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Detection and analysis of oscillations in a mineral flotation circuit

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ABSTRACT

A novel mineral slurry assaying method combining visible and near-infrared reflectance spectroscopy and traditional X-ray fluorescence has recently been developed. The improved method is capable of high-frequency assaying of slurry grades in zinc and copper flotation, thus being able to reveal possible rapid oscillations in the grades. In this study, the new measurements are used to detect oscillations in a copper flotation circuit. In addition, a recursive extension of a data-based signal analysis method, singular spectrum analysis, is introduced and applied to process data measured from the flotation circuit. The aim of the study is to pinpoint and eliminate the cause of the oscillations on-line in order to improve the performance of the circuit. It is shown that the oscillations can effectively be detected by the proposed method and that the structural similarity between the oscillatory signal and the other process signals can be quantified in real time.

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1. Introduction

Monitoring and control of mineral flotation plants is largely based on the on-line chemical analysis of the process streams (McKee, 1991; Wills & Napier-Munn, 2006). Traditionally, automatic slurry analyzers have been using X-ray fluorescence (XRF) to detect the elements in the slurry. A recent improvement of the analyzer technique by visible and near-infrared reflectance spectroscopy (Haavisto & Kaartinen, 2009; Haavisto, Kaartinen, & Hyötyniemi, 2008) has reduced the assaying interval from the common XRF range of 10–20 min to 10 s when applied to zinc and copper flotation. As a result, monitoring of the grade changes especially in the concentrates has improved: it is now possible to detect disturbances and oscillations of the grades faster and in higher frequencies than before.

Oscillations in industrial processes are unwanted phenomena, since all deviations of the process variables from the optimal values reduce the process performance. In mineral flotation the optimum operating condition is a trade-off between the concentrate grade and recovery of the valuable mineral (Wills & Napier-Munn, 2006). If the grade of the final concentrate is increased by control actions, more of the valuable mineral is typically lost in the tailings. On the other hand, if recovery is improved, more gangue minerals are lifted to the concentrate and the grade drops. Oscillations in flotation circuits alter the intended balance between the grade and recovery and usually shift the operating point down from the best possible grade–recovery curve, thus

reducing the performance of the mineral processing plant. Since the material flows in mineral processing are typically large, even a small decline of the performance may result in considerable economic losses.

The detection and analysis of faults and disturbances in industrial processes is a widely studied subject. The extensive review by Venkatasubramanian, Rengaswamy, Yin, and Kavuri (2003), Venkatasubramanian, Rengaswamy, and Kavuri (2003) and Venkatasubramanian, Rengaswamy, Kavuri, and Yin (2003) classifies the different approaches into three categories: qualitative model based methods, quantitative model based methods and process history based methods. Accordingly, this study concentrates on the statistical process history data based methods, including also the on-line use of the process data. The main emphasis is on oscillation detection and comparison of process signals in order to find the cause of the oscillations.

Techniques for oscillation detection based on the data have been introduced in the literature (Thornhill, 2007, chap. 6). One common method is to analyze the zero crossings of either the error signals or mean-centered process signals (Hägglund, 1995; Thornhill & Hägglund, 1997). Slightly more reliable results are obtained by using the autocovariance function zero crossings (Miao & Seborg, 1999; Thornhill, Huang, & Zhang, 2003), since the amount of high-frequency changes and signal noise is reduced. The analysis of oscillations in the spectral domain is a natural way of avoiding problems caused by phase shifts and time delays between the signals. Furthermore, the power spectra of the signals can be classified, for example, by principal component analysis (PCA) (Thornhill, Shah, Huang, & Vishnubhotla, 2002) or by independent component analysis (ICA) (Xia, Howell, & Thornhill, 2005) to group the different oscillations present in the

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process. Recently, the analysis of autocovariance functions and power spectra has been combined by Karra and Karim (2009).

In this study a slightly different approach is taken by applying and developing singular spectrum analysis (SSA) for characterization of the oscillations. SSA is a data-based technique for analyzing the structure of time series (Broomhead & King, 1986; Golyandina, Nekrutkin, & Zhigljavsky, 2001), and has mainly been applied in climatic, meteorological and geophysical studies (Robertson & Mechoso, 2000; Vianna & Menezes, 2000). In the process industry SSA has not been so popular, even though it is a general algorithm and can readily be used for industrial time series (Aldrich & Barkhuizen, 2003; Jemwa & Aldrich, 2006). In oscillation analysis, SSA provides a time delay and phase shift invariant method to smoothen the signals, to extract the oscillations and to find the similarities in the signal structures.

The aim of this study is to utilize the novel high-frequency copper grade measurements to detect harmful oscillations in a flotation circuit. To serve this purpose, a recursive extension to the basic SSA method (rSSA) is introduced. rSSA enables the continuous on-line calculation and similarity measurement of signal structures. The results can be used to detect the plant-wide oscillations on-line and to pinpoint or narrow down the underlying cause.

The article is structured as follows: In Section 2, properties of the flotation circuit and data collected are described briefly, and Section 3 provides an introduction to SSA and derives the new rSSA method. Section 4 illustrates the results obtained and finally Section 5 sums up the work in the form of conclusions.

2. Materials

2.1. Process

The oscillation analysis presented in this study was conducted at the concentration plant of Pyhäsalmi mine (Inmet Mining Corporation) in central Finland. The main products of the mine are copper, zinc and sulfur, and the concentration is performed in this order in three consecutive flotation circuits. The concentration process starts with grinding, where the crushed ore is mixed with water and ground to small particle size in order to liberate the different minerals. The resulting slurry is pumped to the flotation circuits.

Each circuit consists of flotation cells used to separate the specific mineral particles from other particles. In a flotation cell, slurry is mixed with air bubbles that rise to the surface and form a froth layer. Because of the flotation chemicals added to the slurry, the specific mineral particles attach to the rising bubbles and are carried with the froth over the edge of the cell as a concentrate flow. The rest of the slurry flows out from the bottom of the cell as a tailing flow. By repeating this separation in several interconnected stages, the final concentrate is obtained.

The structure of the copper flotation circuit at Pyhäsalmi is shown in Fig. 1. The circuit contains one bank of flotation cells for the rougher stage (CuRF) and one for the scavenger stage (CuSF), two banks for middlings flotation (CuMFI and CuMFII) and five for cleaning (CuCF0–CuCFIV). To improve recovery, the tailings from several cells are circulated back to the beginning of the process for re-processing, as typical for flotation circuits. The banks are connected by slurry pipes with pump pits, the pumps being individually controlled based on the slurry level in the pit. Generally, both the slurry re-circulation and the unit controllers of the pumps are possible causes for oscillations.

2.2. Data

The starting point for this study was the improved assays of the copper content of the final concentrate, obtained by the slurry spectroscopy prototype (see Haavisto & Kaartinen, 2009) in combination with the XRF analyzer (Outotec Courier[®] 6 SL). The spectroscopy measures the reflectance spectrum of the slurry with a 10 s sampling interval, and data-based modeling is used to carry out the corresponding elemental content analysis. The sparse XRF assays are used to recursively calibrate the regression model between the spectra and the elemental values.

The oscillations detected in the copper content were compared to the process history data. The data consisted of the slurry levels in all the cell banks and the flow rates of the final tailing (CuT), final concentrate (CuC) and scavenger concentrate (CuSC), which were measured after the corresponding pump pits and pumps.

Data were collected for 72 h in September 2008. Since the process history data are stored with a 1 min sampling interval at Pyhäsalmi, the assay signal was sampled with the same interval. Thus the data contained 4320 samples of each of the 13 signals.

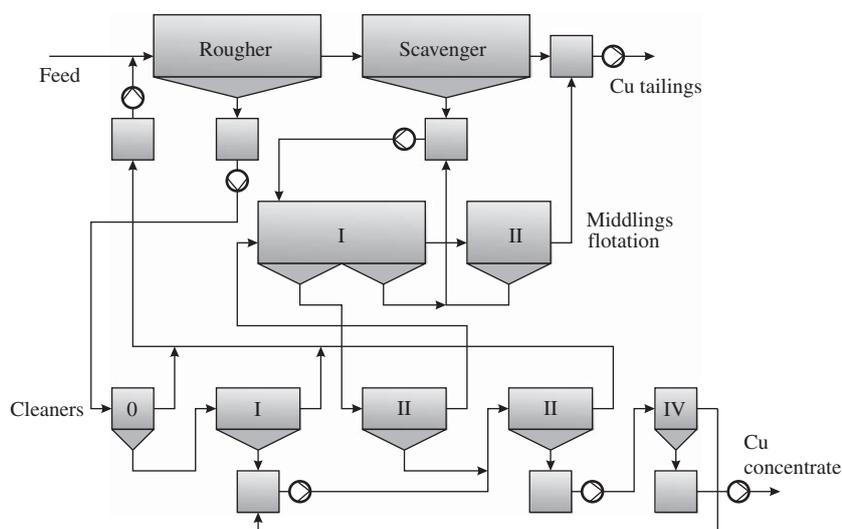


Fig. 1. The copper circuit at Pyhäsalmi consists of rougher, scavenger, middlings flotation and cleaner stages. The main pump pits and pumps are indicated in the figure in addition to the flotation cell banks.

3. Methods

The first part of this section briefly describes the calculation and properties of the basic SSA, mainly based on the book by Golyandina et al. (2001). In the second part, a novel recursive extension of SSA (rSSA) is introduced that can be used to monitor and analyze the structure of process signals on-line in real time.

3.1. Basic singular spectrum analysis

The two main phases of singular spectrum analysis are decomposition and reconstruction. In decomposition the original time series is windowed into a set of overlapping fixed length vectors consisting of consecutive samples of the series. By applying multivariate data-based methods to these vectors the data are then decomposed into different structural components representing the trends, oscillations and noise of the original time series. In the reconstruction phase a subset of the components is selected to reproduce an estimate of the signal in which the interesting signal properties are emphasized. Both the reconstructed signal and the extracted structure information can be used for further analysis.

Decomposition: Assume that an N sample long signal $\mathbf{f} = [f(1), f(2), \dots, f(N)]$ has been collected and the window length is selected as an integer L , $1 < L < N$. Embedding, the first operation of SSA, is performed by collecting all the possible lagged vectors \mathbf{z}^T , that is, sets of L consecutive samples in \mathbf{f} as the $K = N - L + 1$ rows of the trajectory matrix $\mathbf{Z}(N)$:

$$\mathbf{Z}(N) = \begin{bmatrix} \mathbf{z}^T(1) \\ \mathbf{z}^T(2) \\ \mathbf{z}^T(3) \\ \vdots \\ \mathbf{z}^T(K) \end{bmatrix} = \begin{bmatrix} f(1) & f(2) & \dots & f(L) \\ f(2) & f(3) & \dots & f(L+1) \\ f(3) & f(4) & \dots & f(L+2) \\ \vdots & \vdots & \ddots & \vdots \\ f(K) & f(K+1) & \dots & f(N) \end{bmatrix}. \quad (1)$$

It is worth noting that the trajectory matrix is a Hankel matrix, meaning that the values on each skew diagonal are constant.

The trajectory matrix is then treated as any data matrix containing multivariate data samples: each column corresponds to a variable, and each row to a sample of an L -dimensional data vector. By using PCA (or equally, singular value composition, SVD, Basilevsky, 1994), it is possible to write matrix $\mathbf{Z}(N)$ as a sum of d elementary matrices:

$$\mathbf{Z}(N) = \mathbf{Z}_1(N) + \mathbf{Z}_2(N) + \mathbf{Z}_3(N) + \dots + \mathbf{Z}_d(N), \quad (2)$$

where d equals the rank of $\mathbf{Z}(N)$, and each elementary matrix is the outer product of the corresponding PCA loading $\mathbf{p}_i(N)$ and score $\mathbf{t}_i(N)$ column vectors:

$$\mathbf{Z}_i(N) = \mathbf{t}_i(N)\mathbf{p}_i^T(N). \quad (3)$$

The loadings (or principal components) are obtained as the eigenvectors of the sample covariance matrix

$$\mathbf{R}(N) = \frac{1}{K}\mathbf{Z}(N)^T\mathbf{Z}(N), \quad (4)$$

and the corresponding eigenvalues describe the importance of each principal component. As usual, the principal components are assumed to be arranged in a descending order according to the eigenvalues.

As a result of PCA, each loading vector $\mathbf{p}_i(N)$ describes a typical L sample long structure present in the signal \mathbf{f} and can be interpreted as an 'eigensignal'. By the nature of PCA these eigensignals are orthogonal, and arranged in a descending order according to the data variation they represent. Moreover, each lagged vector $\mathbf{z}(k)$ in $\mathbf{Z}(N)$ can be reconstructed by a linear combination of the eigensignals.

Reconstruction: As discussed by Golyandina et al. (2001), the eigensignals in SSA typically represent trend components, oscillatory components, or noise components of the original time series. In the reconstruction phase of SSA the interesting components $\{i_1, \dots, i_p\}$ (say, the main oscillations) are grouped and used to reconstruct the trajectory matrix:

$$\hat{\mathbf{Z}}(N) = \mathbf{Z}_{i_1}(N) + \dots + \mathbf{Z}_{i_p}(N). \quad (5)$$

Since only a subset of all elementary matrices in the original decomposition (2) is used, an estimate of $\mathbf{Z}(N)$ is obtained. To calculate the reconstruction $\hat{\mathbf{f}}$ of the original signal \mathbf{f} , the values on every skew diagonal in the estimated trajectory matrix $\hat{\mathbf{Z}}(N)$ (i.e. all the different estimates of the same sample of signal \mathbf{f}) are further averaged, and the averages are collected as the samples of the reconstructed signal $\hat{\mathbf{f}}$.

The basic version of SSA as presented here is only suitable for a signal that is measured before the analysis, and thus has a fixed and known length. Moreover, the structure of the signal has to remain roughly constant in time so that the captured eigensignals are capable of describing the whole time interval. If the signal contains major structural changes, it is possible to apply the basic SSA separately to different parts of the signal (Golyandina et al., 2001) in order to detect and characterize the changes. This idea is further developed in the following.

3.2. Recursive singular spectrum analysis

To be able to apply SSA in real-time detection of oscillations of industrial processes, an on-line version of the algorithm that describes the current signal structure at each time instant is required. This can be accomplished by emphasizing the most recent data and recursively updating the PCA model.

Data weighting: The covariance matrix of the lagged vectors (4) essentially contains the information that is used in SSA. To emphasize the latest samples over the old ones, exponential weighting is used to obtain a weighted covariance matrix:

$$\mathbf{R}_w(N) = \frac{1}{\sum \text{diag}(\mathbf{W})}\mathbf{Z}^T(N)\mathbf{W}\mathbf{Z}(N), \quad (6)$$

where \mathbf{W} is a $K \times K$ diagonal matrix with the values $\{\lambda^{K-1}, \lambda^{K-2}, \dots, \lambda^1, 1\}$ on the diagonal. The constant forgetting factor $0 \ll \lambda < 1$ is a parameter determining the weight of a sample based on its age. If $\mathbf{R}_w(N)$ is now used instead of $\mathbf{R}(N)$ in SSA, the last (K :th) lagged vector $\mathbf{z}(N)$ is taken into account as it is, and the older ones with exponentially reducing weights; the older the vectors are, the less effect they have on the final SSA.

Recursive formulation: So far SSA has been applied to a fixed signal with N samples. In the on-line version, however, the data analysis must be performed concurrently with the data collection: after each new (k :th) sample of f is measured, the trajectory matrix is augmented with the new lagged vector

$$\mathbf{z}(k) = [f(k-L+1), \dots, f(k)]^T, \quad (7)$$

and the weighted covariance matrix $\mathbf{R}_w(k)$ is calculated. The easiest way to realize this is to use a recursive formula for updating the weighted covariance matrix (see e.g. Ljung & Söderström, 1983):

$$\mathbf{R}_w(k) = \lambda\mathbf{R}_w(k-1) + (1-\lambda)\mathbf{z}(k)\mathbf{z}^T(k). \quad (8)$$

In fact, when (8) is utilized, it is not necessary to construct the trajectory matrix or store the old lagged vectors; all the relevant information of the past is stored in the covariance matrix estimate $\mathbf{R}_w(k)$. The same approach has been used also in adaptive data-based process analysis (see e.g. Dayal & MacGregor, 1997).

The eigensignals $\mathbf{p}_i(k)$ and their weights (scores) $\mathbf{t}_i(k)$ describe the current structure of the signal and are obtained by the

eigenvalue decomposition of $\mathbf{R}_w(k)$. The reconstruction phase is performed as in basic SSA by grouping a set of eigensignals $I = \{i_1, \dots, i_p\}$:

$$\hat{\mathbf{z}}(k) = t_{i_1}(k)\mathbf{p}_{i_1}(k) + \dots + t_{i_p}(k)\mathbf{p}_{i_p}(k) = \mathbf{P}_I^T(k)\mathbf{t}_I(k), \quad (9)$$

where $\mathbf{t}_I(k) = [t_{i_1}(k), \dots, t_{i_p}(k)]^T$ is a column vector containing the scalar score values related to the grouped eigensignals in the rows of matrix $\mathbf{P}_I(k) = [\mathbf{p}_{i_1}(k), \dots, \mathbf{p}_{i_p}(k)]^T$.

However, the diagonal averaging operation can be omitted since, at time k , the latest sample $f(k)$ would be present only once in the trajectory matrix. Thus, the value $\hat{f}(k)$ is obtained directly as the last element of $\hat{\mathbf{z}}(k)$.

As a result of rSSA, for every k the selected eigensignals $\mathbf{P}_I(k)$ and scores $\mathbf{t}_I(k)$ that capture the interesting parts of the current signal structure are obtained. Based on these, the reconstructed signal value $\hat{f}(k)$ is calculated.

Preprocessing: Even though preprocessing is typically not used before applying SSA (Golyandina et al., 2001), it can improve the results in certain applications. In this study, oscillations are considered to be the only important structural property extracted from the signals, whereas trends and noise components are discarded. This could be accomplished by correct grouping of the eigensignals in the reconstruction phase. In practice, however, the automatic classification of eigensignals could be difficult. Instead, least-squares fitted linear trend is subtracted from each new lagged vector $\mathbf{z}(k)$ before updating the covariance matrix (8). The trend removal is a standard preprocessing step used for example in system identification (Ljung, 1999), and in this case it nicely prevents the trend components from appearing among the extracted eigensignals.

Parameters: The two parameters of the rSSA algorithm are window length L and forgetting factor λ . In the case of oscillation detection, the window length should be selected to be at least as long as the wavelength of the slowest expected oscillation. The window length also affects the trend removal property of the algorithm, since a long window leads to removal of the long-term trends only.

The forgetting factor, on the other hand, describes the amount of change that is to be expected in the signal structure. If rapid changes are likely, a smaller forgetting factor will force the algorithm to adapt quicker to the new structure. On the other hand, λ near to 1 will average the changes and let rSSA capture only the long-term structural variation.

A third choice is related to the grouping of the latent variables in the reconstruction phase, that is, how to select the indices $I = \{i_1, \dots, i_p\}$. It is difficult to automatically detect all the oscillatory eigensignals and leave out the noise components. However, if one is only interested in the most important structural properties of the signal, it should suffice to select a couple of the main eigensignals and leave the rest out in the reconstruction phase. As seen by the results and discussed by Golyandina et al. (2001), each oscillation typically results to two equally strong eigensignals; thus in most cases it is sufficient to select an even number of eigensignals for the reconstruction when dealing with oscillations.

Analysis: The reconstructed signal $\hat{\mathbf{f}}$ can readily be used for further analysis. As a result of the selected eigensignals in the grouping and reconstruction phase, the reconstruction only contains the interesting phenomena of the original signal \mathbf{f} . In the case of oscillation detection and characterization, both the trend and high frequency noise components are eliminated and the main oscillations remain. To analyze the frequency content of the signal, discrete Fourier transformation (DFT) can be applied either to the reconstructed signal $\hat{\mathbf{f}}$, to the latest reconstructed lagged vector $\hat{\mathbf{z}}(k)$, or to the captured eigensignals in $\mathbf{P}_I(k)$. The DFT

of the reconstructed signal or the lagged vectors includes the amplitude information, whereas the eigensignals are normalized and indicate the oscillations present in the signal structure as captured by rSSA. However, the scores in $\mathbf{t}_I(k)$ contain the information on the importance of each eigensignal at the particular time instant.

A convenient way to estimate the structural similarity of a lagged vector \mathbf{z} and an extracted signal structure (a group of eigensignals) \mathbf{P} is to calculate heterogeneity (slightly modified from Golyandina et al., 2001):

$$g(\mathbf{z}, \mathbf{P}) = \frac{(\text{dist}(\mathbf{z}, \mathbf{P}))^2}{\|\mathbf{z}\|^2} = \frac{\|\mathbf{z} - \mathbf{P}^T\mathbf{P}\mathbf{z}\|^2}{\|\mathbf{z}\|^2}, \quad (10)$$

where $\|\cdot\|$ is the Euclidean norm. Heterogeneity measures the normalized squared distance of the lagged vector from the subspace spanned by the eigensignals (rows of \mathbf{P}), thus equalling 0 if \mathbf{z} lies completely in the subspace and 1 if \mathbf{z} is orthogonal to the subspace. Heterogeneity can be used to compare the structure of different parts of the same signal or the structures of two separate signals.

4. Results and discussion

The results obtained by the rSSA analysis of the mineral flotation circuit data (Section 2) are described and discussed as follows. At first, the copper grade signal of the final copper concentrate is analyzed and the oscillation period in the signal is characterized. Then the analysis is extended to other process variables in order to find possible oscillating parts of the flotation circuit.

4.1. rSSA of the copper content

The copper content signal of the final copper concentrate (CuC) during the analysis period is shown in the upper part of Fig. 2. The normal level of the copper content at the plant is around 29%. A clear drop of the content due to a process failure is visible around 14–15 h from the start. After that, the signal gradually turns into a steady oscillation with a peak-to-peak amplitude of about two percentage points. The oscillation then lasts for more than 30 h.

The recursive SSA was first applied to the copper content signal with window length $L = 90$ (1.5 h) and forgetting factor $\lambda = 0.95$. The middle pane of Fig. 2 shows the on-line reconstruction of the signal with two main eigensignals ($I = \{1, 2\}$) used in the reconstruction phase. Due to the removal of trends of each lagged vector before analysis, there are no low frequency trends in the reconstruction and the signal varies around zero. Correspondingly, the exclusion of the less important eigensignals smooths the high-frequency noise, thus highlighting the main oscillations in mid-frequencies.

The six main eigensignals $\mathbf{p}_i(k)$, $i = 1, \dots, 6$, extracted by SSA during the oscillation phase at 45 h (i.e. $k = 2700$) are shown in Fig. 3. Also the amount of variance explained by each eigensignal and the frequency of the strongest oscillation (calculated by DFT) is given for each eigensignal. It is evident that the basic form of the oscillation is captured by the two main eigensignals which are approximately sinusoidal but have different phases. As a linear combination of these two eigensignals, a sinusoid with the same frequency but an arbitrary phase can be produced.

The second eigensignal pair in Fig. 3 has clearly captured the first harmonic of the main oscillation with double frequency. However, this second pair is less important, since already almost 80% of the variance is explained by the first two eigensignals. Accordingly, the eigensignals 5 and 6 are even less important representing only less than 3% of the total variance. Also, the form

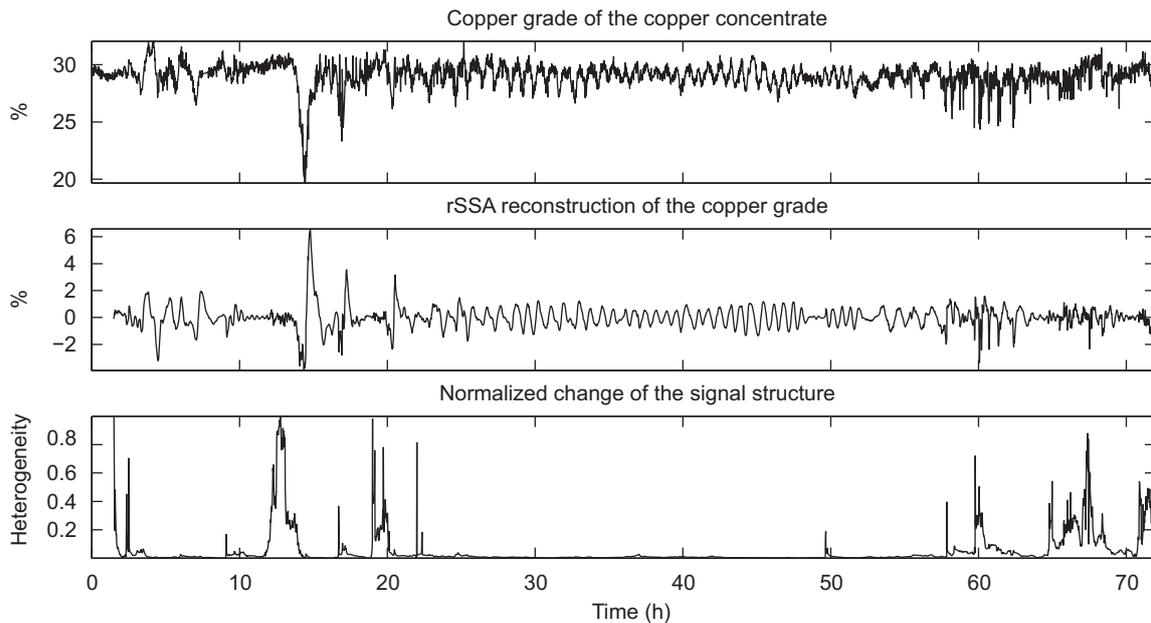


Fig. 2. Copper grade of the final copper concentrate, the recursive singular spectrum analysis reconstruction and the heterogeneity describing the change of the signal structure.

of these eigensignals differs more from a pure sinusoid, making the frequency analysis less meaningful.

The presence of steady oscillations in a signal can be assumed to keep the signal structure approximately constant, whereas the structure of a non-periodic signal (after trend removal) is more likely to change. Based on this, the heterogeneity (10) was applied to determine the structural changes in the copper content signal in order to detect the oscillating periods. For each time index k , the heterogeneity $g(\hat{\mathbf{z}}(k), \mathbf{P}_1(k-1))$ between the newest lagged vector estimate and the *previous* structure subspace was calculated. The aim is to measure the structural difference between the recent past and the newest sample of the signal.

The lowest pane in Fig. 2 shows the heterogeneity obtained describing the momentary change in the copper content signal structure. Clearly, during the long oscillation period (hours 23–57) the changes are small, whereas the non-periodic regions (e.g. hours 12–14, 19–20 and 65–68) correspond to large heterogeneity values. The less regular oscillations between hours 4 and 9 are characterized by a low change in the signal structure, and the large peak caused by the process failure at 14–16 h is smooth enough to keep the heterogeneity small.

It seems that the presented heterogeneity measure offers a potential way to automatically detect the presence of oscillations in a signal. If the analysis was performed for a stable signal (with noise), the main SSA eigensignals would capture only random variations caused by the noise, and the signal structure would change very quickly. On the other hand, the presence of any periodicity in the signal keeps the heterogeneity low.

4.2. Circuit-wide analysis

After detecting the unwanted oscillations in the process signal, the next step is to find out what causes them. To obtain this, rSSA was applied to all signals collected from the process with the same parameters as for the copper grade signal, and the heterogeneity measure (10) was further utilized. Even though heterogeneity was originally presented as a method to analyze the structural changes between different parts of one signal (Golyan-

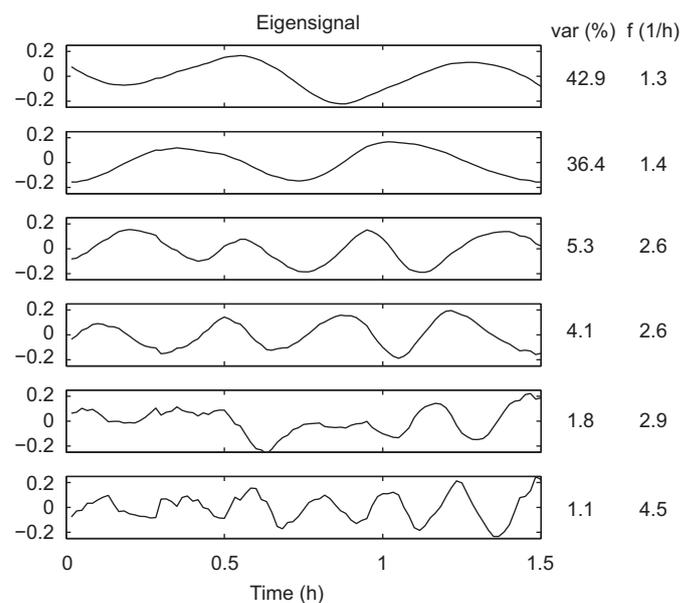


Fig. 3. The six main eigensignals of the copper concentrate copper content signal at 45 h. Also the variance explained by each eigensignal and the frequency value having the largest amplitude in the discrete Fourier transformation of the eigensignal are given.

dina et al., 2001), it can easily be applied to measure the differences between the structures of two signals. One simply has to calculate the heterogeneity between the structure subspace $\mathbf{P}_1^1(k)$ of signal 1 and the latest lagged vector $\hat{\mathbf{z}}^2(k)$ of signal 2, $g(\hat{\mathbf{z}}^2(k), \mathbf{P}_1^1(k))$. Again, this can be performed for every new k , so that a sliding heterogeneity value between the signals is obtained in real time. The advantage of the method over, for example, cross correlation is that the effects of noise and delays between the signals are reduced, and that the analysis can readily be calculated on-line.

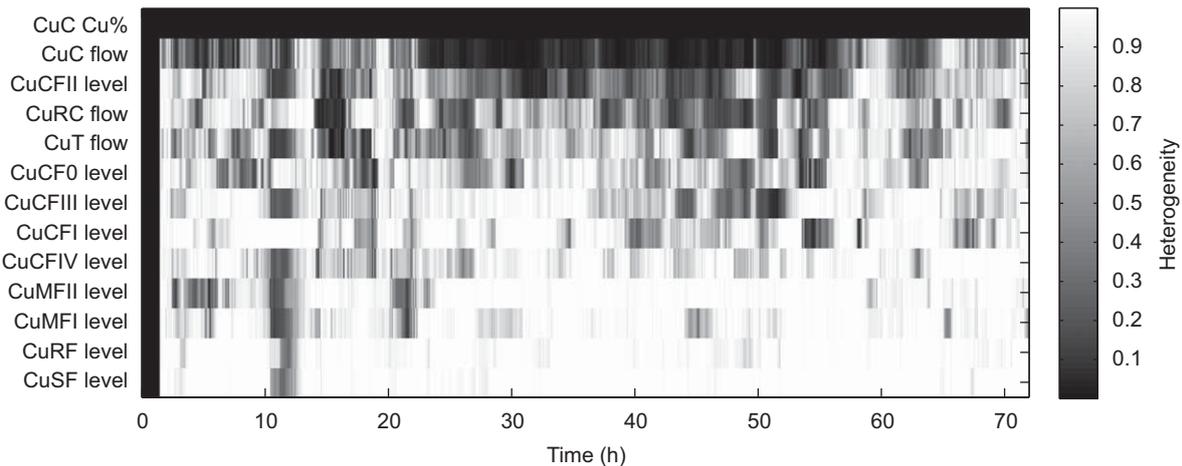


Fig. 4. The sliding heterogeneity values between the copper content of the copper concentrate and the other measured process signals: the lighter the value, the less the signal structure resembles the copper content signal at that particular time. The signals are ordered according to the average similarity with the copper content. The values for the initial 1.5 h are omitted as they are used to initialize the rSSA algorithm.

Fig. 4 shows the calculated sliding heterogeneity values for the reconstructed lagged vectors of each process signal compared to the subspace of the copper content of the copper concentrate (CuC Cu%) for each $k = 1, \dots, 4320$. Clearly, after the copper content itself, the lagged vectors of the copper concentrate flow are very similar with the structure of the copper content signal. This is natural since the amount and grade of the concentrate in a flotation circuit are typically tightly connected: if more slurry is forced to the concentrate the grade drops and vice versa. Thus the grade oscillations are clearly seen also in the flow rate measurements.

The structure of the slurry level signal of the second cleaning cell bank (CuCFII) is also quite similar to the copper grade. Especially during the oscillation period the similarity is quite obvious, thus suggesting that the slurry level in CuCFII is oscillating as well. In addition, the rougher concentrate (CuRC) flow is showing signs of the same oscillation, whereas the copper tailing (CuT) flow is more or less similar with the grade signal during the whole time period. The slurry levels in the other cleaning cell banks also have some similarities with the grade signal. The middlings, rougher and scavenger levels, however, have a different structure at least during the oscillation period.

The results of the heterogeneity analysis can be confirmed by visual inspection of the rSSA reconstruction of the signals. In **Fig. 5**, a short period (hours 43–45) of all the signals and the rSSA reconstructions during the oscillation period are shown. As indicated by the heterogeneity values, the three uppermost signals show clear oscillations with the same frequency. It is also important to notice that the level changes in the cleaner stage II (CuCFII) have a significantly larger amplitude than in the other flotation stages. The oscillations are barely visible in the rougher concentrate (CuRC) and final tailing (CuT) flows as well as in the level signal of the third cleaner cell bank (CuCFIII). The other variables do not show similar oscillations.

The frequency contents of the signals are visualized by showing the DFTs of the first eigensignals $\mathbf{p}_1(k)$ as a function of k (**Fig. 6**). The amplitude of the oscillations is knowingly discarded by using the normalized eigensignals. This is because, for example, in the case of the copper grade signal the large process failure peak would dominate the frequency analysis and subdue all the other oscillations in the figure.

The oscillation period between the hours 23 and 57 is clearly seen in the DFT of the signals of copper grade, concentrate flow

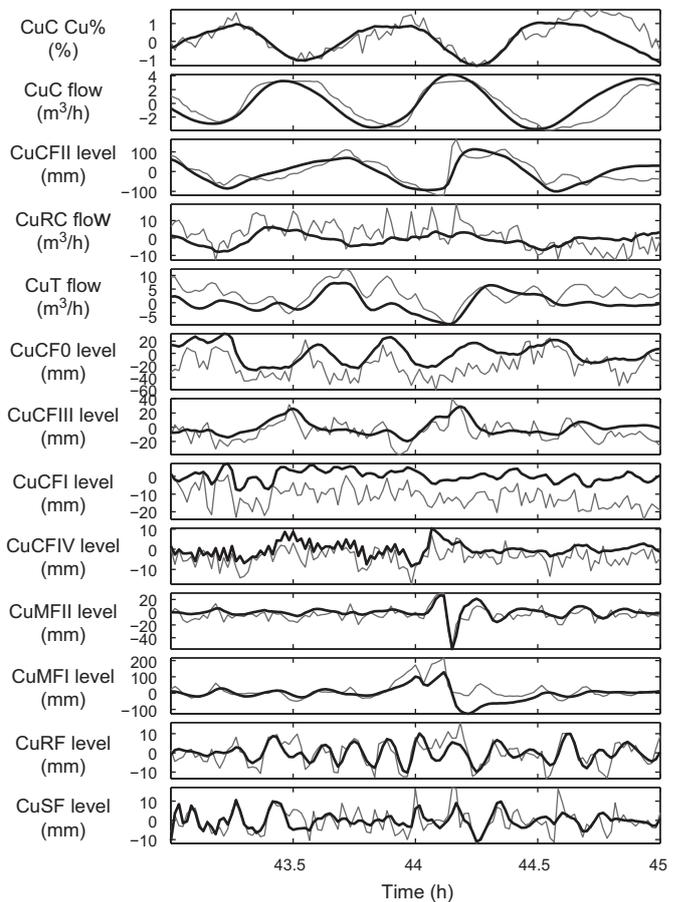


Fig. 5. The mean-centered original signals (thin line) and the rSSA reconstructions (thick line) for the period between 43 and 45 h.

and level of the cleaning cell bank II, as already detected by the heterogeneity analysis. A slight increase in the frequency is detected during the period as the main frequency shifts from around 1.5 to almost 2 oscillations per hour. The frequencies of the rest of the signals show progressively less similarities with the copper grade.

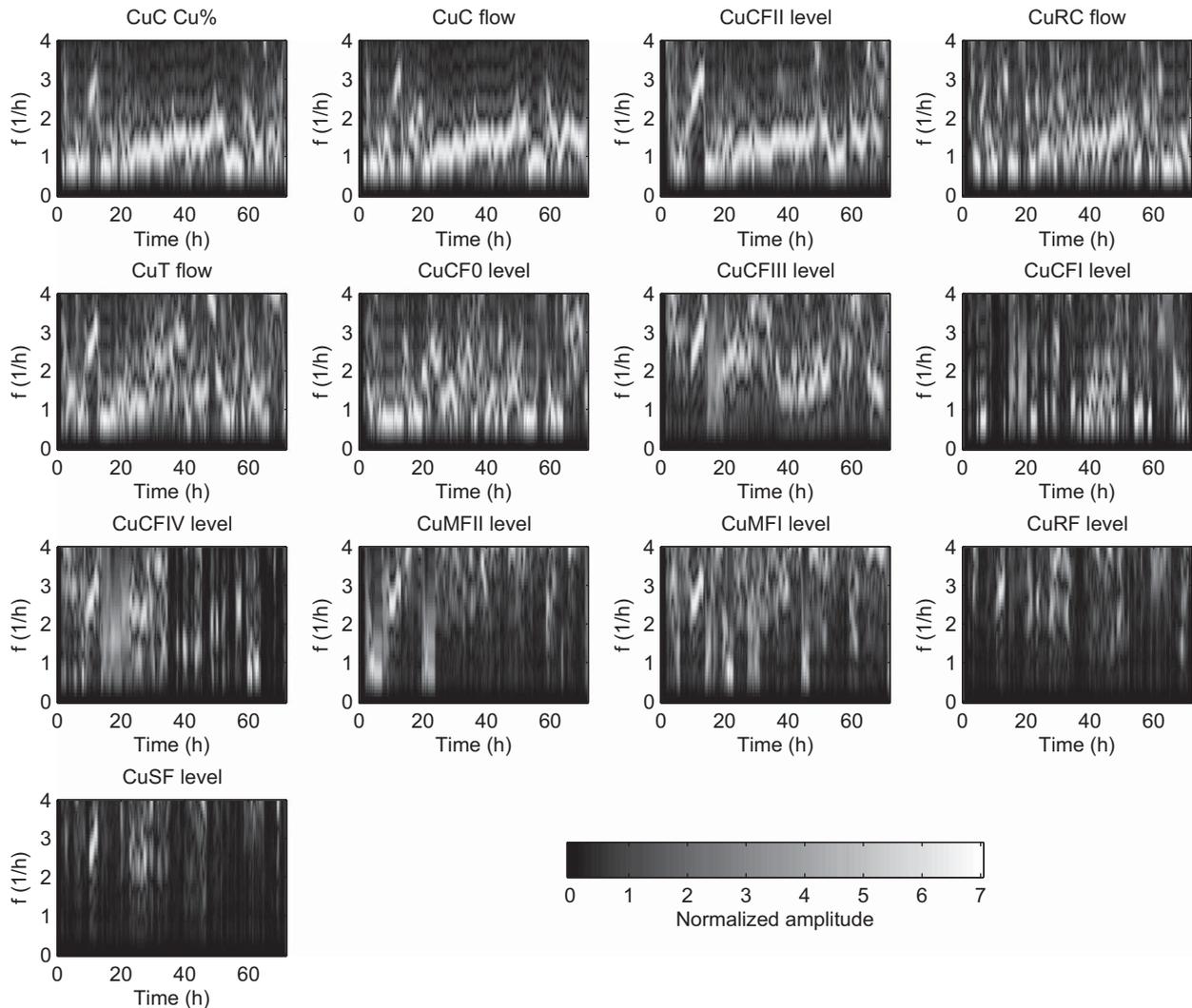


Fig. 6. The discrete Fourier transformations of the first eigensignals extracted by rSSA for each process variable during the whole data collection period. Since the eigensignals are normalized, the amplitude is normalized as well. Only small frequencies are shown for clarity.

The source of the copper concentrate grade oscillations can be narrowed down by the analysis presented. Clearly, the earliest signal in which the oscillations are found is the rougher concentrate flow, thus indicating that the rougher bank, or at least the rougher concentrate pump pit, belongs to the oscillating part of the circuit. The cleaner stages 0, I, III, and IV merely pass the oscillations on to the final concentrate (see Fig. 1). However, since the oscillations are so clearly seen in the slurry level of the cleaner stage II and partly in the final tailing flow, the other parts of the circuit must be influenced by them as well.

In fact, the heavy oscillation of the slurry level in the cleaner stage II could itself be the cause of the oscillation of the final concentrate grade. Based on laboratory measurements, it is known that the copper grade of the cleaner II concentrate is generally lower than the grade of the slurry flow that it is combined with just before the cleaner stage III. An oscillating slurry level in the cleaner bank II results in oscillating concentrate flow over the edge of the bank. Thus, a periodically varying amount of lower-grade concentrate from the cleaner stage II is mixed with the higher-grade slurry from the pump pit of the cleaner stage I, which in turn means that the grade of the mixture is oscillating. This grade oscillation could further be seen in the final concentrate, and could also affect the other parts of the circuit through the feedback connections.

Additional visual inspection of the frequencies in Fig. 6 reveals other similarities between the signals. For example, around 11–14 h, most of the signals oscillate with a frequency increasing from 2.5 to 3.5 oscillations per hour. Only the levels of the two initial cleaning stages (CuCF0 and CuCFI) and the rougher concentrate (CuRC) flow signal do not show this oscillation. The structural similarity of the signals during the period is further visible as a dark vertical band in Fig. 4.

5. Conclusions

Oscillations in mineral flotation reduce the concentration performance and cause economic losses since the optimal grade–recovery–relation is not constantly maintained. Recently the detection of rapid oscillations has become possible due to the improved mineral slurry assaying method based on the combination of reflectance spectroscopy and traditional X-ray fluorescence analysis.

The rapid oscillations of the copper concentrate grade measured by the new measurement from a typical copper flotation circuit were studied. A new recursive singular spectrum analysis method capable of on-line analysis of the signal structures was introduced. The oscillatory structure of the copper

grade was compared to the structure of the other process signals in order to find the possible cause of the oscillations. A strong similarity was found between the flow rate and the grade of the final concentrate. The slurry level in the second cleaner flotation cell and the concentrate flow rate of the rougher concentrate were shown to clearly reveal the same oscillations. Some traces of the oscillations were detected in other process signals (e.g. in the final tailing flow rate).

The most likely source for the oscillations is, however, the oscillating level of the cleaner stage II. There the amplitude of the level change is the highest, and the periodically varying mixing of the low-grade concentrate from the cleaner stage II with another higher-grade slurry flow can easily result in an oscillating slurry grade in the final concentrate.

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