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The topology of interbank payment flows

Kimmo Soramäki[a], Morten L. Bech[b,*,1], Jeffrey Arnold[b,1],
Robert J. Glass[c,2], Walter E. Beyeler[c,2]

[a]*Helsinki University of Technology, Finland*
[b]*Federal Reserve Bank of New York, New York, NY, USA*
[c]*Sandia National Laboratories, Albuquerque, NM, USA*

Abstract

We explore the network topology of the interbank payments transferred between commercial banks over the Fedwire® Funds Service. We find that the network has both a low average path length and low connectivity. The network includes a tightly connected core of banks to which most other banks connect. The degree distribution is scale free over a substantial range. We find that the properties of the network changed considerably in the immediate aftermath of the events of September 11, 2001.
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1. Introduction

At the apex of the US financial system is a network of interconnected financial markets by which domestic and international financial institutions allocate capital and manage their risk exposures. The events of September 11, 2001 and, to a lesser extent, the North American blackout of August 14, 2003 underscored that these markets are vulnerable to wide-scale disruptions. The inability of one of these markets to operate normally can have wide-ranging effects not only for the financial system but potentially for the economy as a whole. Consequently, the financial industry and regulators are devoting considerable resources to strengthen

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*Corresponding author.

E-mail address: morten.bech@ny.frb.org (M.L. Bech).

the resiliency of the US financial system [1,2]. Critical to the smooth functioning of these markets are a number of wholesale payments systems and financial infrastructures that facilitate clearing and settlement.

Despite the importance of these systems and infrastructures little empirical research on the impact of disruptions to these systems and infrastructures is available. One branch of the literature has focused on simulating the effects from a default of a major participant [3–6]. Another branch has done detailed case studies of disruptions to the US financial system, e.g., the 1987 stock market crash and the events following September 11th [7–10].

However, the payment system can be treated as a specific example of a complex network. In recent years, the physics community has made significant progress towards understanding the structure and functioning of complex networks [11–15,40,42]. The literature has focused on characterizing the structure of networked systems and how the properties of the observed topologies relate to stability, resiliency and efficiency in case of perturbations and disturbances [11,16].

A few recent papers [17–19] have started to describe the actual topologies observed in the financial system using this methodology. This paper adds to this literature by describing the network topology of the interbank payment flows in the US. Moreover, we add new insight to the response of complex networks to perturbations by analyzing the effects of September 11th on the network.

The paper is organized as follows. Section 2 describes the interbank payment system and the data. Section 3 presents a visualization of the data. Section 4 presents the topological characteristics of the interbank payment flow network and discusses how they are important for analysis of payment systems. Section 5 looks at the impact of September 11th on these characteristics. Section 6 concludes.

2. Interbank payment system

We use transaction data from the Fedwire® Funds Service (Fedwire) to create the interbank payment network. Fedwire is a real-time gross settlement system, operated by the Federal Reserve System, in which more than 7500 participants initiate funds transfers that are immediate, final, and irrevocable when processed. Participants use Fedwire to process large-value, time-critical payments, such as payments for the settlement of interbank purchases and sales of federal funds; the purchase, sale, and financing of securities transactions; the disbursement or repayment of loans; and the settlement of real estate transactions. Fedwire also is used by ancillary payment systems such as automated clearing houses (ACH) and other large value payment systems e.g. the Clearing House Interbank Payment System (CHIPS) and Continuous Linked Settlement (CLS) Bank. The processing of payments requires liquidity, i.e., funds at the bank's reserve account at the Federal Reserve or daylight overdrafts that the Federal Reserve extends to the system participants. Typically the amount of these balances and the total credit extended is only a small fraction of the total value of payments transacted during a day and most of the payments are settled with liquidity received by the banks from incoming payments. Hence, an undisturbed flow of funds in the network is critical for smooth operations.

We analyze the network of the actual payments flows transferred over Fedwire. We model the payment flows as a directed network and establish a link from the sender of payment to the receiver of payment on the basis of payments sent. While Fedwire participants include a variety of entities, including government agencies, we consider only the subset of payments between commercial banks. Thus, commercial banks constitute the nodes in the network, and a directed link from one bank to another is present in a day if at least one transaction debits the account of former and credits the account of the latter. We use banks' primary accounts, called master-accounts, and exclude loops that result from banks making transfers across their own sub-accounts. Additional transactions between any two banks add to the associated link weights in terms of value and volume (number) of payments settled. The network could be defined in alternative ways. From a technical perspective, Fedwire is a star network where all participants are linked to a central hub, i.e., the Federal Reserve, via a proprietary telecommunications network. From a payment processing perspective Fedwire is a complete network as all nodes (participants) are linked in the sense that they can send payments to and receive payments from any other participant. However, these representations do not represent the actual behavior of participants and the flow of liquidity in the system.

We consider data for the first quarter of 2004. Each day is modeled as a separate network, for an ensemble of 62 daily networks. The average daily value of funds transfers between commercial banks amounts to $1.3

trillion and the daily number of payments to 345,000. On the peak activity day, 644,000 payments worth more than $1.6 trillion were processed. The average value per payment is $3 million, but the distribution is highly right-skewed with a median payment of only $30,000. Both the daily value and volume of payments show periodicity around the first and last days of the month as well as on mid month settlement days for fixed income securities (see Fig. 4).

3. Visualizing the network

An intuitive way to analyze a network is to draw it as a graph, as in Fig. 1. The interbank payment network on the first day of our sample is illustrated in Fig. 1. The figure includes over 6600 nodes and more than 70,000 links. Each link is shaded by the associated weight, with darker shades indicating higher values. Despite resemblance to a giant fur ball, the graph suggests the existence of a small group of banks connected by high value links. To gain a clearer picture of this group, we graph a subset of our network in Fig. 2 where we focus on high value links. This graph shows the undirected links that comprise 75% of the value transferred. This network consists of only 66 nodes and 181 links. The prominent feature of this network is that 25 nodes form a densely connected sub-graph, or clique, to which the remaining nodes connect. In other words, we find that only a small number of banks and the links between them constitute the majority of all payment values sent over the network.
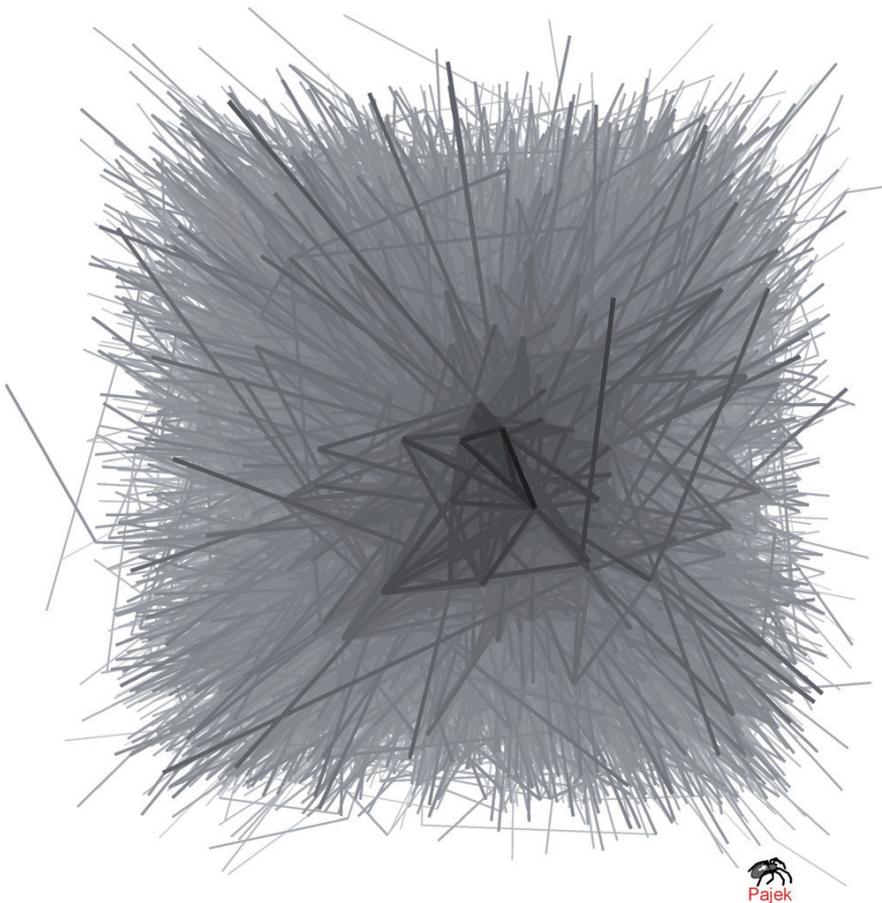

Fig. 1. Fedwire interbank payment network. First day of Sample. 6600 nodes and over 70,000 undirected links [39].
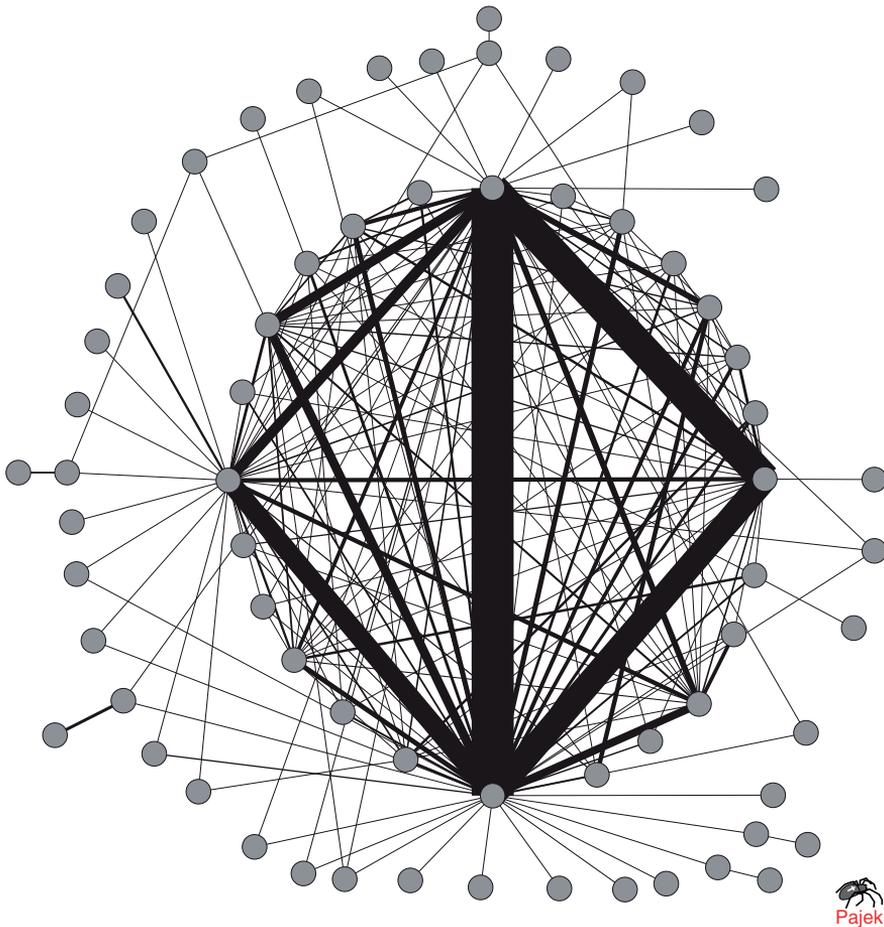
Fig. 2. Core of the Fedwire interbank payment network; largest undirected links totaling 75% of daily value transferred [39].

4. Topology characteristics

The large number of nodes and links makes detailed analysis of the structure by visualization difficult and comparisons across time and between networks almost infeasible. The complexity of our network leads us to consider statistical measures. In this section, we describe a series of commonly used statistical measures of topological characteristics for the interbank payment network.

4.1. Components

Definition: A starting point for the quantitative analysis of a network is to partition the set of nodes into components according to how they connect with other nodes. Dorogovtsev et al. [20] divide a network into a single *giant weakly connected component* (GWCC) and a set of *disconnected components* (DCs). The GWCC is the largest component of the network in which all nodes connect to each other via undirected paths. The DCs are smaller components for which the same is true. In empirical studies the GWCC is found to be several orders of magnitude larger then any of the DCs [21]. This is also the case here.

The GWCC consists of a *giant strongly connected component* (GSCC), a *giant out-component* (GOUT), a *giant in-component* (GIN) and *tendrils*. The GSCC comprises all nodes that can reach each other through a directed path. A node is in the GOUT if it has a path from the GSCC but not to the GSCC. In contrast,
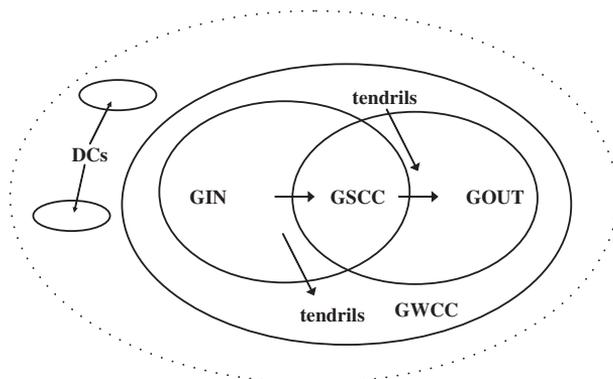
Fig. 3. Disconnected components. GSCC = giant strongly connected component. GIN: giant-in component. GOUT: giant-out component.

a node is in GIN if it has a path to the GSCC but not from it. Tendrils are nodes that have no directed path to or from the GSCC. They have a path to the GOUT or a path from the GIN (see Fig. 3).

Discussion: Over our sample period a total of 7584 different banks are part of the network. We find that the network's GWCC is composed of on average 6490 ± 83 (mean ± standard deviation over the 62 days) banks. On 36 of the 62 days we also find a small number of DCs consisting of between two to eight banks. Over the sample period 6854 different banks were part of the GSCC. Of these, 2578 were present in the network on all days. The GSCC contains 78% of the nodes in the GWCC on average whereas the GIN, GOUT and tendrils contain 8%, 12% and 2%, respectively. In terms of value transferred, 90% occurs within the GSCC and 7% is from GIN to GSCC. We will focus on the GSCC component in the analysis below.

The component analyses provide insights on the structure of liquidity flows in the payment system and give clues with respect to the relative importance and vulnerability of banks in the system in case of disruptions. As banks in GOUT only receive funds from other banks in the GSCC, a disruption by a bank in GOUT would only affect other banks in that component. Banks in GIN are affected only by disruptions in the same component, and not by banks in other components as their payment processing is not dependent on incoming liquidity from these banks. Banks outside the GSCC tend to be smaller whereas all money center banks belong to the GSCC.

4.2. Size, connectivity and reciprocity

Definition: The most basic properties of a network are the number of nodes n and the number of links m. The number of nodes defines the *size* of a network while the number of links relative to the number of possible links defines the *connectivity* of a network. The connectivity (p) is the unconditional probability that two nodes share a link. For a directed network, the connectivity is $p = m/n(n - 1)$. It ranges from $1/n$ for a tree network to 1 for a complete network. Reciprocity is the fraction of links for which there is a link in the opposite direction in the network.

Discussion: The average size of the daily network formed by the GSCC was 5086 ± 128 nodes. Almost 710,000 different links were found between banks over the sample period, with only 11,000 of them present on all days. On average the network had $76,614 \pm 6151$ directed links. In comparison, a complete network of similar size has over 25 million links. The connectivity is only $0.30 \pm 0.01\%$. In other words, the interbank payment network is extremely sparse as 99.7% of the potential links are not used on any given day. The reciprocity averages $22 \pm 0.3\%$. Hence, less than a quarter of the relationships that exist between banks had payments going in both directions. Fig. 7 shows that the reciprocity of a link increases with the gross value and volume of payments settled over the link. All undirectional links with more than 100 payments or more than 100 million dollars were reciprocal. Increased reciprocity on large links is the likely the result of either complementary business activity or the risk management of bilateral exposures.

The number of nodes and links are almost perfectly correlated across days and are both highly correlated with the value and volume of payments settled (see Tables 1 and 2). In particular, the size and number of links spike on the high value and volume days identified in Section 2 (see Figs. 4–7). Interestingly, the connectivity is also highly correlated with value and volume. On high payment activity days, the network not only grows but it also becomes denser as the level of interactions between banks increases faster than size. However, the same is not true for reciprocity. Reciprocity is uncorrelated with value and volume and connectivity. The extent to which the relationships between banks are bilateral does not appear to depend on either the overall level of payment activity or the connectivity between banks.

The periodicity in the size and number links suggests that it might be insightful to model high payment activity days separately. For simplicity, we ignore this and treat the networks as coming from the same data generating process.

4.3. Distance and diameter

Definition: The *distance* from node i to node j, d_{ij}, is the length of the shortest path between the two nodes. If node i has a link to node j then $d_{ij} = 1$. The average distance from a node to any other node in a strongly connected network, commonly referred to as the *average path length* of a node, is $\ell_i = 1/(n-1)\sum_{j \neq i} d_{ij}$.

Table 1
Correlations of basic network properties

ρ_{xy}	Value	Volume	n	m	p	r
Value						
Volume	0.88					
n	0.73	0.84				
m	0.78	0.92	0.96			
p	0.76	0.91	0.80	0.93		
r	0.06	0.09	−0.32	−0.18	−0.003	

n = size, m = number of links, p = connectivity, r = reciprocity.

Table 2
Turnover, component and network statistics for the Fedwire interbank payment network, fourth quarter 2004

	Mean	Median	Min.	Max.	SD
Payments					
Volume (000)	436	411	371	644	60.3
Value ($tr)	1.30	1.27	1.13	1.64	0.11
Mean value ($mn)	3.01	3.06	2.48	3.35	0.20
Components					
GWCC	6460	6484	6355	6729	83
DC	2	2	0	8	2
GSCC (n)	5086	5066	4914	5395	123
GIN	527	528	404	645	49
GOUT	774	782	595	916	67
Tendrils	103	103	88	116	7
Connectivity and reciprocity					
m	76,614	75,397	69,077	94,819	6151
p (%)	0.3	0.29	0.28	0.33	0.01
r (%)	21.5	21.5	21	23	0.03

$tr: $trillion, $mn: $million, GWCC: giant weakly connected component, GSCC: giant strongly connected component, GIN: giant in component, GOUT: giant out component, DC: disconnected component. All network statistics are calculated for GSCC. n = size, m = number of links.
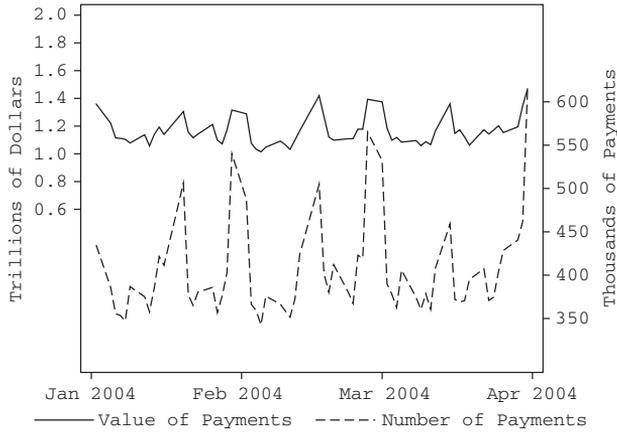
Fig. 4. Value and volume of interbank payments transferred over Fedwire in fourth quarter 2004.
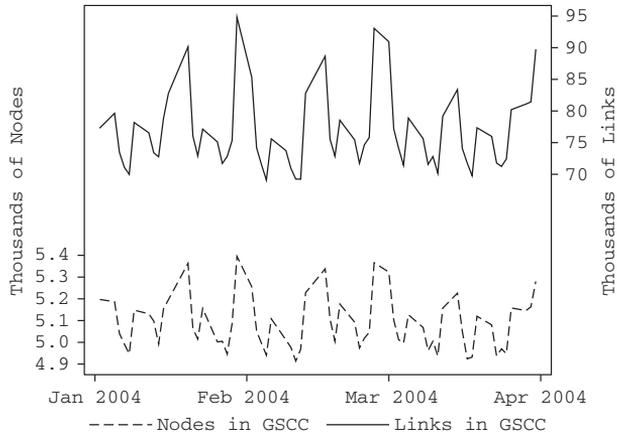

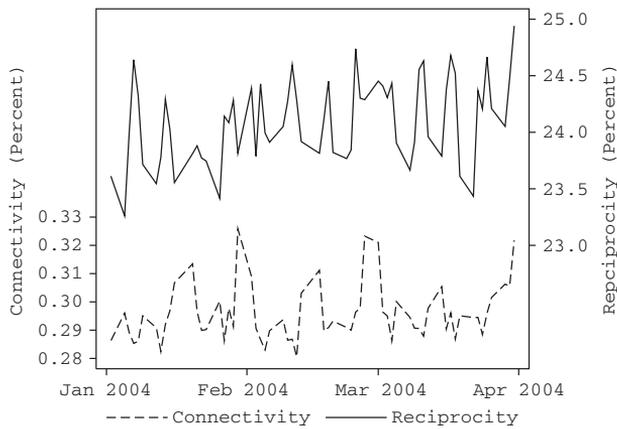Fig. 5. The number of nodes and links in the giant strongly connected component (GSCC).

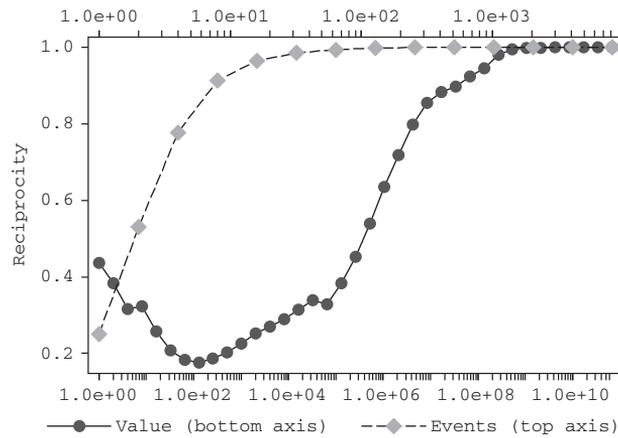Fig. 6. Connectivity and reciprocity in GSCC.

Fig. 7. Probability that a link is reciprocal based on gross value and volume of the link.

Table 3
More network statistics for the Fedwire interbank payment network, fourth quarter 2004

	Mean	Median	Min.	Max.	SD
Distance measures					
$\langle \ell \rangle$	2.62	2.63	2.56	2.66	0.02
$\langle \varepsilon \rangle$	4.67	4.63	4.18	5.74	0.33
D	6.6	7	6	7	0.5
$M(2)$ (%)	41.6	41.3	38.9	47.3	2.0
$M(3)$ (%)	95.9	95.8	95.1	97.1	0.5
$M(4)$ (%)	99.9	99.9	99.8	100	0.0
Clustering					
$\langle C \rangle$	0.53	0.53	0.51	0.55	0.01
Degree distribution					
$\langle k \rangle$	15.2	14.8	13.9	17.6	0.8
Max k^{out}	1922	1913	1772	2269	121
Max k^{in}	2097	2070	1939	2394	115
$\hat{\gamma}_{\text{MLE}}^{\text{out}}$	2.11	2.11	2.09	2.14	0.01
$\hat{\gamma}_{\text{MLE}}^{\text{in}}$	2.15	2.15	2.15	2.18	0.01

$\langle \ell \rangle$ = average path length, $\langle \varepsilon \rangle$ = average eccentricity, D = diameter, $M(x)$: mass distance function, $\langle C \rangle$: clustering coefficient, $\langle k \rangle$ = average degree, k^{in} = in-degree, k^{out} = out-degree, γ = power law coefficient.

The average path length of a network is defined as $\langle \ell \rangle = (1/n)\sum_i \ell_i$. In a classical random network $\langle \ell \rangle \approx \ln(n)/\ln(p \cdot n)$. The *eccentricity* of a node, ε_i, is the maximum distance to any other node in the network, i.e., $\varepsilon_i = \max_j d_{ij}$. The *diameter* of a network (D) is the maximum eccentricity (or distance) across all nodes, i.e., $D = \max_i \varepsilon_i$. Goh et al. [22] defines the *mass distance function* as the fraction of nodes within a certain distance of a node. At the network level it is defined as $M(x) = 1/n(n-1)\sum_i \sum_{i \neq j} 1(d_{ij} \leqslant x)$, where $1(\cdot)$ is the indicator function taking the value one if true and zero otherwise.

Discussion: The average path length is $\langle \ell \rangle = 2.6 \pm 0.2$ across our sample. In comparison, the average path length of a same size classical random network is 3.2. The mean eccentricity is $\langle \varepsilon \rangle = 4.7 \pm 0.33$, and the diameter ranges between 6 and 7 across days (see Table 3). The interbank payment network exhibits the *small-world* phenomenon common for many complex networks. Informally, small world means that any node can be reached from any other node in only a few steps. A short path length to any other node is a common property

for all nodes in the network as illustrated by the mass distance function shown in Table 3. It shows that it is possible for a sparse network with low connectivity to be extremely compact. Although few nodes directly connect, 41% are within two links, and 95% are within three links from each other. This further reflects the notion that the interbank payment network is comprised of a core of hubs with whom smaller banks interact. The compact nature of the network is interesting from the perspective of efficiency and resiliency of the payment system. The shorter the distances between banks in the network, the easier liquidity can be recirculated among the banks. On the other hand, a payment system where liquidity flows rapidly is also likely to be more vulnerable to disruptions in these flows.

4.4. Degree distribution

Definition: Two important characteristics of a node in a directed network are the number of links that originate from the node and the number of links that terminate at the node. These two quantities are referred to as the *out-degree*, k_i^{out}, and *in-degree*, k_i^{in}, of a node respectively. The average degree of a node in a network is the number of links divided by the number of nodes, i.e., $\langle k \rangle = (1/n)\sum_i k_i^{out} = (1/n)\sum_i k_i^{in} = m/n$. Networks are often categorized by their degree distributions, $P(k_i = x)$. The degree distribution of a classical random network is a Poisson distribution. Many real networks have fat-tailed degree distributions and a large number have been found to follow the power law $P(k_i = x) \backsim k_i^{-\gamma}$ in the tail. A power-law distribution is also sometimes called a *scale-free* distribution and networks with such a degree distribution are referred to as scale-free networks.

Discussion: The average degree in the network is $\langle k \rangle = 15.2 \pm 0.8$. However, most banks have only a few connections and a small number of "hubs" have thousands of links to other banks. Almost half of all banks have 4 or fewer outgoing links and 15% have only a single outgoing link. The banks with the largest in- and out-degrees averaged 2097 ± 115 and 1922 ± 121 links respectively. The out-degree distribution of the interbank payment network is shown in Fig. 8 together with the degree distribution for a classical random network with the same connectivity. For degrees greater than 10, the distribution follows a power law. The maximum likelihood estimate of the coefficient is $\hat{\gamma} = 2.11 \pm 0.01$ with a standard error of $\hat{\sigma}_\gamma = 0.03 \pm 0.001$. Appendix A describes the estimation procedure. The correlation between in and out degrees across nodes is 0.97 and the in-degree distribution looks similar to the out-degree distribution. It has a lower power law coefficient of $\hat{\gamma} = 2.15 \pm 0.01$ with the same standard error. Inaoka et al. [19] and Boss et al. [17], both find evidence of scale-free distributions for the Japanese interbank payment system (BoJ-NET) and the Austrian interbank market, respectively. The BoJ-NET has a power law tail of 2.3 for $k > 20$ and the Austrian interbank market has a coefficient of 3.1 for $k^{out} > 40$ and 1.7 for $k^{in} > 40$. A degree distribution following a power law distribution with a coefficient between 2.1 and 2.3, is not unique to payment systems, and appears to be a
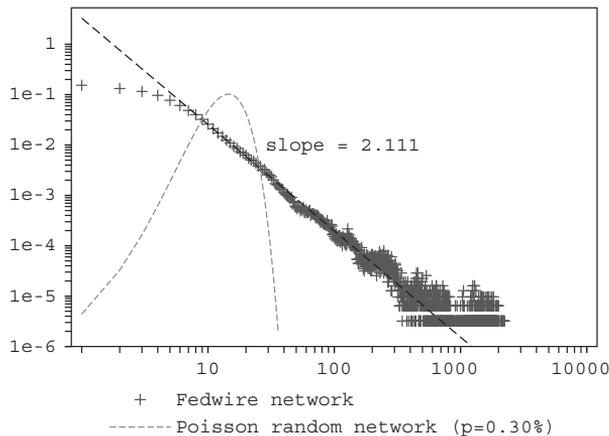

Fig. 8. Out-degree distribution.

common feature in complex networks. Albert et al. [23], and Crucitti et al. [16] find that scale-free networks are robust to random failures, but vulnerable to targeted attacks. Banks that have a low in-degree and high weights for these links are likely to be more vulnerable to disturbances than other banks as the removal of one link will severely limit the amount of incoming funds. Conversely, banks with high out degree have ceteris paribus the potential to affect more counterparties if their payment processing is disrupted.

4.5. Degree correlations

Definition: The probability that a node connects to another node may depend on the characteristics of the respective nodes. One characteristic of a node that is endogenous to the structure of the network is its degree. In an *uncorrelated* network there is no dependence between the degree of a node and the degrees of its neighbors. A network is said to be *assortative* if nodes with a given degree are more likely to have links with nodes of similar degree. A network is said to be *disassortative* if the opposite is true, i.e., nodes with low degrees are more likely to be connected to nodes with high degrees, and vice versa (e.g. Ref. [24]). The sign of the Pearson correlation coefficient between the degree of each node at the end of a link shows the direction of the dependency: zero for uncorrelated networks, positive for assortative networks and negative for disassortative networks. Another method is to compute the average degree of the nearest neighbors of a node as a function of the node degree. This is known as the average nearest neighbor degree (ANND) function, $\langle k_{nn} \rangle(k)$ [25].

Discussion: Both the correlation coefficients and the ANND functions show that the interbank payment network is disassortative. For example, the correlation of out degrees is -0.31 and the ANND defined for the out-degree of successors is inversely related to the out degree of a node as shown in Fig. 9. Using other combinations of neighbors and degree direction, we get similar coefficients. Disassortivity is common among technological and biological networks, as opposed to social networks which tend to exhibit assortative properties [14]. Moreover, Newman [26] shows that assortative networks percolate more easily than disassortative networks and that they are more robust to node removal.

4.6. Clustering coefficient

Definition: Another common correlation measure between nodes is the probability that two nodes which are the neighbors of the same node, themselves share a link. This is equivalent to the observation that two people, each of whom is your friend, are likely to be friends with each other. One way of measuring the tendency to cluster is the ratio of the actual number of directed links between the neighbors of a node, $m_{nn,i}$, over the
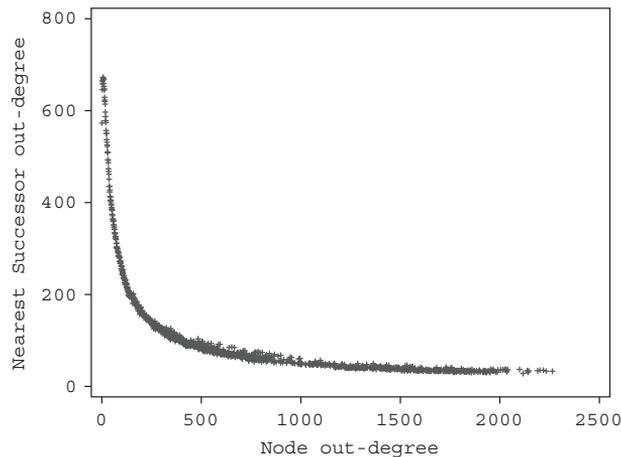

Fig. 9. Average nearest successor out-degree as a function of out-degree.

number of potential links among them

$$C_i = \frac{m_{nn,i}}{k_i(k_i - 1)}. \tag{1}$$

The clustering coefficient for the entire network, $\langle C \rangle$, is given by the average of all individual coefficients [27]. A tree network has a clustering coefficient of zero, and a complete network a coefficient of one. In a classical random network, the clustering coefficient is the unconditional probability of connection, i.e., $\langle C \rangle = p$.

Discussion: The average clustering coefficient of the networks calculated using the successors of a node is 0.53 ± 0.01. As such the observed clustering coefficient of the network is 90 times greater than the clustering coefficient of a comparable random network. However, the clustering coefficient for the network as a whole conceals the fact that the clustering across nodes is highly disperse, as illustrated in Fig. 10. More than 35% of the nodes have either a coefficient of zero or one. This is largely the result of low degree nodes [28]. Ignoring nodes with a degree smaller than three increases the average clustering coefficient to 0.62. A high level of clustering is observed in many other real world networks [27]. In a payment network, the clustering coefficient measures the degree of payments between a bank's counterparties. In terms of resilience one could hypothesize that disturbances in banks with a higher clustering coefficient might have a larger impact on their counterparties, as some of the disturbance may be passed on by the bank's neighbors to each other—in addition to the direct contagion from the source of the disruption.

4.7. Link weights and node strength

Definition: Weights, w_{ij}, are assigned to the links in a network to show the importance of each link. Barrat et al. [29] define *strength*, s_i, of a node as the sum of the weights of all the links attached to it, i.e., $s_i = \sum_j w_{ij}$, and suggest calculating the average strength as a function of the degree of a node, $s(k)$, in order to investigate the relationship between these two node characteristics. For a directed network, strength can be defined over both the incoming and outgoing links.

Discussion: The average weight per link in terms of value and volume are 15.2 ± 0.8 million and 5.2 ± 0.3 payments, respectively. The distribution of link weights follows a power law when weighted by the volume of payments but when weighted by the value of payments, the distributions of the link weights is closer to a lognormal distribution. The same is true for the distribution of node strengths (see Fig. 11).

For both weight measures, strength increases faster than the degree of a node. Like Barrat et al. [29], we see a power-law relationship between (out) strength and the degree of a node, $s(k) \sim k^\beta$. The coefficient is $\hat{\beta}_{\text{volume}} = 1.2 \pm 0.01$ when volume is used as weight and it is $\hat{\beta}_{\text{value}} = 1.9 \pm 0.01$ when value is used (see Fig. 12). More connected nodes transact a higher value and volume of payments than would be suggested by their degree
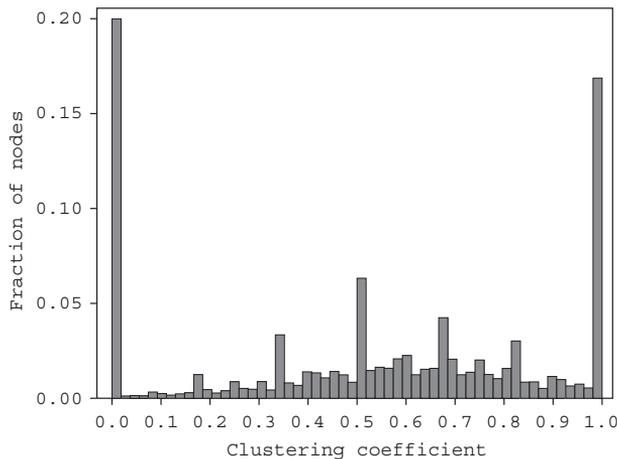

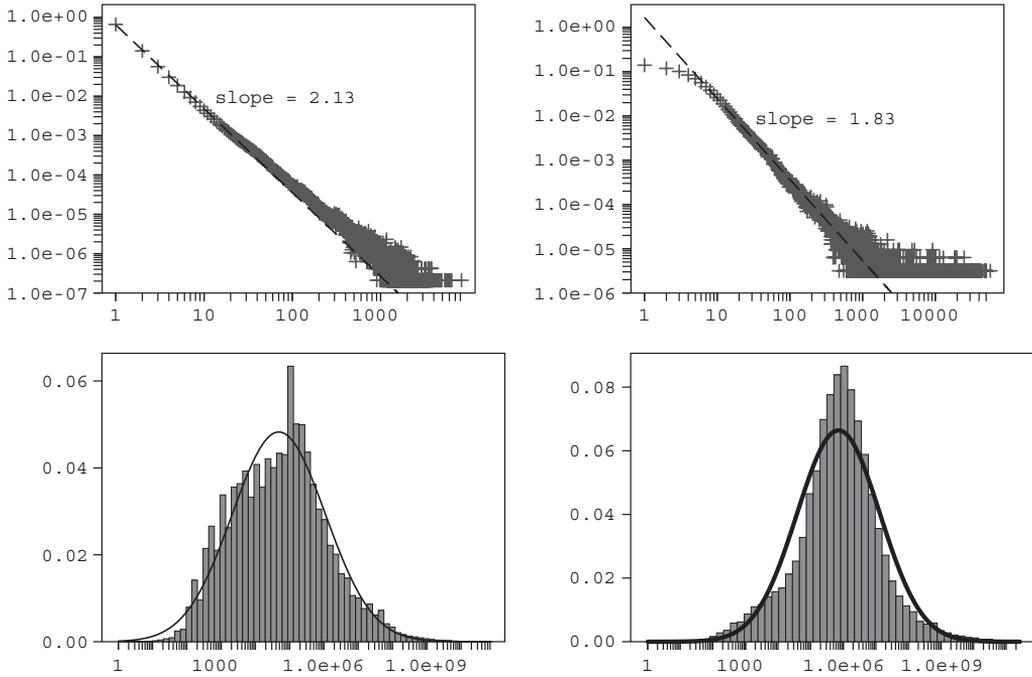Fig. 10. Distribution of the local clustering coefficients.

Fig. 11. Top left: Volume link weight distribution. Top right: Volume out-strength distribution. Bottom left: Value link weight distribution. Bottom right: Value out-strength distribution.
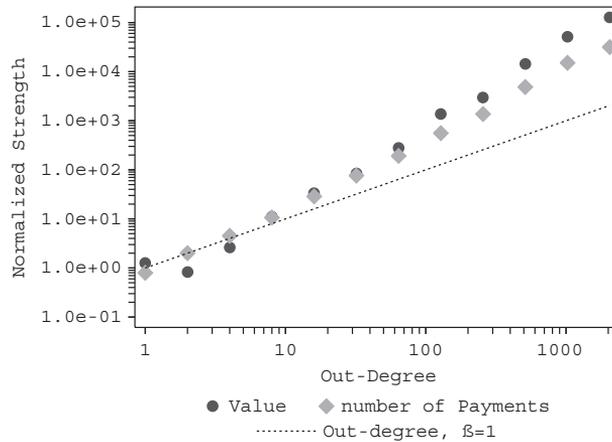

Fig. 12. Node out-strength as a function of out-degree.

alone. For example, if a bank has twice as many out links as another bank, it would be expected to send 2.3 times the number of payments, and 3.7 times the value of payments. The fact that the weights of the links are dependent on the degree of the banks has a bearing on the analysis of liquidity flows in the network. While the degree distribution and other unweighted measures of the network reveal interesting information on the structure of business relationships between banks in the payment system, accurate analysis and modelling of liquidity flows necessitates taking the weights of the links into consideration.

5. The impact of September 11th on network topology

The terrorist attacks of September 11th, 2001 disrupted the financial systems of the United States, including the interbank payment system. The attacks affected the structure of the interbank payment network in two ways. First, the massive damage to property and communications systems in lower Manhattan made it more difficult, and in some cases impossible, for many banks to execute payments to one another [9,30], i.e., some nodes were removed from the system or had their strength reduced. Second, the failure of some banks to make payments disrupted the payment coordination by which banks use incoming payments to fund their own transfers to other banks. Once a number of banks began to be short of incoming payments, others became more reluctant to send out payments themselves, i.e., links were either removed or had their weight reduced [10,30].

Both effects reduced the circulation of funds and collectively banks grew short of liquidity. The Federal Reserve recognized this trend toward illiquidity and provided funds through the discount window and open market operations in unprecedented amounts in the following week [30]. The impact on the interbank payment network is summarized in Table 4 and Figs. 13–16. The number of nodes in the giant weakly

Table 4
Network statistics for giant strongly connected component

	Non 9/11		9/11-2001
	Mean	Std. Dev.	
n	5325	137	4795
m	84,786	6638	59,640
p (%)	0.30	0.01	0.26
r (%)	24.3	0.4	22.3
$\langle k \rangle$	15.9	0.8	12.4
$\langle \ell \rangle$	2.65	0.03	2.80
$\langle \varepsilon \rangle$	4.84	0.30	5.28
$\langle C \rangle$	0.52	0.012	0.465
$M(2)$ (%)	39.3	2.1	29.3
$M(3)$ (%)	95.3	0.6	91.3
$M(4)$ (%)	99.8	0.0	99.5

n = size, m = number of links, p = connectivity, r = reciprocity, $\langle k \rangle$ = average degree, $\langle \ell \rangle$ = average path length, $\langle \varepsilon \rangle$ = average eccentricity, $\langle C \rangle$ = clustering coefficient. Non 9/11-2001 = September 4–21, 2001.
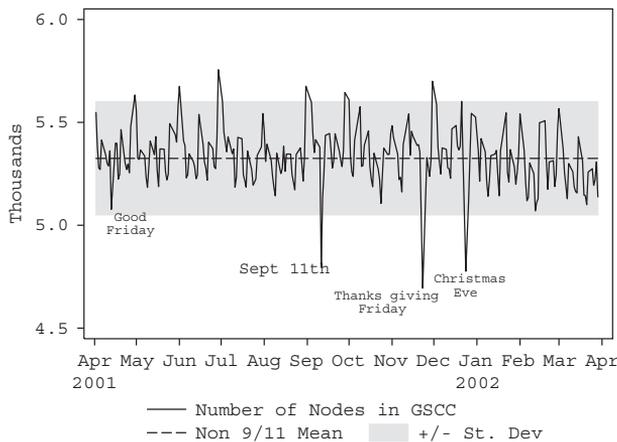

Fig. 13. Number of nodes in giant strongly connected component, April 2001–April 2002.
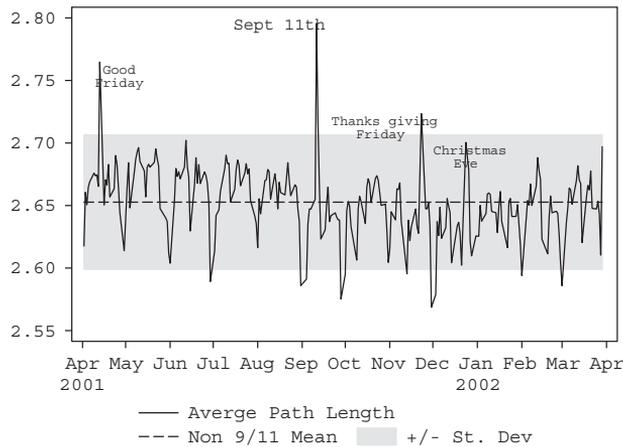
Fig. 14. Average path length of giant strongly connected component, April 2001–April 2002.
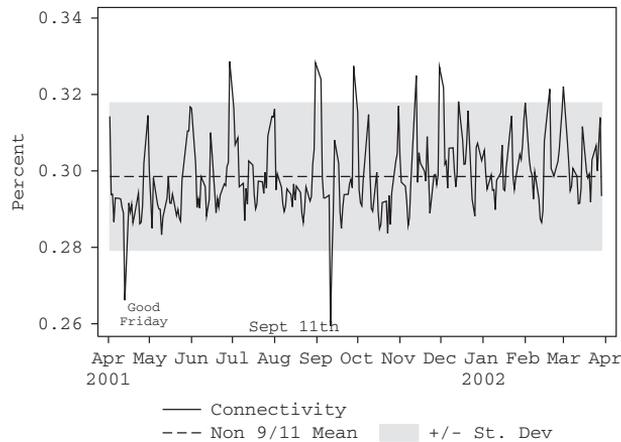

Fig. 15. Connectivity in giant strongly connected component, April 2001–April 2002.

connected component decreased by 5% from 6755 to 6466, while the number of nodes in the giant strongly connected component (GSCC) decreased by 10%, from 5325 to 4795. The relative size of the GSCC on September 11th was 6 percentage points lower than typical, as non-offsetting payment flows placed more nodes in the giant in- and giant out-components. The number of nodes in the GSCC did not return to its normal level until September 14th.

The connectivity on September 11th dropped from 0.30% to 0.26%. After two days of low connectivity, the connectivity shot up to over 0.30% from the 14th to 17th. This "overshooting" is likely the result of banks settling payments they had delayed in the days after September 11th.

As the connectivity decreased and key nodes were removed from the system, the average distance between nodes increased. The mass distance function for September 11th was well below its normal range. Thus both the average path length and the average eccentricity rose (see Table 4). The local structure of the network was also disrupted. Reciprocity fell from 24% to 22%, and the clustering coefficient from 0.52% to 0.47%. These results are indicative of the breakdown in the coordination between banks found in Ref. [10].

The size of the September 11th network was similar to the Friday after Thanks-giving and Christmas Eve (see Fig. 13). In terms of connectivity and average path length the September 11th network was more similar
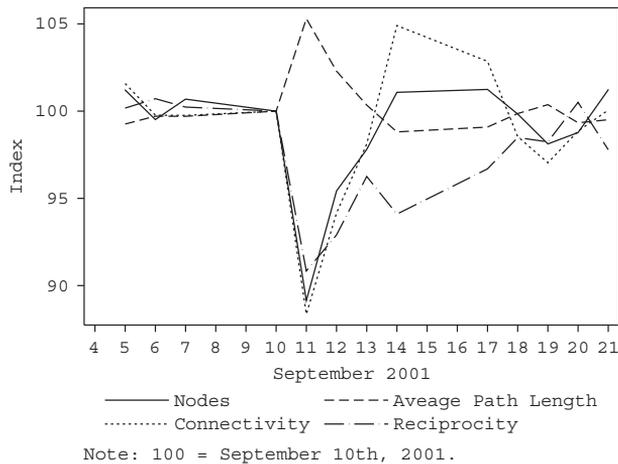
Fig. 16. Size, average path length, connectivity and reciprocity, September 4–21, 2001. September 10, 2001 = 100.

to Good Friday of 2001, when the New York Stock Exchange was closed. However, none of these semi holidays can capture all the changes to the topology that occurred on September 11th.

6. Conclusion

In this paper, we analyzed the topology of the 62 daily networks formed by the payment flows between commercial banks over Fedwire. These networks share many of the characteristics commonly found in other empirical complex networks, such as a scale-free degree distribution, high clustering coefficient and the small world phenomenon. The network is disassortative like many other technological networks. We also found that, apart from a few holidays, the statistics characterizing the network are quite similar from day to day. Moreover, we found that the topology of the network was significantly altered by the attacks of September 11th, 2001. The number of nodes and links in the network and its connectivity was reduced, while the average path length between nodes was significantly increased.

Because scale-free networks are found in many areas, the performance of such networks under ordinary and disrupted conditions is receiving increasing attention. Static scale-free networks, for example, have been found to preserve their connectivity under random node removal yet to be vulnerable to disconnection following removal of high-degree nodes [23]. The vulnerability of a particular network depends both on its structure and on the mechanisms of contagion. As these differ across networks, we cannot extrapolate this conclusion to payment networks. In the case of a payment system, understanding the dynamics of the liquidity flows is essential for assessing network robustness. A question for further research is how the degree distribution and other topological measures relate to contagion of disturbances.

Finally, many of the network statistics appear to vary periodically and abruptly. An interesting question is whether the networks of daily payments naturally form distinct clusters, and what processes create such distinctions. The scale-free daily network is constructed from individual payments accumulated over the day. Analysis of the intra-day payment data will give us insights into how the network forms and on the regularity of this process.

Appendix A. Maximum likelihood estimation of the power law exponent

In recent years, a significant amount of research has focused on showing that the distributions of many physical and social phenomena follow a power-law, i.e., $P(k) \sim k^{-\gamma}$. Maximum likelihood estimators for slope coefficient γ for the discrete and continuous case are derived in Refs. [31,32], respectively. Since, many empirical distributions are found only to follow a power law in the right hand tail of the distribution, i.e., $P(k) \sim k^{-\gamma}$ for $k > a$, we derive the maximum likelihood estimator for a truncated power law distribution. The probability distribution of a truncated random variable is given by

$$P(k|k>a) = \frac{P(k)}{1 - P(k \leqslant a)}.$$

In the discrete case the power law distribution is given by

$$P(k) = \frac{k^{-\gamma}}{\zeta(\gamma)},$$

where $\zeta(\gamma)$ is the Riemann Zeta function. Hence, we have

$$P(k|k>a) = \frac{k^{-\gamma}/\zeta(\gamma)}{1 - \sum_{s=1}^{a} s^{-\gamma}/\zeta(\gamma)} = \frac{k^{-\gamma}}{\zeta(\gamma,a)},$$

where $\zeta(\gamma,a)$ is the Hurwitz Zeta function. Analogous to Goldstein et al. [31] we have that the maximum likelihood estimator, $\hat{\gamma}_{\mathrm{MLE}}$, equates the negative logarithmic derivative of the Hurwitz Zeta function with the average logarithm of the data in the sample, i.e.,

$$\frac{\zeta'(\gamma,a)}{\zeta(\gamma,a)} = \frac{1}{n}\sum_{i} \log(x_i).$$

The equation can be solved numerically for $\hat{\gamma}_{\mathrm{MLE}}$. An estimate of the standard error of the maximum likelihood estimator can be computed by evaluating the second derivative of the log-likelihood function at $\hat{\gamma}_{\mathrm{MLE}}$

$$\hat{\sigma}_{\mathrm{MLE}} = \frac{1}{\sqrt{n}} \cdot \left(\frac{\zeta(\hat{\gamma}_{\mathrm{MLE}},a)^2}{\zeta''(\hat{\gamma}_{\mathrm{MLE}},a)\zeta(\hat{\gamma}_{\mathrm{MLE}},a) - \zeta'(\hat{\gamma}_{\mathrm{MLE}},a)^2} \right)^{1/2}.$$

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