

## Non-equilibrium supercurrent through mesoscopic ferromagnetic weak links

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**Abstract.** – We consider a mesoscopic normal metal, where the spin degeneracy is lifted by a ferromagnetic exchange field or Zeeman splitting, coupled to two superconducting reservoirs. As a function of the exchange field or the distance between the reservoirs, the supercurrent through this device oscillates with an exponentially decreasing envelope. This phenomenon is similar to the tuning of a supercurrent by a non-equilibrium quasiparticle distribution between two voltage-biased reservoirs. We propose a device combining the exchange field and non-equilibrium effects, which allows us to observe a range of novel phenomena. For instance, part of the field-suppressed supercurrent can be recovered by a voltage between the additional probes.

Externally controlled weak links in mesoscopic superconducting circuits have been at the focus of interest in recent years [1]. The possibility to control the quasiparticle distribution by external voltage probes allows tuning the supercurrent through the device (mesoscopic SNS transistors). It has been predicted that devices with tunnel junctions [2] and systems with good metallic contacts [3] can enter a peculiar mesoscopic non-equilibrium state at low temperatures, which even allows reversing the supercurrent, turning the system into a  $\pi$ -junction. This phenomenon has been verified experimentally [1].

Another phenomenon of high interest in superconducting mesoscopics is the combination of ferromagnetic (F) elements with superconductors (S) [4–8]. A strong exchange interaction  $h$  in the ferromagnet is expected to suppress the superconducting proximity effect, and hence also the supercurrent. (Several recent experiments [9–11] do not confirm this expectation, a fact which, at this stage, is not understood.) For weak fields, the supercurrent through a SFS weak link and the transition temperature of a SF multilayer are predicted to oscillate [12–14] as a function of the field, or of the width  $d$  of the ferromagnet. The latter defines a characteristic

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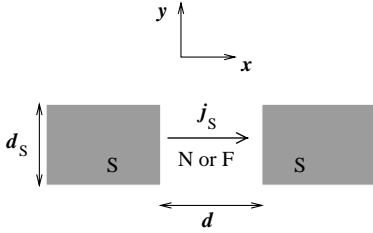


Fig. 1

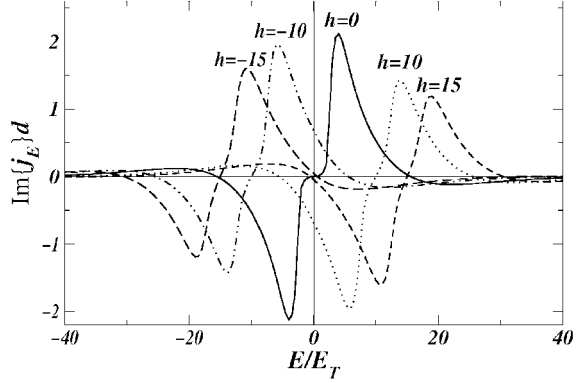


Fig. 2

Fig. 1 – Schematic picture of the studied S-F-S structure.

Fig. 2 – Spectral supercurrent for different exchange fields  $h$  at  $\phi = \pi/2$  as a function of energy. The exchange fields are expressed in units of the Thouless energy  $E_T$ . The variation in the peak heights is due to a finite magnitude of the order parameter  $\Delta = 50E_T$ .

energy scale, the Thouless energy, which in the diffusive limit is  $E_T = \hbar D/d^2$ , proportional to the diffusion constant  $D$ .

In this article, we show that the non-equilibrium-controlled supercurrent in mesoscopic SNS transistors [3] and the supercurrent in SFS weak links are formally equivalent, although one is tuned by varying the distribution function, while the other is controlled by modifications of equilibrium spectral functions. Combining the two phenomena, we can recover by an applied voltage part of the supercurrent which is suppressed by the exchange field. Thereby, one can measure the exchange field in the weak link.

For definiteness we consider a quasi-one-dimensional system depicted schematically in fig. 1 assuming a three-dimensional system with structural changes only in one direction. The magnetism in the weak link, or the Zeeman splitting, is accounted for by the energy  $\sigma h$  of a spatially homogeneous exchange field coupling to the electron spin  $\sigma = \pm 1$ . In the diffusive limit, the system can be described by the Usadel equation for quasiclassical Green's functions [5, 7, 15]. While the equilibrium results of the present work can also be obtained in the imaginary-time Matsubara formalism, we have chosen to use the real-time Keldysh technique in order to include also non-equilibrium processes. Then we have

$$D\partial_x^2\theta = -2i(E - \sigma h)\sinh\theta + 2\Delta\cosh\theta + \frac{D}{2}(\partial_x\chi)^2\sinh 2\theta, \quad (1)$$

$$j_E(E, h) = \sinh^2\theta\partial_x\chi, \quad \partial_x j_E = 0, \quad (2)$$

where  $\theta(E, h, x)$  and  $\chi(E, h, x)$  are complex variables parametrising the quasiclassical diagonal and off-diagonal Green's function  $G(E, h, x) = \cosh(\theta)$  and  $F(E, h, x) = \sinh(\theta)\exp[i\chi]$ . For a system of length  $d$ , eq. (1) introduces a natural energy scale  $E_T = D/d^2$ . Hence, one way to tune the relevant energies is by varying the length  $d$ . Deep in the superconducting electrodes the exchange field or Zeeman splitting vanishes. For simplicity, we assume a bulk BCS solution up to the interfaces,  $\theta_S = \arctan(\Delta/E)$ ,  $\chi_S = \pm\phi/2$  in the superconducting electrodes with amplitudes  $\Delta$  and phase difference  $\phi$  of the order parameters of the two superconductors. Furthermore, we assume clean interfaces, and neglect the reduction of Andreev reflection

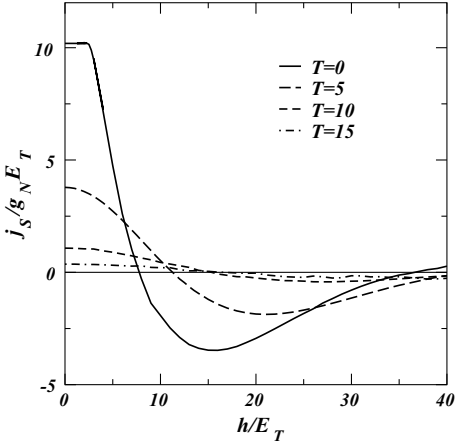


Fig. 3

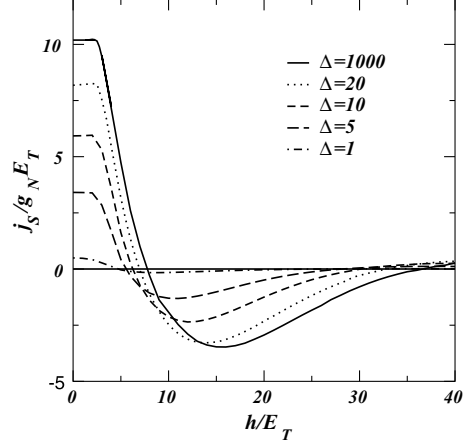


Fig. 4

Fig. 3 – Supercurrent  $j_S(\phi = \pi/2)$  as a function of exchange field  $h/E_T$  through the structure depicted in fig. 1 for different temperatures  $T/E_T$ . The superconducting order parameter  $\Delta = 1000E_T$ .

Fig. 4 – SFS supercurrent  $j_S(\phi = \pi/2)$  as a function of exchange field  $h/E_T$  for different values  $\Delta$  of the superconducting order parameter at  $T = 0$ .

expected in spin-polarised systems [16–18]. We expect the error due to these approximations to be only quantitative (for the latter point, see the discussions below).

The imaginary part of the conserved “spectral supercurrent”,  $j_E$ , in eq. (2) enters into the observable supercurrent as

$$j_S(\phi) = \frac{d}{4} \sum_{\sigma=\pm 1} g_{N\sigma} \int_{-\infty}^{\infty} dE (1 - 2f(E)) \text{Im}\{j_E(E, \sigma h)\}. \quad (3)$$

Here  $f(E)$  is the distribution function of quasiparticles in the weak link, which in the absence of applied voltages reduces to the equilibrium Fermi distribution  $f^{\text{eq}}$ . Furthermore,  $g_{N\sigma} = 2e^2 N_{0\sigma} D_\sigma$  is the normal-state conductivity for spin  $\sigma$ , and  $N_{0\sigma}$  is the corresponding normal-state density of states at the Fermi level. Our approach (eqs. (1), (2)) assumes spin-independent densities of states and diffusion constants. It is valid at low fields  $h$ , when the variation in the densities of states is small,  $N_{0\uparrow} - N_{0\downarrow} \ll (N_{0\uparrow} + N_{0\downarrow})/2$ . In this case we may put  $g_{N\uparrow} = g_{N\downarrow} \equiv g_N$ . The distribution function  $f$  in general is obtained from kinetic equations [15], but for the moment, we assume thermal equilibrium.

It is instructive to see how the spectral supercurrent  $\text{Im}\{j_E\}$  depends on energy  $E$  and exchange fields  $h$ . It is plotted in fig. 2 for a phase difference  $\phi = \pi/2$  between the superconducting electrodes. For  $h = 0$ , the function  $\text{Im}\{j_E\}$  is antisymmetric around the Fermi surface. At low energies  $E \lesssim E_T$ , it vanishes until some phase-dependent  $E_c(\phi)$ . At larger energies it increases sharply, and then decreases exponentially, oscillating between positive and negative values. The exchange field shifts the position of the symmetry point from  $E = 0$  to  $E = \sigma h$  and for a superconducting gap  $\Delta$  of the order of  $h$ , distorts the symmetry. Since  $\Delta$  serves as an upper cutoff, which is not shifted, the overall magnitude of the spectral supercurrent decreases when  $h$  becomes comparable to  $\Delta$ .

In equilibrium we have  $1 - 2f(E) = \tanh(E/2T)$ . This term and the sum of the spectral

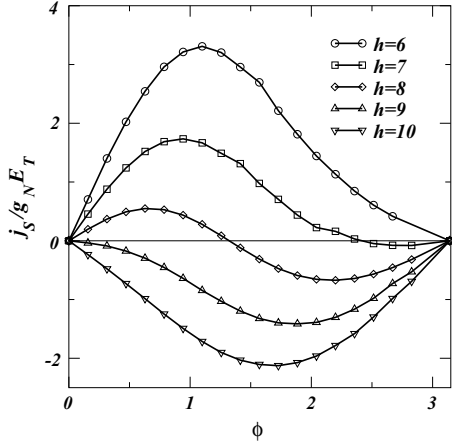


Fig. 5

Fig. 5 – Supercurrent  $j_S(\phi)$  as a function of phase for different exchange fields  $h/E_T$  in the regime where the crossover from the ordinary 0-state to the  $\pi$ -state occurs for the first time. Here,  $T = 0$  and  $\Delta = 1000E_T$ .

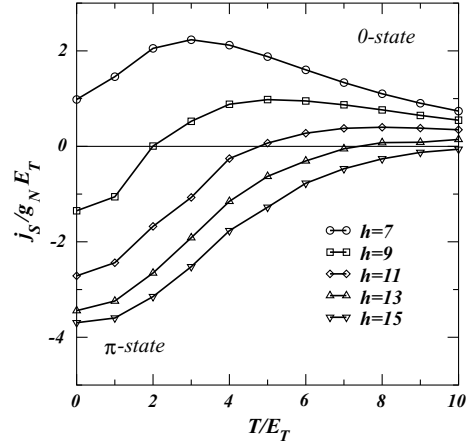


Fig. 6

Fig. 6 – Crossover from the  $\pi$ -state ( $j_S(\phi = \pi/2) < 0$ ) to the 0-state ( $j_S(\phi = \pi/2) > 0$ ) as a function of temperature  $T$  for a few values of the exchange field  $h/E_T$ .

supercurrent  $\sum_{\sigma} j_E(E, \sigma h)$  are antisymmetric around  $E = 0$ . Hence, for the discussion of the total supercurrent  $j_S$  we can concentrate on the part  $E > 0$ . At low  $T \lesssim E_T$  the supercurrent  $j_S$  is given by an alternating sum over the decreasing areas under the oscillating function  $\text{Im}\{j_E\}$  measured from the  $E$ -axis (see fig. 2). At  $h = 0$ , the positive first term dominates the sum and yields a large supercurrent  $j_S$ . Increasing  $h$  shifts the negative peak from  $E < 0$  to  $E > 0$ , hence decreasing  $j_S$ , and even reversing its sign. At finite temperature, the low-energy part,  $E \lesssim T$ , is effectively cut off, hence  $j_S$  decreases in amplitude. This result is illustrated in figs. 3 and 4, where  $j_S(\phi = \pi/2)$  is plotted as a function of different exchange fields at different temperatures and for different bulk order parameters  $\Delta$ . Analogous results can be obtained for a constant exchange field by varying the distance  $d$  of the superconducting reservoirs and through it the Thouless energy  $E_T$ .

In the regime where  $j_S(\phi = \pi/2)$  is negative, the junction forms a so-called “ $\pi$ -junction” [12], since the ground state of the system, with no supercurrent flowing between the two superconductors, is reached for a phase difference equal to  $\pi$ . The supercurrent-phase relation for different exchange fields is plotted in fig. 5, showing the crossover from an ordinary behaviour to a  $\pi$ -state. A closer analysis of fig. 3 shows that the precise value of  $h/E_T$  where the crossover occurs depends weakly on temperature, since higher values of  $T$  smoothen the oscillations between positive and negative contributions to  $j_S$ . At  $h = 0$ ,  $j_S(\phi = \pi/2)$  is positive for any  $T$ . It remains positive as long as the thermal energy dominates over the exchange,  $T \gg h$ . With increasing  $T$  the cross-over to a  $\pi$ -junction is shifted towards higher fields. This dependence was probably observed in ref. [19]. It is an alternative way to verify the current reversal to what has been discussed in previous proposals, where typically one requires many different samples with varying widths [4] but otherwise equal parameters. The crossover is illustrated in fig. 6 for a few values of  $h/E_T$ .

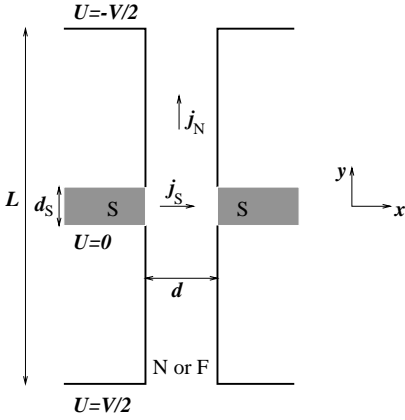


Fig. 7

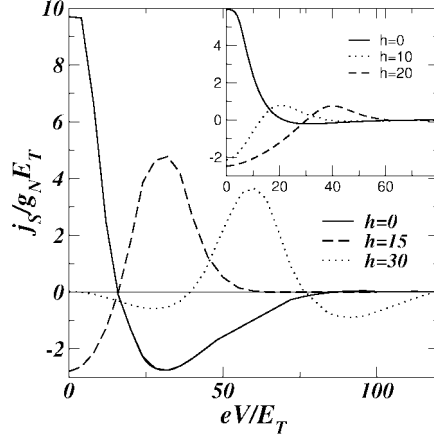


Fig. 8

Fig. 7 – Four-probe setup for studying the non-equilibrium effects on the supercurrent. It is assumed that  $L \gg d_S$  and that the superconductors lie in the middle of the normal wire ( $y = 0$ ) so that the distribution function has the two-step form between the superconductors. Furthermore, we expect that the four-probe setup does not notably alter the spectral supercurrent obtained from a quasi-one-dimensional calculation (for a more detailed discussion, see refs. [3, 23]).

Fig. 8 – Supercurrent  $j_S(\phi = \pi/2)$  of the four-probe structure at different fields as a function of the voltage  $V$  between the normal probes. In the calculations for the main picture, the magnitude of the order parameter was set to  $\Delta = 100E_T$ , and at the inset,  $\Delta = 10E_T$ , thereby showing that even when  $\Delta$  is of the order of  $h$  and  $eV$ , a local maximum is obtained at  $eV = 2h$ .

By shifting the variable of integration  $E$  in eq. (3) by  $\sigma h$ , one finds

$$j_S(\phi) = \frac{dg_N}{2} \sum_{\sigma} \int_{-\infty}^{\infty} dE (1 - f^{\text{eq}}(E + \sigma h)) \text{Im}\{j_E(E)\}. \quad (4)$$

The shifted distribution function  $f = 1/2 \sum_{\sigma} f^{\text{eq}}(E + \sigma h)$  has the same form as the two-step distribution function measured in ref. [20]. There it appeared as the solution of a kinetic equation in the centre of a diffusive metal between two normal probes with voltage  $eV = \pm 2h$  in the limit where the inelastic scattering length is longer than the distance between the two normal reservoirs. The spectral supercurrent in general still depends on the exchange field via the boundary conditions. However, if the superconducting gap  $\Delta$  is much larger than the exchange field,  $\Delta \gg h$ , this dependence can be neglected. In this limit, the supercurrent in the presence of an exchange field is the same as for a non-equilibrium distribution four-probe structure described in ref. [3] (see fig. 7 for a schematic picture).

It is interesting to note that this behaviour of the diffusive-limit supercurrent as functions of the exchange field and the external potential is very similar to the supercurrent through a multiprobe structure in the clean limit. This limit has been described by Dobrosavljević-Grujić *et al.* [21] for the ferromagnetic two-probe case and by van Wees *et al.* [22] including a voltage in a non-magnetic three-probe setup. In this case, the supercurrent is carried by the Andreev levels, whose energies are controlled by the exchange field [21], and whose occupation can be tuned by the voltage [22]. With both parameters, for example, the system can be driven into a  $\pi$ -state.

We can combine the effects of exchange field and non-equilibrium distribution [3, 23] by considering the structure in fig. 7, where the magnetic material is placed between superconductors and normal voltage leads. Here, the distribution function is

$$f(E, y) = \left(\frac{1}{2} - \frac{y}{L}\right) f^{\text{eq}}(E + eV/2) + \left(\frac{1}{2} + \frac{y}{L}\right) f^{\text{eq}}(E - eV/2), \quad (5)$$

exhibiting the two-step form observed by Pothier *et al.* [20]. In this case, if the superconducting reservoirs are located around  $y = 0$  and provided  $\Delta \gg h, eV$ , the observable supercurrent can be written as a sum of four terms,

$$j_S(\phi) = \frac{dg_N}{8} \int_{-\infty}^{\infty} dE (1 - 2f^{\text{eq}}(E)) \text{Im}\{j_E(E - h - eV/2) + j_E(E - h + eV/2) + j_E(E + h - eV/2) + j_E(E + h + eV/2)\}. \quad (6)$$

For example, if the potential is exactly twice the exchange field,  $eV = 2h$ , due to the anti-symmetry of  $\text{Im}\{j_E\}$ , we have

$$j_S(\phi) = \frac{1}{2} (j_S^{\text{SFS}}(\phi, 0) + j_S^{\text{SFS}}(\phi, 2h)) \approx \frac{1}{2} j_S^{\text{SFS}}(\phi, 0). \quad (7)$$

The latter approximate equality holds if  $h \gg E_T$ . Here,  $j_S^{\text{SFS}}(\phi, h)$  is the supercurrent through the SFS structure with the exchange field  $h$  in the weak link. Hence, one can use the external potential to recover half of the zero-field supercurrent. This is illustrated in fig. 8, where the supercurrent of the four-probe structure is plotted as a function of voltage with a few magnitudes of fields.

The results summarised by eq. (7) provide a way to measure the exchange field and at the same time to explore the applicability of the simplified model for the ferromagnet used here and previously [7, 8, 12]. When the voltage-dependent supercurrent  $j_S(V)$  reaches a maximum,  $eV$  should equal  $2h$ . Deviations could occur as, for instance, this model neglects the band structure effects [24] important in the ferromagnets. Also, to be able to measure the actual supercurrent through a typical ferromagnet with Curie temperature  $T_{\text{Cu}} \gg \Delta$ , the ratio  $h/E_T$  has to be made small by fabricating very thin weak links. Moreover, our assumption of the diffusive regime requires  $d \gg l_{\text{el}}$ , and a quantitative agreement cannot be expected for thin structures. Finally, due to the strong electron-electron interactions in ferromagnets, producing a short inelastic relaxation length, the normal probes should be fabricated rather close to each other to obtain the two-step form for the distribution function. For conventional ferromagnets the exchange field is large, which would correspond to enormous voltages. However, we expect that our model is approximately valid for setups constructed from ferromagnetic alloys with  $T_{\text{Cu}}$  of the order of the superconducting critical temperature [19], or in situations where  $h$  can be related to the Zeeman splitting in magnetic fields much weaker than the superconducting critical field.

In summary, we have calculated the supercurrent through a ferromagnetic weak link as a function of the exchange field in the ferromagnet. In the calculations, the Keldysh technique was used to provide a description of non-equilibrium effects. We found that when  $\Delta \gg h$ , the problem is formally equivalent to the four-probe measurement of the supercurrent through a normal-metal weak link. Furthermore, we showed that applying a non-equilibrium potential in the transverse direction, one can recover half of the supercurrent of a ferromagnet with an exchange field  $h \gg E_T$ , as compared to the supercurrent in the absence of  $h$ .

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