

WEB-HIPRE: GLOBAL DECISION SUPPORT BY VALUE TREE AND AHP ANALYSIS

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ABSTRACT

Web-HIPRE is a Java applet for multiple criteria decision analysis. Being located on the WWW, it can be accessed from everywhere in the world. This has opened up a completely new era and dimension in decision support. Web-HIPRE provides a common platform for individual and group decision making. The models can be processed at the same or at different times and the results can be easily shared and combined. There is a possibility to define links to other WWW addresses. These links can refer to any other kind of information such as graphics, sound or video describing the criteria or alternatives. This can improve the quality of decision support dramatically. The most common weighting methods including AHP, SMART, SWING, SMARTER and value functions are supported. Web-HIPRE is located on <http://www.hipre.hut.fi/>

RÉSUMÉ

Web-HIPRE est une applet Java pour l'analyse de décision multicritère. Située sur le web, elle est donc accessible de n'importe où sur la terre, ouvrant ainsi une ère et une dimension complètement nouvelles dans l'aide à la décision. *Web-HIPRE* offre une plate-forme commune à la décision individuelle et de groupe, où les modèles peuvent être traités simultanément ou en différé. Les résultats peuvent être aisément partagés et combinés. La possibilité de définir des liens à d'autres sites internet est aussi offerte. Ceux-ci peuvent guider vers des types d'information de tous genres, tels que des graphiques, des sons ou des vidéos décrivant les critères ou les alternatives. La qualité de l'aide à la décision peut ainsi drastiquement augmenter. Les méthodes de pondération les plus communes telles que l'AHP, SMART, SWING, SMARTER et les fonction valeur sont supportées par *Web-HIPRE* (l'adresse du site est www.hipre.hut.fi).

1. INTRODUCTION

Web-HIPRE (Hierarchical Preference analysis on the World Wide Web) (Hämäläinen and Mustajoki, 1998) is a WWW software for multicriteria decision analysis based on the well-known decision support software HIPRE 3+ (Hämäläinen and Lauri, 1995). It provides an implementation of multiattribute value theory (MAVT) (Keeney and Raiffa, 1976) and the analytic hierarchy process (AHP) (Saaty, 1980, 1994; Salo and Hämäläinen 1997) to support the different phases of decision analysis, i.e. structuring of the problem (see e.g. French et al., 1998), prioritization and analyzing the results. Individual models can be integrated into a group model via the Internet. Figure 1 shows how Web-HIPRE can be used in different areas of decision support. It lies in the intersection of *decision making*, *group collaboration*, and *computer support*.

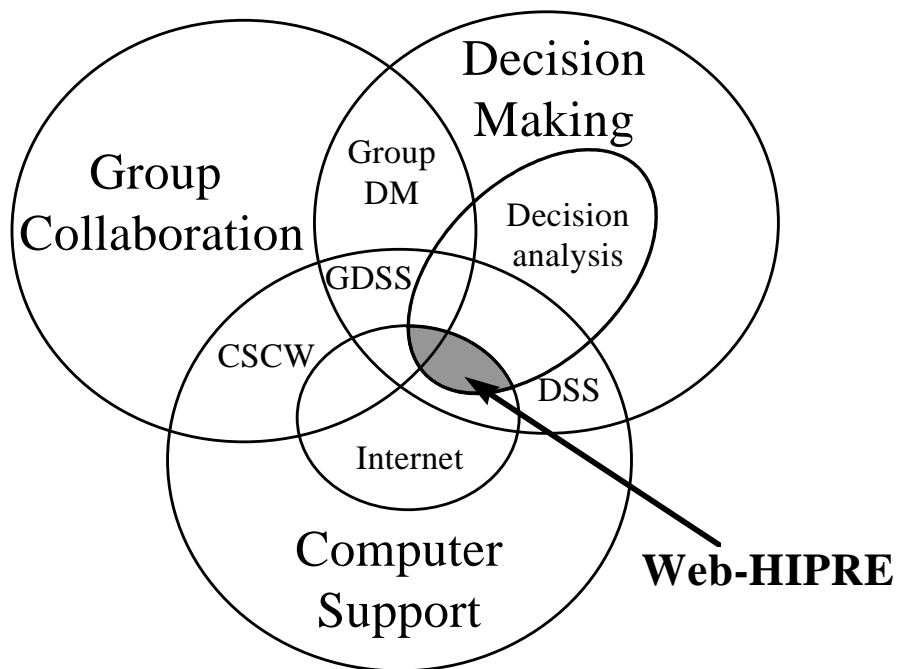


Figure 1: Areas related to Web-HIPRE

Web-HIPRE is the first globally available general purpose decision analytical software on the WWW. For a review of other decision analytical software available, see e.g. Buede (1998) or the listing on the web page of the Decision Analysis Society. The research group of the Systems Analysis Laboratory has also developed other web based decision support systems, such as Joint Gains (Kettunen and Hämäläinen, 1999) for negotiation support and Opinions-Online (Hämäläinen and Kalenius, 1999) for voting, surveys and group collaboration. So far, there are only very few other related software. We can only mention the INSPIRE system for negotiation support (Kersten, 1996) and the NIMBUS for multiobjective optimization (Miettinen and Mäkelä, 1998). The number of implementations is, however, likely to soon increase very rapidly.

The Internet provides new possibilities to support decision making, and especially group decision making. From one perspective, the Internet is a channel for electronic communication (see e.g. Bhargava and Krishnan, 1998). It fulfills the requirements of the communication network in the group support systems concept (Nunamaker, 1997). It is a global network to share information and it provides easy access to external data in multimedia formats at any time and at any place. Interactive applications can be implemented, for example, with the Java programming language.

From another perspective, the Internet can be seen as a digital information library (see e.g. Bhargava and Krishnan, 1998). Information can be published for a wide audience at high speed and low cost. For the users, the Internet is thus a distributed hypermedia repository available all the time.

Web-HIPRE makes use of both the electronic communication possibilities and the easy retrieval of existing information from the Internet. In a group process, members can give preferences in different locations and they can combine and share the results easily. The distributed working mode has already been successfully tested in a student exercise (Kersten et al., 1999). External information on related WWW pages can be used by direct links, which are loaded by clicking the elements of the hierarchy. These can contain any kind of multimedia material such as videos, pictures, sound, virtual tours and so on, which can help the decision maker to more accurately evaluate his/her attributes and preferences. Also, we can create libraries of decision models and preferences describing stakeholders opinions in public policy issues. For example, Figure 2 presents the WWW page for a problem of selecting a cellular phone. This page explains the backgrounds of the problem and introduces the phone alternatives. The alternative elements in the hierarchy (Figure 3) have also direct links to the home pages of the cellular phones.



Figure 2: WWW page for the cellular phone selection problem

Web-HIPRE can be accessed from any location having a Java-enabled WWW browser (e.g. Netscape or Internet Explorer). The applet is loaded in the browser's local memory and when the browser is closed, no files remain on the user's computer. Models can be saved on a server to a public or personal password protected directory. The use of HIPRE 3+ models is supported.

Web-HIPRE can also be installed in an independent computer and run locally or via a local area network (LAN). A local server might be useful, for example, when Internet connections are slow. Also, although the models in private directories are password protected, some organizations might still want to have increased security. It is easy to set up the system even on a portable computer. Group collaboration can then be easily supported also in locations having no access to the Internet. Today, there exists light systems for portable LANs, which gives us the possibility to create a portable group decision support facility.

2. PRIORITIZATION METHODS

The prioritization methods available in Web-HIPRE are based on MAVT and AHP. In both methods the decision problem is visually structured into a value tree of objectives/attributes (the MAVT terminology), or a hierarchy of criteria (the AHP terminology) (see Figure 3). In the sequel, we use both of these terms without a difference in the meaning.

In this description, we focus on properties of Web-HIPRE which are not found in other decision analysis software. The possibilities provided by the Internet come first, but on the methodological side, we also have unique features and solutions. These include the possibility to combine the methods in an individual model and the flexible use of scales in the AHP as well as the important group modeling facilities.

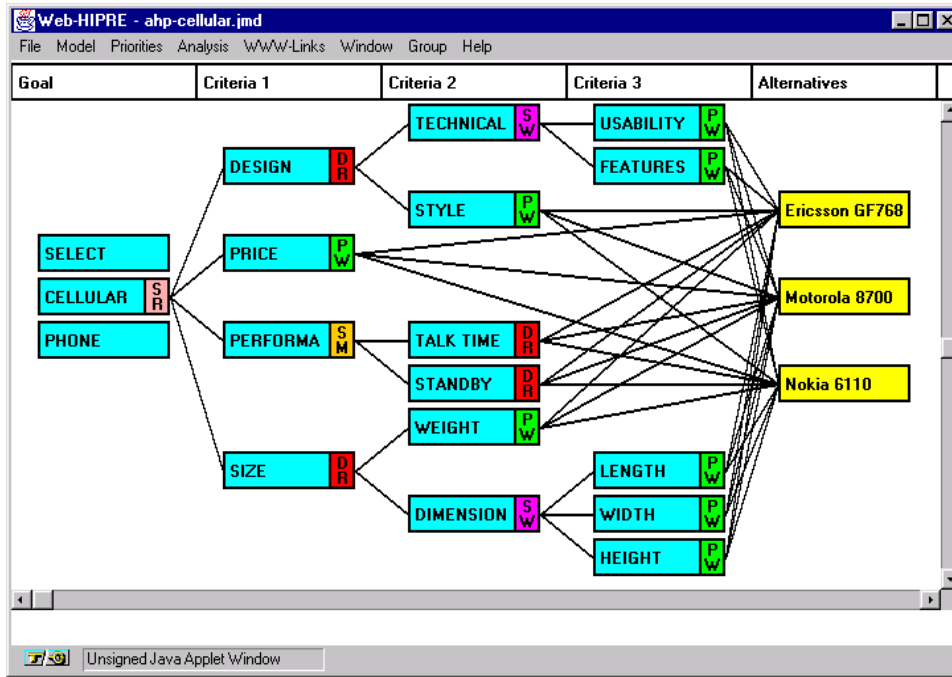


Figure 3: The window for structuring the value tree. Different weighting/rating methods can be used in the local subtrees, and the selected methods are shown by abbreviations (DR=Direct, SM=SMART, SW=SWING, SR=SMARTER, PW=AHP(Pairwise Comparisons) and VF=Value Functions)

2.1 Multiattribute value theory (MAVT)

In MAVT, the decision maker's preference statements are elicited and reflected in the overall value scores derived for each alternative. The value scores are composed of the ratings of the alternatives with respect to each of the lowest level attribute and of the relative weights of the attributes.

In Web-HIPRE, we assume that an additive value function can be used (for details on the assumptions under which this can be done, see e.g. Keeney and Raiffa, 1976). Then, the overall value score for an alternative x is

$$v(x) = \sum_{i=1}^n w_i v_i(x_i), \quad (1)$$

where $v_i(x_i)$ is the component value score of an attribute rating x_i and w_i is the weight associated with the attribute i . The component value functions $v_i(\bullet)$ and the weights w_i get values between 0 and 1, and the weights are normalized to sum up to one. The weight w_i indicates the relative importance of the change in an attribute i from its worst level to its best level compared to the corresponding changes in the other attributes.

The weights of the attributes can be defined directly by point allocation, or with one of the specific procedures, such as SMART (Edwards, 1977; von Winterfeldt and Edwards, 1986; Edwards and Barron, 1994) (see Figure 4), SWING (von Winterfeldt and Edwards, 1986) or the AHP (Saaty, 1980). The AHP is often presented as a completely different method. However, it can be shown that the AHP can be considered to be a variant of MAVT when the questions in the weight elicitation are presented in terms of value differences (Salo and Hämäläinen, 1997). This also justifies the use of the different weighting procedures in different local comparisons in the hierarchy. Web-HIPRE is the first software implementation supporting the simple rank based SMARTER technique (Edwards and Barron, 1994). On the lowest level, value scores can be given directly or value functions can be used to transform the ratings of the alternatives into the value scores.

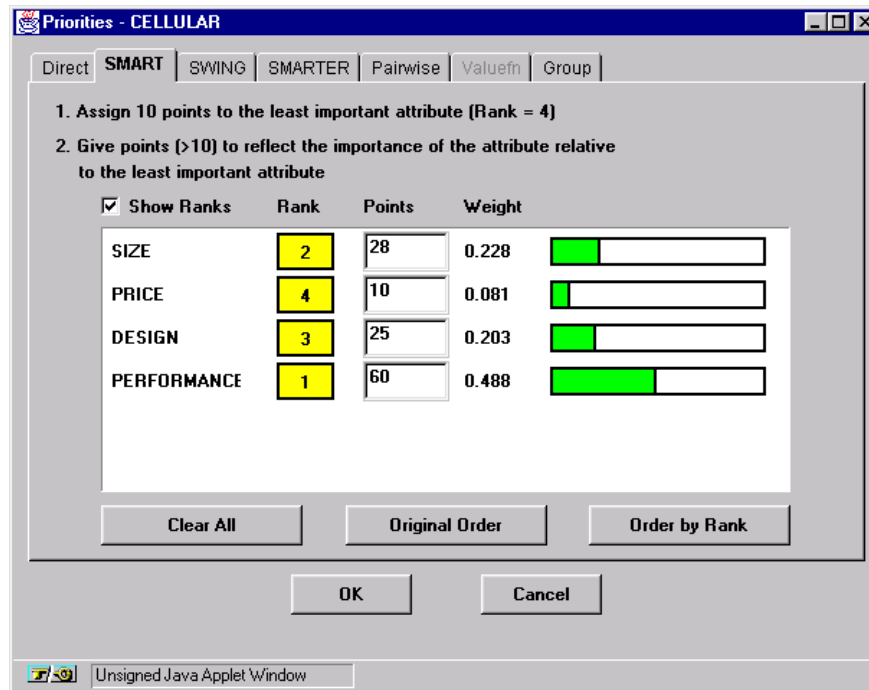


Figure 4. Weighting with the SMART method

2.2 The Analytic Hierarchy Process (AHP)

Saaty (1980) has developed the AHP as a theory for ratio scale decision models. This is a more general setting than the interval scale approach of MAVT (see Salo and Hämäläinen, 1997; Saaty, 1997). A general ratio scale approach assumes the existence of a zero point for the criteria outcomes. This is quite problematic with intangibles. What is the zero point of beauty, for example? In practice, anchor points can be used to help the situation. There is very little behavioral research on the AHP, but it can indeed be the case that decision makers implicitly adhere to the value difference type of interpretation, which makes the AHP results compatible with MAVT (see Salo and Hämäläinen, 1997). In the original AHP (Saaty, 1980), the preference statements are selected from a set of integers between 1 and 9. However, this scale has problems because of the lack of steps, for example, between 1 and 2. It has been shown that, for example, the use of the 9/9–9/1 scale (Ma and Zheng, 1991) or the balanced scale (Salo and Hämäläinen, 1997) can give better results (see Pöyhönen et al., 1997a). Web-HIPRE supports both the original and the balanced scale. Verbal statements can be assigned to these discrete scales by choosing an appropriate expression from the list. Preferences can also be given graphically with the slider or numerically by typing in a value (Figure 5). Then one, in fact, uses a continuous scale. When all the pairwise comparisons have been given, the resulting weights for each attribute are immediately shown by numbers and bar graphs.

The original *consistency ratio* (CR) of the AHP (Saaty, 1980) is only applicable with the original scale. When using a different discretization of the scale or a continuous scale, a scale-invariant consistency measure should be used. The *consistency measure* (CM) of Salo and Hämäläinen (1997) is used in Web-HIPRE. It is derived by transforming the inconsistent replies into an extended set of feasible preference statements, and using the properties of this set to measure the inconsistency of the pairwise comparison matrix. The consistency measure is defined as

$$CM = \frac{2}{n(n-1)} \sum_{i>j} \frac{\bar{r}(i,j) - \underline{r}(i,j)}{(1 + \bar{r}(i,j))(1 + \underline{r}(i,j))} \quad (2)$$

where $\bar{r}(i,j) = \max_k a(i,k)a(k,j)$, $k \in \{1, \dots, n\}$ stands for the extended bound of the comparison matrix element $a(i,j)$ (the element in the i th row and the j th column) (Salo, 1993), and $\underline{r}(i,j)$ is the

inverse of $\bar{r}(j,i)$. Thus, the consistency measure is an indicator of the size of this extended region formed by the set of local preferences such that $w_i \leq \bar{r}(i,j)w_j$ for all $i,j \in \{1,\dots,n\}$. This measure ranges from 0 to 1 and its value increases as the inconsistency of the elements of the comparison matrix increases.

The use of the consistency measure is similar to the consistency ratio. If the measure indicates a high inconsistency, the decision maker should examine the possible reasons for it, and possibly redefine the pairwise comparison judgments. At this point, we do not give a practical rule or limit for the acceptable level of inconsistency. This is an issue which needs practical testing and the recommendations are not likely to be independent of the size of the model. A fixed rule, such as the ten per cent level proposed for the CR (Saaty, 1980), is problematic and should not be enforced. We think that the analyst should use the CM primarily as a warning indicator against unintentional judgmental errors.

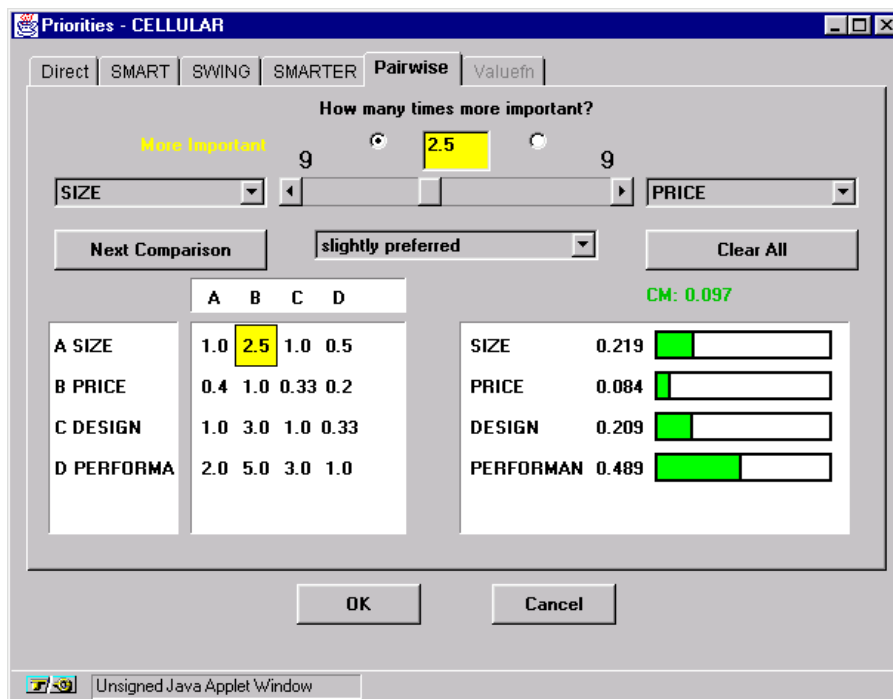


Figure 5. AHP weighting window

2.3 Combined use of the methods

A unique feature of Web-HIPRE is the possibility to use different weighting methods in one hierarchy. Thus, under each element of the hierarchy the decision maker can select the most suitable method. Locally, the decision maker can apply different methods, but naturally only one of them will be active when the results are computed. All the prioritizations entered are always stored, which allows the easy testing of the applicability and user acceptance of the prioritization methods. This also allows to study the convergence of the results. In each criterion element of the hierarchy, the method used in the corresponding local comparisons can be visualized (Figure 3).

Web-HIPRE leaves the responsibility of choosing the method to the user. It is designed to be a professional tool, and thus it assumes that the user has enough knowledge of decision analysis to be able to use the methods correctly. Of course, for example, an AHP oriented user can restrict the AHP to be the only weighting method, and thus use Web-HIPRE as an AHP only software. Due to this generality, the software is particularly suitable for research and teaching purposes.

Certain conversions need to be carried out, when AHP and MAVT methods are combined. When comparing alternatives, the AHP scores can be converted into a compatible 0–1 value scale by setting the lowest priority score to zero, the highest priority score to one, and scaling the

intermediate scores proportionally to this scale (Dyer, 1990). The converted score w_{ci} for the alternative i is then defined as

$$w_{ci} = \frac{w_i - w_{\min}}{w_{\max} - w_{\min}}, \quad (3)$$

where $w_{\max} = \max(w_1, \dots, w_n)$ and $w_{\min} = \min(w_1, \dots, w_n)$ are the original maximum and minimum AHP scores. These converted AHP scores can now be treated as value scores. Alternatively, value scores can be normalized to sum up to one to make them compatible with the AHP scores. Both of these conversions are available in Web-HIPRE. Again, the decision maker should be aware of the impacts of these different normalizing options when interpreting the results. For example, normalization of the overall scores introduces the possibility of rank reversal (see e.g. Pöyhönen et al., 1997b).

The emergence of generally available tools such as ours emphasizes the need for more behavioral research for the best practice procedures (see Pöyhönen and Hämmäläinen, 2000a). This includes understanding of the conditions when we can expect to see the convergence of the results elicited by different methods (see Pöyhönen and Hämmäläinen, 2000b). Although the aim of all the methods is to describe the preferences of the decision maker as well as possible, different elicitation methods might be perceived differently and give different results. With a software providing several methods, such as Web-HIPRE, it is possible to compare the overall priorities obtained by different methods.

2.4 Analyzing the results

In Web-HIPRE, the composite priorities, i.e. the overall scores for the alternatives are shown both by numerical values and by bar graphs (Figure 6). Bars can be divided into segments in different ways indicating the relative importance of the criteria and subcriteria. Any criterion can be chosen as the goal. Then, Web-HIPRE calculates the composite priorities from the subhierarchy under selected goal element. Sensitivity with respect to the weights of the criteria or the ratings of the alternatives can be analyzed (see Figure 7).

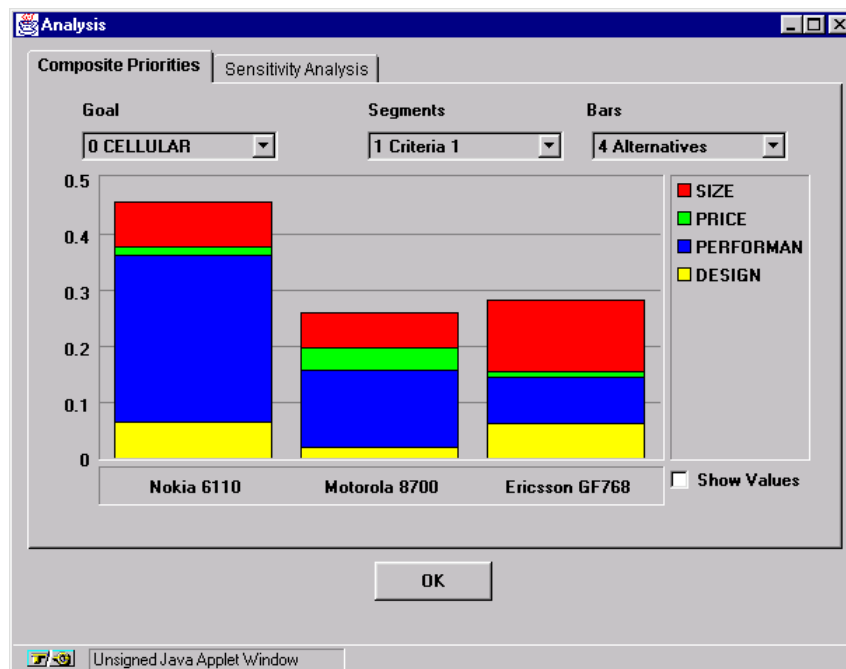


Figure 6. Composite priorities window

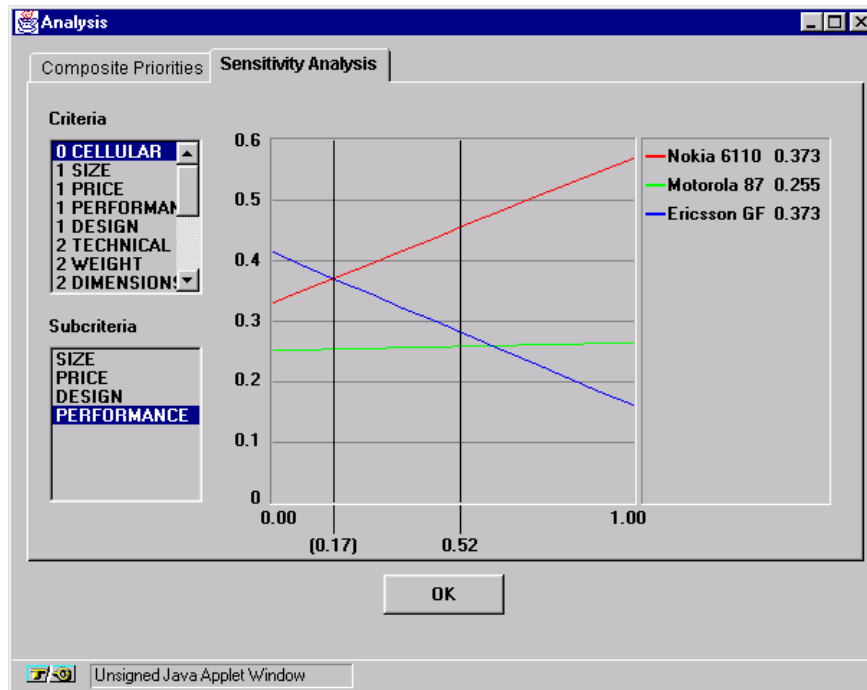


Figure 7. Sensitivity analysis window

3. GROUP DECISION MAKING

There are different ways of approaching multiattribute preference aggregation and combination in group decision making (see e.g. Salo, 1995). In methods allowing incomplete information, such as preference programming (Salo and Hämäläinen, 1995), PAIRS (Salo and Hämäläinen, 1992a) and PRIME (Salo and Hämäläinen, 1992b), it is possible to define intervals for the weight ratios instead of exact number estimates. The intervals can be interpreted to denote the decision maker's preferential uncertainty. Interval models can also be used in group decision making by forming intervals so that they include the opinions of all the group members (see e.g. Hämäläinen et al., 1996; Hämäläinen and Pöyhönen, 1996). As the local weights are presented as intervals, the overall priorities are also intervals. Currently, these methods are not yet supported by Web-HIPRE, but there are two general purpose software available for preference programming type approaches: WINPRE (Hämäläinen and Helenius, 1997) and Prime Decisions (Salo et al., 1999).

Another way to combine individual models is to use a direct aggregation method. In MAVT, weighted sum of individual values can be taken to get the group values, but generally this requires the explicit comparison of interpersonal preferences (Keeney and Raiffa, 1976; Salo, 1995). The AHP literature proposes two aggregation methods: the geometric mean method (Aczel and Saaty, 1983) and the weighted arithmetic mean method (see e.g. Dyer and Forman, 1992). The weighted arithmetic mean method satisfies the commonly accepted axioms for social choice except for the independence of irrelevant alternatives, which is also violated by AHP itself (Ramanathan and Ganesh, 1994). However, it again requires the explicit weighting of group members.

In Web-HIPRE, individual weights can be aggregated into a group model with the weighted arithmetic mean method. This naturally includes the weighted sum of MAVT as a special case. In the group hierarchy each decision maker is graphically represented by an element, which actually presents the whole hierarchy of the individual decision maker. The composite group priorities are generated as a weighted sum of individual priorities, which are obtained from the individual models.

Figure 8 presents an example of a group model for the cellular phone example. The decision makers consist of a family, which is divided into two subgroups, parents and children. These groups can be weighted according their relative importance, and members of both groups can be weighted inside the group as well. In Figure 8, the decision maker elements (Alice, Bob, Carol and

David) represent the entire model of the corresponding decision maker. One should note that these models need not be similar as long as the alternatives are the same. The outcome of these group member elements is the overall scores for the alternatives. All of the models are located in the server and thus they can be combined. One should note that if necessary, the individual members can work in an asynchronous distributed mode, i.e. in different locations and different times.

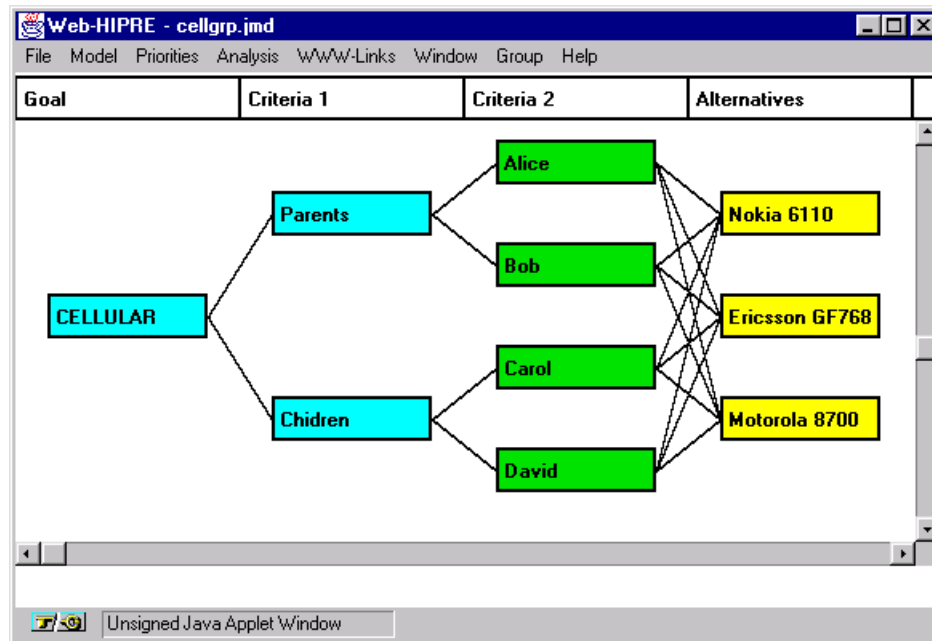


Figure 8. A group model for cellular phone example

The overall weights in the group model are represented similarly as in an individual model. The bar graphs can be divided according to the relative importance of the decision makers and sensitivity analysis can be performed to examine the sensitivity of the weights given to the decision makers or the subgroups.

We want to emphasize that Web-HIPRE can also be used to support group decision making by simply using it as a collaboration platform. All the decision makers' preferences can be made available to the others, and by analyzing them a better understanding of their objectives can be achieved. This can often be the kind of group process needed to proceed towards a consensus.

4. PRACTICAL USE OF WEB-HIPRE

The development of a new regulation policy for Lake Päijänne in Finland (see e.g. Hämäläinen et al., 1999) is the first real life case, where Web-HIPRE was used. The old policy focused on flood protection and the needs of the hydroelectric plants in the Kymijoki River beneath the lake. The development project was launched to study possibilities to improve the regulation policy by including the goals of other interest groups as well. Decision analysis was used to elicit stakeholder opinions about the impacts of the new alternatives for the regulation policy. The value tree of the Lake Päijänne case was constructed on the basis of stakeholder interviews. The project held separate meetings where the prioritizations were analyzed together with the representatives of all the interest groups. For details, see the home pages of the project (Hämäläinen, 1999). The value tree used is also one of the example models on Web-HIPRE, and the interested reader can open it for examination. The benefit of having a WWW based tool is an essential improvement for citizens' participation. The stakeholders can learn the facts by first studying the website's project reports as well as visual and photo material of the environmental topics. Then, they can independently analyze and weight the value tree from their own points of view, and modify the value tree by adding objectives which are important to them. The extent to which these opportunities were taken is not, however, known. When the opinions of the other stakeholders are examined, a better understanding of their view will be achieved. In these kinds of regional

problems, stakeholders often live in geographically distant locations, and thus the benefit of having remote connection to the software and home pages is obvious.

Web-HIPRE is very suitable for studies on cross-cultural differences (see e.g. Kersten and Noronha, 1999). A recent application is the joint teaching project for international web decision support (Kersten et al., 1999). In this project, students from Austria, Canada and Finland formed international teams, who worked with given decision problems. With the help of Web-HIPRE, the teams could globally evaluate a model for the problem and analyze the results.

5. CONCLUSIONS

The Internet has become an enormous distributed hypermedia information system. In this work, we have presented an Internet based decision support system, Web-HIPRE, the first general purpose decision analytical tool available in the WWW. So far, we have only seen the very first steps in using the Internet in group collaboration and real life decision making. However, in the future many traditionally face-to-face interactions between people will increasingly be performed on the web. As visiting the web becomes an everyday practice, the step to start using decision aids will be lower and they could soon become a necessity on any site. One can easily envision many new possibilities. One natural area of future applications is to support consumers' product comparison in electronic commerce. There could be ready made tailored value tree templates for different kinds of purchase decisions with preset links to the product home pages and consumer reports, for example. Teledemocracy is also likely to be a field of rapid growth. The Internet is a way to provide citizens interactive decision support by means of decision analytical tools like Web-HIPRE and by on-line voting platforms like the Opinions-Online system (Hämäläinen and Kalenius, 1998). There are also important social benefits from using such systems in justifying public policy decisions, for example, in environmental or global issues. The explicit open consideration of values in such cases increases the transparency of decision making and enhances the related political processes.

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Web based tools:

- Web-HIPRE: <http://www.hipre.hut.fi>
- Joint Gains: <http://www.jointgains.hut.fi>
- Opinions-Online: <http://www.opinion.hut.fi> or <http://www.opinions-online.com>
- INSPIRE: <http://www.business.carleton.ca/inspire/>
- NIMBUS: <http://nimbus.mit.jyu.fi>

Downloadable software:

- WINPRE: <http://www.hut.fi/Yksikot/SAL/Downloadables/>
- Prime Decisions: <http://www.hut.fi/Yksikot/SAL/Downloadables/>

The Lake Päijänne project:

- <http://www.paijanne.hut.fi>

The joint teaching project for international Web decision support:

- http://www.business.carleton.ca/~gregory/teaching/42.541/web_dss_project.html

Decision analysis software listings:

- <http://faculty.fuqua.duke.edu/daweb/dasw.htm>