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Comparison and analysis of experimental and virtual laboratory scale punch through tests

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A B S T R A C T

Laboratory scale punch through tests on floating rubble consisting of plastic blocks were conducted and simulated with a 3D discrete numerical model. The purpose was to analyse the experimental method and to validate the model. The motivation of using plastic blocks instead of ice was to simplify the interpretation of results as the plastic blocks do not freeze together. The indentor force and the lateral force induced by the rubble on one of the basin walls were recorded as a function of indentor penetration. Further, the experiments were recorded with a video camera and a motion tracking software was used to analyse the rubble deformation. The force records and deformation patterns from the experiments and simulations were in agreement. The evolution of the deformation patterns could be closely linked to the indentor force records, which demonstrates the need for the numerical model to correctly represent the rubble deformation. The experiments and the simulations showed, that the lateral force within the pile increased considerably during a punch through experiment. This makes the interpretation of punch through experiment results for material modelling challenging: the friction angle of the rubble can become overestimated making the punch through test unsuitable for achieving accurate values for friction angle. Consequently, no value for the rubble friction angle was derived here.

1. Introduction

Punch through experiments are an important method for testing ice rubble properties. In a punch through experiment, a flat indentor platen penetrates through the rubble mass while the force applied by the rubble on the indentor is measured. From the indentor force-displacement records and the dimensions of the experimental set up, some rubble properties can be derived.

First punch through experiments were performed by Leppärenta and Hakala (1992) in the Baltic Sea using a loading platform and concrete blocks. Since then, the experimental equipment has been improved and the method has been used in full scale as reported in e.g. Bruneau et al. (1998), Heinonen and Määttänen (2000, 2001a, b), Croasdale et al. (2001) and Heinonen (2004). Even if efforts have been made to measure and visually observe the rubble deformation in full scale experiments (Croasdale et al., 2001; Heinonen, 2004), the possibilities to study the actual failure mechanisms of the ice rubble have been rather limited.

On the other hand, the knowledge on the failure mechanism of the rubble is of crucial importance in the development of material models for the ice rubble. For the more detailed analysis on the behaviour and failure mechanism of rubble, laboratory scale punch through experiments have been performed by e.g. Leppärenta and Hakala (1992), Bruneau et al. (1998), Azarnejad et al. (1999), Azarnejad and Brown (2001), Jensen et al. (2001), Lemee and Brown (2002), and Serré (2011).

While the results from the laboratory scale experiments have increased the understanding on the failure of a rubble, they have also yielded results that differ from those achieved in the field (Croasdale et al., 2001; Liferov and Bonnemaire, 2005). One of these differences is the loading rate dependency of the rubble behaviour first found in laboratory experiments by Azarnejad and Brown (2001). The loading rate dependency has been earlier discussed in a review by Liferov and Bonnemaire (2005) and analysed in Polojärvi and Tuhkuri (2012).

Another important difference is that in the laboratory the indentor displacements at maximum loads are relatively high when compared to those in the full scale. This is believed to indicate difference in rubble behaviour between laboratory and full scale experiments. In full scale, the maximum loads are believed to be dominated by the failure load of the cohesive skeleton formed by freeze bonds (Heinonen, 2004; Liferov and Bonnemaire, 2005). Further, it has been assumed that it is only after the breakage of this skeleton that the frictional resistance of the pile becomes mobilized. In the laboratory, the rubble usually goes through considerable deformation before maximum indentor force is reached and it is believed that also the frictional resistance contributes significantly to the maximum load (Liferov and Bonnemaire, 2005).
This latter difference motivates the punch through experiments on non cohesive rubble presented in this paper. In this work, laboratory scale punch through experiments were performed using rubble consisting of plastic blocks. The motivation of using plastic blocks instead of ice blocks was to simplify the interpretation and analysis of the results. When plastic blocks are used, freeze bonds between the blocks and other features, e.g. sintering, characteristic to the ice rubble are avoided (Ettema and Schaefer, 1986; Kuroiwa, 1961).

The understanding on failure mechanisms is also important for the modelling efforts of the ice rubble. The modelling of granular material such as rubble is challenging. Earlier work in modelling of ice rubble has included analytic soil mechanics models (Ettema and Urroz, 1989), continuum models (Heinonen, 2004; Heinonen and Määttänen, 2001b; Liferov et al., 2002, 2003; Serré, 2011), discrete models (Polojärvi and Tuhkuri, 2009; Tuhkuri and Polojärvi, 2005) and pseudo-discrete models (Liferov, 2005). In the work presented in this paper, rubble pile was numerically modelled as 3D discontinuous media. In this type of modelling, the rubble consists of individual blocks which interact through contacts. Further, the deformation of the rubble is due to displacements of the blocks within it. Two techniques commonly used in numerical modelling of discontinuous media are the discrete element method (DEM) and combined finite–discrete element method (FEM–DEM), first introduced by Cundall and Strack (1979) and Munjiza et al. (1995), respectively. In ice mechanics, DEM and FEM–DEM have earlier been used e.g. in studies of ice ridging (Hopkins, 1992, 1998; Hopkins et al., 1999), ice pile-up against structures (Haase et al., 2010; Paavilainen and Tuhkuri, 2012; Paavilainen et al., 2006, 2009, 2011) and punch through experiments (Polojärvi and Tuhkuri, 2009, 2010; Tuhkuri and Polojärvi, 2005).

Here, the individual blocks within the simulated rubble were assumed rigid. The contact forces between the blocks were solved using penalty functions and potential contact force method (Munjiza, 2004). This modelling technique for contact forces was chosen as it gives robust platform for solving 3D contacts of polyhedral blocks.

The purpose of this paper is two-fold. First the experimental results are presented and compared to the results from the numerical model. Through the comparison the model is validated and its applicability to simulate ice mechanics related problems thus demonstrated. Then the experimental and modelling results are analysed in parallel. The scope of the analysis is to gain further understanding on the failure process of the rubble and to briefly discuss the implications of the results to the material modelling of the rubble. No rubble material properties apart from the rubble shear strength are derived using the results.

The article is organized as follows. First the experimental set up and the results from the experiments are presented in Section 2. Then, the numerical modelling of the experiments is introduced in Section 3. After this in Section 4, the results from the experiments and the simulations are presented, compared and analysed. The implications of the results on the interpretation of punch through experiment results and on the material modelling of the rubble results are briefly discussed in Section 5. Finally, the conclusions are presented in Section 6.

2. Experiments

In this section, the experimental set up and the conducted experiment types are described. Then a run through of punch through experiments is presented with a description of indenter load records. In the end of the section, the derivation of the load records due to rubble material is presented. The experimental set up is here described only briefly, but more data on it can be found from Polojärvi and Tuhkuri (2012).

2.1. Experimental set up

The test basin illustrated in Fig. 1 was made of transparent acrylic glass (PMMA) and was supported by a steel frame. The basin dimensions were 2.5 × 0.5 × 1.0 m. The indenter platen was 0.5 × 0.49 × 0.01 m and made of polyethylene (PE). The basin had movable PE walls attached to the basin frame which enabled to change its width w1. The lateral force Fx applied by the rubble on the instrumented movable wall at x = − w0/2 was measured to investigate the lateral confinement within the rubble. In some of the experiments, covers were installed into the basin. The covers were cut from a 0.01 m thick PE plate and reached from the edges of the indenter platen to the movable walls.

The main properties of the rubble blocks are given in Table 1. The blocks had a shape of a rectangular cuboid and were sawn out of 0.02 m thick PE plate by the supplier. The PE plate unfortunately had different surface roughness on its opposing faces, hence the surface roughness on opposing block faces differed. The sliding friction coefficient μ for wet blocks was measured as described in Polojärvi and Tuhkuri (2012). Value of μ depended on the pair of contacting block faces and varied between 0.04 and 0.12.

2.2. Experiment types

The summary of the tests reported in this paper is given in Table 2. As the table shows, two different types of punch through experiments were conducted: uncovered and covered basin experiments. The experiment types differed by the boundary conditions on the top of the rubble. In the covered basin experiments, the rubble besides the indenter platen was covered by PE-plates whereas in the uncovered basin experiments these plates were not used. In addition to actual punch through tests, open water experiments with no rubble in the

<table>
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<th>Property</th>
<th>Symbol</th>
<th>Unit</th>
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<td>Material density</td>
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<tr>
<td>Friction coefficient</td>
<td>μ</td>
<td></td>
<td>0.04...0.12</td>
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</table>

Table 1

Properties of the rubble blocks.
With high indentor velocities, this type of factoring becomes challenging. Using the results from the open water experiments as shown below, indentor load into the contribution of the experimental setup and the relatively slow indentor platen buoyancy as the platen started its water entry. Further, all $F_I$ records show a local peak which always occurred after the indentor submergence and was related to hydrodynamics (Polojärvi and Tuhkuri, 2012). After the peak, $F_I$ corresponded to the rubble resistance and to the combined load of the rubble and the indentor platen buoyancies. In the uncovered basin experiments, $F_I$ decreased after the peak until the end of the experiment. Instead, in the covered basin experiments, $F_I$ still increased after the peak and started to decrease later in the experiment.

The $F_I$ records from the punch through and the open water experiments were used to derive load records due to rubble material for the analysis presented in Section 4. As illustrated in Fig. 2, the rubble load $F_R$ records were derived by subtracting $F_I$ records from the open water experiments from those from the punch through experiments. The derivation of $F_R$ records by subtraction was applicable, as $F_I$ records from the open water and the punch through experiments showed similar features: the rate of $F_I$/basin changes after the indentor water entry in them was approximately equal. Thus, both showed the figures, experiments with both basin widths $w_b$ yielded to very similar $F_I$ records.

In both experiment types, $F_I$ first started to increase in the beginning of the experiment as shown in Fig. 2a and b. The rate of $F_I$ changes then changed due to the indentor platen buoyancy as the platen started its water entry. Further, all $F_I$ records show a local peak which always occurred after the indentor submergence and was related to hydrodynamics (Polojärvi and Tuhkuri, 2012). After the peak, $F_I$ corresponded to the rubble resistance and to the combined load of the rubble and the indentor platen buoyancies. In the uncovered basin experiments, $F_I$ decreased after the peak until the end of the experiment. Instead, in the covered basin experiments, $F_I$ still increased after the peak and started to decrease later in the experiment.

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<table>
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<tr>
<th>Test type</th>
<th>$w_b$ [m]</th>
<th>$h$ [m]</th>
<th>Number of experiments</th>
<th>$v_I$ = 2.5 mm/s</th>
<th>$v_I$ = 5 mm/s</th>
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<tr>
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<td>5</td>
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<td>5</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<tr>
<td>Covered basin</td>
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<tr>
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<td>2</td>
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<tr>
<td></td>
<td>0.5</td>
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<td>4</td>
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<tr>
<td>Open water</td>
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<td>N.A</td>
<td>3</td>
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</tbody>
</table>

2.3. Indentor and rubble force records from the experiments

The experiments were found to be well repeatable, thus the indentor force $F_I$ records from the repeated experiments were used to derive the mean indentor force $F_I$ records as presented in Polojärvi and Tuhkuri (2012). Typical $F_I$ records from both punch through experiment types are shown in Fig. 2a and b. As seen from

Fig. 2. Mean indentor force $F_I$ records from punch through experiments with $v_I$ = 5 mm/s, $h$ = 0.5 m, and with both basin widths $w_b$: (a) uncovered and (b) covered basin experiments.
through flow could be used to derive mean rubble load in the case of a sudden failure of a cohesive network, a peak followed by a drop in the rubble resistance was due to the rubble buoyancy and friction: the lack of peak force indicates, that also similar. Therefore, the position in relation to the water surface in both experiments is equal. In experiments were similar, the records derived from them were run in 3D in a domain with dimensions of the basin used in the experiments (see Fig. 1). The basin walls in the simulations were rigid and were not allowed to move.

The simulations were performed in two phases: (1) in the initial phase the initial configuration of the rubble was generated and (2) in the punch through phase the indentor penetrated the rubble. In the first phase, the initial configuration of the pile was generated by releasing blocks under water with random orientations and velocities. After releasing, the blocks were allowed to float until their kinetic energy was dissipated due to friction, drag and damping in contacts and a quasi static pile had formed. In the covered basin simulations, the covers and the indentor were lowered on top of the quasi static rubble pile in the end of the initial phase.

Typical rubble piles at the beginning of punch through phase of the simulations are illustrated in Fig. 4a and b. In the punch through phase, the indentor penetrated into the rubble with constant indentor velocity. During the penetration, the force applied on the indentor platen and on basin walls due to rubble blocks was gauged. The hydrodynamics, besides the simplified drag force acting on rubble blocks, were not modelled. Hence, the indentor load records from the punch through phase of the simulations could be directly compared to the experimental \( F_R \) records (see Fig. 3).

The main parameters for the simulations presented in the next section are given in Table 3. The block shape, size and mass properties in the simulations were chosen after the experiments. As mentioned above, the friction coefficient \( \mu \) in the experiments depended on the pair of contacting block faces: in the simulations \( \mu = 0.08 \), chosen after the mean value of friction coefficients measured in the experiments, was used. Reference simulations with friction coefficient depending on the pair of contacting faces were performed with no change in results presented below. This shows, that the number of contacting faces was high enough to make the effect of varying friction coefficient to disappear. The drag coefficient in the simulations had value \( \zeta = 1.05 \) chosen after a shape of a cube (Granger, 1995). The bulk porosity \( \eta \) given in the table is defined as \( \eta = (V_r - V_b)/V_r \), where \( V_r \) and \( V_b \) are the volume of rubble and the volume of blocks within the rubble, respectively. \( V_b \) was readily available in the simulations and \( V_r \) was determined by post processing the simulation data.

The results in the next section are from simulations with indentor velocity \( v_I = 10 \text{ mm/s} \). The results from the simulations are compared to the experiments with \( v_I = 2.5 \text{ mm/s} \) and \( 5 \text{ mm/s} \). The comparison of the results from the simulations and the experiments with

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3. Simulations of experiments

In the discrete numerical model used here, the rubble pile consisted of numerous individual rigid blocks interacting through contacts. In addition to forces due to contacts, the block forces were due to buoyancy, gravitation and water drag. All forces applied on blocks were determined as presented in Appendix A. The simulations were run in 3D in a domain with dimensions of the basin used in the experiments (see Fig. 1). The basin walls in the simulations were rigid and were not allowed to move.

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![Fig. 3](image-url)  
Fig. 3. The derivation of the rubble load \( F_R \): (a) indentor load records from a punch through \( (F_{pt}) \) and an open water \( (F_{ow}) \) experiment and (b) the rubble load \( F_R = F_{pt} - F_{ow} \). In (a), the indentor starts its entry into the water at (1) and the peak related to the hydrodynamic force occurs at (2) after the indentor submergence (Polojärvi and Tuhkuri, 2012). The data from the open water experiment is shifted right so that the indentor position in relation to the water surface in both experiments is equal. In (b), \( F_{pt} \) and \( F_{ow} \) from (a) are shown in gray.

![Fig. 4](image-url)  
Fig. 4. Examples of typical initial rubble configurations before indentor penetration in simulated punch through experiment: simulation of (a) an uncovered and (b) a covered basin experiment. The basin walls are not shown in the figure for clarity.
different values of \( v_1 \) was applicable, as additional simulations with \( v_1 = 5 \text{ mm/s} \) showed negligible change in results. The indentor load histories from these additional simulations are presented in Appendix B. Limited use of low \( v_1 \) values was related to the length of the simulation times: the decrease in \( v_1 \) leads to an increase in simulated time period and thus to an increase in simulation time.

### 4. Results, model validation and analysis

#### 4.1. Comparison of rubble load records

As mentioned above, the rubble load \( F_R \) records derived from indentor load records (see Fig. 3) could be directly compared to the indentor load records from the simulations. Therefore, the experimental \( F_R \) records and indentor load records from the simulations are both referred to with the same symbol \( F_R \) below. The repeated simulations with different initial rubble configurations yielded to similar \( F_R \) records and were used to derive mean load records, which are referred to with symbol \( F_R \) similarly to the experimental mean rubble load records.

The load records and levels in the simulations and experiments were in good agreement as illustrated by Figs. 5 and 6. The \( F_R \) records shown in Fig. 5a and b from the experiments and simulations are similar; in both \( F_R \) increased with the same rate in the beginning of the indentor penetration and reached its maximum with approximately equal indentor displacement. In the covered basin simulations \( F_R \) was underestimated towards the end of the indentor stroke, whereas in the uncovered simulations \( F_R \) records were in good agreement until the end of the experiment. The maximum rubble load \( F_R \) values were also approximately equal in experiments and simulations as shown by Fig. 6a and b. The basin width \( w_0 \) had no major effect on \( F_R \) records nor \( F_R \) values as further shown by Figs. 5 and 6.

As illustrated by Fig. 5a and b, in both experiments and simulations, the indentor displacement as \( F_R \) was reached depended on the experiment type. When the basin was uncovered, \( F_R \) was reached close to the indentor submergence, but when the basin was covered, \( F_R \) was in general still increasing at indentor submergence and \( F_R \) was reached later in the experiment. The reason for this behaviour was a difference in the evolution of the rubble deformation patterns between experiment types, as shown in the next section.

#### 4.2. Deformation of the rubble

In addition to load records, the deformation patterns in the simulations and experiments were found to be in good agreement as presented below. Further, the evolution of the deformation patterns and the mass flow of the rubble material appear to have a close relation with the rubble load \( F_R \) records. This demonstrates the need for a numerical model to correctly represent mass flow of the rubble.

##### 4.2.1. Visualization of the deformation patterns

The visual analysis on the rubble deformation in the experiments was based on the video recordings. The recordings were post processed using a motion tracking software which detected and tracked the block corners throughout the experiment giving their trajectories. The trajectories were then used in the analysis on deformation patterns as follows.
For each block corner $j$, its coordinate $x^j = [x^j_1, y^j_1]$ (coordinate system given in Fig. 1) at the time instant of frame $k$ of the recording was first used to derive displacement of the corner from

$$\Delta x^j = \Delta x^j_i + \Delta y^j = (x^j_1 - x^j_0)_i + (y^j_1 - y^j_0)j$$

where $x^j_0$ and $y^j_0$ are the corner coordinates in the initial rubble configuration.

These displacements were then used to estimate displacing rubble volume as illustrated in Fig. 7a and b. First, the indentor displacement $y^k_1$ for frame $k$ was determined. Then, tracking points with $y^j_1 > R y^j_1$, where $R$ is a constant, were picked from the data (Fig. 7a). After this, the points with $\Delta y^j > R y^j_1$ were used to define a convex hull (Fig. 7b). The area of the hull was used to estimate the volume of rubble with $\Delta y^j > R y^j_1$. By varying $R$, a presentation for the deformation pattern of the rubble was achieved.

The trajectories of the block corners in the simulations were achieved in similar manner by post processing the simulation data in MATLAB (MATLAB, 2009). As the analysis based on the video in the experiments was limited to the visible part of the rubble, only the block vertices within the distance of block length (0.09 m) from the long basin wall at $z = -0.25$ m (see Fig. 1) were used when analysing the data from the simulations.

4.2.2. Comparison of the deformation patterns in experiments and simulations

Snapshots in Fig. 8 illustrate rubble deformation during covered basin punch through experiments and corresponding simulations. In the snapshots of Fig. 8, hulls for the rubble volumes with $\Delta y^j > R y^j_1, R = 0.90, 0.75, 0.50$ and 0.10 are superposed on top of the images from experiments and simulations. The hull with the highest $R$ value (0.90) estimates the volume of rubble moving with the indentor, whereas the hull with the smallest $R$ (0.10) gives an approximation for the total volume of the downwards moving rubble mass.

The first snapshot in Fig. 8 with $y_1 = 35$ mm shows the rubble at the point, where indentor platen has reached the water surface. The rubble mass under the indentor is observed to move with the platen (hulls with $R > 0.90$ in the figure). The volume of this mass has an approximate shape of a wedge with one corner reaching downwards into the $y$-direction and one side aligning with the indentor platen. The corner reaching downwards stayed within the indentor edges in $x$-direction.

The next snapshot in Fig. 8 with $y_1 = 80$ mm shows the deformation pattern at the maximum rubble load $F_{R}^{w}$. The volume of displaced rubble has increased from the previous snapshot, whereas the volume of the rubble moving with the indentor ($R > 0.90$) is observed to have stayed similar. The increase in the volume of the hulls with $R < 0.90$ was due to increase in their width.
The last snapshot in Fig. 8 with \( y_I = 240 \text{ mm} \) taken towards the end of the indentor stroke still shows that the deformation patterns are similar in the simulations and experiments. Lateral movement of the rubble towards the indentor center line above the indentor platen is observed.

4.2.3. Relation of deformation patterns and \( F_R \) records

The simulation data was used to analyse the relation between the \( F_R \) and rubble deformation in more detail as the analysis was not limited only on the visible part of the rubble. The displacing rubble volume during simulated experiments is again illustrated by Fig. 9a and b using the velocities of the blocks. The figures show the total volume of blocks with their centroids having the downward velocities \( v_y > 2 \text{ mm}/\text{s}, v_y > 5 \text{ mm}/\text{s}, \text{ and } v_y > 9 \text{ mm}/\text{s} \) in simulations with indentor velocity \( v_I = 10 \text{ mm}/\text{s} \). The basin width \( w_b \) did not affect these volumes, thus the data from the simulations with both \( w_b \) is included in the data shown in the figures.

Fig. 8 shows two distinct features: the experiment type has an effect on the evolution of the displacing rubble volumes and that the volume of displacing rubble is at its maximum close to the maximum rubble load \( F_R^m \). The latter feature can be seen by comparison of \( F_R \) records in Fig. 5a and b to Fig. 9a and b: the maxima for the rubble volumes with \( v_y > 5 \text{ mm}/\text{s} \) and \( v_y > 9 \text{ mm}/\text{s} \) are reached approximately simultaneously with \( F_R^m \). As a consequence of downwards moving rubble volume increasing during longer displacement interval in the covered than in the uncovered basin experiments, the maximum rubble load \( F_R^m \) is reached later in covered experiments.

As Fig. 9 further shows, the volume of solid material moving downwards decreased in all simulations towards the end of the indentor stroke. The main reason for this decrease was the movement of the blocks above the indentor platen illustrated by Fig. 10a and b, which show block centroid velocities into lateral and vertical directions towards the end of a simulated experiment. Fig. 10a clearly shows blocks above the indentor platen moving towards the indentor center line resulting into upward movement of rubble blocks through pile thickness seen in Fig. 10b.
The lateral force $F_x$ is not constant during the indentor stroke: this indicates that the confinement pressure within the pile increases during indentor penetration. The increase in $F_x$ from its initial value was considerable and led to maximum lateral force values, $F_{x_{m}}$, which were relatively high in comparison to the vertical rubble load.

### 4.3.1. Lateral force records

Fig. 11 shows the mean lateral force $F_x$ recorded by the Auger on the instrumented wall of the basin as measured as described in Section 2. In the simulations, $F_x$ was similarly to the experiments gauged on the short walls of the basin. As first shown below, $F_x$ records from the experiments and simulations were in fair agreement. Further it is shown, that $F_x$ was not constant, but first increased, during the indentor stroke. This indicates that the confining pressure within the pile increases during indentor penetration. The increase in $F_x$ from its initial value was considerable and led to maximum lateral force values, $F_{x_{m}}$, which were relatively high in comparison to the vertical rubble load.

### 5. Discussion

#### 5.1. Effect of the basin walls

The effect of the basin walls on the experimental results in previous section was studied using the simulations. For this, the vertical frictional forces $F_I$, on rubble due to the basin walls were monitored in the simulations. Same friction coefficient value ($\mu = 0.08$) was used for block-wall and block-block contacts (see Table 3). The results below show, that the long basin walls (at $z = \pm 0.25$ m in Fig. 1) somewhat increased the maximum rubble load ($F_{R_{m}}$), and that the basin likely was wide enough for the short walls (at $x = \pm w_b/2$) to not affect the results presented above.

Typical $F_x$ records from simulations of both experiment types are given in Fig. 16a and b. In the figure, negative (positive) $F_x$ values indicate a wall resisting downwards (upwards) rubble movement. Hence, the friction at long walls of the basin resisted downwards rubble movement, whereas at the short walls, the upward rubble movement was resisted by friction.

Further Fig. 16a and b illustrates how the covers affected the $F_x$ records at the long walls: the maximum absolute value of frictional force in uncovered simulations was reached in the beginning of the indentor penetration, whereas in the covered simulations it was reached later. This phenomenon is likely related to the differences in the evolution of the deformation patterns between different experiment types (see Section 4.2.3).

The friction at the long walls somewhat increased the measured maximum rubble load $F_{R_{m}}$. This can be seen from the values of $F_{R_{m}}$ at the instant of the maximum rubble load $F_{R_{m}}$ given in Fig. 17a and b. As the figure shows, $F_{R_{m}}$ values increased with $h$. This is due to the influences to the material modelling as briefly discussed next.
increase in the number of frictional contacts between rubble mass and the walls with \( h \). The data in the figure together with the maximum rubble load \( F^R \) values in Fig. 6 was used to estimate the total contribution of \( F^m \) on \( F^R \); sum of \( F^m \) on opposing long walls accounted for 2 (with \( h = 0.3 \) m) to 6 (\( h = 0.5 \) m) % of \( F^R \) in uncovered and for 3 (\( h = 0.3 \) m) to 8 (\( h = 0.5 \) m) % of \( F^R \) in covered basin simulations.

As Fig. 17 further shows, the values of \( F^m \) at short walls were low in all simulations and virtually vanished in covered simulations due to the covers restricting the upward movement of the rubble. Low \( F^m \) values in Fig. 17 indicate, that the rubble in the vicinity of the walls did not move. This is in accordance with the observations during the experiments and was also shown by the deformation patterns in Section 4.2. In addition and more importantly, the low \( F^m \) values on short walls together with approximately equal \( F^m \) values on long walls with both \( w_0 \) indicate, that the rubble deformation patterns were not affected by the basin width in the experiments.

5.2. Derivation of material parameters

The results presented in the previous section can be used to discuss the derivation of the material parameters from the experimental results. For this, the Mohr–Coulomb material model commonly used in interpretation of punch through experiment results is considered. In the model, the shear strength \( \tau \) is given by

\[
\tau = c + \sigma \tan \phi, \tag{2}
\]

where \( \sigma \) is the confining pressure and material parameters \( c \) and \( \phi \) are the cohesion and Mohr–Coulomb friction angle of the material, respectively.

An estimate for the experimental \( \tau \) can be derived from the results using a formula similar to that in Azarnejad and Brown (2001). In the estimate, the maximum rubble load \( F^R \) is assumed to include components due to rubble buoyancy \( \langle F_b \rangle \), friction due to basin walls \( \langle F_w \rangle \) and shearing of the rubble \( \langle F_s \rangle \), hence \( F^R = F_c + F_w + F_s \). Further it is assumed that the rubble fails on two distinct failure planes leading to \( F_c = 2\tau bh \), where \( b \) is the basin width and \( h \) the rubble depth. With these assumptions, the rubble shear strength can be derived from

\[
\tau = \frac{F_c}{2bh} = \frac{F^m - F_w - F_s}{2bh}. \tag{3}
\]

Here, the shear strength values for the rubble were estimated by using a common first approximation of indentor supporting only the rubble mass directly under it at the instant of \( F^R \). In this case, the buoyant component in previous equation can be approximated from \( F_b = (1 - \eta) (\rho_w - \rho_0) w_0 bh \), where \( w_0 \) is the indentor width. The \( \tau \) values derived from the previous expression using this value of \( F_b \) are shown in Fig. 18 for the simulations and experiments. The \( \tau \) values from covered experiments and all simulations in the figure clearly suggest, that \( \tau \) increased with \( h \). The \( \tau \) values given in the figure are fairly close to those measured for unconsolidated ice rubble in similar experiments by (Azarnejad and Brown, 2001). It should also be noticed, that if the friction on the long walls is not taken account, Eq. (3) yields \( \tau \) values up to 10% higher than given in the figure.

The effect of increasing confining pressure on the derivation of the Mohr–Coulomb friction angle \( \phi \) of the rubble can be studied using the equations above. For this, Eq. (2) is substituted into Eq. (3) and \( \tau = 0 \) is assumed, leading to following dependency between \( \sigma \) and \( F_c \):

\[
\sigma \tan \phi = \frac{F_c}{2bh}. \tag{4}
\]

The results in Section 4.3.1 indicate, that \( \sigma \) increases during the indentor penetration: the lateral force \( F_w \) was observed to increase up to roughly 50% from its initial value. Therefore, even if a
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reasonable estimate for the initial value of $\sigma$ was available, e.g. derived from the initial hydrostatic pressure within the rubble, it is not necessarily an accurate estimation for $\sigma$ during an experiment. If this estimate was anyhow used to solve $\phi$ for the rubble from Eq. (2), $\phi$ could become overestimated as demonstrated in the following using a simplified example.

Assuming, that a value for shear strength $\tau$ has been derived from the experimental results (see e.g. Azarnejad and Brown (2001) for derivation of $\tau$) and $\gamma = 0$ in Eq. (2), the $\phi$ value for some ratio $\sigma/\tau$ is given by $\phi = \arctan(\gamma/\sigma)$. First, let $\phi_0 = \arctan(\tau/\sigma_0)$, where $\sigma_0$ is the initial confining pressure within the pile, and $\phi_h = \arctan(\tau/\sigma_h)$, where $\sigma_h$ is the actual confining pressure. Then, the change in the estimate of $\phi$ due to change in $\sigma$ can be studied using ratio $\phi_h/\phi_0$.

Fig. 19 shows the values of $\phi_h/\phi_0$ for different ratios of $\sigma_h/\sigma_0$ as function of $\phi_h/\tau$. As the figure shows, with low $\phi_h/\tau$ ratios the values of $\phi$ are likely estimated well. On the other hand, large $\phi_h/\tau$ ratios lead to an overestimation of $\phi$ if actual $\sigma$ value is underestimated. The 50% increase in $\sigma$ ($\sigma_h/\sigma_0 = 1.5$), suggested by the increase in $F_x$ (see Section 1), would lead to clear overestimation of $\phi$ even with moderate $\phi_h/\tau$ ratios. It should be noticed, that $\phi_h/\tau = 1$ would lead to $\phi = 45^\circ$ ($\phi$ decreases with increase in $\phi_h/\tau$), which is already in the high end of values suggested for ice rubble (Ettema and Urroz, 1986; Liferov and Bonnemaire, 2005).

Hence, an overestimation of $\phi$ can occur due to increasing $\sigma$ and the derivation of $\phi$ from the results is thus challenging. Further, the use of Mohr–Coulomb model is difficult as even $\phi_0$ is not constant through the rubble thickness (Ettema and Schaefer, 1986). It should also be noticed, that if the assumption $\gamma = 0$ for the rubble was incorrect even further overestimation of $\phi$ would result (Liferov and Bonnemaire, 2005). Further, the control on $\gamma$ in experiments is difficult as the freeze bonds between the ice blocks start to form within very short time periods (Ettema and Schaefer, 1986; Repetto-LLamazares et al., 2011). Due to these reasons, some other experimental technique should be utilized to measure $\phi$ for the rubble, as also suggested by Serré (2011).

5.3. Boundary conditions and material modelling

The results in Sections 2.3 and 4.1 demonstrate, that the occurrence of the measured maximum load does not necessarily correspond to the occurrence of maximum load due to rubble. Still, the maximum measured load is usually first of interest when material properties of the rubble are derived from the results of a punch through test. For this derivation of the material properties, the measured load has to be divided into components due to rubble resistance and due to other sources, e.g. hydraulics and experimental equipment buoyancy.

![Fig. 11. Mean lateral force $F_x$ records from covered basin experiments with both basin widths $w_b$. In the experiments and simulations shown, rubble thickness $h$ was 0.5 m. The indenter velocity $v_i$ in the experiments was 2.5 mm/s and 10 mm/s in the simulations.](image)

![Fig. 12. Mean values of initial lateral force $F_x^0$ with their standard deviations for all rubble thicknesses $h$ used and both basin widths $w_b$ in (a) covered basin experiments and in (b) simulations of uncovered (un) and covered (co) basin experiments. In (a), $F_x^0$ values from the experiments with $v_i = 2.5$ mm/s and 5 mm/s and in (b) simulations with $v_i = 10$ mm/s were included. In addition in (a), linear fits for both basin widths $w_b$ are shown. The fits in (b) are for simulations of uncovered and covered basin experiments with both $w_b$ included.](image)
As presented in Section 4, maximum rubble load $F_{Rm}^b$ was in general reached with higher indentor displacement in the covered basin than in the uncovered basin experiments. This difference between test types was related to the evolution of the displacing rubble mass (see Figs. 8 and 9). The maximum measured indentor load occurred instead immediately after the indentor submergence and was related to hydrodynamic force acting on the indentor platen (Polojärvi and Tuhkuri, 2012) i.e. the displacement at maximum measured load was related to the experimental setup rather than rubble load.

In the aspect of material parameters in e.g. Mohr–Coulomb model (Eq. (2)), the division of the measured load into components due to cohesion, rubble buoyancy and frictional resistance is important. Out of these, the buoyant and frictional components are changed by the evolution of the deformation patterns and the change in confining force (Sections 4.2.3 and 4.3.1). In addition, the change in the boundary conditions makes the division even more complicated: the results are not only altered by rubble properties, but also by the boundary conditions of the experiment (see Figs. 8 and 9).

6. Conclusions

Laboratory scale punch through tests on floating rubble consisting of plastic blocks were conducted and simulated with a 3D discrete numerical model. The purpose was to analyse the experimental method and to validate the model. The motivation of using plastic blocks instead of ice was to simplify the interpretation of results as the plastic blocks do not freeze together. The results from the simulations were found to be in agreement with the experimental results. The main results presented in this paper are:

• The ability of the discrete numerical method to simulate laboratory scale punch through experiments on unconsolidated rubble was demonstrated: the experimental force records and the deformation patterns of the rubble during indentor stroke were in good agreement with the simulation results.

• The evolution of the deformation patterns was shown to be closely related to the load due to the rubble material. This relation was

![Fig. 13. Mean values of maximum lateral force $F_{x}^{m}$ with their standard deviations for all rubble thicknesses $h$ used and both basin widths $w_b$ in (a) covered (co) basin experiments and in (b) simulations of uncovered (un) and covered (co) basin experiments. In (a), $F_{x}^{m}$ values from the experiments with $v = 2.5$ mm/s and 5 mm/s and in (b) simulations with $v_{I} = 10$ mm/s were included. In addition, linear fits for both $w_b$ in (a) are shown. In (b), linear fits for different boundary conditions are given, as both $w_b$ yielded to approximately equal value of $F_{x}^{m}$.]

![Fig. 14. Mean values of ratio $F_{x}/F_{Rm}$ with their standard deviations of maximum lateral force $F_{x}^{m}$ to maximum rubble load $F_{Rm}^{b}$ for all rubble thickness $h$ and both basin widths $w_b$ in (a) covered (co) basin experiments and in (b) simulations of uncovered (un) and covered (co) basin experiments. The data used in the figures is given in Figs. 6 and 13.]
shown to effect the load curve during the whole indentor stroke. This demonstrates the importance of the numerical model to correctly represent the rubble deformation and its mass flow in simulations.

- The lateral force and thus the confining pressure were in general still increasing when the maximum rubble load was reached. This could lead to an overestimation of the friction angle of the rubble if the increase in confining pressure is not properly taken into account when interpreting the results.

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Appendix A. Mechanics

In the discrete numerical model for the rubble used here, the pile consisted of individual rigid blocks which interacted through contacts. Due to the relatively low force levels in the experiments, the effect of deformation of individual blocks on results was likely negligible and the assumption of rigid blocks thus applicable. The forces and moments...
acting on the simulated blocks were due to contacts, buoyancy, gravitation, and water drag. The determination of these forces is presented below. The moments can be derived from the equations below.

Contact forces between the blocks were derived using penalty function and potential contact force method described in detail in Munjiza and Andrews (2000) and Munjiza (2004). The blocks were here meshed into tetrahedrons and contact forces were solved for tetrahedrons. The total contact force acting on a block was then achieved by superposition of the forces acting on its tetrahedrons.

In the potential contact force method, a potential \( \psi \) with continuous first partial derivatives with respect to spatial coordinates is defined in every point \( P \) of each finite element volume \( \Omega \). Further, \( \psi = \psi(P) \) vanishes on finite element boundaries \( \Gamma \) for smooth collision response. Hence

\[
\psi(P) > 0, \quad P \in \Omega \backslash \psi(P) = 0, \quad P \in \Gamma. \tag{A.1}
\]

As the blocks here were each divided into tetrahedrons, the volume coordinates of these tetrahedrons were used as function for \( \psi(P) \) (Munjiza, 2004).

The contact force \( d_\psi \) applied to an infinitesimal volume element \( d\Omega \) penetrating into \( \psi \) is determined from the gradient of \( \psi \) as (Fig. A.1 a)

\[
\frac{d_\psi(P)}{d\Omega} = -s \nabla \psi(P), \tag{A.2}
\]

where \( s \) is a positive constant penalty term. The contact force \( f_c \) due to \( \psi(P) \) was determined by integration over the overlap volume \( \Omega_o \) of two colliding elements from

\[
f_c = -s \int_{\Omega_o} \nabla \psi(P) d\Omega = -s \int_{\Gamma_c} \psi(P)n d\Gamma,
\]

where Gauss’s theorem is used and \( n \) is the unit outer normal of \( \Gamma_c \) (Fig. A.1 b). The distributed load due to \( \psi \) is thus reduced to a force acting on a single point on \( \Gamma_c \) which is solved and used as a point of contact.

The inelasticity in collisions is modelled by using viscous damping relative to the rate of change in overlap volume. The viscous component of the normal force \( f_\nu \) is defined as

\[
f_\nu = c f(\nu \cdot n)n,
\]

where \( \nu \cdot n \) is the normal component of the relative velocity of the contacting blocks at the point of application of \( f_\nu \) and \( c \) is the viscous damping constant.

---

Fig. 17. Frictional force \( F_f \) as maximum rubble load \( P_f \) is reached in (a) uncovered and (b) covered simulations. Negative (positive) \( F_f \) indicates a wall resisting downwards (upwards) rubble movement. Long and short walls in the legend refer to the opposing walls at \( x = \pm 0.25 \) m and \( x = \pm 0.5 \) m, respectively (see Fig. 1).

Fig. 18. Mean values of rubble shear strength \( \tau \) with its standard deviations derived using Eq. (3) for all rubble thicknesses \( h \). The data from experiments (exp.) and simulations (sim.) with both basin widths \( w_b \) is included in the data shown in the figure.

Fig. 19. The overestimation of friction angle \( \phi \) from Eq. (2) with zero cohesion (\( c = 0 \)). In the figure \( \phi_0 \) is the friction angle yielded by the results and initial confining pressure \( c_0 \) within the rubble. Further, \( \phi_0 \) and \( \phi \) are the actual friction angle and confining stress, respectively. Shear stress \( \tau \) is assumed to have been derived from the experiments. On lateral load records (Section 4.3.1) suggested values up to \( \phi/\phi_0 = 1.5 \) shown with bold black line.
No tensile forces between colliding blocks are allowed and thus the total contact force \( f_c \) should always act on the direction of \(-\mathbf{n}\) in contacts. Hence the following condition was used:

\[
f_c = \begin{cases} 0, & \text{if } |f_c| < 0 \\ f_c - f_{\text{fr}} & \text{else}
\end{cases}
\]

(A.5)

Sliding friction was modelled by using dynamic Coulomb friction model. Frictional force \( f_{\text{fr}} \) was solved from

\[
f_{\text{fr}} = -\mu |v_c - v_s| \cdot \mathbf{n}
\]

(A.6)

where \( \mu \) is the friction coefficient and \( v_c - v_s \cdot \mathbf{n} \) is the tangential component of the relative velocity of contacting blocks at the point of contact.

The determination of the buoyant force due to the water and gravitational force acting on the blocks in the simulations was fairly straightforward. For a partially submerged block shown in Fig. A.2, the buoyant force was determined by integrating hydrostatic pressure over the submerged part \( A_s \) of the block surface \( A \). For a block with its total volume submerged or above the water level, buoyant force was applied on block centroid. The gravitational force was always applied on the block centroid as further shown in Fig. A.2.

The external force \( f_e \) due to buoyancy and gravitation, for a block was determined from the following equation derived in three parts according to the block position (see Fig. A.2 for directions):

\[
f_e = \begin{cases} \rho_b V \mathbf{g} \cdot \mathbf{j}, & \text{if above water level.} \\
\rho_w A_s (y - y_w) \cdot (-\mathbf{n}) \cdot \mathbf{j} \cdot d\mathbf{j} + \rho_b V \mathbf{g} \cdot \mathbf{j}, & \text{if partially submerged.} \\
\rho_b A_s \mathbf{g} \cdot \mathbf{j}, & \text{if submerged.}
\end{cases}
\]

(A.7)

In the previous equation \( V \) is the volume of the block, \( \rho_b \) and \( \rho_w \) are the material densities of the blocks and water, respectively, \( g \) is the gravitational acceleration, \( y_w \) the water line, \( \mathbf{n} \) the block surface unit normal and \( \mathbf{j} \) is a unit vector parallel to global y-axis.

Rigorous modelling of the hydrodynamics in the experiments would be challenging and was not attempted here. Instead, only a simplified model for the water drag acting on the blocks was implemented and used in the simulations. The model was derived to approximate the pressure drag on the blocks. The pressure drag is commonly solved from \((\text{Granger, 1995})\)

\[
D = C_d \left( \frac{1}{2} \rho w U^2 \right) A_p
\]

(A.8)

where \( C_d \) is the drag coefficient, \( U \) the velocity and \( A_p \) the projected area (area projected into the direction of \( U \)) of a block. The direction of \( D \) in the previous equation is opposite to \( U \).

For the approximation of \( D \) in Eq. (A.8), the drag force was modelled as a distributed load acting on part of the submerged block surface \( A_s \) with surface velocity \( \mathbf{u}_s \) having \( \mathbf{u}_s \cdot \mathbf{n} > 0 \) i.e. on the part of \( A_s \) with \( \mathbf{u}_s \) having a positive normal component. Total drag force \( f_d \) acting on the block was then achieved from the integral

\[
f_d = \left\{ \begin{array}{ll}
\frac{1}{2} C_d & 0 \\
0 & \text{if } \mathbf{u}_s \cdot \mathbf{n} > 0 \\
\text{otherwise.}
\end{array} \right.
\]

(A.9)

In the case of pure translation of the block \((\mathbf{u}_s = \text{const})\) on the part of \( A_s \) into the direction \( \mathbf{u}_s \), the previous equation yields to \( f_d \) with absolute value equal to \( D \) in Eq. (A.8).

Appendix B. Effect of loading rate

The simulations presented in the paper were run with indenter velocity \( v_i = 10 \text{ mm/s} \), whereas in the experiments the indenter velocities \( v_i = 2.5 \text{ mm/s} \) and \( v_i = 5 \text{ mm/s} \) were used. For reference, simulations with \( v_i = 5 \text{ mm/s} \) were run with negligible change in results. This is shown by mean rubble load records \( F_{RB} \) from simulations with \( v_i = 10 \text{ mm/s} \) and \( 5 \text{ mm/s} \) in Fig. B.1. The figure clearly shows, that the \( F_{RB} \) records with these indenter velocities yielded virtually equal \( F_{RB} \) records.

![Fig. A.1](image1.png)  
Fig. A.1. (a) An infinitesimal volume element \( d\Omega_o \) at point \( P \) penetrating into a finite element with volume \( \Omega \) and (b) the overlap volume \( \Omega_c \) of two elements.

![Fig. A.2](image2.png)  
Fig. A.2. The gravitational and buoyant force acting on a partially submerged block. Whereas the gravitational force is always constant and applied on block centroid \( \mathbf{x} \), the buoyant force is dependent on submerged part of the block. Submerged part of the block surface \( A_s \) is shown in gray. The figure also shows the directions of gravitational acceleration \( g \) and unit vector \( \mathbf{j} \) both aligning with global y-axis.

![Fig. B.1](image3.png)  
Fig. B.1. Mean rubble load \( F_{RB} \) records from the simulations with indenter velocities \( v_i = 5, 10 \) and \( 40 \text{ mm/s} \) of (a) uncovered and (b) covered experiments. The data is from simulations with rubble thickness \( h = 0.5 \text{ m} \) and basin width \( w_b = 1.5 \text{ m} \).
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