Combined experimental and numerical study on ice block breakage

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Abstract

Sea ice features made of ice rubble, such as rubble fields and ice ridges, are the most difficult ice condition encountered by offshore structures or vessels operating in first-year ice. Thus, scientific understanding of the mechanical behaviour of ice rubble is crucial to correctly estimate ice loads on offshore structures. Ice rubble is a granular material and its mechanical properties are often studied assuming that the ice blocks constituting the rubble do not break. It has, however, been observed in laboratory experiments that the ice blocks within deforming ice rubble may break due to compressive ice-to-ice contact forces. This occurs as the load is transmitted within the rubble through force chains, sequences of contacting ice blocks under high compressive stress. The present thesis studies ice block breakage using three approaches: 1) an experimental study on ice block breakage under compressive ice-to-ice contacts, 2) high resolution bonded particle modeling (BPM) of the aforementioned breakage experiments, and 3) the development of a simplified ice block breakage model for DEM simulations.

In the experiments, three ice blocks were set to form two ice-to-ice contacts and compressed until the failure of the three block system. The experiment results revealed that shear failure was the primary failure mode of the ice blocks in compressive ice-to-ice contacts and that the force transmitted by the contact was limited by the bulk strength of the ice. BPM simulations further confirmed the hypothesis that the failure of ice blocks occur due to shearing governed by the bulk strength of the material. Moreover, the stress distribution within the ice blocks, obtained from the simulations, showed that the Mohr-Coulomb failure criterion can be used to model the observed shear failure in a simple yet reliable manner. Following that, a simplified block breakage model based on shear failure and the Mohr-Coulomb failure criterion could be developed. The breakage model is for DEM simulations of ice rubble. The breakage model was integrated into an existing DEM tool and direct shear box experiments on ice rubble were simulated. The simulation results revealed that block breakage is one of the modes of force chain failure within deforming ice rubble. Simulations further showed that block breakage reduces the shear resistance and the shear strength of ice rubble.

The findings of the present thesis indicate that the common approach of modeling contact failure of ice blocks in DEM simulations by using a contact force limit based on local crushing of ice is not always accurate. Instead, this thesis suggests that the shear failure of ice blocks has to be accounted as well, since block breakage limits the load transmitted by force chains. Moreover, the reduction of the shear strength of the ice rubble due to block breakage implies that the ice load estimates obtained from DEM simulations of ice-structure interaction processes without a block breakage model may be overly conservative.

Keywords  Ice mechanics, Discrete element method, Material failure, Breakage, Numerical modeling

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Dedicated to mom and dad,
who gave the little they had,
to ensure I would have the opportunity of an education.
Preface

The research presented in this thesis was funded by the Academy of Finland through the project (309830) Ice Block Breakage: Experiments and Simulations (ICEBES). I was also financially supported by the Jenny and Antti Wihuri foundation, the Finnish Maritime Foundation and the Doctoral Program of the Aalto University School of Engineering during the thesis work. I acknowledge all financial support with profound gratitude. Computational resources for the numerical simulations were provided by the CSC – IT Center for Science, Finland, under the project (2000971) Mechanics and Fracture of Ice.

I would like to express my deepest gratitude to my supervisor Associate professor Arttu Polojärvi for providing me the opportunity of working on this research and for the continuous guidance, support and encouragement during the thesis process. I would also like to thank my thesis advisor Professor Jukka Tuhkuri for the insightful comments and suggestions which had valuable impact on the thesis. I am also thank full to Dr. Jan Åström for the guidance in numerical modeling and for providing me the opportunity of using the numerical tool HiDEM. Special thanks are extended to Dr. Mingdong Wei for his tremendous support during the ice block breakage experiments. I am also thankful to Dr. David M. Cole for the insightful discussions and suggestions which shaped the research presented in this thesis. I would like to extend my sincere thanks to all my colleagues at the Ice Mechanics research group for their support and for creating a positive working atmosphere at the university. I would also like to acknowledge the pre-examiners of the thesis and the anonymous reviewers of the publications led to this thesis for their time and efforts spent on reviewing the scientific work.

Words cannot express my gratitude to my mother, father and brother for their love, nurture and encouragement throughout the years. This endeavor would not have been possible without their support. Lastly, many thanks to my friends, family, loved ones in Finland and Sri Lanka for being there for me in the ups and downs during this thesis work.

Helsinki, June 29, 2023,

Malith Prasanna Rupasingha Arachchige
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This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I Malith Prasanna, Mingdong Wei, Arttu Polojärvi, David M. Cole. Laboratory experiments on floating saline ice block breakage in ice-to-ice contact. *Cold Regions Science and Technology*, 189, 103315, 2021.


Author’s Contribution

Publication I: “Laboratory experiments on floating saline ice block breakage in ice-to-ice contact”

This publication describes an experimental study on breakage of floating saline ice blocks under compressive ice-to-ice contacts. The author participated in designing the experiments and performed them with Mingdong Wei. Further, the author processed the data from the experiments, analysed the results and wrote the manuscript. All authors discussed the results and commented on the manuscript. Arttu Polojärvi and David M. Cole provided writing support.

Publication II: “Modeling ice block failure within drift ice and ice rubble”

This publication presents a numerical analysis of the ice block breakage experiments of Publication I using a bonded particle model HiDEM. The author further developed the existing bonded particle model HiDEM by integrating a cohesive softening model into it. The author also prepared and performed the simulations, processed the data from them, analysed the results and wrote the manuscript. All authors discussed the results and commented on the manuscript. Arttu Polojärvi and Jan Åström provided writing support.

Publication III: “Breakage in quasi-static discrete element simulations—an application to ice rubble”

This publication presents a simplified block breakage model for discrete element method (DEM) simulations based on the findings of Publication
Author's Contribution

I and Publication II and, a numerical study of block breakage within shearing ice rubble. The author developed the ice block breakage model and implemented it into an existing DEM tool. In addition, the author prepared and performed the simulations, processed the data from them, analysed the results and wrote the manuscript. Arttu Polojarvi discussed the results, commented on the manuscript and provided writing support.
1. Introduction

1.1 Background

Ice rubble, a collection of ice blocks and ice fragments, forms as a result of the deformation and failure of intact ice sheets. Sea ice features made of ice rubble, such as rubble fields and ice ridges, are the most difficult ice condition encountered by offshore structures or vessels operating in first-year ice. Insight on the mechanical behaviour of ice rubble and its failure is important in order to correctly estimate ice loads on ships and offshore structures operating on ice covered waters. Ice loads exerted by a rubble are limited by the force transmission capacity of rubble [1]. Ice rubble has the characteristics of a granular material, thus, force transmission in a rubble pile occurs through force chains, that is, a series of ice blocks under high compressive contact forces [2]. The failure of the force chains limits the force transmission capacity of ice rubble. A common failure mode for a force chain is buckling. Force chain failure due to buckling and its influence on ice loads on an inclined structure were studied by Paavilainen and Tuhkuri [3] and Ranta et al. [4]. In addition to buckling, a force chain can fail due to failure of one or several of the ice blocks belonging to it [5], that is, a force chain can fail due to block breakage. Ice block breakage was observed in direct shear box experiments on ice rubble by Pustogvar et al. [6]. The mechanics of ice block breakage under compressive ice-to-ice contacts has not been thoroughly explored, yet understanding of ice block breakage could be essential for understanding ice rubble behaviour. This provides the motivation for the present thesis, which focused on ice block breakage.

With the advances of computational techniques, discrete element method (DEM) [8] has become a common tool for modeling ice-structure interaction processes due to its ability to simulate the motions and interactions of individual ice blocks. DEM simulations have been used previously to study the mechanical properties of ice rubble by numerically modeling direct
shear box experiments \cite{7, 9, 10} (Fig. 1.1), punch through tests \cite{11–14} and flexural failure tests \cite{15} of ice rubble. Moreover, DEM has also been used in simulating processes where an intact ice sheet fails into ice rubble, such as ice ridge formation \cite{9, 16–18}. In addition, there have also been previous DEM studies of ice loads on a ship passing through ice ridges \cite{19, 20} and brash ice \cite{21–23}. DEM can be used in conjunction with the finite element method (FEM) to create combined FEM-DEM simulations of ice-structure interactions. In such simulations, ice sheet failure is modeled by using FEM, while the interactions between broken ice blocks are modeled using DEM \cite{24}. FEM-DEM simulations have also been used previously to study the ice loading processes on inclined structures \cite{3, 25–31}, conical offshore structures \cite{31, 32} and ships \cite{32, 33}. Bonded particle models (BPM) \cite{34} are another variant of DEM models, where the ice is modeled as a lattice of spheres joined together by bonds, while the failure of the bonds mimics material failure. BPM simulations have been used previously to estimate ice loads on ships and conical structures \cite{35–38} as well as to study the underlying mechanics of the failure processes in an ice-structure interaction process \cite{39–41}. A comprehensive review of the applications of DEM in simulating ice-structure interaction processes can be found in Tuhkuri and Polojärvi \cite{42}.

A block breakage model for DEM simulations should consist of a relevant failure criterion and a method to produce progeny fragments following breakage. For materials other than ice rubble, one common approach has been to compute the average principal stress for each block based on contact forces acting on it and compare that with the compressive strength of the material of the block \cite{43–47}. This was the approach...
chosen by Hocking [33] for ice blocks. Often in this approach, once the failure criterion is met, the block is replaced by its fragments, whose dimensions are obtained from a predefined fragment size distribution [43-45, 48, 49]. One of the drawbacks in this approach is that the blocks are assumed to have uniform stresses which is not true even for moderately elongated block geometries. The boundary element method has been used in DEM simulations to overcome the aforementioned limitation. In this case, the stress state, the onset of breakage, and the path of the failure can be defined more accurately. However, the computational burden caused by such an approach makes it infeasible for simulations with a large number of blocks. Another common modeling technique for breakage, used in dynamic simulations of granular materials, is to compute the cumulative impact energy of the blocks and compare that with the fracture energy of the material of the block to determine if breakage occurs [48–51]. Nevertheless, such techniques are not applicable for ice rubble deformation due to the quasi-static nature of the process. In addition to aforementioned work by Hocking [33], only Hopkins and Hibler [9] has earlier included breakage into simulations of ice rubble. This was done by using a bending failure-based block breakage model for ice blocks with high aspect ratios and was not based on any experimental evidence. Hence, there exists a knowledge gap related to the development of a simplified block breakage model that can be used in large scale DEM simulations of ice rubble. This is where the efforts of the present thesis are aimed at.

An ice specimen subjected to compression may fail through various modes such as splitting, shearing, spalling and crushing [52, 53]. The failure mode depends on, for example the direction of forces relative to the grain direction and confinement of the ice specimen [52]. Moreover, experimental observations suggest that the boundary conditions, the frictional forces at the contacts, and the geometry of the contacts, may have an influence on the failure mode as well [54, 55]. Kuehn et al. [56] argue that there could be local perturbations at the ice contact surfaces, which could flatten without causing specimen failure if the ice is deformed sufficiently slowly. Therefore, it is apparent that the failure of ice blocks within deforming rubble is influenced by compressive ice-to-ice contacts and the quasi-static nature of the process, and the underlying mechanics of the failure process is not intuitive. There has, however, not been any previous systematic experimental or numerical study on the breakage of ice blocks under compressive ice-to-ice contacts, which brings forth novelty to the present thesis.
1.2 Objectives and scope

The present thesis concentrates on investigating ice block breakage under compressive ice-to-ice contacts using experiments and numerical modeling. The underlying observation-based assumption is that some of the blocks within ice rubble break due to compressive ice-to-ice contact forces exerted on them by their neighbouring blocks when the rubble deforms and transmits loads through force chains. The main objectives of the thesis are:

- Generate experimental data on ice block breakage under compressive ice-to-ice contacts for simple contact geometries.
- Investigate the underlying mechanics of ice block breakage under compressive ice-to-ice contacts by using high resolution BPM modeling.
- Develop a simplified ice block breakage model that can be used in quasi-static DEM simulations with non-spherical discrete elements.
- Study the influence of block breakage on force transmission within deforming ice rubble and the mechanical behaviour of rubble.

1.3 Research approach

Fig. 1.2 illustrates an overview of the research approach utilized in the present thesis. First, laboratory scale experiments were performed to investigate ice block breakage under compressive ice-to-ice contacts using a unique three ice block setup consisting of two ice-to-ice contacts (Publication I). Then the breakage experiments were modeled using an existing high resolution bonded particle model (BPM) HiDEM \[57–59\] to reproduce the failure modes and force records obtained from the experiments (Publication II). BPM was further used to investigate the underlying mechanics and applicable failure criterion for ice block breakage. Next, a simplified breakage model was developed based on the evidence from the experiments and the high resolution modeling (Publication III). The breakage model was then integrated into an existing discrete element method (DEM) tool \[31\]. Then direct shear box experiments on ice rubble were simulated and compared with the experimental results of Pustogvar et al. \[6\] for validation. Simulations were further used to study the effect of breakage on the mechanical behaviour of ice rubble.
Introduction

Combined experimental and numerical study on ice block breakage (Doctoral thesis)

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Figure 1.2. An illustration of the research approach utilized in the present thesis.

1.4 Dissertation structure

The present thesis is structured according to the research approach illustrated in Fig. 1.2. Chapter 2 describes the ice block breakage experiments, their results and analysis. Chapter 3 presents the high resolution BPM modeling of the breakage experiments together with a detailed analysis on the mechanics of ice block breakage. In Chapter 3, the newly developed block breakage model for DEM simulations is introduced. This chapter further presents simulations of direct shear box experiments and their results. Finally, Chapter 5 presents the main original features and conclusions of the present thesis and remarks on future work.
2. Ice block breakage experiments

2.1 Overview of experiments

In the first part of this research, laboratory scale experiments were performed to study the failure of floating saline ice blocks under compressive ice-to-ice contacts. Publication I describes these experiments in detail. The main objective of the experiments was to obtain fundamental understanding of the maximum force transmitted through contacts and the failure modes of ice blocks under compressive ice-to-ice contacts for simple contact geometries with varying areas. Boundary conditions created by ice-to-ice contacts may have a significant effect on the failure mode of the blocks [54, 55], but according to the best knowledge of the author, there has not been any previous experimental work on breakage of ice blocks under ice-to-ice contacts. In addition to this, a key highlight of these experiments was the use of relatively warm floating ice blocks with a naturally occurring temperature gradient, which mimics in-situ testing of sea ice in nature. Often, laboratory experiments on ice are performed using relatively cold, dry samples whose mechanical behaviour is different from natural sea ice [60–63].

In the experiments, three ice blocks were set to form two ice-to-ice contacts of given length, $c$, as shown in Fig. 2.1. Then they were compressed until the failure of the three block system. The total force applied on the three block system, $F_p$, and the force transmitted through two contacts, $F_1$ and $F_2$, were recorded. The test matrix is presented in Table 2.1. Six contact lengths, $c = 25, 50, 75, 100, 125$ and $150$ mm, were used throughout the series of 32 experiments.
Ice block breakage experiments

2.2 Ice properties and experimental setup

A detailed description of the experimental setup and procedures, ice growing tank and process, and ice characterization are presented in Publication I and by Wei et al. [61]. In brief, the ice blocks used in the experiments had the dimensions of 300 mm × 300 mm × 110 mm (length × width × thickness) and they were grown inside high-density polyethylene molds to achieve high dimensional accuracy. The ice had a density of 886 kgm⁻³ and 5 ppt melt water salinity. The microstructure of the ice was type S2 columnar with 3 mm average grain size. The compressive strength of ice was 780 kPa.

The experiments were conducted using a force controlled, uni-axial compression testing rig with a saline water basin as illustrated in Fig. 2.2. The
The testing rig was equipped with a hydraulic cylinder to apply the compressive force, three sets of load cells to measure the force applied on the three block system and the reaction forces on the platens behind the two blocks away from the cylinder (the resultant of reaction forces on each platen were equal to the force transmitted through each contact), and a displacement gauge to measure the linear displacement of the loading platen attached to the cylinder. A constant loading rate of 5 kNs\(^{-1}\) was used. Ice blocks were harvested from the growing tank and transferred immediately to the saline water basin of the testing rig, and hence there was no noticeable brine drainage nor temperature change of ice. The saline water basin of the rig was filled by transferring water from the ice growing tank after harvesting the ice blocks, and thus the salinity and the water temperature in the test basin were the same as in the growing tank. The floating ice blocks had a through thickness temperature gradient with an average of -2.5 \(^\circ\)C. The air temperature in the cold room was -10 \(^\circ\)C.
2.3 Experimental results

2.3.1 Contact force records

Fig. 2.3 presents a typical record of the compressive force, $F_p$, applied on the three block system, and the force transmitted by the two contacts, $F_1$ and $F_2$, against time, $t$, from a breakage experiment with $c=100$ mm. As shown by the record, after initial settling at the contacts, $F_p$ increased linearly until near the maximum load, then reached a peak value, and dropped nearly instantaneously due to the failure within the system. Both $F_1$ and $F_2$ records followed a similar trend to $F_p$. The sum of $F_1$ and $F_2$ was on average less than 5% of $F_p$, which means that the setup was able to measure the force transmitted by each contact accurately. Further, the difference between $F_1$ and $F_2$ was about 10% during loading, which indicates that the loading on the contacts was virtually symmetrical. It was typical that the drop in $F_1$ and $F_2$ was not simultaneous, but occurred sequentially indicating that one of the contacts failed first followed by the failure of the other. The maximum contact force, $F_c$, was defined as the maximum force transmitted by the first contact to fail. The analysis was focused on $F_c$, since the loading became asymmetric following the first failure. It is worth emphasizing that the contact force records showed

![Figure 2.3. Typical $F$–$t$ curve for contact length, $c = 100$ mm. Marker ‘a’ in the figure indicates the time instance of 90% of maximum contact force, $F_c$. Figs. 2.5a and b present snapshots of the experiment at instances of markers ‘a’ and $F_c$. Figure reproduced from Publication I.](image-url)
a brief non-linear portion near the maximum load, implying material softening occurring upon failure. Moreover, $F_c$ was reached slightly before the sudden force drop. This indicates that the failure can be characterized as quasi-brittle.

Fig. 2.4 presents $F_c$ data from all 32 experiments plotted against the nominal contact area, $A_c$, which was calculated by multiplying $c$ by the thickness of the ice specimens tested. As expected, $F_c$ shows an increasing trend with $A_c$. The figure further shows a trend-line fitted for $F_c$ data, the slope of which, 712 kPa, can be regarded as the nominal compressive contact stress. In the figure, markers indicate the failure mode of the ice blocks: crushing, shearing, splitting and Y-type failures. A detailed illustration of these failure modes are presented in Publication I. As seen in the figure, shear failure was the most common failure mode as it occurred in about 75% of experiments. Thus, the analysis was focused on the shear failure.

### 2.3.2 Failure process

A typical failure process of the ice blocks observed in the experiments is presented in a four-photo sequence in Fig. 2.5. The first two photos of the sequence correspond to the time instances of 90% of $F_c$ (marker ‘a’) and $F_c$ as shown in Fig. 2.3, while the final two photos show the first and the second failure of the contacts. In Fig. 2.5a, at 90% of $F_c$, there was no visible damage in the ice blocks. As the contact force increased further, visible material damage started to appear near contacts and accumulated into a thick band of interconnected flaws (bright white bands in Fig. 2.5b). The first contact failure occurred slightly after $F_c$ due to a macroscopic

![Figure 2.4](image-url)  

**Figure 2.4.** Maximum contact force, $F_c$, plotted against nominal contact area, $A_c$. Failure modes of the blocks are indicated in the legend. Figure reproduced from Publication I.
Ice block breakage experiments

Figure 2.5. Snapshots of the failure process of ice blocks in a $c = 100$ mm experiment at instances of (a) $90\%$ of $F_c$, (b) $F_c$, (c) first failure and (d) second failure. The $F-t$ curve of the experiment is presented in Fig. 2.3. Figure reproduced from Publication I.

failure plane as shown in Fig. 2.5c. The second contact failure also followed a similar process (Fig. 2.5d). It is important to note that the shear failure observed in the experiments can be characterized as a material failure due to the bulk strength of the material rather than instantaneous crack propagation caused by stress concentrations at the contacts. Propagating cracks tend to curve towards the free boundaries of specimen [64], which would have resulted in curved final failure planes. In addition, crack propagation governed failure would not be likely to yield a clear linear dependency between $F_c$ and $A_c$ as in the case of Fig. 2.4.
2.4 Analysis

2.4.1 Shear failure of ice blocks

The shear failure observed in the ice block breakage experiments has the characteristics of ‘Coulombic shear faults’ [65]: the failure planes consist of accumulated and interconnected flaws and have a 20-30° angle with respect to the loading direction. They are a common failure mechanism of ice under low confinement compressive loading.

The maximum contact force, $F_c$, can be resolved onto the failure plane as

$$F_r = F_c \cos \theta \quad \text{and} \quad F_n = F_c \sin \theta,$$

where $F_r$ and $F_n$ are the nominal shear and normal forces acting along and perpendicular to the surface of the failure plane, respectively. $\theta$ is the angle of the shear plane with respect to the direction of contact force (Fig 2.6) and was obtained from photos of the experiments. Furthermore, the area of the shear plane, $A_r$, could be estimated by multiplying the length of the shear plane by the thickness of the ice block.

Fig. 2.7 presents $F_r$ plotted against $A_r$ and grouped according to the contact length, $c$, of the respective experiment. $F_r$ shows an increasing trend with $A_r$. The trend line fitted to the data has a relatively high coefficient of determination, $R^2=0.73$, and physically sound zero intercept. The linear trend between $F_r$ and $A_r$ suggests that the failure was due to the bulk strength of the material, which has the value of the gradient, 279 kPa, of the trend line.

![shear plane](image)

**Figure 2.6.** Nominal normal and shear forces, $F_n$ and $F_r$ respectively, resolved onto the failure plane for quasi-static force equilibrium at the critical moment of shear failure. $F_c$ is the maximum contact force while $\theta$ is the failure angle. Figure reproduced from Publication I.
2.4.2 Discussion on the experimental results

The results and analysis of the ice block breakage experiments show that the failure of ice blocks under quasi-static compressive ice-to-ice contact forces primarily occurs due to shear failure of the blocks. The failure pattern resembles ‘Coulombic shear faults’. Moreover, the failure is due to the bulk strength of the material rather than instantaneous crack propagation caused by stress concentrations at the contacts. This result stands in contrast with the common approach of modeling the contact failure of ice blocks in DEM simulations by using a contact force limit based on the crushing or compressive strength of the ice [3, 24, 66–68]. As shown in Fig. 2.4, crushing failure at the contacts is not the primary failure mode, but the shear failure of blocks is. Thus, importantly, the experimental results here propose that the shear failure of ice blocks has to be accounted as well when modeling contacting ice blocks, or ice rubble, in DEM simulations.
3. High resolution modeling

3.1 Overview of simulations

In the second part of this research, ice block breakage experiments were simulated by using a bonded particle model (BPM) HiDEM [57–59]. This research led to Publication II. The main objective of these simulations was to investigate the underlying mechanics of ice block breakage under compressive ice-to-ice contacts by numerically reproducing the breakage experiments using a high resolution numerical model. In BPM, material is modeled as a lattice of densely packed particles connected by bonds. Failure of the bonds mimics material failure. Unlike traditional numerical modeling techniques such as the finite element method, BPM does not require a material model when simulating material response and failure. In the work of this thesis, HiDEM was developed further by implementing features such as material softening and accounting for the anisotropic microstructure of columnar ice.

3.2 Numerical model

3.2.1 HiDEM

In HiDEM, ice is modeled as a lattice of dense packed spherical particles connected by Euler-Bernoulli beam elements. Contact interaction between the spheres is modeled using the Hertzian contact model, where the contact force is proportional to the overlap volume between spheres (Fig. 3.1c). Dissipation of elastic energy due to deformation is modeled by applying a viscous damping force on the particles. The equation of motion for particle $i$ in the lattice is
High resolution modeling

\[ M\ddot{r}_i + C\dot{r}_i + \sum_j \gamma_{ij} C'\dot{r}_{ij} + \sum_j \gamma'_{ij} K r_{ij} = F_i, \]  
(3.1)

where \( r_i \) is the position vector of particle \( i \) and \( r_{ij} \) is the relative position vector of particle \( i \) with respect to particle \( j \). Further, \( M \) is the diagonal matrix including the mass properties of the particle, matrix \( C \) contains damping coefficients for viscous drag, and matrix \( C' \) contains damping coefficients for inelastic collisions between particles. Matrix \( K \) is the stiffness matrix of Euler-Bernoulli beam connecting particle pair \( i - j \). Forces due to the deformation of beam elements \( (Kr_{ij}) \) are summed over all the beams particle \( i \) is connected to. Parameter \( \gamma_{ij} \) is unity if particle pair \( i - j \) is in contact and zero otherwise, while parameter \( \gamma'_{ij} \) is unity if particle pair \( i - j \) is connected by a beam and zero otherwise. \( F_i \) is the sum of external forces acting on particle \( i \). According to Eq. 3.1, an intact particle-beam lattice exhibits a granular visco-elastic material response.

The model is described in detail by Åström [57] and Åström et al. [58].

### 3.2.2 Material softening

In the standard HiDEM model, available at the start of this research, beams had been set to fail instantaneously upon reaching the failure strain mimicking a brittle failure. Preliminary simulations of the breakage experiments using the standard HiDEM model yielded localized crushing at the contacts and shattering of the blocks, caused by the instantaneous energy release from the failing beams. This was contradictory to the experiment results and indicated that a failure model with material softening is re-
required to capture the quasibrittle failure of ice. Thus, a cohesive softening failure model was implemented in HiDEM in order to dissipate the elastic energy stored in the beams. The softening model used was originally developed by Paavilainen et al. [24] and it is based on the fictitious crack model by Hillerborg et al. [69]. In the model, it is assumed that there exist a cohesive zone at the crack tip. When the crack propagates, stresses in this zone are decreased linearly with increasing crack opening displacement δ (Fig. 3.2a). This dissipates the elastic energy stored in the material.

In the cohesive softening model implemented in HiDEM, it is assumed that a beam can break by forming a cohesive crack at one of its ends. Based on the status of the cohesive crack, each beam has three modes: a linear-elastic mode, a softening mode, and a failed mode. In the linear-elastic mode, the beam is intact, and thus, its strains are governed by the Euler-Bernoulli beam theory. The $\sigma - \epsilon$ curve OA of Fig. 3.2b represents this behaviour. In the softening mode, a cohesive crack has formed. This is represented by point B of Fig. 3.2b, where the strains of the beam consists of elastic and plastic strains. In the failed mode, cohesive softening ends as a full crack has formed. Point C of Fig. 3.2b corresponds to the instance where the beam does not transmit stresses and is considered broken.

In the cohesive mode, the tensile stress of the beam at its outer-most fiber, $\sigma_t$, is calculated as

$$\sigma_t = E_b \left( \epsilon_t - \frac{\delta}{l} \right),$$

where $E_b$ is the Young’s modulus of the beam material, δ is the crack opening displacement and $l$ is the undeformed length of the beam. $\epsilon_t$ is the elastic tensile strain at the outmost fiber, given by

![Figure 3.2. The cohesive softening model used. (a) Linear softening function of a cohesive crack. (b) Stress-strain response of a beam with a cohesive crack. Figure reproduced from Paavilainen et al. [24].](image-url)
\[ \varepsilon_t = \varepsilon_a + \varepsilon_l + \varepsilon_r, \quad (3.3) \]

where \(\varepsilon_a\), \(\varepsilon_l\), and \(\varepsilon_r\) are the strains due to axial, lateral and rotational deformations, respectively. These strain components are calculated by using the shape functions of the Euler-Bernoulli beam [70].

The failure criterion, \(\Lambda_c\), of the cohesive crack is

\[ \Lambda_c = \sigma_t - \sigma_c \left(1 - \frac{\delta}{\delta_c}\right), \quad (3.4) \]

where \(\sigma_c\) and \(\delta_c\) are the tensile strength and the critical crack opening displacement, respectively. When linear softening is assumed, \(\delta_c\) is related to the fracture energy, \(G_c\), by

\[ \delta_c = \frac{2G_c \hat{l}}{\sigma_c \hat{l}^*}, \quad (3.5) \]

where \(\hat{l}\) is the average length of a beam in the lattice. \(\hat{l}^*\) is a length scale calibration parameter used for scaling \(\delta_c\) for different particle sizes, which makes the softening model scale invariable (Publication II).

When \(\Lambda_c < 0\), the stress in the beam is permissible and the cohesive crack is closed (the beam is in elastic mode) or, the cohesive crack has formed and the beam is being unloaded or reloaded (the beam is in softening mode). The crack starts to open if \(\Lambda_c \geq 0\), which in turn reduces the stresses in the beam. In this instance, a new permissible tensile stress, \(\sigma_e\), can be calculated as

\[ \sigma_e = \sigma_c \left(1 - \frac{\delta}{\delta_c}\right), \quad (3.6) \]

where,

\[ \delta = \frac{\varepsilon_t - \sigma_c}{E_b} \frac{\sigma_c}{1 - \frac{\sigma_c}{E_b \delta_c}}. \quad (3.7) \]

The reaction forces and the moments at the ends of the beam are scaled down proportionally as the tensile stress reduces during the cohesive crack growth. This leads to material softening behaviour in the sphere-beam lattice. It is worth noting here that using the total strain of the beams (Eq. 3.3) enables modeling mixed mode failure, since a beam can break due to relative translations and rotations of its ends into any direction.

### 3.2.3 Modeling columnar ice

The topology of the particle-beam lattice in BPMs has an effect on the failure paths [71]. In the case of columnar ice, this is important as the failure...
mode and strength of ice depend on the direction of loading with respect to the columns. Preliminary simulations here with a random lattice resulted in numerous simultaneously occurring local failures with randomly oriented failure planes. Therefore, a particle-beam lattice replicating the columnar grain structure of S2 columnar ice used in the breakage experiments was utilized. The lattice was created by depositing a single layer of particles mimicking the horizontal cross section of ice, which was then replicated into vertical direction in a hexagonal closed packing to mimic the columnar grains. Fig. 3.3 presents a comparison of particle lattices in HiDEM with the grain structure of ice. With the lattice mimicking the columnar structure of ice, simulations yielded material failure which resembled that of ice.

![Figure 3.3](image.png)

**Figure 3.3.** Grain structure of ice compared to the planar random particle-beam lattice structure. (a) Horizontal thin section of natural ice showing a cross section of columnar grains. (b) Horizontal cross section of the particle-beam lattice. (c) Vertical thin section of natural ice showing the columnar grains. (d) Vertical cross section of the particle-beam lattice. On the right, the blue and grey particles have 3 mm and 4 mm diameters respectively. Figure reproduced from Publication II.
of the experiments. The three block systems simulated consisted of about $6 \times 10^5$ particles.

### 3.3 Breakage simulation results

All contact lengths, $c$, used in the experiments were modeled using HiDEM. Each $c$ was simulated using six different sphere packings to take into account potential scatter in the results. Fig. 3.4 summarizes the simulation results: failure patterns and $F - t$ curves, for $c = 50$, 100 and 150 mm are presented. The figure shows how the simulated ice blocks failed in a manner similar to that in the experiments where the predominant failure mode was shear failure due to bulk material failure. This type of shear failure can be distinguished from the figures by the relatively straight failure paths having a 20-30° angle with respect to the direction of the compressive force. Analysis was focused on the first contact to fail similar to the analysis of the breakage experiments.

The $F - t$ curves of Fig. 3.4 present force transmitted by two contacts, $F_1$ and $F_2$, and typical experimental $F$ records for the first contact to fail. The simulated $F - t$ curves agree well with the experimental curves for $c = 100$ and 150 mm, but less well for $c = 50$ mm. The experimental $F - t$ curve for $c = 50$ mm has an initial nonlinear portion due to ice blocks locally deforming at the contacts, which was not captured by the simulation. The significance of these deformations vanishes when $c > 50$ mm and then the $F - t$ curves from the simulations tend to follow those from the experiments.

Since the simulations yielded shear failure patterns alike to the breakage experiments, a similar analysis as in Section 2.4.1 was performed on the simulation results. Nominal shear and normal forces acting on the shear plane, $F_\tau$ and $F_n$, respectively, were calculated by using Eq. 2.1. The area of the shear plane, $A_\tau$, was determined by multiplying the failure plane length with the thickness of the ice block. Fig. 3.5 presents $F_\tau$ plotted against $A_\tau$ for the simulation data, experimental $F_\tau - A_\tau$ results already earlier presented in Fig. 2.7, and linear fits on both data sets. The simulation results agree well with the experiments, which confirms that the simulations were able to reproduce the breakage experiments. It is worth emphasizing again that the linear trend between $F_\tau$ and $A_\tau$ suggests that the shear failure observed was governed by the shear strength of the material, which in this case is related to the gradient of the fitted line.
Figure 3.4. Failure patterns and $F - t$ curves of three block breakage experiment simulations. The black line in the failure pattern figures are failure planes predicted by using Mohr-Coulomb failure criterion as explained in Section 3.4.1. Figure reproduced from Publication II.
3.4 Analysis

3.4.1 Mechanics of ice block breakage

Material failure due to bulk strength can be analysed using pre-failure stress distributions and a suitable failure criterion. Thus, the stress distribution within the simulated ice blocks was investigated by computing the Cauchy stress tensor $\sigma_i$ for each particle [72]. For particle $i$

$$\sigma_i = \frac{1}{2\Omega_i} \left( \frac{1}{2} \sum_j r_{ij} \otimes f_{ij} + f_{ij} \otimes r_{ij} \right), \quad (3.8)$$

where $\Omega_i$ is the volume of particle $i$, $f_{ij}$ is the force on $i$ due to the beam connecting it to particle $j$, $r_{ij}$ is the position vector of particle $j$ with respect to particle $i$, and $\otimes$ is the tensor product between the two vectors. The summation here is over all the particles connected to $i$ by beams. Fig. 3.6 shows the stress distribution of ice blocks in the $c = 100$ mm simulation earlier presented in Fig. 3.4b immediately before the failure. The figure shows how $\sigma_{zz}$ aligning with the loading direction is the dominant stress component. Moreover, there is a zone of high shear stress, $\sigma_{xz}$, near to the edges of contacts.

As discussed in Section 2.4.1, the shear failure observed here has the characteristics of Coulombic shear faults. Thus, the applicability of the Mohr-Coulomb failure criterion for predicting the shear failure in the simulation was tested. According to this criterion, the admissible stress state of a failure plane is
Figure 3.6. Stress distribution in the three block system: (a) $\sigma_{zz}$, (b) $\sigma_{xx}$, and (c) $\sigma_{xz}$. Contact length $c = 100$ mm. Figure reproduced from Publication II.

\[
\Lambda = |\tau| - (c_i + \mu_i \sigma_n) < 0, \quad (3.9)
\]

where $\tau$ and $\sigma_n$ are the nominal shear and normal stresses on the failure plane, while $c_i$ and $\mu_i$ are the internal cohesion and friction of the material, respectively. The direction of $\tau$ and $\sigma_n$ are same as the directions of $F_\tau$ and $F_n$, respectively, in Fig. 2.6.

It was observed in the experiments that the shear planes passed through the edges of contacts (Publication I Fig. 9). Thus, only the orientation of the failure plane had to be determined to confirm the applicability of the Mohr-Coulomb failure criterion. The orientation of the shear plane should be the plane with the maximum value for $\Lambda$. The shear and normal stresses, $\tau$ and $\sigma_n$, respectively, acting on a plane having an angle $\theta$ (Fig. 3.7a) can be solved as
\[ \sigma_n = \frac{1}{2}(\sigma_{xx} + \sigma_{zz}) - \frac{1}{2}(\sigma_{xx} - \sigma_{zz})\cos2\theta - \sigma_{xz}\sin2\theta \]  

(3.10)

and

\[ \tau = -\frac{1}{2}(\sigma_{xx} - \sigma_{zz})\sin2\theta + \sigma_{xz}\cos2\theta. \]  

(3.11)

As \( c_i \) in Eq. 3.9 must be positive due to being a material parameter, the maximum of \( \Lambda \) occurs when \( |\tau| - \mu_i\sigma_n \) is at its maximum. Thus, \( \Lambda \) can be calculated partially \( (|\tau| - \mu_i\sigma_n) \) for planes having arbitrary values of \( \theta \). Fig. 3.7b presents \( \tau, \sigma_n \) and \( |\tau| - \mu_i\sigma_n \) deduced from the stress distribution of Fig. 3.6 as a function of \( \theta \). Here, \( \mu_i = 0.75 \) and the average value of each stress component acting on a plane with given \( \theta \) was used. As shown in Fig. 3.7b, the maximum of \( |\tau| - \mu_i\sigma_n \) occurs at \( \theta = 26^\circ \), thus, this would be the failure plane angle according to the Mohr-Coulomb failure criterion. This matches with the simulated failure plane of Fig. 3.4b. Moreover, the

![Arbitrary plane](image)

\( \theta \)

(a)

![Graph](image)

max at \( \theta = 26^\circ \)

(b)

**Figure 3.7.** (a) Plausible shear failure planes for arbitrary \( \theta \) values. (b) The Mohr-Coulomb failure criterion applied to the \( c = 100 \text{ mm} \) simulation of Fig. 3.6. The figure shows the stress components of the Mohr-Coulomb failure criterion vs. \( \theta \). Figure reproduced from Publication II.
maximum value of $|\tau| - \mu_i \sigma_n$, 280 kPa, corresponds to $c_i$. This is because failure will occur when $\Lambda = 0$, which means $c_i = |\tau| - \mu_i \sigma_n$ in Eq. 3.9.

Similar failure plane predictions were conducted for all simulation results. The black lines of Fig. 3.4 present the failure planes predicted using the Mohr-Coulomb theory. They match the simulated failure planes well. Fig. 3.8a shows the $\theta$ values obtained from the Mohr-Coulomb theory compared with $\theta$ values from the simulations for all contact lengths, $c$. The figure presents the mean values and standard deviations of $\theta$ from the simulations and the experiments. $\theta$ calculated from the Mohr-Coulomb theory matches well with both the simulation and experimental results. In addition, Fig. 3.8b presents $c_i$ calculated from the Mohr-Coulomb theory.

**Figure 3.8.** (a) Failure plane angles, $\theta$, calculated using the Mohr-Coulomb failure criterion and from the simulations. (b) Values of internal cohesion, $c_i$, calculated using the Mohr-Coulomb failure criterion. Figure reproduced from Publication II.
plotted against $c$. The figure also shows $c_i$ calculated from simulations of an ice block failing under pure shear loading. According to the figure, $c_i$ can be considered constant for all values of $c$ used in the experiments, which is a physically sound result, since $c_i$ is a material parameter.

### 3.4.2 Discussion on the simulation results

The BPM simulation results confirmed that the ice block failure in the breakage experiments (Chapter 2) occurred due to shear failure governed by the bulk strength of the material. Moreover, the analysis of the simulations indicates that the Mohr-Coulomb failure criterion can be used to predict the shear failure of ice blocks. It should be emphasized that the Mohr-Coulomb failure criterion was not used as a failure criterion in HiDEM, but instead, the stress distribution at the onset of block failure agrees with the criterion. In other words, a Mohr-Coulomb failure criterion-based shear failure can be used in DEM simulations to model the breakage of ice blocks under compressive ice-to-ice contacts.

In addition, the results and the analysis of the breakage experiment simulations show that HiDEM is capable of modeling quasi-brittle failure of ice under compressive ice-to-ice contacts. Thus, BPMs can be a powerful tool in numerical ice mechanics research, since modeling ice failure is challenging using other simulation techniques, such as the finite element method. This is partly due to lack of accurate material models and partly due the challenges in modeling fragmentation using simulation techniques based on a continuum approach. Moreover, BPM was used here in the context of analysing laboratory scale experiments to investigate the underlying mechanics of material failure. This demonstrates the advantages of using experiments and simulations in combination to study the complex material behaviour of ice.
4. Simplified breakage model for DEM

4.1 Overview of the breakage model

In the final part of the research presented in this thesis, a simplified block breakage model for DEM simulations was developed based on the findings of the block breakage experiments and high resolution simulations reported in the previous chapters. This research lead to Publication III. According to the simplified breakage model, the ice blocks break by shear failure governed by the Mohr-Coulomb failure criterion. The breakage model was integrated into an existing in-house DEM tool of Aalto University, developed by the ice mechanics research group [31], and direct shear box experiments on ice rubble were simulated to investigate the effect of block breakage on ice rubble behaviour. The key highlight of the breakage model is that it is based on detailed experimental observations and high resolution modeling of ice block breakage under compressive ice-to-ice contacts. To the knowledge of the author, this is the first breakage model based on experimental evidence aimed specifically at DEM simulations of ice rubble. It is worth emphasizing that breakage models that are based on direct experimental observations are rare for granular materials in general. In addition, it is also worth noting here that there are existing analytical models on ice edge failure based on shear and the Mohr-Coulomb failure criterion [73, 74], but not for other ice features than an ice sheet.

4.2 The block breakage model

The developed breakage model is based on checking each discrete element on every time step of the simulation for potential failure planes meeting the Mohr-Coulomb failure criterion. This is done using so-called base planes. Fig. 4.1a presents a polyhedral ice block, the base plane of which aligns with the face that has the largest surface area. The actual plane of
shear failure is assumed to be perpendicular to the base plane. In the case of ice with columnar grains, an intuitive choice for the base plane could be a plane across the columnar grains, since the shear strength of ice is lowest in the direction perpendicular to such a plane.

On each time step, the contact forces acting on the block and the block geometry are projected onto the base plane as shown in Fig. 4.1a. This transforms the three-dimensional discrete element into a two-dimensional polygon with contact forces acting on its edges as presented in Fig. 4.1b. Then, the quasi-static force equilibrium of the polygon is studied to find the nominal shear and normal forces, and the related nominal stresses, acting along potential failure planes as illustrated in Fig. 4.1b. This approach is similar to what was used in Section 3.4.1 to find the failure plane of ice blocks in the BPM simulations of the breakage experiments. The contact forces are projected to the base plane by

\[ f^*_i = f_i - (f_i \cdot e_b) e_b, \]  

where \( f^*_i \) is the projection of contact force \( f_i \), and \( e_b \) is the unit vector perpendicular to the base plane. The points of application of all contact forces are transformed similarly onto the base plane.

The breakage model assumes that each failure plane passes through a point of application of one of the contact forces. Thus, for each contact point, the polygon boundary is discretized into a number of mesh points. Then, planes with different orientations are tested if they satisfy the failure criterion. Fig. 4.1b shows a mesh created for checking if \( f^*_1 \) leads to breakage. In the figure, a potential failure plane spanning from the point of application of \( f^*_1 \) and mesh point \( K \) is tested.
Each potential failure plane is tested by analysing the quasi-static equilibrium to solve the nominal shear and normal forces, $f_s^*$ and $f_n^*$, respectively, acting on the plane. These can be calculated as

$$\begin{align*}
f_s^* &= \sum_{i=1}^{p} f_i^* \cdot e_s^* \\
f_n^* &= \sum_{i=1}^{p} f_i^* \cdot e_n^*,
\end{align*}$$

(4.2)

where $p$ is the number of contact forces acting on the fragment and $e_s^*$ and $e_n^*$ are the tangential and normal unit vectors of a potential failure plane, respectively. Then, the nominal shear and normal stresses, $\tau$ and $\sigma_n$, respectively, are

$$\begin{align*}
\tau &= \frac{f_s^*}{A^*} \\
\sigma_n &= -\text{sign}(f_n^*) \left| \frac{f_n^*}{A^*} \right|
\end{align*}$$

(4.3)

where $A^*$ is the area of the potential failure plane calculated by multiplying the length of the line spanning between the mesh point and the point of application of the contact force by the nominal thickness of the block on the failure surface.

Nominal stresses $\tau$ and $\sigma_n$ can then be used in the Mohr-Coulomb failure criterion of Eq. 3.9 to test if a failure would occur along the plane being tested. If the criterion is met, the element is divided into two fragments along the failure plane found. In the case of there exists more than one plane fulfilling the criterion, plane with the minimum $\Lambda$ is chosen as the failure plane. The breakage model can then be applied on new fragments in the following time steps to simulate further fragmentation. It is worth mentioning here that the simulation time step has to be sufficiently small so that $\tau$ and $\sigma_n$ would not develop into very large values exceeding $\Lambda$. A pseudo code of the breakage model is presented in Algorithm 1.

### 4.3 Simulation setup

#### 4.3.1 Discrete element tool

The breakage model was integrated into the three-dimensional, parallelized DEM code of the ice mechanics research group of Aalto University. The code is described in detail by Polojärvi [31]. In brief, the code uses polyhedrons to describe arbitrarily shaped ice blocks interacting through pairwise contact forces. The simulations are explicit with the central difference method used for time stepping. A soft contact force model is used, and thus, the contact force between two interacting blocks is calculated based on a small overlap between them. The application point of the contact force is at the centroid of the overlap volume. The contact force between two interacting blocks, $f$, is given by
Algorithm 1 block breakage algorithm

1: Choose a base plane for the breakage model
2: Project ice block boundary onto the base plane
3: Project contact forces and their points of application onto the base plane: $f^*$ contact force resolved onto the base plane (Eq. 4.1)
4: $p$: No. of contact forces acting on the ice block
5: for $i = 1$ to $p$ do
6: Discretize the projected ice block boundary of the element
7: $k$: No. of mesh points on the projected ice block boundary
8: for $j = 1$ to $k$ do
9: Define a plausible failure plane between the point of application of $f^*_i$ and the mesh point $K_j$
10: Find the nominal shear and normal forces, $f^*_s$ and $f^*_n$, acting on the failure plane (Eq. 4.2)
11: Find the nominal shear and normal stresses, $\tau$ and $\sigma_n$, acting on the failure plane (Eq. 4.3)
12: Apply the Mohr-Coulomb failure criterion and calculate the critical value for the failure criterion, $\Lambda$ (Eq. 3.9)
13: end for
14: end for
15: if max($\Lambda$) $\geq 0$ then
16: Failure will occur along the plane with max($\Lambda$)
17: Split the ice block along the failure plane
18: Define two new ice blocks in the DEM simulation for the fragments
19: end if

\[ f = f_n + f_t, \quad (4.4) \]

where $f_n$ is the normal component of the contact force calculated by using an elastic-viscous-plastic contact force model, while $f_t$ is the tangential component of the contact force due to the tangential compliance and the friction between the blocks [66]. The elastic and viscous components of $f_n$ are calculated by using the gradient of overlap volume and its rate of change, respectively [75, 76]. The plastic component of $f_n$ is based on the area of contact and, importantly, describes local yielding or crushing of the material of the block at the contact.

4.3.2 Direct shear box setup

Fig. 4.2a illustrates the set up of the direct shear box experiments simulated in Publication III. The actual experimental work was performed by Pustogvar et al. [6] with the setup shown in Fig. 4.2b. The ice rubble specimen modeled was 600 mm $\times$ 400 mm $\times$ 40 mm (length $\times$ width $\times$ height).
thickness). Two rubble types were used, one consisting of ice blocks of 30 mm × 20 mm × 40 mm and another consisting of ice blocks of 60 mm × 40 mm × 40 mm, below referred to as small and large blocks, respectively. The thickness of the ice blocks was equal to that of the shear box, and thus, the experiments could be considered pseudo two-dimensional. In the simulations, the shear box was filled with the ice blocks by gravity deposition and then by applying a vibrating motion to the box. The initial porosity of the ice rubble in the simulations and the experiments was about the same. A constant confining pressure, \( P \), was applied on the rubble via the shear box cover, which was also allowed to rotate. Simulations were ran with \( P \) varying in the range 5.75...22.06 kPa, while the experiments were only performed with \( P = 5.75 \) and 11.03 kPa. The velocity of the upper half of the shear box, \( d \), was 0.02 ms\(^{-1}\).

The shear force exerted on the ice rubble by the top part of the shear box, \( S(d) \), was recorded as a function of its displacement, \( d \) (Fig. 4.2a). Then, the instantaneous shear stress within the ice rubble specimen, \( \tau_{R}(d) \), could be defined as

\[
\tau_{R}(d) = \frac{S(d)}{A_{R}(d)},
\]

where \( A_{R}(d) \) is the area of the shear plane. Using \( \tau_{R}(d) \), the mean shear resistance and maximum shear strength of the rubble, \( \tau_{R} \) and \( \tau_{R}^{m} \), respectively, could be calculated. \( \tau_{R} \) was defined here as the average of \( \tau_{R}(d) \) for a given interval of \( d \).
4.4 Simulation Results

4.4.1 Shear force records

Fig. 4.3 shows typical shear force-displacement, $S - d$, records from numerical direct shear box experiments with and without the breakage model. The records are from simulations with small blocks and large blocks as stated in the figure. The confining pressure, $P$, was 5.75 kPa in both simulations. The internal cohesion of the ice blocks, $c_i$, was 250 kPa in simulations with the breakage model. The figure also presents $S - d$ records from the corresponding experiments of Pustogvar et al. [6]. Experimental records reach only to $d \approx 0.4$ m due to technical difficulties with the setup [6].

Including breakage improved the simulation results as demonstrated by the $S - d$ records in Fig. 4.3. As the figure shows, the $S - d$ records from the simulations and the experiments show similar general features. They include several distinct load peaks, below referred to as peak load events. However, the magnitude of the peak loads in the simulations without the breakage model are significantly higher than in the experiments. On the other-hand, $S - d$ records from the simulations with the breakage model show similar peak load levels than the experimental records. It is also interesting to notice that the $S - d$ records from the simulations with the breakage model initially follow the $S - d$ records from the simulations without the breakage model. The two records deviate from each other as $S$ builds up towards the first peak load event. The magnitude of peak loads are in general lower in simulations with the breakage model. This means...
that breakage has a clear effect on the mechanisms that limit peak load values.

Fig. 4.4 presents the mean and maximum of the shear force, $S$ and $S^m$, respectively, from the simulations compared with those from the experiments with confining pressures $P = 5.75$ and $11.03$ kPa. The figure shows data from five repeated simulations and three repeated experiments with different initial configurations of ice blocks. $S$ and $S^m$ were here defined for each simulation and experiment by taking the average and the maximum of $S(d)$ for the displacement interval $d = 0...0.3$ m. This interval was chosen since the block breakage was more prominent after the first quarter of the experiment [6, 7]. As shown in Fig. 4.4, $S$ and $S^m$ results from the simulations with the breakage model agree better with the experimental results than the simulations without the breakage model. In the simulations with the breakage model, simulated and experimental results for $S$ and $S^m$ differed on average by 11% and 14%, respectively. Simulations without the breakage model led to 23% and 41% errors for $S$ and $S^m$, respectively. Thus, the accuracy of DEM simulations increases when ice blocks in the simulation are allowed to break.

### 4.4.2 Block strength and rubble strength

The results from the previous section indicate that block breakage affects the shear force measured in shear box experiments, which in turn affects

![Figure 4.4](image-url)  
**Figure 4.4.** Mean shear force, $S$, and maximum shear force, $S^m$, in simulations and experiments in the case of small and large blocks. Values are for shear box displacement of 0.0 to 0.3 m. Figure reproduced from Publication III.
the measured shear stress for the ice rubble, $\tau_R$. Fig. 4.5 demonstrates the change in the mean shear resistance and the maximum shear strength of ice rubble, $\tau_R$ and $\tau^m_R$, respectively, against the internal cohesion of ice blocks, $c_i$. The results are from simulations with small and large blocks and confining pressures of $P = 5.75$ and 11.03 kPa. Again, each case was simulated using five different initial rubble configurations to take into account scatter in the results, and thus, the figure also presents means and standard deviations for the results. Here $\tau_R$ results are calculated for $d = 0...0.3$ m to reduce the effect of the shear box walls on the results [7]. The figure also presents an asymptotic fit for each data set. Curve fitting was done using the MATLAB curve fitting toolbox with the function, $y = a' - (a' - b')\exp^{-x/c'}$, where $a'$, $b'$ and $c'$ are the parameters. For each data set, curves were forced to go through the mean value for the simulations without the breakage model, indicated by $c_i = \infty$ kPa in the figure.

As shown in Fig. 4.5, both $\tau_R$ and $\tau^m_R$ increase with $c_i$ towards the respective $\tau_R$ and $\tau^m_R$ from the simulations without the breakage model. This means that the effect of block breakage on the shear strength of ice rubble decreases rapidly with increasing strength of the ice blocks. This behaviour was consistent with all $P$ values tested. The effect of breakage on $\tau_R$ seems to disappear with $c_i > 400$ kPa, as the $\tau_R$ values are then about 90% of those from the simulations without the breakage model. However, for $\tau^m_R$, breakage seems to effect still at $c_i = 500$ kPa. Moreover, the effect of $c_i$ on $\tau^m_R$ is more notable with higher $P$.

**Figure 4.5.** Effect of block strength on the shear strength of ice rubble for two different confining pressures, $P$: results of mean shear resistance and maximum shear strength, $\tau_R$ and $\tau^m_R$, are presented. Block strength was varied by changing the value of internal cohesion, $c_i$ (Eq. 3.9). Values from the simulations without the breakage model ($c_i = \infty$ kPa) are also presented. Figure reproduced from Publication III.
Simplified breakage model for DEM

The $\tau_R$ and $\tau_R^m$ results from the simulations with the breakage model can be normalized by the results from the corresponding simulations without the breakage model. Fig. 4.6 combines the normalized data from the simulations with different values of $P$. Data from the simulations with different values of $P$ could be combined here, as normalized $\tau_R$ and $\tau_R^m$ yielded by the simulations with different values of $P$ behaved very similarly. This was confirmed by performing additional simulations with $P = 16.45$ and 22.06 kPa. In other words, the effect of $P$ on breakage is independent of $c_i$. The figure also presents an asymptotic fit for each data set. As shown by the figure, normalized $\tau_R$ and $\tau_R^m$ results for small and large blocks nearly overlap, which means that the effect of breakage on the shear resistance and the instantaneous maximum shear strength of ice rubble is also independent of block size.

4.5 Analysis

4.5.1 Effect of breakage on peak load events

As demonstrated in Section 4.4.1, the magnitudes of the peak loads are significantly lower in simulations with the breakage model. Peak load events occur due to formation and subsequent buckling failure of force chains \[7\]. However, when breakage is allowed, ice blocks constituting force chains can break before the load transmitted by the force chain builds up to the force required for its buckling. This behaviour is demonstrated in Fig. 4.7 showing two sequences of three images from the first peak load event of the simulations with small blocks with the $S-d$ record presented in Fig. 4.3. The left and the right columns of Fig. 4.7 present simulations without and with the breakage model, respectively. Ice blocks in the figure are colored according to the first principal stress of their particle stress

![Figure 4.6](image-url)  
**Figure 4.6.** Normalized ice rubble shear strength values $\tau_R$ and $\tau_R^m$ plotted against internal cohesion, $c_i$, of the blocks. Here, values were normalized by the mean $\tau_R$ and mean $\tau_R^m$ of the corresponding simulations without the breakage model. The figure further shows curves fitted to the mean values of normalized results. Figure reproduced from Publication III.
Simplified breakage model for DEM tensor [77, 78]. This is to demonstrate the main compressive force and its direction within the ice blocks. The principal stress values are normalized by the maximum principal stress value at the peak load event in the simulation without the breakage model and only shown for the blocks whose normalized principal stress values are higher than 0.1.

Fig. 4.7a shows that a force chain consisting of highly compressed ice blocks starts to form in both simulations at the beginning of the peak load event. As the force increases, several ice blocks within the force chain break.

**Figure 4.7.** Snapshots from the simulations, without the breakage model (left column) and with the breakage model (right column) with $S - d$ records in Fig. 4.3. Snapshots are from the first peak load event. First (a), in both simulations, a force chain forms near the shear plane. Then, (b) blocks within the chain break when breakage is allowed. After this, (c) blocks within force chain get further compressed when breakage is not allowed, while the force chain has disappeared and new fragments have formed in the simulation with the breakage model.
in the simulation with the breakage model (right column of Fig. 4.7b) and then the force chain collapses due to the breakage of blocks (right column of Fig. 4.7c). However, in the simulation without the breakage model, blocks in the force chain get compressed further (left column of Fig. 4.7c) until the eventual buckling of the force chain. This confirms that breakage is one of the force chain failure modes and explains why the magnitude of the first peak load in the simulation with the breakage model is significantly lower than that in the simulation without the breakage model in Fig. 4.3. As described above, the $S-d$ records of the two simulations deviated from each other after the first peak load event. This is due to the new fragments in the simulation with the breakage model. Similar behaviour was observed consistently in the simulations with the breakage model. However, some force chains in the simulations with the breakage model also collapsed due to buckling, and thus, breakage was not the only failure mode in them.

### 4.5.2 Discussion on the simulation results

The simulation results of this chapter demonstrate that a Mohr-Coulomb failure criterion based breakage model can be used successfully to model the shear failure of ice blocks in DEM simulations. Importantly, the results also indicate that the breakage model may improve the overall accuracy of DEM simulations in terms of ice rubble behaviour and force transmitted through rubble. In DEM simulations of ice, local failure of ice blocks at contacts is usually modeled by setting a plastic limit for the contact force \[3, 24, 66–68\]. The results of the present work, including the experiments, the high-resolution modeling, and the DEM simulations with breakage suggest that such a model is not sufficient, but that breakage has to be accounted as well. This is due to the force transmission capacity of the force chains being often limited by the breakage of ice blocks.

The shear box simulation results show that the force chain collapse due to block breakage often occurs at lower force levels compared to force chain buckling. Therefore, when blocks are allowed to break, the force carrying capacity of the ice rubble reduces, which in turn decreases the shear resistance, $\tau_R$, and the maximum instantaneous shear strength, $\tau_{\text{m}R}$, of ice rubble. As Fig. 4.6 demonstrates, normalized $\tau_R$ and $\tau_{\text{m}R}$ showed about 20% reduction for $c_i = 250$ kPa, chosen following the experiments of Publication I. This means that the DEM simulations of ice-structure interaction processes without a block breakage model \[3, 24, 68\] may overestimate peak ice load magnitudes, since force chain buckling governs the maximum ice loads on the structure in such simulations. Thus, ice load estimates from such simulations have to be used cautiously when interpreting limit loads on structures. In addition, the normalized $\tau_R$ and $\tau_{\text{m}R}$ results of Fig. 4.6 showed that the effect of breakage on the shear strength of ice rubble is independent of block size. This means that using
smaller elements in DEM simulations in order to mitigate the effects of force chains is not effective, but that block breakage has to be considered. This is because of the distinct role of block breakage as a load limiting mechanism in deforming ice rubble.

It is worth mentioning here that although the developed breakage model is based on experimental observations on ice, it can be applied to other granular materials given that the grain failure in them is due to the strength of the material rather than instantaneous crack propagation caused by stress concentrations. This is because the Mohr-Coulomb failure criterion utilized in the breakage model is known to be applicable to other materials such as rocks and concrete [79].
5. Concluding remarks

In the present thesis, ice block breakage under compressive ice-to-ice contacts was studied. Novel ice block breakage experiments were performed to investigate the failure modes of ice blocks. High resolution bonded particle method (BPM) simulations were then performed to analyse the underlying failure mechanisms of ice blocks. A simplified ice block breakage model for DEM simulations was developed based on the findings of the experiments and the high resolution modeling. Finally, the newly developed ice block breakage model was integrated into the Aalto University ice mechanics research group’s in-house DEM tool and the effect of ice block breakage on the mechanical behaviour of ice rubble was investigated by simulating direct shear box experiments on ice rubble. The main original features and findings of this thesis are presented below.

Original features:

1. The ice block breakage experiments were performed using a unique set-up allowing investigating and gathering data on the breakage of laboratory grown floating ice blocks under compressive ice-to-ice contacts (Publication I).

2. The columnar grained, saline, floating ice specimens used in the experiments resembled sea ice in nature with a naturally occurring temperature gradient (Publication I), contrary to more regular testing on dry and cold ice specimens.

3. High resolution BPM simulations of the breakage experiments utilized a particle lattice mimicking the columnar grain structure of naturally grown ice to take into account the anisotropic material properties of ice (Publication II).

4. A cohesive softening model for beam failure was integrated into the BPM simulation tool to dissipate the elastic energy stored in the material
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and, further, to achieve quasi-brittle failure (Publication II).

5. The simplified ice block breakage model developed in this thesis was based on detailed experimental observations, which is rare for any type of granular material (Publication III).

Main findings:

1. The results of the ice block breakage experiments showed that the failure of ice blocks under quasi-static compressive ice-to-ice contact forces primarily occurs due to the shear failure of the blocks. The shear failure resembles ‘Coulombic shear faults’ and is governed by the shear strength of ice (Publication I).

2. The experiments and the BPM simulations together showed that the common approach of modeling contact failure of ice blocks in DEM simulations by using a contact force limit based on the crushing or compressive strength of ice is not always accurate, since the load transmitted through contact is limited by shear failure when the blocks are under quasi-static compressive loading (Publication I and Publication II).

3. BPM simulations are able to successfully reproduce the quasi-brittle shear failure of ice given that the energy dissipation at the failure and the columnar grain structure of ice are mimicked in the model (Publication II).

4. BPM simulations further revealed that the Mohr-Coulomb failure criterion can be used to model the failure of ice blocks under compressive ice-to-ice contacts in a simple yet reliable manner (Publication II).

5. The block breakage model developed in the thesis captures the failure of ice blocks under compressive ice-to-ice contacts in DEM simulations of ice rubble. (Publication III).

6. Simulated direct shear box experiments showed that ice block breakage is one of the failure modes of force chains within ice rubble. Thus, accounting for block breakage improved the overall accuracy of DEM simulations of direct shear box experiments of ice rubble (Publication III).

The present experimental and numerical efforts on investigating ice block breakage under compressive ice-to-ice contacts were limited to quasi-static loading conditions. Therefore, the developed breakage model has to be applied cautiously in DEM simulations of dynamic ice failure processes.
Ice exhibits loading rate dependent failure characteristics, thus the failure modes at higher loading rates could be different than the shear failure observed in the ice block breakage experiments. Therefore, further experimental and numerical research is required to gather more data on ice block failure in dynamic conditions. Moreover, other failure modes such as crushing and splitting were also observed in the ice block breakage experiments in about 25% of the cases. Therefore, further research work is needed to understand the underlying mechanics of such failures and to model them in DEM simulations. Such work could be performed using the techniques presented in this thesis.

The breakage model developed in this thesis relies on a base plane to check if the Mohr-Coulomb failure criterion is fulfilled. Moreover, the failure planes are always assumed to be perpendicular to the base plane. Therefore, implementing the model on discrete elements with non-prismatic geometries could be challenging. In such cases, the use of other alternative failure techniques, such as the boundary element method, could be more suitable even if computational burden might limit their use. Nevertheless, sea ice in nature often exhibits prismatic geometries as ice grows inwards into the water body from the free surface. Thus, the model is applicable in most cases of ice rubble simulations. Moreover, for highly elongated discrete elements, the dominant failure mode could be bending failure instead of shear. Therefore, the block breakage model could be used in conjunction with a bending failure model in DEM simulations containing highly elongated elements. Further, the simplified block breakage model can be applied to other granular materials than ice, given that the failure of the grains is governed by the strength of material related failure rather than instantaneous crack propagation. The failure criterion used in the model (Mohr-Coulomb criterion) can be replaced with other strength based failure criteria to suit the material being modeled, without having to modify the other parts of the breakage model. It is also worth highlighting here that the systematic research approach of combining experiments with DEM simulations utilized in the present thesis can be expanded to investigate the failure mechanisms in other research areas such as geosciences and bulk material handling.

In the present thesis, the effect of block breakage on ice rubble behaviour was investigated using pseudo two-dimensional direct shear box experiments. Thus, future research is needed to study the effect of breakage in a three-dimensional shear box setup. The significance of force chains on rubble behaviour might diminish in a three-dimensional setup due to the additional degrees of freedom of elements. Moreover, it is also worthwhile to study ice-structure interaction problems, for example, an intact ice sheet interacting with an inclined structure, using a DEM tool with the block breakage model. DEM has to be combined with the finite element method (FEM) to simulate such problems due to the need to model a continuous in-
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tact ice sheet in addition to ice rubble. Therefore, integrating the breakage model into a FEM-DEM tool is a natural extension of the present work. Nevertheless, this is a challenging task. Often intact ice is described by bonding several discrete elements together with, for example, deformable beam elements. In this case, modeling breakage occurring through several discrete elements bonded together may be required.
References


References


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References


