Publication III

M. Ojanen, M. Shpak, P. Kärhä, R. Leecharoen, and E. Ikonen. 2009. Uncertainty evaluation for linking a bilateral key comparison with the corresponding CIPM key comparison. Metrologia, volume 46, number 5, pages 397-403.

http://www.iop.org/journals/met
http://stacks.iop.org/met/46/397

Metrologia **46** (2009) 397–403 doi:10.1088/0026-1394/46/5/002

Uncertainty evaluation for linking a bilateral key comparison with the corresponding CIPM key comparison

M Ojanen[1], M Shpak[1], P Kärhä[1], R Leecharoen[2] and E Ikonen[1,3]

[1] Helsinki University of Technology (TKK), Metrology Research Institute, PO Box 3000, FI-02015 TKK, Finland
[2] National Institute of Metrology, Thailand (NIMT), Kluang Luang Patumthanee 12120, Thailand
[3] Centre for Metrology and Accreditation (MIKES), PO Box 9, FI-02151 Espoo, Finland

Received 8 January 2009, in final form 18 May 2009
Published 11 June 2009
Online at stacks.iop.org/Met/46/397

Abstract
A method for evaluating the uncertainty in linking a bilateral key comparison to another key comparison with several participants is presented theoretically and demonstrated with an actual comparison. Equations are derived for the uncertainties of the unilateral and mutual degrees of equivalence for the linked participant in the bilateral comparison. It is shown that the uncertainty components related to uncorrelated effects in the measurements of the linking participant play a critical role in determining the additional uncertainties due to the linking process. As a practical example, the results are applied to a bilateral comparison of the spectral irradiance scales of MIKES (Finland) and NIMT (Thailand) in the spectral range from 290 nm to 900 nm.

S This article has associated online supplementary data files.

1. Introduction

The quality of the calibration services of the national metrology institutes (NMIs) is routinely assessed and assured with a series of comparison measurements. To obtain a full coverage of all NMIs in the world, different types of comparisons are needed. Key comparisons of the CIPM (International Committee of Weights and Measures) have a few participants from each regional metrology organization. It has been agreed that only the CIPM key comparison provides a key comparison reference value (KCRV). All NMIs are offered a possibility to link their measurement results to the KCRV through bilateral and regional key comparisons with linking laboratories that have taken part in the CIPM key comparison.

Combining the results of two comparisons is a topic that needs further research. The linking uncertainties have been studied earlier in the metrology fields of, e.g., electricity [1], acoustics [2] and length [3]. It appears that every metrology field has its own specific features, although the theory behind linking is general as presented in the above references. Quantitative evaluation of the various uncertainty components due to the linking process would help in optimization of the linkage. Such uncertainties include contributions due to the instability of the transfer standards and the reproducibility of the results of the linking NMIs.

In the spectral irradiance key comparison CCPR-K1.a, the participants reported two types of uncertainties, which are related either to correlated or uncorrelated effects in the measurements [4]. The contributions related to the correlated effects reproduce their (unknown) values systematically from measurement to measurement over an extended period of time required for the comparison, whereas the uncorrelated contributions vary randomly, either between the individual measurements or between the measurement rounds. The separation of the uncorrelated effects from the combined uncertainty is useful for the uncertainty evaluation in linking the results of different comparisons [5, 6]. In addition, the uncertainty components nominally related to correlated effects may need to be considered if the linking comparison takes place after a time interval considerably longer than the duration of the CIPM key comparison [5].

The importance of the division into the uncertainties related to uncorrelated and correlated effects in linking comparisons has also been recognized in other metrology fields. Delahaye and Witt linked two electricity comparisons using the uncertainty related to the imperfect reproducibility of the results of a participant in the time period spanning the two comparison measurements [1]. The uncertainties related to correlated effects in the measurements produce only a negligible contribution in gauge block measurements by optical interferometry [3]. Also in accelerometer calibrations the correlations are not critical in the sense that they are expected to have a small influence on the resulting degrees of equivalence [2]. The methods presented in [1–3] have been applied to comparisons in electricity, acoustics and length, but the theory behind those methods is not restricted to a particular field of metrology.

This work presents quantitatively several uncertainty contributions which need to be taken into account in linking comparisons in photometry and radiometry. The main results consist of equations for the uncertainties of the unilateral and mutual degrees of equivalence for the linked participant in a bilateral comparison. These equations quantify the additional uncertainty due to the linking process, including contributions of the uncertainty of the KCRV, the transfer uncertainties of the comparisons and the uncertainties related to the uncorrelated effects in the measurements of the linking NMI. As explained in appendix A, it is straightforward to extend the analysis of the bilateral comparison to a regional comparison by assuming that each participant of the regional comparison has carried out a bilateral comparison with the link NMIs. The results are applied to a practical example of a bilateral comparison EURAMET.PR-K1.a.1 between the Centre for Metrology and Accreditation (MIKES), Finland, and the National Institute of Metrology (NIMT), Thailand [7]. One of the main conclusions of this work is that the uncertainty components related to uncorrelated effects in the measurements of the linking NMIs dominate the additional uncertainty due to the linking process. This finding is of special importance if NMIs with low uncertainties need to seek linkage to the KCRV via bilateral or regional key comparisons.

2. Method for linking a bilateral comparison to the key comparison

2.1. CIPM key comparison

We consider a star-like CIPM key comparison consisting of cycles where the pilot and the participant measure a set of artefacts. Each participant measures a designated set of artefacts specific to it and the pilot, and at least two cycles are carried out over a period of several years. In comparisons of spectral quantities, measurements at different wavelengths are considered as separate comparisons. The symbols and uncertainties are described here in the same way as in the *Guide to the Expression of Uncertainty in Measurement* (GUM) [8]. We denote by capital letters X_i, Ξ_i, $E_{\mathrm{kc},i}, \ldots$, random variables, whereas the corresponding expectation values are denoted by lower case letters x_i, ξ_i, $e_{\mathrm{kc},i}, \ldots$. Corresponding to the error contributions identified by capital letters $E_{\mathrm{subscript}}$, we denote their expectation values by lower case characters $e_{\mathrm{subscript}}$ and the associated standard uncertainties by $u(e_{\mathrm{subscript}})$. All error contributions are unknown and thus these types of model equations are useful only for the uncertainty evaluation.

The result X_i of NMI i in the key comparison can be modelled as

$$X_i = \Xi_i + E_{\mathrm{kc},i} = \Xi_{\mathrm{true},i} + E_{\mathrm{uc},i} + E_{\mathrm{cor},i} + E_{\mathrm{kc},i}, \qquad (1)$$

where $E_{\mathrm{kc},i}$ is a zero-mean random variable corresponding to the transfer error of the key comparison for participant i ($i = 1, 2, \ldots, N$). The error contribution $E_{\mathrm{kc},i}$ is related to the transfer uncertainty of the comparison, such as the artefact instability uncertainty in CCPR-K1.a [4]. Random variable $\Xi_i = \Xi_{\mathrm{true},i} + E_{\mathrm{uc},i} + E_{\mathrm{cor},i}$ includes all the error contributions related to uncorrelated and correlated (systematic) effects in the measurements at NMI i. Subscripts 'uc' and 'cor' refer to uncorrelated and correlated contributions, respectively. We also use the idealized concept of 'true quantity value' $\Xi_{\mathrm{true},i}$, with zero uncertainty, to better explain our approach to count all uncertainty components, as suggested in the *International Vocabulary of Metrology* (VIM) [9]. The standard uncertainties corresponding to equation (1) are related by

$$u^2(x_i) = u_i^2 + u_{\mathrm{kc}}^2, \qquad (2)$$

where the transfer uncertainty $u_{\mathrm{kc}} = u(e_{\mathrm{kc},i})$, as determined by the analysis of the CIPM key comparison results, is assumed to be the same for all participants and

$$u_i^2 = u_{\mathrm{uc},i}^2 + u_{\mathrm{cor},i}^2 \qquad (3)$$

is the measurement uncertainty reported by participant i, consisting of the uncertainty components related to uncorrelated $[u_{\mathrm{uc},i} = u(e_{\mathrm{uc},i})]$ and correlated $[u_{\mathrm{cor},i} = u(e_{\mathrm{cor},i})]$ effects. The underlying errors causing the uncertainty components due to correlated effects reproduce their unknown values systematically from measurement to measurement over an extended period of time, whereas the uncorrelated effects vary randomly between different measurements.

The KCRV X_{ref} is calculated as the weighted mean of the individual results X_i with appropriately determined weights w_i. Finally, the unilateral and mutual degrees of equivalence are obtained as

$$D_i = X_i - X_{\mathrm{ref}} = \Xi_{\mathrm{true},i} + E_{\mathrm{uc},i} + E_{\mathrm{cor},i} + E_{\mathrm{kc},i} - X_{\mathrm{ref}} \qquad (4)$$

and $D_{ij} = X_i - X_j$, respectively. The unilateral degree of equivalence describes the difference between the measurement result of the NMI and the KCRV, while the mutual degrees of equivalence D_{ij} specify the differences of the measurement results of two participants of the CIPM key comparison.

2.2. Bilateral key comparison

Let us next consider a situation where another NMI α that has not taken part in the CIPM key comparison wishes to use a bilateral comparison to link its measurements to the KCRV through NMI i. A set of transfer standards is then calibrated by both NMI i and NMI α in such a way that the stability of these transfer standards can be checked. When the link NMI carries out measurements before and after NMI α, the unilateral degree of equivalence for NMI α via NMI i is

$$D_{\alpha(i)} = D_i + \Delta_{\alpha i}, \tag{5}$$

where

$$\Delta_{\alpha i} = \Xi_\alpha - \left(\Xi'_i + \Xi''_i\right)/2 + E_{\mathrm{bc}} \tag{6}$$

is the difference between the results of NMI α and NMI i in the bilateral comparison. Equation (6) takes into account the transfer error E_{bc} of the bilateral comparison and a potential difference in the results Ξ_i, Ξ'_i and Ξ''_i of NMI i in the CIPM and bilateral comparisons. The primed symbols denote the results of the link NMI during the two measurement rounds of the bilateral comparison. If instead NMI α carries out the repeat measurement, equation (5) remains unchanged when $\left(\Xi'_i + \Xi''_i\right)/2$ is replaced by Ξ'_i and Ξ_α is replaced by $\left(\Xi'_\alpha + \Xi''_\alpha\right)/2$ in equation (6).

Substituting equations (4) and (6) into equation (5) gives (see appendix B, available from the online version of this journal, for a more detailed derivation of the equation)

$$D_{\alpha(i)} = \Xi_\alpha + I_i - X_{\mathrm{ref}} + E_{\mathrm{kc},i} + E_{b,i} + E_{r,i}, \tag{7}$$

where

$$I_i = \Xi_{\mathrm{true},i} - \left(\Xi'_{\mathrm{true},i} + \Xi''_{\mathrm{true},i}\right)/2 \tag{8}$$

corresponds to the linking invariant defined in [3]. The combined error due to the transfer and uncorrelated effects in the measurements of NMI i in the bilateral comparison is given by

$$E_{b,i} = E_{\mathrm{bc}} - E'_{\mathrm{uc},i}/2 - E''_{\mathrm{uc},i}/2 \tag{9}$$

and

$$E_{r,i} = E_{\mathrm{uc},i} + E_{\mathrm{cor},i} - E'_{\mathrm{cor},i} \tag{10}$$

is the error related to the reproducibility between the results of NMI i in the bilateral comparison and in the CIPM key comparison assuming that $E''_{\mathrm{cor},i} = E'_{\mathrm{cor},i}$. Note that the error components related to correlated effects in equation (10) are cancelled in $E_{r,i}$ if the bilateral comparison is carried out shortly after the CIPM key comparison, when the division used in the latter comparison into uncertainty components due to correlated and uncorrelated effects is still valid.

2.3. *Uncertainty of the unilateral degree of equivalence for the linked participant*

Calculation of the uncertainty of the degree of equivalence in equation (7) needs to take into account that the terms $-X_{\mathrm{ref}}$ and $+E_{\mathrm{kc},i}$, with expectation values $-x_{\mathrm{ref}}$ and $+e_{\mathrm{kc},i}$, are not independent in the weighted mean

$$x_{\mathrm{ref}} = \sum_j w_j x_j = \sum_j w_j \left(\xi_j + e_{\mathrm{kc},j}\right), \tag{11}$$

where the sum of weights w_j is equal to 1 ($j = 1, 2, \ldots, N$). Using equation (11) and grouping terms we obtain [10]

$$-x_{\mathrm{ref}} + e_{\mathrm{kc},i} = -\sum_j w_j \xi_j - \sum_{j\neq i} w_j e_{\mathrm{kc},j} + (1 - w_i)\, e_{\mathrm{kc},i} \tag{12}$$

for the expectation values of the coupled terms in equation (7). All terms corresponding to the defined random variables on the right-hand side of equation (12) are now related to uncorrelated effects. Thus the standard uncertainty can be calculated as

$$\begin{aligned} u^2(-x_{\mathrm{ref}} + e_{\mathrm{kc},i}) &= \sum_j w_j^2 u_j^2 + \sum_{j\neq i} w_j^2 u_{\mathrm{kc}}^2 + (1 - w_i)^2 u_{\mathrm{kc}}^2 \\ &= u^2(x_{\mathrm{ref}}) + (1 - 2w_i) u_{\mathrm{kc}}^2, \end{aligned} \tag{13}$$

where the equation

$$u^2(x_{\mathrm{ref}}) = \sum_j w_j^2 (u_j^2 + u_{\mathrm{kc}}^2) \tag{14}$$

resulting from equation (11) is used.

When the KCRV is calculated as an ordinary weighted mean, it follows that $w_i = u^2(x_{\mathrm{ref}})/u^2(x_i)$. For key comparisons in photometry and radiometry, it is agreed by the CCPR (Consultative Committee for Photometry and Radiometry) that the weighted mean with cut-off is used as the KCRV. The cut-off value for the uncertainty u_{co} is determined as the average of the standard uncertainties u_i of those participants that reported standard uncertainties less than or equal to the median standard uncertainty of all participants. For the KCRV calculation and for the use of equation (13), the weight w_i is then proportional to $1/[u_{\mathrm{kc}}^2 + \max(u_i^2, u_{\mathrm{co}}^2)]$. When using the ordinary weighted mean, the value of w_i can in principle approach 1, whereas $w_i < 0.325$ when using the weighted mean with cut-off in a comparison with ten or more participants (see appendix C, available from the online version of this journal). With the result of equation (13), the standard uncertainty of the unilateral degree of equivalence of equation (7) is (see appendix B online for detailed derivation)

$$u^2(d_{\alpha(i)}) = u_\alpha^2 + u^2(x_{\mathrm{ref}}) + (1 - 2w_i) u_{\mathrm{kc}}^2 + u_{b,i}^2 + u_{r,i}^2, \tag{15}$$

where u_α is the uncertainty of the measurements at NMI α, the uncertainty of the linking invariant I_i is fully described by the various error contributions and

$$u_{b,i}^2 = u_{\mathrm{bc}}^2 + u_{\mathrm{uc},i}^2/2 \tag{16}$$

is the uncertainty of the bilateral comparison consisting of the uncertainty u_{bc} corresponding to the transfer error E_{bc} of equation (6) and of the contribution by the uncertainty components related to uncorrelated effects in the measurements of the link NMI. The uncertainty component related to the reproducibility of the results of the link NMI in the two comparisons is denoted by $u_{r,i}$. If the time interval between the linked comparisons is short, it can be argued that the systematic components in the results of NMI i are cancelled and $u_{r,i}$ is equal to $u_{\mathrm{uc},i}$. On the other hand, if the time interval between the comparisons is considerably longer than the duration of the CIPM key comparison, an estimate $u_{r,i} > u_{\mathrm{uc},i}$ may be better justified, taking into account the potentially non-zero expectation value of $E_{\mathrm{cor},i} - E'_{\mathrm{cor},i}$ (see equation (10)).

The difference $\Xi'_i - \Xi''_i$ of the results of NMI i in the bilateral comparison may be used to give an estimate of the instability of the transfer standards, where the standard uncertainty of this estimate is $\sqrt{2}\,u_{\mathrm{uc},i}$. The simple average of the results of NMI i in equation (6) provides a straightforward way to account for this instability. In some cases, the uncertainty contribution by insufficient correction of this instability may be included in u_{bc}, where it contributes to $u_{b,i}$ in equations (15) and (16). If NMI α determines the stability of the transfer standards by a repeat measurement and NMI i measures only once,

$$u_{b,i}^2 = u_{\mathrm{bc}}^2 + u_{\mathrm{uc},i}^2 \tag{17}$$

should be used in equation (15) instead of equation (16).

2.4. Mutual degrees of equivalence and optimization of uncertainties due to linking

The mutual degree of equivalence between NMI α and any participant $j \neq i$ of the CIPM key comparison is

$$\begin{aligned} D_{\alpha j} &= D_{\alpha(i)} - D_j \\ &= \Xi_\alpha - \Xi_j + I_i - E_{\mathrm{kc},j} + E_{\mathrm{kc},i} + E_{b,i} + E_{r,i}, \end{aligned} \tag{18}$$

where the terms $\pm X_{\mathrm{ref}}$ in equations (4) and (7) for $D_{\alpha(i)}$ and D_j cancel each other. Since all the terms of equation (18) are related to uncorrelated effects, the standard uncertainty of the mutual degree of equivalence can be calculated in a straightforward way,

$$u^2(d_{\alpha j}) = u_\alpha^2 + u_j^2 + 2u_{\mathrm{kc}}^2 + u_{b,i}^2 + u_{r,i}^2. \tag{19}$$

When $j = i$, several contributions in equation (18) cancel each other, resulting in

$$u^2(d_{\alpha i}) = u_\alpha^2 + u_{\mathrm{cor},i}^2 + u_{b,i}^2. \tag{20}$$

The additional uncertainty due to linkage in the degrees of equivalence of NMI α is mainly determined by the sum $u_{b,i}^2 + u_{r,i}^2$ in equations (15) and (19). The lower limit for both of these terms is determined by the uncertainties related to uncorrelated effects in the measurements of NMI i because $u_{b,i} \geqslant u_{\mathrm{uc},i}/\sqrt{2}$ and $u_{r,i} \geqslant u_{\mathrm{uc},i}$ (see equations (9) and (10)). For minimization of the additional uncertainties due to linking, it is thus important that the uncertainties of the link NMIs related to uncorrelated effects are small. The lower limit for $u_{b,i}$ can be reduced by repeated measurements in the bilateral comparison, whereas the lower limit for $u_{r,i}$ is fixed by the CIPM key comparison. It is a basic property of the division into uncertainties related to the correlated and uncorrelated effects that the contribution by the uncorrelated components can be reduced by investing more effort in the comparison and repeating the measurement cycles several times.

3. Application of the uncertainty evaluation to a real comparison

The main results of the uncertainty evaluation in linking a bilateral comparison to the CIPM key comparison are given in equations (15)–(17) and (19). A practical example is useful to demonstrate the application of the equations presented. Such an example is provided by a recent bilateral comparison of

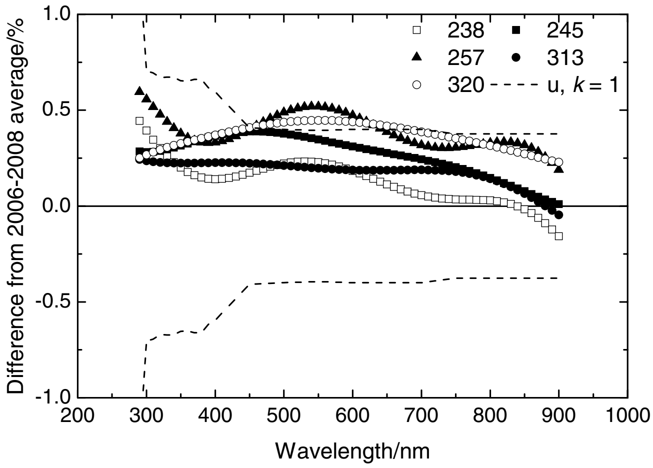

Figure 1. Relative difference of the MIKES calibrations of 2006 from the average of two subsequent calibrations based on spectral irradiance scale realizations in 2006 and 2008. The numbers 238, 245, etc indicate the labels of the measured lamps. The lamps marked with filled symbols were transported to NIMT and back, while the ones marked with open symbols stayed at MIKES. The dashed lines indicate the standard uncertainty related to uncorrelated effects in the measurements of MIKES.

spectral irradiance scales between MIKES and NIMT in the spectral range from 290 nm to 900 nm [7]. The pilot and link NMI i of the bilateral comparison is MIKES. The linked NMI α is NIMT. The results of NIMT are linked through MIKES to the spectral irradiance key comparison CCPR-K1.a [4], carried out in 2000–2003.

In CCPR-K1.a, the uncertainties of the participants were divided into three categories: uncertainties related to uncorrelated effects, to correlated effects within a round and to fully correlated effects. For our purposes, the uncertainties related to correlated effects within a round can be treated as uncorrelated effects between rounds. We therefore combined the uncertainties related to uncorrelated effects and the uncertainties related to correlated effects within a round quadratically to obtain the values of $u_{\mathrm{uc},i}$ for MIKES.

The bilateral comparison was carried out through calibrations of a group of three transfer standard lamps prepared by MIKES. The lamps were labelled as BN-9101-245, -257 and -313. After measurements at MIKES in December 2006, the lamps were carried by hand to NIMT for calibration. After the calibration at NIMT in November 2007, the lamps were carried by hand back to MIKES for repeat measurement in February 2008 to monitor for possible drifts.

3.1. Uncertainties due to linking

Figure 1 shows the difference of the MIKES calibration results of 2006 from the average of the results of 2006 and 2008 [7]. The difference is comparable to the value of $u_{\mathrm{uc},i}$ of MIKES [4] indicated by the dashed lines. The two monitor lamps (open symbols) that stayed at MIKES show a similar change in their spectral irradiance as the three transfer standard lamps used in the comparison. Thus, there are no indications that the lamps would have changed during the comparison measurements. The relative transfer uncertainty $u_{\mathrm{bc}} = 0.0017$

Table 1. Standard uncertainties of NIMT (u_α), uncertainties related to uncorrelated effects in the measurements of MIKES ($u_{\mathrm{uc},i}$), uncertainties of the bilateral comparison ($u_{b,i}$), uncertainties of MIKES including the long-term reproducibility of the scale ($u_{r,i}$), uncertainties of KCRV [$u(x_{\mathrm{ref}})$] and transfer uncertainties of the CIPM key comparison (u_{kc}) at a few representative wavelengths.

	Relative standard uncertainty × 100					
Wavelength/nm	u_α	$u_{\mathrm{uc},i}$	$u_{b,i}$	$u_{r,i}$	$u(x_{\mathrm{ref}})$	u_{kc}
290	2.55	1.24	0.90	1.86	0.30	0.25
300	2.53	0.70	0.52	1.05	0.28	0.26
400	2.49	0.60	0.45	0.89	0.20	0.18
500	1.49	0.40	0.33	0.40	0.16	0.20
600	1.86	0.40	0.33	0.40	0.15	0.18
800	1.84	0.38	0.32	0.38	0.15	0.13
900	3.02	0.38	0.32	0.38	0.15	0.14

of the bilateral comparison includes a component due to the presence/absence of the stray-light reducing baffle in the MIKES/NIMT measurements.

Columns two and three in table 1 show examples of the standard uncertainty u_α of the NIMT measurements and of the uncertainty $u_{\mathrm{uc},i}$ related to uncorrelated effects in the measurements of MIKES [4]. The standard uncertainty $u_{b,i}$ of the bilateral comparison in table 1 consists of the uncertainty u_{bc} and of the uncertainty related to uncorrelated effects in MIKES measurements as shown in equation (16). For this bilateral comparison with NIMT, it was possible to obtain information on the MIKES scale reproducibility, $u_{r,i}$, with the help of an informal comparison with A*STAR, Singapore [11]. In the wavelength range from 450 nm to 900 nm, the results of the informal comparison [11] and key comparisons CCPR-K1.a [4] and CCPR-K1.a.1 [6] are well in agreement within the uncertainties calculated on the basis of the uncertainties related to uncorrelated effects in the measurements of the NMIs. For the wavelength range from 290 nm to 400 nm, the average difference of 2% is somewhat larger than the average standard uncertainty of 1.3%. Since the deviation is systematic over almost the whole wavelength range, agreement within coverage factor $k = 2$ was not considered satisfactory and the uncertainties related to uncorrelated effects were multiplied by a factor determined as the ratio of 1.5 of the average difference to the average standard uncertainty. This process restored approximate agreement with the assumption that the scales of MIKES and A*STAR have not changed between 2003 and 2008 by more than expected on the basis of random uncorrelated effects. Accordingly, column five of table 1 lists values $u_{r,i} = 1.5 \times u_{\mathrm{uc},i}$ for the spectral range from 290 nm to 400 nm and $u_{r,i} = u_{\mathrm{uc},i}$ for the longer wavelengths.

Table 1 also shows the approximate uncertainty of the KCRV, $u(x_{\mathrm{ref}})$, and the transfer uncertainty of the key comparison, u_{kc}. The latter uncertainty is obtained from the final report of the CCPR-K1.a as the artefact instability uncertainty [4]. The weight of MIKES is $w_i \approx 0.1$. With the parameters of table 1, equations (15) and (19) can be used to calculate the uncertainties of the degrees of equivalence of the linked NMI α.

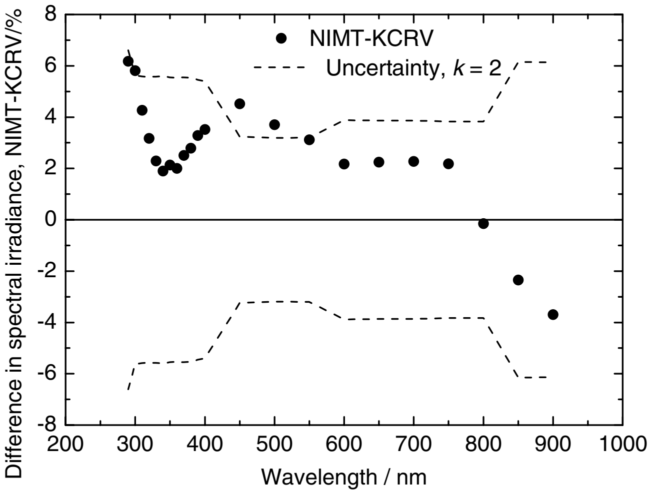

Figure 2. Degrees of equivalence of NIMT at different wavelengths. Calculation of the expanded uncertainty is based on equation (15).

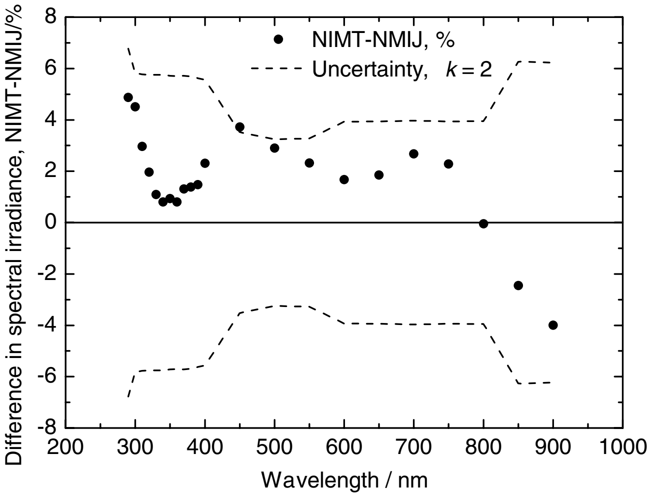

Figure 3. Mutual degrees of equivalence between NIMT and NMIJ at different wavelengths. Calculation of the expanded uncertainty is based on equation (19).

3.2. *Results of the comparison*

The difference between the results of NIMT and MIKES in the bilateral key comparison was calculated as the average of the results obtained with the three transfer standard lamps [7]. Figure 2 shows the relative difference between the NIMT results and the KCRV as given by the expectation values corresponding to equation (5). The expanded uncertainty in figure 2 is calculated using equation (15). Figure 2 illustrates the main results of the bilateral key comparison quantifying the NIMT unilateral degrees of equivalence $d_{\alpha(i)}$ and their uncertainties at 22 different wavelengths.

Use of equation (19) is illustrated by calculating the uncertainty of the degrees of equivalence between NIMT and NMIJ (Japan) as shown in figure 3. NMIJ is the source of traceability for NIMT and thus u_j in equation (19) is taken as the uncertainty related to uncorrelated effects in the measurements of NMIJ as listed in the final report of CCPR-K1.a [4]. This avoids double counting of the uncertainty components related to correlated effects in the measurements of NMIJ since they are already included in the spectral irradiance scale uncertainty of NIMT.

4. Conclusions

As a specific conclusion related to the example comparison we repeat the findings of [7]: the results of the bilateral comparison between NIMT and MIKES emphasize the importance of comparison measurements even in the case where an NMI takes traceability from another NMI. NIMT found a deviation at the wavelength of 450 nm, which may indicate an unknown effect related to the change of the detector of the spectroradiometer close to that wavelength. At all other wavelengths, the NIMT results were acceptable within the uncertainty of the mutual degrees of equivalence between NIMT and NMIJ.

A general method was presented to quantify the uncertainty contributions in linking results of two related comparisons—a CIPM key comparison and a corresponding bilateral key comparison. The method takes into account the contribution of the uncertainty of the KCRV, the transfer uncertainties of the comparisons and the uncertainty related to uncorrelated effects in the measurements of the linking NMI. As the main result, equations for the uncertainties of the unilateral and mutual degrees of equivalence for the linked participant in the bilateral comparison are given. Dividing the uncertainties of the linking NMI into uncertainties due to correlated and uncorrelated effects serves three purposes. (1) The additional uncertainty due to linking can be realistically estimated, (2) when selecting the linking NMIs for bilateral and regional key comparisons, NMIs with small uncorrelated effects in the measurements should be favoured and (3) it can be ensured that the uncertainties related to correlated effects are only taken into account once with a linked NMI taking traceability from a participant in the CIPM key comparison. It is straightforward to extend the uncertainty evaluation to a regional key comparison with multiple participants and link NMIs (see appendix A). The method should also be applicable to all comparisons regardless of the metrology field.

The uncertainty evaluation was demonstrated here with a bilateral spectral irradiance comparison between NIMT and MIKES. The measurement results of NIMT were compared with the KCRV of CCPR-K1.a using MIKES as the link NMI. The uncertainty related to uncorrelated effects in the measurements of the link NMI was found to account for more than 80% of the additional uncertainty due to the linking process in the degrees of equivalence of the linked NMI. In this case, the NIMT uncertainty typically contributed 90% of the uncertainty of the unilateral degrees of equivalence, but this cannot be taken as a general rule in the future, when the participation in CIPM key comparisons is limited in almost all metrology fields. If an NMI with uncertainty below the cut-off uncertainty in CCPR-K1.a [4] were the linked NMI, the uncertainties of their degrees of equivalence would be largely affected by the uncertainty related to uncorrelated effects in the measurements of the linking NMI. This important conclusion is valid with any linking NMI selected from the participants of CCPR-K1.a.

It is emphasized that the above considerations and the related theory have not yet been presented in the literature for the cases where the uncertainties related to uncorrelated and correlated effects in the measurements of the link NMI have comparable magnitudes. In [3], a general approach is presented which is extended here to include a detailed description of the linking uncertainty evaluation with an example of a linking comparison in photometry and radiometry. Although uncertainties related to uncorrelated effects, such as those caused by wringing of gauge blocks [3] in length metrology, may dominate in some cases, justifying simpler analysis methods, the exploitation of correlations to obtain more reliable linking, using the methods presented here, is recommended for those comparisons where a reliable classification into uncertainties related to correlated and uncorrelated effects is available.

Appendix A. Linking a regional key comparison

Extension of the uncertainty evaluation method to a regional key comparison is presented in this appendix. The regional comparison is carried out with participants $\alpha, \beta, \ldots$ and $i, j, \ldots$, where the link NMIs $i, j, \ldots$ are also participants of the CIPM key comparison. The regional key comparison provides similar differences $\Delta_{\alpha i}$ and Δ_{ij} between the results of the participants as would be obtained in a set of bilateral comparisons. The link to the CIPM key comparison can then be analysed by assuming that each NMI $\alpha, \beta, \ldots$, has carried out a bilateral comparison with the link NMIs.

An auxiliary reference value can be formed using the measurement result Ξ_p of the pilot NMI p of the regional key comparison. Then the results of the comparison can be listed as deviations

$$\Delta_{\alpha p} = \Xi_\alpha - \Xi_p + E_{\mathrm{rc},\alpha} \tag{A1}$$

of the participants, where $E_{\mathrm{rc},\alpha}$ is a random variable corresponding to the transfer error of the regional key comparison for participant α. The bilateral differences between the participants' results are obtained as

$$\Delta_{\alpha i} = \Delta_{\alpha p} - \Delta_{ip} = \Xi_\alpha - \Xi_i' + E_{\mathrm{rc},\alpha} - E_{\mathrm{rc},i}, \tag{A2}$$

where the auxiliary reference value Ξ_p disappears, and it is assumed that only the pilot performs the measurements twice to ensure the stability of the transfer standards. Using equations (4), (5) and (A2) and taking into account the random error $E_{r,i}$ of equation (10) due to lack of stability between the NMI i results in the regional key comparison and in the CIPM key comparison, the unilateral degree of equivalence for NMI α through link NMI i is obtained as

$$D_{\alpha(i)} = \Xi_\alpha + I_i - X_{\mathrm{ref}} + E_{\mathrm{kc},i} + E_{\mathrm{rc},\alpha} - E_{\mathrm{rc},i} - E'_{\mathrm{uc},i} + E_{r,i}, \tag{A3}$$

where $I_i = \Xi_{\mathrm{true},i} - \Xi'_{\mathrm{true},i}$. As compared with equation (7), the term E_{bc} of a bilateral key comparison is replaced by $E_{\mathrm{rc},\alpha} - E_{\mathrm{rc},i}$ in equation (A3).

The values $D_{\alpha(i)}, D_{\alpha_{(j)}}, \ldots$, are combined with weights $\omega_i, \omega_j, \ldots$, to obtain the unilateral degree of equivalence D_α

of NMI α. For two link NMIs ($\omega_i + \omega_j = 1$), the unilateral degree of equivalence is

$$D_\alpha = \omega_i D_{\alpha(i)} + \omega_j D_{\alpha(j)} = \Xi_\alpha + \omega_i I_i + \omega_j I_j - X_{\rm ref} + E_{{\rm rc},\alpha} + \omega_i(E_{{\rm kc},i} - E_{{\rm rc},i} - E'_{{\rm uc},i} + E_{r,i}) + \omega_j(E_{{\rm kc},j} - E_{{\rm rc},j} - E'_{{\rm uc},j} + E_{r,j}). \quad \text{(A4)}$$

Neglecting the common parts in $-X_{\rm ref}$ and $\omega_i E_{{\rm kc},i} + \omega_j E_{{\rm kc},j}$ terms, the uncertainty of the unilateral degree of equivalence is given by

$$u^2(d_\alpha) = u_\alpha^2 + u^2(x_{\rm ref}) + (\omega_i^2 + \omega_j^2)u_{\rm kc}^2 + (1 + \omega_i^2 + \omega_j^2)u_{\rm rc}^2 + \omega_i^2(u_{{\rm uc},i}^2 + u_{r,i}^2) + \omega_j^2(u_{{\rm uc},j}^2 + u_{r,j}^2), \quad \text{(A5)}$$

where $u_{\rm rc}$ is the transfer uncertainty related to the results of the regional comparison. Comparing equation (A5) with $\omega_i = \omega_j = 1/2$ with the result of equation (15), it is seen that the uncertainty components due to the CIPM key comparison transfer and instability of the link NMI results are somewhat reduced, whereas the regional/bilateral transfer uncertainty term is somewhat increased in equation (A5) (assuming that $u_{\rm rc} = u_{b,i}$). In deriving equations (A2)–(A5) it has been assumed that none of the NMIs α, β, i, j is the pilot of the linked comparisons.

Appendices B and C are available from the online version of this journal.

References

[1] Delahaye F and Witt T J 2002 Linking the results of key comparison CCEM-K4 with the 10 pF results of EUROMET project 345 *Metrologia* **39** (Tech. Suppl.) 01005

[2] Elster C, Link A and Wöger W 2003 Proposal for linking the results of CIPM and RMO key comparisons *Metrologia* **40** 189–94

[3] Decker J E, Steele A G and Douglas R J 2008 Measurement science and the linking of CIPM and regional key comparisons *Metrologia* **45** 223–3

[4] Woolliams E R, Fox N P, Cox M G, Harris P M and Harrison N J 2006 Final report on CCPR-K1.a: Spectral irradiance from 250 nm to 2500 nm *Metrologia* **43** (Tech. Suppl.) 02003

[5] Ikonen E, Hovila J and Park S N 2006 Uncertainties in linking bilateral and regional key comparisons to the CCPR key comparison *Presented at the 2nd CIE Expert Symp. on Measurement Uncertainty (Braunschweig, Germany, 11–17 June 2006)*

[6] Wilkinson F, Xu G and Liu Y J 2006 Final report on CCPR-K1.a.1: Bilateral comparison of spectral irradiance between NMIA (Australia) and SPRING (Singapore) *Metrologia* **43** (Tech. Suppl.) 02002

[7] Ojanen M, Shpak M, Kärhä P, Leecharoen R and Ikonen E 2009 Report of the spectral irradiance comparison EURAMET.PR-K1.a.1 between MIKES (Finland) and NIMT (Thailand) *Metrologia* **46** (Tech. Suppl.) 02001

[8] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML 2008 *Evaluation of Measurement Data—Guide to the Expression of Uncertainty in Measurement* JCGM 100:2008 (GUM 1995 with minor corrections) http://www.bipm.org/en/publications/guides/gum.html

[9] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML 2008 *International Vocabulary of Metrology—Basic and General Concepts and Associated Terms* JCGM 200:2008

[10] Cox M G 2002 The evaluation of key comparison data *Metrologia* **39** 589–95

[11] Liu Y J, Xu G, Ojanen M and Ikonen E 2009 Spectral irradiance comparison using a multi-wavelength filter radiometer *Metrologia* **46** S181–5

Appendix B. Derivation of equations (7) and (15)

Detailed derivation of Eqs. (7) and (15) is described in this Appendix. The unilateral degrees of equivalence in Eq. (5) are calculated as follows: Replacing the term D_i in Eq. (5) with Eq. (4) and the term $\Delta_{\alpha i}$ with Eq. (6), we obtain

$$D_{\alpha(i)} = \Xi_{\text{true},i} + E_{\text{uc},i} + E_{\text{cor},i} + E_{\text{kc},i} - X_{\text{ref}} + \Xi_\alpha - (\Xi_i' + \Xi_i'')/2 + E_{\text{bc}}. \quad \text{(B1)}$$

Using the analogies of Eq. (1) for Ξ_i' and Ξ_i'' results in

$$\Xi_i' = \Xi_{\text{true},i}' + E_{\text{uc},i}' + E_{\text{cor},i}' \quad \text{(B2)}$$

and

$$\Xi_i'' = \Xi_{\text{true},i}'' + E_{\text{uc},i}'' + E_{\text{cor},i}''. \quad \text{(B3)}$$

Applying Eqs. (8), (B2) and (B3), assuming $E_{\text{cor},i}'' = E_{\text{cor},i}'$, and grouping terms in Eq. (B1) gives

$$D_{\alpha(i)} = \Xi_\alpha + I_i - X_{\text{ref}} + E_{\text{kc},i} + E_{\text{bc}} - (E_{\text{uc},i}' + E_{\text{uc},i}'')/2 + E_{\text{uc},i} + E_{\text{cor},i} - E_{\text{cor},i}'. \quad \text{(B4)}$$

Applying Eqs. (9) and (10), we obtain Eq. (7).

The uncertainty of the unilateral degrees of equivalence, Eq. (15), is derived from Eq. (7) as follows: The standard uncertainty corresponding to random variable Ξ_α is denoted by u_α. The uncertainty corresponding to the term I_i is zero, as the term is related to the true quantity values with zero uncertainties. The standard uncertainties corresponding to $E_{b,i}$ and $E_{r,i}$ are denoted by $u_{b,i}$ and $u_{r,i}$. The uncertainty of the key comparison related terms $-X_{\text{ref}} + E_{\text{kc},i}$ is given in Eq. (13). Equation (15) is obtained by combining the above mentioned uncertainties.

Appendix C. Values of the largest weights in the KCRV calculation

The value of the largest weight for the KCRV calculation in a comparison of $N \geq 10$ participants is considered in this Appendix, when using the CCPR method for calculation of the cut-off value u_{co} of the uncertainty. Assumption $u_{\text{kc}} \approx 0$ leads to the largest value of the weights

$$w_i = \min\{1/u_i^2, 1/u_{\text{co}}^2\} / \sum_{j=1}^{N} \min\{1/u_j^2, 1/u_{\text{co}}^2\} \quad \text{(C1)}$$

where

$$u_{\text{co}} = \frac{1}{m}\sum_{j=1}^{m} u_j \quad \text{(C2)}$$

is the cut-off uncertainty and uncertainties u_j, $j = 1, 2, \ldots, m$, are less than or equal to the median of all the uncertainties in the comparison of N participants. The largest weight is obtained for such a case, where those uncertainties that are larger than the median, are so large that their contribution to the sum of Eq. (C1) can be neglected, reducing the summation to run from $j = 1$ to $j = m$.

Special cases with $N = 10$ participants and $m = 5$ are analyzed in table 2 using $N = m$ in Eq. (C1). The largest weight of $w_1 = w_2 = 25/77 \approx 0.3247$ is obtained for the case III. It is straightforward to show that if one of the uncertainties approximated by zero is increased, then the corresponding weight is reduced. In case III, the value of u_3 can be changed by a small amount to $u_3 + \varepsilon$, while all other uncertainties remain unchanged. Then the weights

$$w_1 = w_2 = (25/77) / (1 + 18\varepsilon^2/77) < 25/77 \quad \text{(C3)}$$

are obtained, when terms of higher order than ε^2 are neglected. Simultaneous variation of uncertainties u_3 and u_4 also leads to the conclusion that the largest weights are reducing when u_3 and u_4 deviate from u_5. These results show that case III is a local maximum for the value of the largest weight.

When $m = 6$, a table corresponding to table 2 can be calculated for the respective weights. The largest of these weights is 0.2667 and it is obtained for $u_1 \approx 0$, $u_2 \approx 0$, $u_3 \approx 0$ and $u_4 = u_5 = u_6$. Thus the maximum achieved for $m = 5$ is significantly larger than that for $m = 6$. These analyses show that the upper limit of $w_i < 0.325$ given in Sec. 2.3 holds for all values $N \geq 10$.

Table 2. Largest weight calculated for the participants whose uncertainty is replaced by the cut-off uncertainty. Equations (C1) and (C2) are used with $N = m = 5$.

Case	Uncertainties	u_{co}/u_5	Largest weight
I	$u_1 = u_2 = u_3 = u_4 = u_5$	1	1/5 = 0.2000
II	$u_1 \approx 0$, $u_2 = u_3 = u_4 = u_5$	4/5	25/89 = 0.2809
III	$u_1 \approx 0$, $u_2 \approx 0$, $u_3 = u_4 = u_5$	3/5	25/77 = 0.3247
IV	$u_1 \approx 0$, $u_2 \approx 0$, $u_3 \approx 0$, $u_4 = u_5$	2/5	25/83 = 0.3012
V	$u_1 \approx 0$, $u_2 \approx 0$, $u_3 \approx 0$, $u_4 \approx 0$, u_5	1/5	25/101 = 0.2475