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Analysis and Design of EMI Filters for DC-DC Converters Using Chain Parameters

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Abstract

Switch-mode power converters (SMPCs) are known as one of the greatest sources of electromagnetic interference (EMI) in electronic equipments. In order to comply with the strict electromagnetic compatibility (EMC) requirements, an EMI filter is needed at the input of a SMPC. The design of EMI filters aims at achieving required insertion loss (*IL*), i.e. attenuation of the power of the unwanted electromagnetic emissions (EME) from a switch-mode dc-dc converter. However, the EMI filter affects converter's dynamics.

This paper discusses how chain parameters can be used to calculate the *IL* provided by converter's input filter and also in the input system stability analysis.

The stability analysis shows that relatively small resistances in series with the capacitor and inductor, which are closest to SMPC's input, can compensate the effect of converter's negative input impedance.

Chain parameters are very useful in *IL* calculations as well. Comparison between LC- and π-filter shows that if dc power line's impedance is know to be very low, LC-filter will perform as π-filter. With unknown or high dc source impedance π-configuration should be used.

1. Introduction

SMPC are the most widely used dc power supplies in nowadays because they are significantly smaller, lighter and a lot more efficient than linear power supplies. The main drawback of SMPCs is the current related interference at their inputs and voltage related interference at their outputs [1]. The requirements of the load dictate the design of the output filter, which is an important part of the design of the converter and its controller. The input filter, on the other hand, is not necessary for the operation of the converter itself. Its task is to ensure EMC within the system or with neighbouring systems, and to comply with the

EMC standards. Because the standards limit the power of the EME, EMI filters are designed to provide certain insertion loss (*IL*) [2], but they also affect converter's dynamics and this should be considered in their design as well. An example of EMI filter for SMPC is shown in Fig. 1.

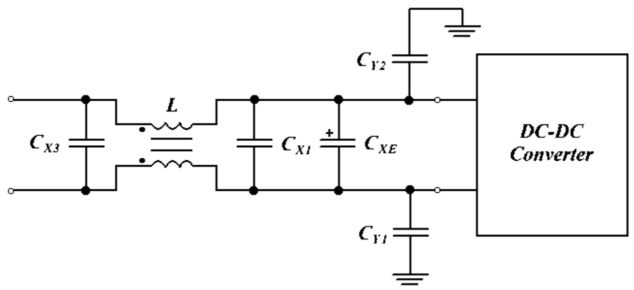

Fig. 1: Typical EMI filter for SMPC [3].

Assuming their components do not vary with time, EMI filters can be considered passive linear electrical circuits. Such circuits can be viewed as linear two-port networks [4]. Two-port networks can be characterized by different sets of two-port parameters, which relate the input and output port variables, i.e. input and output voltages and currents. Each set of two-port parameters can be easily measured or calculated from another known set of two-port parameters [4]. Chain parameters, also known as transmission or ABCD parameters, were used in EMC calculations also before [2], but not specifically in the design of EMI filters for SMPC. This paper shows how they can be used for *IL* calculations and also in the stability analysis of EMI filter terminated with negative impedance.

The paper is organized as follows. The chain parameters of a π-filter as seen from the converter and from the dc power line are derived in Section 2. Section 3 shows how chain parameters can be used to determine whether the EMI filter terminated with negative impedance is a stable system. In Section 4 the *IL* provided by a π-filter is calculated using chain parameters. The last Section 5 summarizes the results.

2. Chain Parameters

A two-port network with voltages and currents defined is shown in Fig. 2a). The input port variables have subscript "*i*", and the output port variables, i.e. output voltage and current have subscript "*o*". The chain parameters are defined as:

$$\begin{bmatrix} U_i \\ I_i \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \cdot \begin{bmatrix} U_o \\ -I_o \end{bmatrix} = \mathbf{C} \cdot \begin{bmatrix} U_o \\ -I_o \end{bmatrix} \quad (1)$$

Cascaded two-port networks, as in Fig. 2b), form a two-port network, with chain matrix, which can be obtained by matrix multiplication of the chain matrices of the cascaded networks:

$$\begin{bmatrix} U_i \\ I_i \end{bmatrix} = \begin{bmatrix} U_{i1} \\ I_{i1} \end{bmatrix} = \mathbf{C_1} \cdot \begin{bmatrix} U_{o1} \\ -I_{o1} \end{bmatrix} = \mathbf{C_1} \cdot \begin{bmatrix} U_{i2} \\ I_{i2} \end{bmatrix} = \mathbf{C_1} \cdot \mathbf{C_2} \cdot \begin{bmatrix} U_{o2} \\ -I_{o2} \end{bmatrix} = \mathbf{C_1} \cdot \mathbf{C_2} \cdot \begin{bmatrix} U_o \\ -I_o \end{bmatrix} \Rightarrow \mathbf{C} = \mathbf{C_1} \cdot \mathbf{C_2} \quad (2)$$

Passive EMI filters can be viewed as cascade connected two-port networks, where each two-port network represents a single component – inductor, capacitor or resistor. The inductors in an EMI filter are always connected in series with respect to the load. Therefore, the two-port network of an inductor is as shown in Fig. 3a) and its chain parameters are:

$$\mathbf{C} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \quad (3)$$

Z in (3) is inductor's impedance, which can be obtained by measurements or from the equivalent circuit of the inductor, used as its model in the frequency diapason of interest [2].

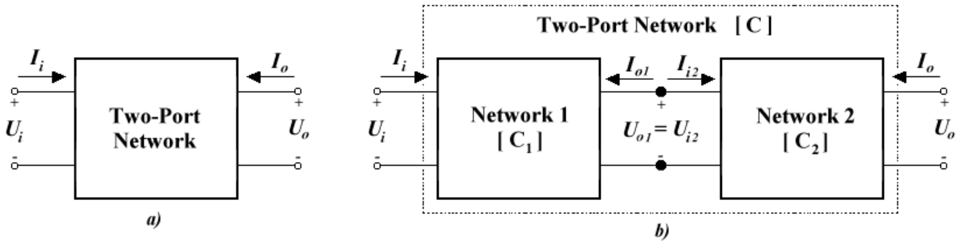

Fig. 2: a) A general two-port network with voltages and currents defined.

b) Cascaded two-port networks.

The use of resistors in EMI filters is not desirable, but sometimes they cannot be avoided – e.g. for damping [1, 5, 6] or matching the source and load impedances [2]. If a resistor is connected in series with respect to the load, like in Fig. 3a), it has the same chain matrix as the inductor (3), but *Z* must be the appropriate expression for resistor's impedance.

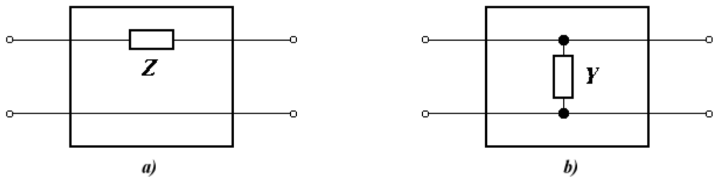

Fig. 3: a) Series connected component; b) parallel connected component.

Unlike inductors, the capacitors are always connected in parallel with respect to the load. Fig. 3b) shows the two-port network of a capacitor. The chain parameters in this case are:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \quad (4)$$

Y in (4) is capacitor's admittance. Again, it can either be measured or calculated from capacitor's equivalent circuit. Some damped EMI filters have a resistor in series with the capacitor, which must be taken into account in the expression for the admittance *Y*.

2.1. Chain Matrix of a π-filter Seen From the DC Power Line

When the disturbance source is on the dc power line side, as it is in Fig. 4a), then the chain parameters as seen from the dc power line are required. For a π-filter they are:

$$\mathbf{C} = \mathbf{C_3 C_2 C_1} = \begin{bmatrix} 1 & 0 \\ Y_3 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 + Y_1 Z_2 & Z_2 \\ Y_1 + Y_1 Z_2 Y_3 + Y_3 & 1 + Z_2 Y_3 \end{bmatrix} \quad (5)$$

This is the chain matrix needed later in the stability analysis.

2.2. Chain Matrix of a π-filter Seen From the Converter

When the converter is the source of electromagnetic emissions (EME), as it is in the example of a π-filter in Fig. 4b), then the chain parameters as seen from the converter's side are needed:

$$\mathbf{C} = \mathbf{C_1 C_2 C_3} = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_3 & 1 \end{bmatrix} = \begin{bmatrix} 1 + Z_2 Y_3 & Z_2 \\ Y_1 + Y_1 Z_2 Y_3 + Y_3 & 1 + Y_1 Z_2 \end{bmatrix} \quad (6)$$

Because matrix multiplication is not commutative, in general the chain matrix of an EMI filter is not same as that seen from the opposite side of the filter. Comparison between (5) and (6) shows that if the chain parameters of a passive π-circuit are known, the chain parameters seen from the opposite side can be obtained by swapping the chain parameters on the main diagonal of the known chain matrix. In addition, if the admittances Y_1 and Y_3 are equal, the chain matrices seen from both sides of the filter are also equal.

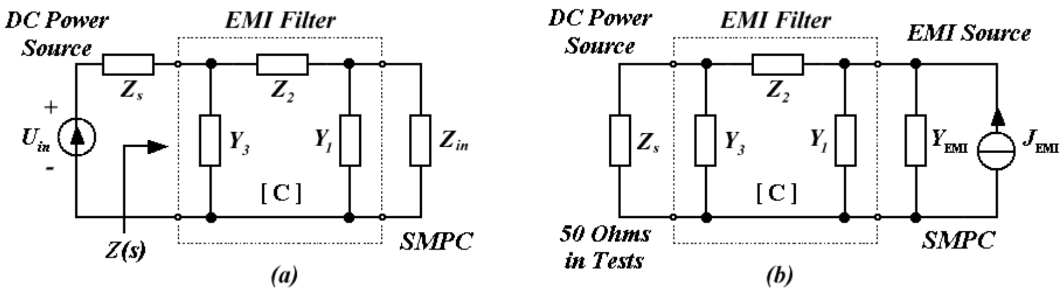

Fig. 4: Circuit to be analysed in: a) stability analysis and b) insertion loss calculations.

3. Stability Analysis

The negative input impedance of a dc-dc switch-mode power converter is considered to be the reason for instability when input filter is added to the converter [7, 1]. Undoubtedly, an EMI filter terminated with a negative impedance might be an unstable system and chain parameters can be used to determine whether such system is stable. As in [8] we call this input system instability. It should be emphasized that the input system instability is not the instability of the converter due to degrading converter's loop-gain – something that is caused by the input filter as well.

3.1. Driving Point Impedance

The driving point impedance Z of a two-port network terminated with impedance Z_{in} can be found easily by using chain parameters. Referring to Fig. 4a), equation (1) and noting that $U_o = -I_o \cdot Z_{in}$, it follows:

$$Z(s) = \frac{U_i}{I_i} = \frac{c_{11} \cdot U_o - c_{12} \cdot I_o}{c_{21} \cdot U_o - c_{22} \cdot I_o} = \frac{c_{11} \cdot Z_{in}(s) + c_{12}}{c_{21} \cdot Z_{in}(s) + c_{22}} \quad (7)$$

After inserting the chain parameters as seen from the dc line (5) in (7) the driving point impedance Z of a π-circuit terminated with impedance Z_{in}, as shown in Fig. 4a), is obtained:

$$Z(s)=\frac{(1+Y_1Z_2)\cdot Z_{in}+Z_2}{(Y_1+Y_1Z_2Y_3+Y_3)\cdot Z_{in}+1+Z_2Y_3} \tag{8}$$

Usually only frequencies within converter's regulation bandwidth raise stability concerns, because that is where the input impedance appears negative. Theoretically, converters employing input voltage feed-forward (IVFF) or current mode control (CMC) with optimal compensation, have negative input impedance regardless of the gain-bandwidth [9]. This extends the frequencies of concern as far as converter's model can be considered accurate, i.e. up to about a third of the switching frequency. Both the gain-bandwidth and a third of the switching frequency are so low that each of the two capacitors Y_1 and Y_3 from the π-filter example in Fig. 4a) can be modelled by a capacitance in series with capacitor's ESR and the inductor Z_2, can be modelled by an inductance in series with inductor's parasitic resistance:

$$Y_1=\frac{sC_1}{1+sr_{C,1}C_1} \quad Z_2=r_L+sL \quad Y_3=\frac{sC_3}{1+sr_{C,3}C_3} \tag{9}$$

Then the driving point impedance of a π-filter terminated with Z_{in} is:

$$Z(s)=\frac{\{s^2L(Z_{in}+r_{C,1})C_1+s[L+(r_LZ_{in}+r_{C,1}Z_{in}+r_Lr_{C,1})C_1]+Z_{in}+r_L\}(1+sr_{C,3}C_3)}{Z_{in}\{s^2C_1C_3(r_L+sL)+s[C_1(1+sr_{C,3}C_3)+(1+sr_{C,1}C_1)C_3]\}+(1+sr_{C,1}C_1)\{1+s[r_{C,3}C_3+(r_L+sL)C_3]\}} \tag{10}$$

3.2. Input System Stability

Consider the following equation:

$$I=Y(s)\cdot U \tag{11}$$

In the language of control theory, (11) means that under voltage excitation the poles of the driving point admittance Y must be in the LHP or simple on the imaginary axis to ensure stability. The poles of Y are the zeros of the driving point impedance Z. Therefore, it is enough to check whether the zeros of Z are in the LHP to ensure that an electrical circuit is stable under voltage excitation.

In case of a π-filter, from (10), it is obvious that the zero due to C_3 is in the LHP because its ESR $r_{C,3}>0$. Thus, C_3 has no effect on the input system stability. From the first term in the numerator in (10), one can conclude that if Z_{in} is a positive real number, i.e. a resistance, all roots will be in the LHP and the system will be stable. If parasitic resistances are zero, i.e. $r_L = r_{C,1} = 0$, and Z_{in} is negative real number, i.e. a negative resistance, the system will be unstable, because at least one of the roots will be in the RHP. The results from the simulations of such an example are shown in Fig. 5a). The unstable driving point admittance is the cause for increasing input current oscillations, causing increasing charge-discharge of C_1, i.e. filter's output voltage, which is the input voltage of the converter, oscillates with increasing amplitude. If such a process develops at converter's input, it will compromise the operation of the SMPC. However, it was not observed during the simulations of a voltage-mode controlled (VMC) buck converter with zero parasitic resistances in the input filter. Input filter's current and converter's input voltage did not oscillate and did not go beyond their operating limits. Small oscillations in converter's output voltage were observed, but they can be due to degrading converter's dynamics, i.e. changes in converter's transfer functions due to the input filter, which can degrade converter's loop gain.

One explanation can be that the input impedance seen during on- and off-times is not negative and that is what the filter "sees". The negative input impedance results from viewing the SMPC as a constant power load (CPL), or from the state-space averaged (SSA) model of the converter, i.e. approximated continous model of a discontinous system. Another explanation could be that the fast switching limits the amplitude of the oscillations so much that they could not become obvious in our simulation.

Another conclusion from (10) is that even very small parasitic resistances r_L or $r_{C,1}$ can stabilize the system. This is why usually inductor's internal resistance and the ESR of C_1 are enough to damp the system. Fig. 5b) shows the simulation results of such a system.

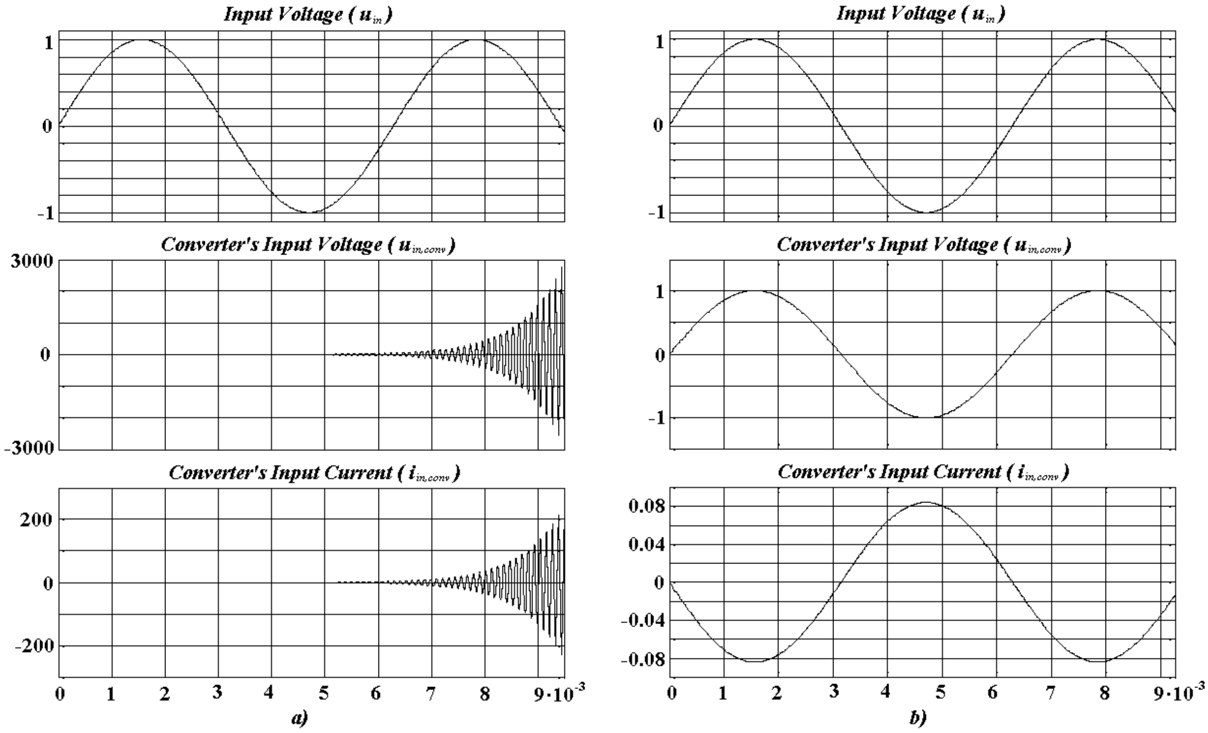

Fig. 5: Simulation of a π-filter with C_1 = 33 μF, L = 10 μH and C_3 = 1 μF, terminated with Z_{in} = –12 Ω.

a) $r_{C,1}$ = 0 Ω, r_L = 0 Ω, $r_{C,3}$ = 10 mΩ. b) $r_{C,1}$ = 10 mΩ, r_L = 0,1 Ω, $r_{C,3}$ = 10 mΩ.

If Z_{in} is not a real constant, but a function of s, the power of the first term in the numerator in (10) increases. For example, Z_{in} of a VMC buck converter is a second order function, dependent on converter's components, the operating point and the load [9].

The positive effect of the ESR of C_1 is one reason for using electrolytic capacitor at the input of the converter (see Fig. 1). The other reason is that the high capacitance of an electrolytic capacitor helps to achieve low output impedance of the filter.

In this analysis only a single stage π-filter was considered. To avoid unnecessary complications the dc-source impedance Z_s in Fig. 4a) was also ignored, but as a positive real function [4] it has stabilizing effect. The inclusion of Z_s, if it is known, will require only one more matrix multiplication or it could be added to the Z seen from filter's input and the same test would apply – whether the zeros of the resulting driving point impedance are in the LHP.

4. Insertion Loss Calculations

EMC standards specify the limits for conducted EME – both common mode (CM) and differential mode (DM). During the design of a SMPC, its EME can be estimated by Fourier analysis of the input current waveform under worst case operating conditions, i.e. minimum input voltage and maximum load. If the converter is ready, its EME can be measured. Subtracting the conducted emissions limits from converter's EME gives the required *IL*, i.e. required attenuation of the power of the conducted EME.

The EMI filter design procedure takes into account many factors - components' limitations, voltage and transient requirements, estimated source and load impedances etc. A full discussion of EMI filter design for SMPC is not possible here, but the application of chain parameters in the most important aspect of EMI filter design, the *IL* calculation, is discussed.

To derive the *IL* refer to Fig. 6. The *IL* is defined as the ratio of the powers (P_1 and P_2) dissipated in the termination impedance Z_s due to the conducted EME before and after the insertion of EMI filter [2]:

$$IL = 10 \cdot \lg \frac{P_1}{P_2} = 10 \cdot \lg \frac{U_1^2 \cdot \mathrm{Re}\{1/Z_s\}}{U_2^2 \cdot \mathrm{Re}\{1/Z_s\}} = 20 \cdot \lg \frac{U_1}{U_2} \qquad (12)$$

U_1 in (12) is the voltage drop over the termination impedance Z_s due to the conducted EME without EMI filter. From Fig. 6a) U_1 is:

$$U_1 = \frac{Z_s}{1 + Y_{\mathrm{EMI}} \cdot Z_s} \cdot J_{\mathrm{EMI}} \qquad (13)$$

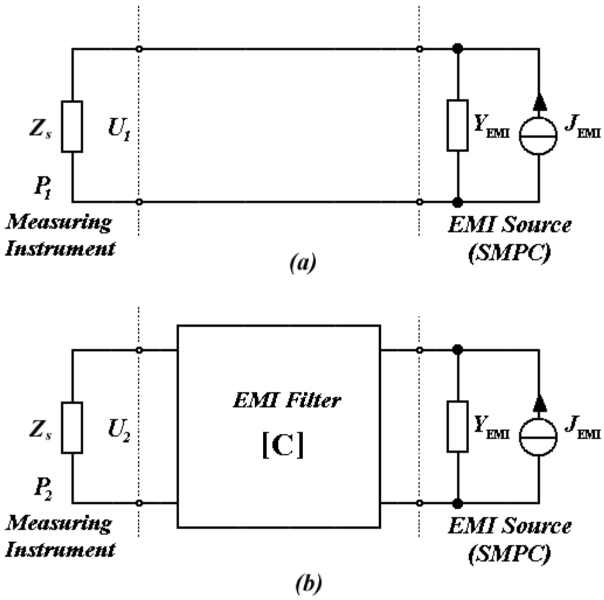

Fig. 6: Definition of the insertion loss.

U_2 is the voltage drop over the termination impedance Z_s, when EMI filter is inserted between Z_s and the EMI source. Referring to Fig. 6b) and using the chain parameters (1), U_2 is:

$$\left.\begin{array}{l}\begin{bmatrix}U_i\\ I_i\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12}\\ c_{21} & c_{22}\end{bmatrix}\cdot\begin{bmatrix}U_o\\ -I_o\end{bmatrix}\\ U_i=\dfrac{J_{\text{EMI}}-I_i}{Y_{\text{EMI}}},U_o=U_2 \text{ and} -I_o=\dfrac{U_o}{Z_s}\end{array}\right\} \Rightarrow U_2=\frac{Z_s}{Y_{\text{EMI}}\cdot(c_{11}\cdot Z_s+c_{12})+c_{21}\cdot Z_s+c_{22}}\cdot J_{\text{EMI}} \quad (14)$$

Inserting (13) and (14) in (12) gives the insertion loss:

$$IL=20\cdot\lg\frac{Y_{\text{EMI}}\cdot(c_{11}\cdot Z_s+c_{12})+c_{21}\cdot Z_s+c_{22}}{(1+Y_{\text{EMI}}\cdot Z_s)} \quad (15)$$

For a π-filter, using the chain parameters as seen from the converter (6), the insertion loss is:

$$IL_\pi=20\cdot\lg\frac{Y_{\text{EMI}}\cdot[(1+Z_2Y_3)\cdot Z_s+Z_2]+(Y_1+Y_1Z_2Y_3+Y_3)\cdot Z_s+Y_1Z_2+1}{(Y_{\text{EMI}}\cdot Z_s+1)} \quad (16)$$

The same EMI filter provides different *IL* for DM and CM noise. When calculating the *IL* for the DM EMI, one should use the chain parameters of the DM components and for the CM EMI – the CM components. For example, the DM components of the EMI filter in Fig. 1 are the leakage inductance of the CM inductor *L* with its parasitic elements, included in Z_2, and the X-capacitors, i.e. Y_1 is the admittance of the parallel connected C_{XE} and C_{X1} with their parasitic elements, and Y_3 is the admittance of C_{X3} with its parasitic elements.

The CM components of the same filter in Fig. 1 are the CM inductor *L* and the Y-capacitors C_{Y1} and C_{Y2}. Another peculiarity is that the π-filter in the same Fig. 1 is effectively a LC-filter at CM. For safety reasons Y-capacitors' values are strictly limited and there are no phase-to-ground capacitors beyond the CM inductor (from converter's point of view). Setting Y_3 = 0 in (6) and (16) gives LC-filter's chain parameters and *IL* respectively. The latter is:

$$IL_{\text{LC}}=20\cdot\lg\frac{Y_{\text{EMI}}\cdot(Z_s+Z_2)+Y_1\cdot Z_s+Y_1Z_2+1}{(Y_{\text{EMI}}\cdot Z_s+1)} \quad (17)$$

The choice of EMI filter configuration – T, LC or π, depends on the source and load impedances [2]. Comparing (16) and (17), reveals that if dc supply's internal impedance Z_s is negligible, i.e. if the dc voltage source is almost ideal, there would be no difference between LC- and π-filter:

$$IL_{Z_s=0}=20\cdot\lg(Y_{\text{EMI}}\cdot Z_2+Y_1Z_2+1) \quad (18)$$

In EMC compliance tests usually Z_s = 50 Ω. In real operation, however, it varies depending on the system, or its mode of operation. The cabling parasitic inductance, in most cases, will make Z_s inductive and a π-filter will perform better.

Although Z_s is often unknown, it is not very difficult to measure. This is not the case with Y_{EMI}, which cannot be measured and is very difficult to estimate. Due to the forced commutation in the SMPC, it appears as a current source, injecting current harmonics with frequencies, which are multiples of converter's switching frequency. For an ideal current source Y_{EMI} = 0. In practice it cannot happen, because snubber circuits and parasitic elements provide some admittance. In addition, some SMPC topologies, such as boost, SEPIC and Ćuk converters, have input inductor, which smoothens the input current. That inductor must be included in Y_{EMI}.

A buck converter with input π-filter, as shown in Fig. 7, was simulated using MATLAB®/Simulink®'s Power System Blockset.

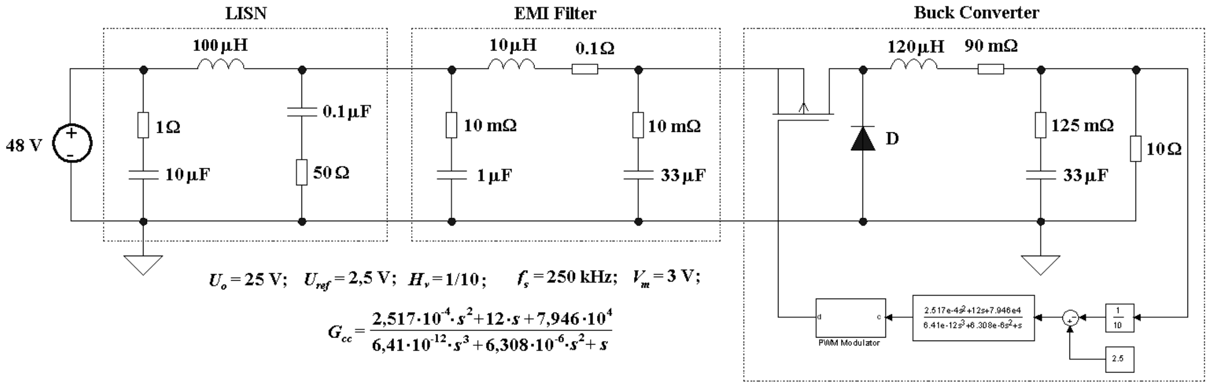

Fig. 7: Values for the components of the VMC buck converter, the EMI filter and the LISN used in the simulations for measuring the insertion loss of the filter.

The *IL* for the first three harmonics of the switching frequency were measured and plotted in Fig. 8 as circles. The lines in the Figure are plots of equation (16) for Z_s = 50 and three different values of Y_{EMI}.

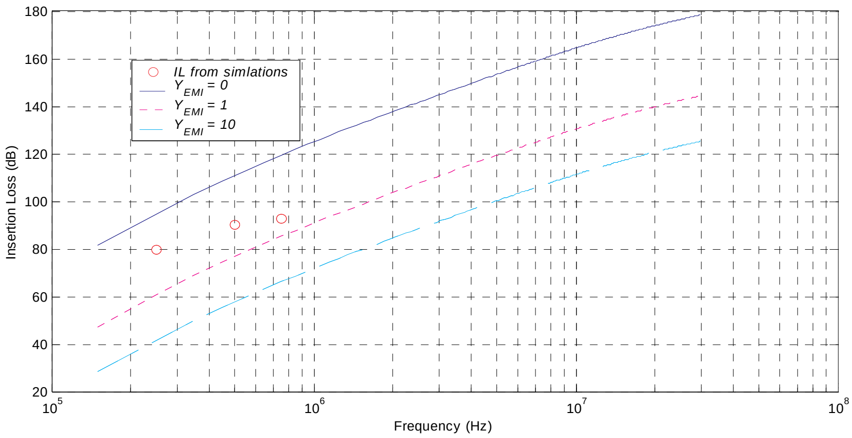

Fig. 8: *IL* of a π-filter, used as input filter of a buck converter.

As expected, the simulations for measuring the *IL* of a buck converter, showed that the *IL* is close to the *IL* curves for very low EMI source admittance, i.e. the converter appears as a very good current source, injecting switching frequency harmonics into the dc line.

5. Conclusions

Chain parameters were used to analyse the input system stability of a SMPC with π-filter. The analysis showed that resistance in series with the converter has stabilizing effect. However, connecting resistors in series with the converter is not recommended because it will reduce significantly the efficiency. Resistance in series with the capacitor, closest to converter's input has stabilizing effect as well. Normally, the ESR of that capacitor and the parasitic resistance of the inductor are enough to achieve input system stability.

Chain parameters prove to be a useful tool in the *IL* calculations as well. One difficulty is to obtain the chain parameters for CM and DC, because of the unknown parasitics and because each filter component contribute differently for the attenuation of the DM and CM noise (a CM inductor is a good example for this). The other difficulty in the EMC analysis is the unknown EMI source admittance. The simulations of a buck converter showed that it appears as very good EMI current source

It was also shown using chain parameters that a π-filter will perform as an LC-filter if the dc line impedance is negligible. Otherwise, π-filter performs better.

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