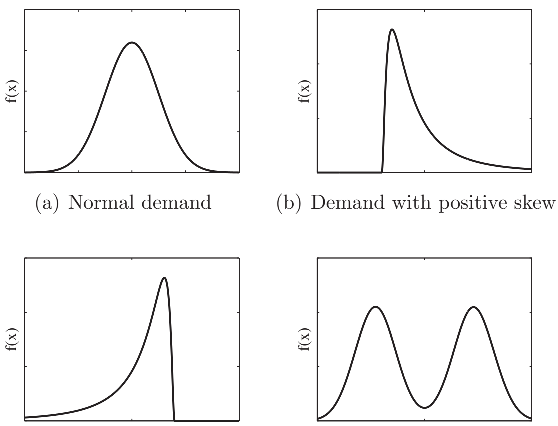
Our second model is for tactical level sourcing with capacity reservation options, introduced by Cachon and Lariviere [5]. They characterize demand with the scaled distribution family, and note that most their results hold for arbitrary distributions. Pasternack [33] also studies sourcing in a two stage setting; he focuses on buyback contracts and gives both generic results and examples based on normal distribution. Barnes-Schuster et al. [2] study various supply contracts and find that their relative value grows along with demand variation, when demand follows normal distribution. Lariviere and Porteus [24] study the impact of demand distribution in a procurement setting, where manufacturer sets the wholesale price of a product. They conclude that pricing is critically dependent of the coefficient of variation of demand. The results hold for a large family of distributions, but this family contains only unimodal distributions. Ben-Tal et al. [3] compare a robust approach with perfect hindsight (deterministic demand) in a simulation study for dynamic inventory management with flexible supply contracts. They find that the mean difference in costs grows from 8% to 38% (the robust setting being more costly) when demand fluctuation changes from 10% to 70%

(with respect to mean demand). While 38% might appear a vast cost difference, its significance is largely theoretical: the comparison is done between a perfect demand forecast and a very conservative forecast using minimum and maximum demand only. Our approach is less conservative (and perhaps more realistic) in the sense that often the decision-maker has some some knowledge about the demand shape, which she arguably should utilize in decision making. Our analysis is also different from the ones mentioned in that we explicitly account for bimodality.

Our third model is for strategic facility location and capacity acquisition under demand uncertainty. Snyder [39] reviews facility location under uncertainty and notes that there are many theoretical papers on both stochastic and robust approaches, but applications are relatively rare due computational burden and difficulties in uncertainty data estimation. Melo et al. [28] also review facility location models and conclude that integrating uncertainty with location decisions is scarce. This might be due to computational and data requirements of these optimization models as noted by Dasci and Verte [9], who propose a complementary framework for facility location with a continuous model. An application of this framework is presented by Dasci and Laporte [8]; this model is also in the focus of our analysis. Dasci and Laporte use normal and exponential demand distributions for illustrative purposes, but they do not focus on the impact of distribution shape. To our knowledge, demand shape has not been studied in this line of literature before – mainly because the majority of the facility location literature focuses on discrete optimization models for determining the exact locations of facilities.

3. Demand distributions

When modeling demand with a probability distribution, one needs to choose an appropriate shape of the distribution and estimate the corresponding parameters or statistical moments. Here, there are relevant questions such as: What is the level of uncertainty that needs to be estimated? Is there high upside or downside potential in the demand? Are there limitations to the demand, due to, e.g., market size? In what follows, we consider the four demand distributions of Fig. 1. These distributions are illustrative (i.e., not estimated from data); they cover most demand types; and can be employed to answer questions such as above.



(a) Normal demand (b) Demand with positive skew

(c) Demand with negative skew (d) Bimodal demand

Fig. 1. Probability density functions of example demand distributions.

The distributions and their key qualitative features are summarized in Table 1. All distributions have identical first and second moments and a coefficient of variation $c_v = \sigma/\mu = 0.5$. Because of the identical mean and variance, *a typical planning system would not differentiate data that would be generated with these distributions*. However, the higher moments – and therefore the shape of distribution – differ significantly, and thus these distributions are qualitatively different from each other.

4. Example models and impact of distribution shape

The models we analyze in this section are summarized in Table 2. They represent typical planning decisions at long-term (strategic), mid-term (tactical), and short-term (operational) planning level. The first two of these models were also particularly interesting to our case company: some of their materials are managed by reorder point based replenishment, and novel procurement contracts such as capacity reservations have become a tempting alternative for their current contracting practices. Fig. 2 links the models to generic planning processes in a typical advanced planning system. For in-depth discussion of planning hierarchies, see also Van Landeghem and Vanmaele [23], Gupta and Maranas [18], Stadtler [41], and Shen [38].

In technical sense, the models are somewhat different: in safety stock calculation there is one decision variable and in the other models there are two. The outcome's dependency on the demand distribution shape varies: in all, the optimal values are based on finding a critical fractile and comparing it to the cumulative distribution function of the demand, but in the latter two, this is only part of the solution. In particular in the facility location model, the quantity of the facilities is independent of the demand shape, whereas the capacity of a facility is not. Finally, we note that the models are not very detailed or complex by purpose; rather we aimed at transparency in the technical analysis.

4.1. Inventory management with safety stock

Continuous replenishment, or the Q,R-model, is common in inventory management. In the model, Q items are ordered when the inventory reaches the reorder point R. The reorder point is often calculated using the concept of safety stock, the calculation of which is subject to some perhaps unrealistic assumptions – we acknowledge that these fallacies have already been pointed out by, e.g., McClain and Thomas [27], further analyzed by Eppen and Martin [12] and, more recently, by Chopra et al. [6].

The reorder point (R) consists of two parts: the lead time demand (*LTD*) and the safety stock (*SS*), which hedges against

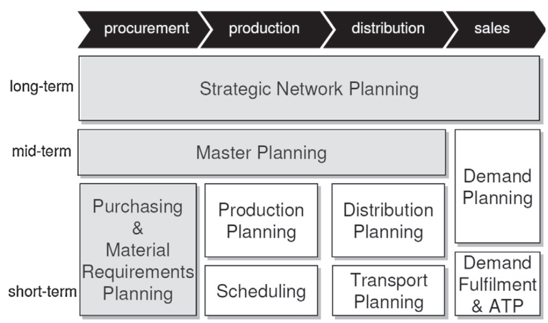


Fig. 2. Typical planning modules of an Advanced Planning System according to Meyr et al. [29]. The analyzed models relate to grayed areas.

Table 1
Distributions for each demand type with equal mean μ and variance σ^2.

Demand	Parameters	Features
Normal $\mathcal{N}(\mu, \sigma^2)$	$\mu = 2\sigma$	Can exhibit negative values. Demand is not limited, yet probability of extreme values approaches zero relatively fast.
Positive skew $\text{Log} - \mathcal{N}(\alpha, \beta) + D_{min}$	$\alpha = \ln(\overline{\mu}^2)/\sqrt{\sigma^2 + \overline{\mu}^2}$, $\overline{\mu} = \mu - D_{min}$, $\beta = \sqrt{\ln(v/(\overline{\mu}^2) + 1)}$	Demand is non-negative – or if shifted as in our examples, has a fixed minimum. Probability mass is concentrated on the left of expected value. Outliers are possible on the right.
Negative skew $D_{max} - \text{Log} - \mathcal{N}(\alpha, \beta)$	See positive skew	Demand has a maximum limit, but is expected to be quite close to this limit. It is possible to end up with a disappointing demand due to extreme values on the left.
Bimodal $\text{B} - \mathcal{N}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, d, p_1)$	$p_1 \overset{1}{=} 0.5$, $\mu_i = \mu \pm d/2$, $d \overset{2}{=} 3\sigma_i$, $\sigma_1^2 = \sigma_2^2 \overset{3}{=} \sigma^2 - d^2 = \sigma^2/10$	The demand is either "low" or "high". The total variation can be divided into two components: within and between the peaks.

1We assume equiprobable peaks.
2We set the difference between peaks $d = 3\sigma_i$, which satisfies the bimodality requirement for normal distributions with identical variances: the difference between the peaks is over twice the peak deviation.
3Based on the (simplified) result from Cohen [7] for estimating parameters of bimodal normal distribution with the method of moments.

demand and lead time uncertainties. Here, D denotes the demand in one time period with expected demand μ_D and variation σ_D^2, and LT is the lead time with expectation μ_{LT} and variation σ_{LT}^2. With these, the expected lead time demand (μ_{LTD}), the safety stock and the reorder point can be calculated with:

$$\mu_{LTD} = \mu_D \mu_{LT} \tag{1}$$

$$SS = z\sqrt{\sigma_D^2 \mu_{LT} + \sigma_{LT}^2 \mu_D^2} \tag{2}$$

$$R = \mu_{LTD} + SS, \tag{3}$$

where z is a service level factor, i.e., a number that corresponds to the desired service level in standard cumulative normal distribution (if the target service level is 95%, $z = 1.645$). These formulas should guarantee that no stock-outs occur during the order lead time with a probability defined by the service level. The Q,R-model is relatively straightforward to implement and in most cases the required attributes (μ_D, σ_D^2, μ_{LT}, σ_{LT}^2) can be estimated from data. Arguably, the biggest flaw of the model is that it assumes that the lead time demand is normally distributed.

In general, the assumption of normality has been challenged in various applications, e.g., in the field of finance, where standard risk management approaches are based on probability distributions. Here, the reorder point corresponds to a widespread risk measure Value-at-Risk (VaR), which is the threshold for a loss that occurs with probability α. In the inventory context, VaR, i.e., the reorder point, is the $(1-\alpha)$%-percentile of the lead time distribution. Similarly, the statistic $\text{VaR}_\alpha^{\text{mean}} := \text{VaR}_\alpha - \mu$ ("mean-VaR"; see, e.g., [11]) corresponds to safety stock. Because percentiles, or quantiles, can be defined with the inverse of cumulative distribution function, the shape of demand distribution is a key determinant in safety stock driven inventory management. And, as in financial applications, false assumptions about the distribution can have undesired consequences, such as high costs (excessive inventory) or unexpectedly high risk level (stockout probability).

For a numerical example, consider a case where lead time is assumed deterministic (5 time units, say, days), and that lead time demand follows the distributions presented in the previous section with expected value $\mu = 500$. For example, the normal lead time demand distribution is thought as a sum of five identical normally distributed variables, with parameters $\mu = 100$ and $\sigma = \sqrt{12,500}$; the sum of which leads to the distribution $\mathcal{N}(500, 250)$. The derivation of other distributions by compounding daily demand distributions would be more complex, but in this example we only consider the distribution of the entire lead time demand. The reorder points can be defined directly using the distributions, by choosing the threshold demand for 95%-percentile as illustrated in Fig. 3.

The results show the impact of demand distribution shape on replenishment policy. For example, the safety stock requirement of positively skewed demand is 137% larger compared to negatively skewed demand (175 vs. 416), even though their expected value and variance are identical. The key difference is the upper limit for demand, which in our example of negatively skewed demand is around 700 items.

4.2. Procurement with capacity reservation options

The following example builds on the capacity reservation option model presented by Cachon and Lariviere [5]. We consider a manufacturer of a product whose demand D is characterized by the cumulative distribution function $F(x) = F_x$. The manufacturer procures a single component from a perfectly reliable supplier, who must install capacity K before demand is observed. The demand is then realized and the supplier fills the final order subject

Table 2
Summary of the decision models.

Application	Objective	Decision variables	Source
Inventory replenishment	Safety stock calculation	Reorder point	Eppen and Martin [12]
Procurement contract design	Procurement cost minimization	Amount of fixed orders and capacity reservations	Cachon and Lariviere [5]
Facility location and capacity acquisition	Minimization of fixed and varying costs	Amount of facilities and capacity of each facility	Dasci and Laporte[8]

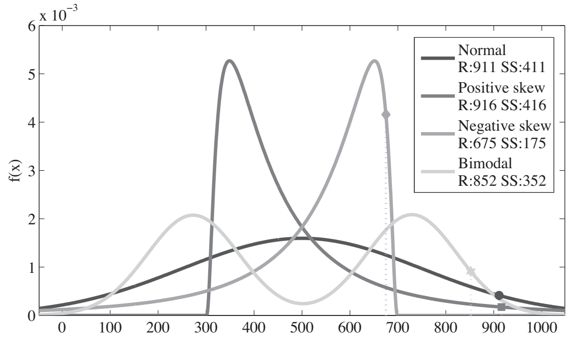


Fig. 3. Reorder points for example distributions with 95% service level.

to the restriction of installed capacity. The expected sales (S) for restricted capacity K is

$$S(K) = E[\min\{D, K\}] = K - \int_0^K F_x \mathrm{d}x. \tag{4}$$

Before the demand is realized, the manufacturer offers the supplier a contract with fixed order commitments $m \geq 0$ for unit price w_m and capacity options $o \geq 0$ for unit price w_o per reserved unit capacity and w_e for each executed option. This sets limits for the manufacturer's final order which must lie between $[m, \ldots, m + o]$. Manufacturer's revenue per fulfilled demand is r and demand D is assumed independent of r.

Following Cachon and Lariviere [5], Section 5.1, we now investigate the forced compliance (manufacturer must build capacity according to contract) and full information scenario, where demand distribution is known for both players. That is, the supplier must cover the largest possible final order $K = m + o$ and prices are reasonable in that they cover costs, so the supplier accepts all contracts offered by the manufacturer. The optimal order policy (m, o) for prices w_m, w_o and w_e can now be defined by maximizing the manufacturer's profit Π:

$$\begin{aligned}\Pi(m, o) = &\underbrace{rS(m+o)}_{\text{expected revenue}} - \underbrace{w_m m - w_o o}_{\text{firm commitments and capacity reservation}} \\ &- \underbrace{w_e[S(m+o) - S(m)]}_{\text{expected option costs}} \\ &= r[m + o - \int_0^{m+o} F_x \mathrm{d}x] - w_m m - w_o o - w_e[(m+o) \\ &- \int_0^{m+o} F_x \mathrm{d}x - (m - \int_0^m F_x \mathrm{d}x)] \\ &= rm + ro - w_m m - w_o o - w_e o \\ &- r\int_0^{m+o} F_x \mathrm{d}x + w_e \int_m^{m+o} F_x \mathrm{d}x. \end{aligned} \tag{5}$$

Our profit function differs from Cachon and Lariviere [5] only in that revenue is calculated from expected sales; this also accounts for the possibility that demand is under the level of firm commitments m. Cachon and Lariviere [5] assume that the manufacturer produces and sells at least m products and hence their expected profit is higher. The maximum of (5) can be found by setting

$$\partial_m \Pi(m, o) = 0 \Rightarrow F(m^*) = 1 - \frac{w_m - w_o}{w_e} \tag{6}$$

$$\partial_o \Pi(m, o) = 0 \Rightarrow F(o^* + m^*) = 1 - \frac{w_o}{r - w_e}, \tag{7}$$

which are the same conditions as in Cachon and Lariviere [5].

Regrouping (5) shows that the optimal profit is dependent on m^*, o^* and two integral terms containing F_x

$$\begin{aligned}\Pi^*(m^*, o^*) = &\underbrace{(r - w_m)}_{>0} m^* + \underbrace{(r - w_o - w_e)}_{>0} o^* \\ &- \underbrace{(r - w_e)}_{>0} \int_0^{m^*+o^*} F_x \mathrm{d}x - w_e \int_0^{m^*} F_x \mathrm{d}x. \end{aligned} \tag{8}$$

If prices and unit revenues are fixed, o^* and m^* are determined by the cumulative distribution function F_x with (6) and (7) and hence the optimal profit depends on only F_x. This dependency is, however, more complex than in the inventory management example. Fig. 4 illustrates the key determinants of the optimal profit. The shape of the distribution (F_x) impacts both the optimal order quantities (in x-axis) and the integral limits (y-limits) and corresponding areas. In some sense, the left-tail of the distribution is now more critical than the right, but the right-side of distribution only becomes irrelevant for values over $m^* + o^*$. Especially if revenue is relatively high compared to costs, a significant part of the distribution becomes meaningful in terms of determining the expected profit.

As a numerical example, we consider the optimal ordering policy and expected profit for demand distributions with μ = 50, 000 and σ = 25, 000, which can now be determined with numerical integration techniques. Here, we calculate the results with the following pricing: $w_m = 1, w_o = 0.1, w_e = 0.95$ and $r = 1.2$, indicating that the capacity reservation option costs 10% of the fixed order price and executed option is 5% more, in total, compared to the fixed order. The optimal strategies are in Fig. 5 and the corresponding values are in Table 3. There are significant differences in both the optimal quantities and the optimal profits: demand with positive skew is the "most profitable" distribution with over 26% margin to the next, which is the bimodal demand. In the optimal fixed order and option quantities, the results differ even more.

Consider now a decision maker, who is responsible for sourcing of a given component. She does not know what the total demand of the component will be and the only possible supplier has agreed to a capacity reservation contract. Demand planners of the company have estimated that the sales of related products will be "around" 50,000 units, but they admit that the forecast can deviate even 50% from this estimate, so that all the example distributions are plausible. The decision maker prefers fixed orders since they are 5% cheaper compared to capacity reservation and execution. But then again, she does not want to end up with too many obsolete components after the demand has been fulfilled. How many fixed orders and capacity reservations should she make?

The answer is crucially dependent on the assumptions about the demand uncertainty: if the decision maker sees that the demand will likely be under 50,000 with some upside potential (positive skew), she should cover most of the demand with fixed orders (m^* = 32,600, o^* = 13,400). Then again, if she believes that the demand will likely be above 50,000, but has some very disappointing scenarios in sight (negative skew), she should hedge against the gloomy scenarios by using the option more (m^* = 9900, o = 50,300).

4.3. Facility location and capacity acquisition

Our final example is facility location and capacity acquisition under uncertain demand. This illustrative model is based on Dasci and Laporte [8], who present an extension to the general optimal market area (GOMA) model presented by Erlenkotter [13]. The GOMA model is used to determine the optimal number (Q^*) of

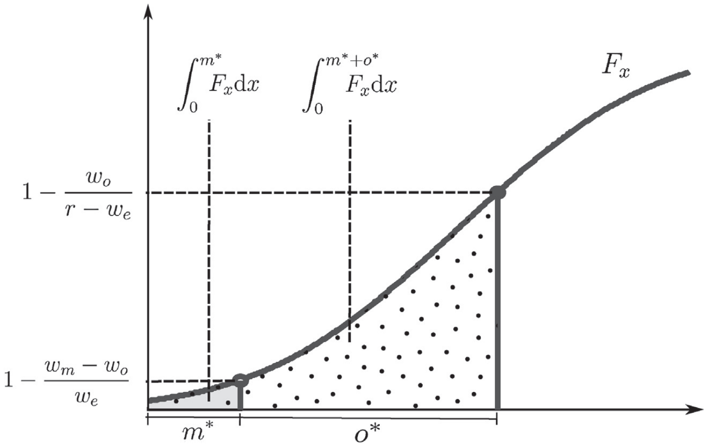


Fig. 4. The determinants of the optimal profit.

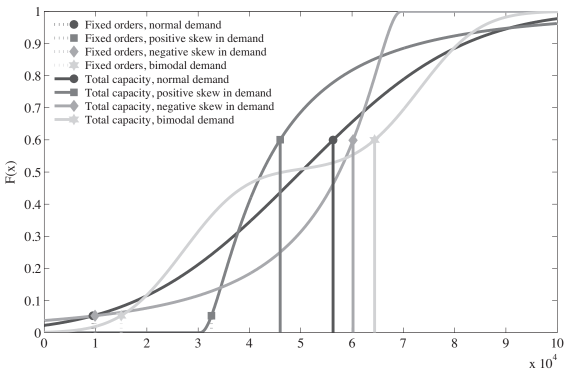


Fig. 5. The optimal quantities for fixed orders and total capacity.

facilities (e.g., retail stores) for a certain market by minimizing costs related to opening and operating a facility and managing the transportation of goods. The central assumption in GOMA models – in contrast to many other facility location models – is that the demand in the market area is uniformly distributed and that facilities are evenly spread and have identical capacities. It can be thought as a high-level model for strategic planning, leaving the exact location of facilities for subsequent analyses. Dasci and Laporte consider a GOMA model with uncertain demand and a linear cost structure.

Let the total market demand D be again characterized with the cumulative distribution function $F(x) = F_x$ and probability distribution function f_x. The decision variables of the model are the market area share of each facility (A) and the capacity of each facility (X). It is assumed that in the case of inadequate total capacity ($D > X/A$), outsourcing (or any other extra capacity) can be used to meet the final demand with an extra cost. Total costs include a fixed cost FC related to opening a facility, a variable cost b per acquired capacity unit (includes all operating costs), a variable cost a related to shortage (acquisition of outsourced capacity, extra transportation costs, etc.) and a transportation cost c per fulfilled demand. The transportation costs are dependent on the average distance between a facility and a customer; mathematically this is expressed with a multiplier $K\sqrt{A}$, where K is a constant "configuration factor" reflecting, e.g., market area shape and chosen metric (see details in [13]). The model minimizes the expected total cost per area (TC):

$$TC(A, X) = \underbrace{\frac{FC}{A}}_{\text{facility openings}} + \underbrace{\frac{bX}{A}}_{\text{capacity}} + \underbrace{E[D]cK\sqrt{A}}_{\text{expected transportation}} + \underbrace{aE[\max\left\{\frac{D-X}{A}, 0\right\}]}_{\text{expected shortage}}, \tag{9}$$

which can be simplified by setting $y := X/A$

$$TC(A, y) = \frac{FC}{A} + by + E[D]cK\sqrt{A} + a\underbrace{E[\max\{D - y, 0\}]}_{\int_y^\infty (x-y) f_x \mathrm{d}x} \tag{10}$$

Now $TC(A, y)$ is separable in A and y, so the cost minimizing area can be found with

$$\partial_A TC(A, y) = 0 \Rightarrow A^* = \left(\frac{2FC}{cKE[D]}\right)^{2/3} \tag{11}$$

and cost minimizing y with

$$\partial_y TC(A, y) = 0 \Rightarrow \partial_y\left(by + \int_y^\infty (x - y) f_x \mathrm{d}x\right) = 0 \Rightarrow b - a\underbrace{\int_y^\infty f_x \mathrm{d}x}_{1-F_y} = 0 \Rightarrow F(y^*) = 1 - \frac{b}{a} \tag{12}$$

In contrast to previous examples, the output of the model is not entirely dependent on the shape of the distribution. Indeed, the optimal market area share of each facility A^* in (11) can be calculated using the expected value of demand only. However, the

Table 3
The optimal profit (Π^*), fixed order (m^*) and capacity reservation (o^*).

Demand distribution	Π^*	m^*	o^*
Normal	5034	9500	46,800
Positive skew	7275	32,600	13,400
Negative skew	4654	9900	50,300
Bimodal	5354	14,800	50,200

optimal size of each facility is dependent on F_x, as can seen from (12). The optimal size is calculated using the relationship between manufacturing and outsourcing (or shortage) costs, which can realistically get almost any value between 0 and 1. This means that the optimal size can be defined exactly, if the quantile function of the distribution is known.

The optimal quantities can be calculated similarly as in the procurement example using numerical integration. This time, we use the demand distributions with $\mu = 5,000,000$ and $\sigma = 2,500,000$. Other parameters used in the examples are $FC = 25,000$, $b = 10$, $a = 25$, $c = 5$ and $K = 0.38$ (K is adapted directly from Dasci and Laporte). Thus, we are looking at a market with expected demand of five million items and want to minimize costs of fulfilling the demand with own capacity (25,000 fixed cost per facility, 10 per produced unit) and possible outsourcing (25 per outsourced unit), taking transportation costs (5 per unit) into account.

For the above parameters, we get $A^* \approx 0.03$, which implies $1/A \approx 33$ facilities. The facility specific capacities for each demand distribution are in Table 4. Here, positive skewness in demand implies almost 20% less capacity than any other demand distribution. The costs, however, vary less than capacity: negative skewness leads to lowest costs – around 15% less than with the bimodal distribution which is the most costly. Negative skewness has the lowest expected shortage, expressed with the last integral term in (10), which is over five times less than with positive skewness.

The results imply that when the decision about how to cover a certain demand area with a mix of own facilities and outsourcing is made, the decision maker should understand the nature of demand uncertainty. If she suspects a long right tail (positive skew), she should start planning relatively low capacity facilities and prepare to cover much of the upside demand with outsourcing because, as noted, the expected shortage (the amount that is outsourced) is over five times more for positive skewness compared to negative skewness. Then again, if she does not see much upside potential in the demand but more vice versa, own facilities should be relatively large and the expected need for outsourcing relatively less.

5. Value of distributional information

The examples above show how the models' output depends heavily on assumption about the distribution. In reality, the decision maker may not know the actual shape of the distribution. She can obtain more information about the distribution, but at a cost. For example, a market study could show whether the distribution is skewed; co-operation with customers or competitors could indicate possible bimodality; and, in general, gathering historical data (i.e., waiting) can help in distribution estimation. To study whether the decision maker should take these kinds of actions, we compare the value of distributional information in the light of these same examples.

We consider the *minimax* regret criterion as presented by, e.g, Perakis and Roels [35]: the decision maker selects the action y that minimizes the maximal regret ρ_i. This regret is defined as the maximal difference of the optimal solution $\Pi_{F_i}{}^*$ when distribution $F_i \in \mathcal{D}$ is known, and the objective function value Π_F when the distribution is unknown (it is only known that the distribution belongs to the set of possible distributions $\mathcal{D}$):

$$\min_y \left\{ \underbrace{\max_{F \in \mathcal{D}} \{\Pi^*_{F_i} - \Pi_F(y)\}}_{\text{maximal regret } \rho_i \text{ for distribution } F_i} \right\}. \tag{13}$$

Again, we assume that the expected value and variance are the same for all distributions, but that the shape of distribution varies, so that $\mathcal{D}$ consists of the four distributions presented earlier.

5.1. Inventory management with safety stock

The inventory management model in Section 4.1 is not based on optimization, so we apply the concept of regret as follows: if the decision maker does not know the true distribution, she selects a safety stock level *SS* that guarantees the desired service level for all four distributions. The regret is the extra inventory compared to the safety stock with the true distribution. In other words, the risk caused by the lack of information is an oversized inventory. This also leads to higher than intended service level, which is here seen as undesirable. It turns out that in this setting, excess inventory is the highest for negative skewness (with high service levels) or for positive skewness (with low service levels).

As an example, if the demand is negatively skewed and service level requirement is 95%, the regret would be 241 because the level that guarantees 95% *among all distributions* is 416 and the requirement for negatively skewed demand is only 175, as presented in Section 4.1. This approach allows the decision maker to evaluate whether an action that "reveals" negative skewness should be taken: if the cost of such action is less than savings from a drop of 241 items in the inventory, the action is recommended.

Regrets for service levels from 50% to 99% are plotted in Fig. 6. The figure shows how the relative value of distributional knowledge changes along with the service level requirement. With high service levels (>86%) the regret is the highest for negative skewness. In this case, the tail on the left is actually meaningless; it is the absence of the right-hand tail that offers value. Similarly with low service levels, positive skewness and concentration of the probability mass on the left offers value, because the positive skew distribution has the highest safety stock regret. Here, the thin right-hand tail allows significant decrease in the safety stock when moving closer to the expected value of the distribution.

5.2. Procurement with capacity reservation options

In the procurement model, maximum regret can be taken as the difference between the optimal profit with a known distribution and the smallest profit when all distributions are possible. The shape of the distribution impacts both the total capacity reserved, and the optimal mix of fixed orders and capacity reservation options. Therefore, when the distribution is not known, there is a risk of both excessive (or insufficient) total reservation, and of too conservative use of cheap fixed orders. As our numerical analysis below shows, both these factors contribute to the value of distributional information.

Computationally, the regret can be determined by simulating different decisions (fixed orders m and capacity reservations o) and selecting the quantities that minimize the difference between the optimal profit and the worst possible profit. For the same parameter values as in Section 4.2, the regret minimizing decision is $m = 10,000$ and $o = 50,000$ regardless of the assumed "true

Table 4
The optimal facility quantity and size and the corresponding costs (in millions).

Demand distribution	Q^*	X^*	TC^* (M)
Normal	33	170,440	76.6
Positive skew	33	139,100	70.6
Negative skew	33	182,230	66.9
Bimodal	33	196,370	78.6

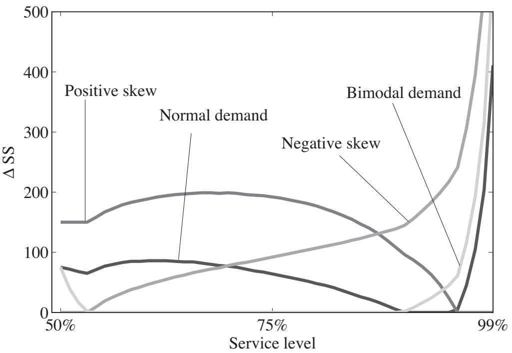


Fig. 6. Regret safety stock levels for different distributions.

distribution" F_i. Here, the highest regret occurs when demand is positively skewed: the expected profit for positively skewed distribution is 1500 (or 21%) less compared to the optimal profit when it is known *beforehand* that demand is positively skewed. The explanation is that if the decision maker were to know about the positive skewness and the concentration of probability mass to the range [30,000, 50,000], her optimal strategy would be to place more cheap fixed orders and use less options. On the other hand, knowledge about normal demand would not have brought any extra profit, i.e., the regret with respect to normality is zero.

Fig. 7 shows the regret minimizing decisions and corresponding regrets when the component revenue varies between 1.20 and 3.60. When the revenue is low, the regret is the highest for positive skewness. This is due to the high share of options in the minmax regret solution, as explained in the example above. When revenue grows, the regret increases for all distributions except for positive skewness. The highest regret occurs with negative skewness: with high markup components, the decision maker should concentrate on finding information about potentially low demand scenarios instead of potentially high demand. Namely, without knowing about the possibility of low demand, it is optimal to make a substantial fixed order and, in addition, to reserve a lot of extra capacity so that the high revenue potential can be fully exploited. If, however, it was known that disappointing demands are possible and that demand has an upside limit, the amount of fixed orders could be reduced to improve profit.

5.3. Facility location and capacity acquisition

In the facility location model, the maximum regret consists of the difference between the maximum cost among all distributions with regret minimizing capacity, and the optimal cost for a known distribution. In our setting, the expected value is fixed and hence the optimal amount of facilities remains unchanged. However, the amount of capacity varies according to the distribution shape, and thus the distributional information helps the decision maker to avoid under- or overinvesting in capacity. In the numerical example below, overinvesting causes most regret when outsourcing cost is relatively low but with high outsourcing costs, underinvesting causes more regret.

Fig. 8 illustrates how the regret minimizing capacity and regret about costs change as the cost of outsourcing varies. As one can expect, the regret minimizing capacity increases along with the outsourcing cost. The changes in the value of distributional information are less straightforward. When outsourcing cost is low, negative skewness has the highest regret because with cheap outsourcing, the other distributions favor low capacity and the flexibility of outsourcing. With the negatively skewed distribution, the probability mass is concentrated on the right side of the expected demand and thus, low capacity implies a lot of outsourcing with high probability which, though relatively cheap, is still more costly than using own capacity.

When outsourcing becomes more expensive, the regret minimizing total capacity quickly grows from 2.5 million to 6.6 million units. Then, it would be most beneficial to find out about positively skewed distribution, because it would optimally lead to much less total capacity as noted in Section 4.3, where optimal total capacity was around 4.6 million with the outsourcing cost $a = 25$. Curiously, with outsourcing cost over 35, bimodality becomes the most valuable information. This is the point where the cost of having too much capacity for positive skewness becomes relatively less significant compared to cost of outsourcing, for which the need is the highest with the bimodal distribution.

6. Managerial implications

Our analysis shows that neglecting the shape of demand distribution can result in non-optimal recommendations, even if the decision maker has (i) a reliable estimate of the expected demand, i.e., a non-biased point forecast, (ii) an estimate of the

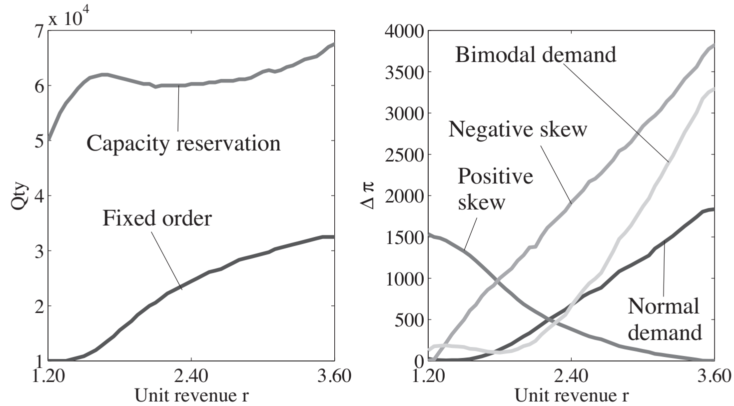


Fig. 7. Regret minimizing procurement quantities and regrets about profit when revenue varies.

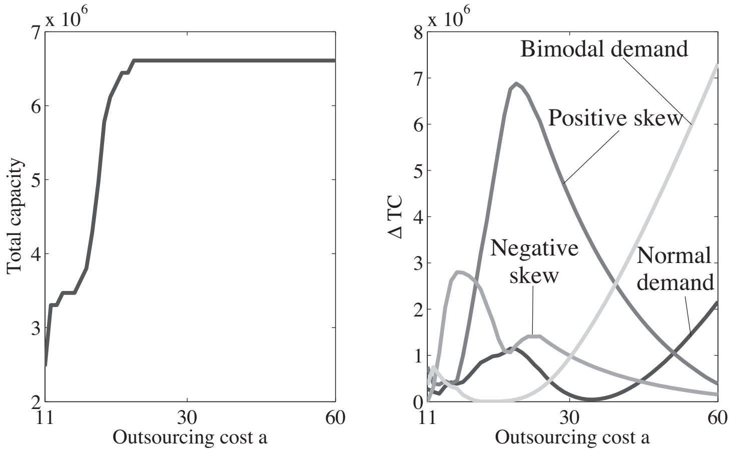


Fig. 8. Regret minimizing capacity and regret about total costs, when outsourcing cost varies.

variation in the forecast, and (iii) a perfectly valid model structure. Furthermore, we have illustrated how the value of distributional information varies between and within models. What, then, does this insight of *distribution sensitivity* mean for model-assisted decision making?

Decision model structure and uncertainty modeling are interdependent. Regardless of their structure, models are often crucially dependent on their inputs. Sensitivity analysis can be used to study how input variation impacts model's output, which is not a trivial task because most models map inputs non-linearly to outputs. In demand context, it is typical to test how, say, a 20% change in demand would affect the optimal strategy, which leads to insight about sensitivity with respect to the expected value. Similarly, to test how improving forecasting accuracy would decrease costs, one can conduct sensitivity analysis with respect to variance in demand. In our examples, however, we have tested how models are affected by *the distribution shape* when expected value and variance are fixed.

The model structure also determines which part of the distribution is relevant. In the inventory management example in Section 4.1, the safety stock quantity depends only on the right tail-quantile of the demand distribution: if one can estimate the value of the quantile function of demand in the point defined by the service level requirement, rest of the distribution can be neglected. In the procurement example in Section 4.2, on the other hand, the distribution impacts the output only in the interval $[0, o^* + m^*]$, so the right tail of distribution is largely irrelevant. The procurement model also illustrated that it may not always be possible to infer how exactly the shape of distribution impacts the model's outcome. Some conclusions can be made about, e.g., how the cost parameters would affect the relative use of options vs. fixed orders, but explicit rules about the impact of distribution shape are not easily derived. In the facility location model in Section 4.3, the output is partially dependent on the distribution shape: the amount of facilities is a straightforward function of expected demand and cost parameters, but the facility size varies based on the distribution shape.

These examples highlight that the structure alone is insufficient to explain the model behavior, wherefore careful modeling of uncertain demand is critical. On the other hand, the model structure partly determines what exactly is important in the probability distribution. Thus, uncertainty modeling and model structure should not be treated in isolation.

Value of distributional knowledge varies between models – and within models when parameters change, as we have illustrated in Section 5. This helps the decision maker to concentrate resources on the most important features about the demand uncertainty. Insight about the value of distributional knowledge has practical implications, e.g., when forecasting demand for the purpose of sales and planning process. While a typical improvement initiative relates to improving forecast accuracy (here, this is equivalent to reducing variance), it might as well be more efficient to assess other distributional characteristics, such as whether there are long tails in the distribution. Also, based on the procurement example in Section 5.2, one could sketch a procurement contract design process where the experts estimate either the possibility of low demands in case of high revenue components, or the possibility of high demands in case of low revenue components.

More generally, one can ask what is the value of distributional knowledge in different decisions, regardless of the decision model. This varies between different levels of decision making:

- *At the strategic level* (long time horizon, largely uncertain demand), distributions are not necessarily used to get direct decision recommendations, because stochastic models can be parameter sensitive and thus unreliable under high uncertainty. But there are other uses for distributions: for example, when planning a launch of a new product, distributions can be used in what-if scenario planning to model upside and downside potential in the demand market with skewness, or the impact of duopoly competition with bimodality.
- *At the tactical level* (shorter time horizon, forecasting is sensible but inaccurate), distributions can give more concrete support. Van Landeghem and Vanmaele [23] argue that the tactical planning level is the most suitable when buffering against demand uncertainty in supply chain planning, where both operational flexibility and the level of uncertainty grow in time. Here, the decision maker has just enough time to prepare for, e.g., demand uncertainty, but she can still establish one single plan instead of preparing for multiple scenarios. Thus, using distributions in tactical level enables robust and risk-adjusted decision recommendations.
- *At the operational level* (short time horizon, accurate forecasting), decisions are characterized by low demand variation and recurrence, such as in the case of daily inventory replenishment.

These decisions are often automated in a typical manufacturing company that has hundreds or thousands of items in the inventory. In many cases, there is plenty of historical data available and thus statistical inference is effective. This makes distribution based optimization possible, as the popularity of the safety stock driven inventory management model illustrates. Even when used with a vague assumption about the demand distribution, it helps companies implement inventory policies where demand variation of each item, probability based risk management, and impact of tail-demand are all accounted for in an automated replenishment process.

Expert knowledge can be used to accumulate distributional information. In many cases there is no historical data available and distributions need to be elicited using expert knowledge. Discussions with our case company suggested that experienced product managers can be capable of, e.g., estimating the shape of demand for a new product, if the task is framed correctly. Thus, we provide some pointers to the literature on the uncertainty elicitation using expert judgments.

Keefer and Bodily [21] study how judgmental assessments of mode, minimum and maximum, or 0.05 and 0.95 fractiles of a random variable can be utilized in distribution estimation. Arguably, an expert (without a background in statistics) can provide more reliable estimates for these statistics instead of assessing a particular distribution and its parameters. Expert-based point forecasts can also be utilized: Gaur et al. [17] study the deviation of demand forecasts among experts and find that the dispersion of expert forecasts is a good estimator for the actual variance of demand. Whether a similar approach could be applied to higher moments is, to our knowledge, an open question. When discrete scenarios are sought, Hoyland and Wallace [19] present an optimization based moment-matching method for scenario generation where arbitrary statistical properties of the uncertainties can be used to create a scenario tree for multistage problems. They illustrate the method using a set of statistical properties (first four moments, worst-case, correlation) to generate scenario trees for returns of four financial assets.

Use of distributions does not require special software. Typical planning systems have poor support for probability distributions. However, the use of spreadsheet tools with a Monte Carlo simulation add-on can already enable analysis that accounts for the distribution shape. Our experience with the case company was that by using existing spreadsheets and simulation, the managers were able to increase their understanding about the impact of demand uncertainty on inventory management related decisions, even though operational tools are yet to be developed.

7. Conclusions

In this paper, we have stressed the importance of uncertainty modeling in risk management models. This was illustrated with examples from inventory replenishment, tactical procurement and strategic capacity planning, which demonstrated how the results of models vary significantly when assumptions of the demand distribution change. The examples were constructed in co-operation with a case company, which found the presented analysis insightful: it helped increase understanding of uncertainties and their implications for various business decisions, before moving ahead with subsequent modeling and implementation activities.

Specifically, we have shown that models are sensitive to the shape of the demand distribution, not only to single parameters such as expected demand or demand variation: skewness, minimum or maximum limits, or bimodality of demand can translate into significant differences in inventory levels, expected profits, or costs in different models. We have also shown that the model design has an impact on what is important in uncertainty modeling: for example, it was noted that in a facility location model, the amount of facilities is only dependent on the expected value of demand, whereas the size of each facility is dependent on the demand distribution shape. Further, we assessed the value of distributional information with a *minmax* regret analysis and concluded that the relative importance of knowledge about the distribution shape varies depending both on the model and its parameters. The analysis helps the decision maker to concentrate on specific features in demand uncertainty, when willing to obtain more information about the demand. It also allows assessing the trade-off between obtaining information and the cost of the required action.

We believe that the utilization of qualitative distribution knowledge has major potential for applications, especially when most software make use of mean and variance only. Many useful methods and tools have already been developed to support the incorporation of qualitative knowledge into decision making models. For existing model-assisted decision processes, the distribution based sensitivity analysis helps in model validation and leads to more robust and risk-adjusted models.

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