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http://www.iop.org/journals/jmm
http://stacks.iop.org/jmm/19/015028
Analysis of vibration modes in a micromechanical square-plate resonator

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Received 4 July 2008, in final form 13 October 2008
Published 17 December 2008
Online at stacks.iop.org/JMM/19/015028

Abstract
Vibration modes of a micromechanical square-plate resonator were studied using a scanning laser interferometer. The resonator consists of a square plate released from a silicon substrate and has its main resonance at 13.1 MHz. The resonator is designed to utilize a square-extensional vibration mode in which the vibrations mainly take place in the direction parallel to the surface of the plate. A special detection scheme was used to allow measurement of the in-plane vibrations in addition to the out-of-plane component detected by the interferometer. The measured vibration modes were compared to numerical finite-element simulations with good agreement. In particular, it was found out that the out-of-plane vibration field measured at the resonance frequency of the main mode is a superposition of a parasitic vertical vibration mode and the out-of-plane component of the main mode. Furthermore, the measurements revealed undesirable vibrations in the anchor regions indicating energy leakage from the resonator causing additional losses. The quality factors of the vibration modes were also determined from the laser-probe measurements and they agree quite well with those determined from electrical measurements.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Increasing functionality and the number of embedded radio systems in modern wireless communication systems, along with the demand for small size and light weight, present a challenge for the device design. As a potential way to reduce the component size and to reach high integrability in these devices, considerable interest has lately been devoted to microelectromechanical systems (MEMS). One bulky component remaining in the modern mobile phone architectures is the quartz reference oscillator. Micromechanical resonators may offer an attractive alternative for these oscillators since they possess compact size, excellent performance, low power consumption and integrability with integrated circuit (IC) electronics [1].

The characterization of vibration modes within MEMS devices is important to validate their design, operation and performance. Optical probing is a powerful tool for the task since the vibrations can be studied without perturbing the device operation. Furthermore, optical measurements provide direct physical information on the acoustic fields within the electro-acoustic devices not obtainable via measuring the electrical response. For instance, optical measurements may reveal sources of acoustic losses and parasitic vibration modes.

Recently, MEMS have been characterized with scanning interferometers [2–5] as well as with feedback interferometry [6, 7]. In addition, the vibration mode shapes of cantilever microbeams, micromachined membranes and microrings have been measured using an interferometric microscope with
A square plate resonator is applied [25–27]. The advantage of vibrations, a method utilizing the surface structure of the plate resonator, is that no heavy image processing tools are needed, and measurements up to the gigahertz range are possible. The measured vibration modes are compared to calculated ones [22–24].

Also, commercial full 3D motion probes based on video microscopy with stroboscopic illumination exist, but they typically operate only at frequencies below 10 MHz. For example, a microscope scanning laser Doppler vibrometer has been used to measure the resonance frequencies and the modal shape of a piezoelectric cantilever [12]. In connection with surface-acoustic-wave (SAW) and bulk-acoustic-wave (BAW) devices, vibrations in the GHz frequency range have recently been visualized, e.g., using scanning Michelson interferometer [13, 14], scanning heterodyne interferometer [15], the knife edge technique [16–18], Mach–Zender laser interferometer [19, 20], dynamic holography with photorefractive interferometric detection [21] and atomic force microscopy [22–24].

In this work, we analyze vibration modes of a micromechanical resonator by measuring both in- and out-of-plane vibrations using a scanning laser interferometer [13]. The Michelson-type interferometer as such detects only out-of-plane vibrations. For detection of the in-plane vibrations, a method utilizing the surface structure of the plate resonator is applied [25–27]. The advantage of the detection method is that no heavy image processing tools are needed, and measurements up to the gigahertz range are possible. The measured vibration modes are compared to finite-element-method (FEM) simulations. We also determine the quality factors \( Q \) values of the vibration modes from the measurement data and compare the results to those obtained from electrical measurements.

2. Square plate resonator

The sample studied is a 13.1 MHz micromechanical resonator described in detail in [1]. These resonators possess a high quality factor of 130,000 (in vacuum, bias voltage of 100 V) and a high power handling capacity of 0.12 mW.

The MEMS resonator, illustrated in figure 1, consists of a square plate released from the silicon substrate and supported at the corners with springs. The dimensions of the resonator plate are 320 \( \mu \text{m} \times 320 \mu \text{m} \times 10 \mu \text{m} \). The device has been fabricated using deep reactive ion etching on a silicon-on-insulator (SOI) wafer. For the sacrificial oxide release etch, a 39 \( \times \) 39 grid of circular holes (diameter 2 \( \mu \text{m} \)) has been fabricated to the resonator plate, see figure 1(c).

The resonator is excited capacitively using transducers placed on all four sides of the square plate. Hence the excitation is mainly in-plane. However, in this work only one transducer was connected to enable excitation of a more versatile selection of vibration modes. The read-out of the resonator is connected to one of the corner supports, see figures 1(a) and 1(b). The substrate around the resonator was grounded to minimize parasitic capacitances. Figure 1(d) depicts the electrical circuit for supplying the radio frequency (RF) signal, \( U_{\text{in}} \), and the bias voltage, \( U_{\text{bias}} \), to the resonator. The following values of the capacitance, \( C \), and the resistance, \( R \), were used: \( C = 100 \text{ nF} \), \( R_1 = 1 \Omega \) and \( R_2 = 100 \text{ k\Omega} \). Resistors \( R_1 \) were used to obtain a well-defined potential at the input and output of the resonator. In the optical measurements, the output port was terminated with a load of 50 \( \Omega \).

The main vibration mode of the resonator, the square-extensional (SE) mode, is described as a two-dimensional expansion preserving the plate’s original square shape, as illustrated in figure 1(a). The dashed and dotted lines indicate the contracted and expanded shapes of the square plate in the SE mode. With a one-electrode drive, the resonator also exhibits the Lamé mode. In the Lamé mode, illustrated in figure 1(b), the edges of the square plate bend in antiphase preserving the volume of the plate. In both cases, the motion is mainly in plane but vibrations perpendicular to the surface are also excited due to the Poisson ratio of the silicon crystal.

To enable laser-interferometric measurements in vacuum, a custom-made sample enclosure was designed and constructed. The resonator was enclosed in a small metal case connected to a vacuum pump. The metal case has a quartz window, through which the measurement beam of the
interferometer is focused by a microscope objective onto the surface of the resonator.

Electrical transmission measurements of the resonator were carried out using the HP 8753E Network Analyzer and the electrical circuit shown in figure 1(d). Figure 2 shows the electrical response of the resonator measured both under ambient air pressure and in vacuum (pressure below 1 mbar). An input power of 0 dBm and a bias voltage of 60 V were used. The resonance frequencies of the Lamé and SE modes are 12.13 MHz and 13.10 MHz, respectively. It should be noted that in vacuum a second resonance appears for the Lamé mode, see figure 2(c).

Electrical Q values can be estimated from the $S_{21}$ response by determining the 3 dB width of the resonance peak. However, since the response of the device is weak in this case, the electrical Q values were determined by fitting a calculated response from an electrical equivalent circuit model to the measured electrical response. The electrical equivalent circuit consists of a series resonant circuit that is connected in parallel to a stray capacitance [1]. The electrical Q value was calculated from the parameters of the fitted equivalent circuit and are listed in table 1.

Mechanical vibrations in the resonator may be approximated with a simple theoretical model [1]. The equation of motion for a lumped mass-spring system is

$$M \frac{\partial^2 x}{\partial t^2} + \gamma \frac{\partial x}{\partial t} + K x = f(t),$$

where $K$ is the effective spring constant, $M$ is the effective mass, $\gamma$ is the damping coefficient, and $f(t)$ is the driving force. The vibration amplitude, $x(t)$, and the driving force, $f(t)$, are assumed to be sinusoidal in time $t$, that is, $x(t) = X e^{i \omega t}$ and $f(t) = F e^{i \omega t}$, where $X$ is the amplitude of the vibration, $F$ is the amplitude of the driving force and $\omega$ is the angular frequency. Substituting $x(t)$ and $f(t)$ into (1), the vibration amplitude is given by

$$|X| = \frac{F/K}{\sqrt{(1 - \omega_0^2 \omega^2)^2 + (2 \omega \omega_0/Q \omega_0)^2}},$$

where $\omega_0 = \sqrt{K/M}$ is the angular resonance frequency and $Q = \omega_0 M / \gamma$ is the mechanical quality factor.

### 3. Optical measurement setup and methods

The optical measurements were carried out using the scanning Michelson laser interferometer described in detail in [13]. With a small modification to the interferometer, also in-plane vibrations can be measured with the same setup in addition to the out-of-plane vibration, though not simultaneously.

#### 3.1. In-plane detection method

The method to detect the in-plane vibrations utilizes the etch-hole structure of the sample. In fact, any edge, such as an edge of the resonator or an edge of a metallized region, resulting in contrast in the optical reflectivity is suitable for this detection method. In figure 3(a), a laser beam focused on the sample surface is partially reflected back from the edge of an etch hole. Figure 3(b) shows the dependence of the back-reflected optical power on the relative position of the laser beam to the etch hole. The crosses in the figure indicate measured values, and the solid line represents a fit of a theoretical curve to the measurement data. The theoretical curve is obtained by considering the reflection of a Gaussian laser beam from a sharp edge in one dimension. The total power in the reflected portion of the beam is proportional to the complementary error function [28],

$$P(x) = \frac{1}{2} \left( 1 - \text{erf} \left( \sqrt{2} \frac{x - x_0}{w} \right) \right),$$

where $x$ is the position of the center of the incident beam, $x_0$ is the position of the edge and $w$ is the waist size of the Gaussian beam. This one-dimensional function matches well to the measured values, even though in our case the beam width (2w) and the hole diameter are of the same order of magnitude, 1.6 $\mu$m and 2 $\mu$m, respectively.
If the laser beam is positioned on top of the edge \( x = x_0 \), and the edge position is made to vibrate with a small amplitude \( u_x \), the reflected light power is modulated at the same frequency and with an amplitude of \( \Delta P \), see figure 3(b). For small vibration amplitudes, the dependence of the measured \( \Delta P \) on the in-plane vibration amplitude \( u_x \) can be assumed to be linear, \( \Delta P = k u_x \). The proportionality constant \( k \) is given by the slope of the power curve in figure 3(b), and hence is determined by the waist size \( w \).

3.2. Simulation of in-plane detection response

The in-plane detection response was simulated numerically for the two-dimensional (2D) case. The circular etch hole was described by a step function

\[
h(r) = \begin{cases} 
0, & r \leq d/2 \\
1, & r > d/2
\end{cases}
\]

where \( d \) is the diameter of the etch hole and \( r \) is the radial coordinate, i.e., \( r^2 = x^2 + y^2 \). The in-plane vibration is described by a vector \( \mathbf{u} = (u_x, u_y) \), where \( u_x \) and \( u_y \) are the amplitudes of the vibration components along \( x \) and \( y \) axes. The vibration amplitudes are assumed to be constant over the small simulated area containing one etch hole. The intensity distribution of the Gaussian beam is described by [29]

\[
I(r) = \frac{2P_0}{\pi w^2} \exp \left( -\frac{2r^2}{w^2} \right),
\]

where \( P_0 \) is the optical power of the laser beam.

To simulate the measurement, the laser beam position was raster-scanned in two dimensions over a surface area containing one etch hole. At each scan point, the power of the reflected light was calculated by determining the overlap integral, \( R(x, y) \), of the Gaussian beam profile and the reflecting plane surface containing the circular hole. It was assumed that any light falling in the hole is lost. The slope coefficient vector \( \mathbf{k} = (k_x, k_y) \) at each scan point was then found by determining the gradient of \( R(x, y) \). Finally, the modulation of the reflected optical power caused by the in-plane vibrations was obtained by taking the scalar product of \( \mathbf{k} \) and \( \mathbf{u} \).

Figure 4 shows a simulation result for an areal scan over one etch hole in the case of in-plane vibrations along the \( x \) axis with \( \mathbf{u} = (1, 0) \) in this case. The dashed circle indicates the edge of the etch hole.
3.3. Measurement of vibration components

The out-of-plane vibration was measured using the scanning laser interferometer [13] in its original Michelson configuration illustrated in figure 5. The measurement is based on detecting the optical phase difference between the two arms of the interferometer, the sample arm and the reference arm. The interferometer signal is detected by a high-speed photodetector at the sample excitation frequency. The sensitivity of the interferometer is $10^{-4}$ Å/√Hz, corresponding to 0.35 pm with the 1 kHz measurement bandwidth used. The waist size of the focused laser beam on the sample surface is 0.8 μm. The minimum step size of the translation stages used to move the sample is 55 nm.

The in-plane vibrations were measured with the same instrument by blocking the reference beam of the interferometer. In this case, only the sample beam reflected from the sample surface propagates to the high-speed photodetector, where its intensity modulation is recorded at the sample excitation frequency. The in-plane vibrations are detected at edges in the resonator. In the case of the straight edge, i.e., the edge of the resonator plate, the vibration component perpendicular to the edge is detected. Utilizing the circular release-etch holes in the resonator plate, a vectorial detection of the in-plane vibration component can be accomplished. It should be noted, however, that the in-plane and out-of-plane vibration amplitudes cannot be directly compared to each other due to the different detection methods.

In the current device, relative amplitudes of the in- and out-of-plane vibrations are measured. However, the absolute amplitudes of the out-of-plane vibrations can be obtained by applying a calibration procedure described in [13]. Also, calibration of the in-plane detection is possible but it requires development of the detection electronics so that the coefficient vector $k$ can be determined. The vector $k$ cannot be obtained from the light-reflection image, since the mean power is recorded with a different photodetector than the intensity modulation. The use of the relative amplitude is justified because the etch holes can be assumed to be identical which implies that the slope, $k$, is the same for each etch hole.

In addition to the relative vibration amplitudes, the mean power of the laser beam reflected from the resonator, i.e., the reflectivity of the resonator surface is recorded at each scan point. The mean light power is measured with a light detector using the faint reflection from the $\lambda$/4 plate. Hence a micrograph-like image of the scanning area (hereafter called light-reflection image) is obtained. The area in the light-reflection image corresponds one-to-one with the area in the amplitude image.

4. FEM simulations

An eigenmode analysis was carried out to simulate the vibration modes of the resonator. The simulations were performed with the stress solver of the multiphysical FEM software Elmer [30]. The simulations were run on a Sun Fire 2K having 96 Ultrasparc IV processors with a memory of 384 GB. The stiffness parameters used in the simulation were $c_{11} = 167.4$ GPa, $c_{12} = 65.2$ GPa, $c_{44} = 79.6$ GPa, and the density was set to $\rho = 2329$ kgm$^{-3}$. The stiffness matrix was rotated 45° to consider the crystal orientation ⟨110⟩. The supporting springs and etch holes were omitted from the FEM analysis as well as the air damping in order to reduce the model complexity and thus the computation time.

A rectangular free body representing the resonator plate was meshed with a varying number of elements, ranging from 6,000 to 200,000. 6,000 elements were sufficient to yield an accuracy of 0.1% for the eigenfrequencies of the lateral modes. However, the remaining uncertainty of the eigenfrequencies for the vertical modes when using 200,000 elements was about 0.5%. This was estimated using a grid convergence analysis. The simulations took 1 h and 7 h and required 10 GB and 40 GB of memory for 70,000 and 200,000 elements, respectively.

The FEM simulations resulted in 15 eigenmodes in the frequency band from 10 MHz to 15 MHz. All the eigenmodes do not necessarily appear in the real resonator since the excitation geometry determines which modes are excited. Table 2 summarizes the resonance frequencies of the modes detected with the laser interferometer. The second column indicates the principal direction of the motion.

Figure 6 shows the simulated out-of-plane vibration components of two selected eigenmodes. Figure 6(a) depicts...
Figure 6. FEM simulation of the out-of-plane vibration amplitude of (a) the SE mode and (b) one of the vertical vibration modes, mode 7.

Table 2. Simulated and measured resonance frequencies of the eigenmodes. The measured values were determined from the optical measurements conducted under ambient air pressure. The second column indicates the principal direction of the motion.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Type</th>
<th>Simulated (MHz)</th>
<th>Measured (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Horizontal</td>
<td>10.53</td>
<td>10.03</td>
</tr>
<tr>
<td>2</td>
<td>Vertical</td>
<td>11.14</td>
<td>10.93</td>
</tr>
<tr>
<td>3</td>
<td>Vertical</td>
<td>11.18</td>
<td>11.10</td>
</tr>
<tr>
<td>4</td>
<td>Vertical</td>
<td>12.27</td>
<td>12.13</td>
</tr>
<tr>
<td>5(1)</td>
<td>Horizontal</td>
<td>12.92</td>
<td>12.13</td>
</tr>
<tr>
<td>6</td>
<td>Horizontal</td>
<td>13.37</td>
<td>12.75</td>
</tr>
<tr>
<td>7</td>
<td>Vertical</td>
<td>12.36</td>
<td>13.11</td>
</tr>
<tr>
<td>8(2)</td>
<td>Horizontal</td>
<td>13.71</td>
<td>13.11</td>
</tr>
</tbody>
</table>

1 Lamé mode.
2 square-extensional (SE) mode.

the out-of-plane component of the SE mode, which is the horizontal main mode having the vibrations mainly in the in-plane direction. Figure 6(b) shows the out-of-plane component of a vertical mode (mode 7) which is considered to be a parasitic mode. According to the simulations, mode 7 also has an in-plane vibration component but the amplitude of that component is much smaller than that of the out-of-plane component. All the simulations are plotted in logarithmic scale using the expression \(20 \log_{10} |u|\), where \(u\) is the amplitude of the eigenmode.

5. Measurement results

Laser-probe measurements were conducted at several frequencies around the resonance frequencies of the Lamé and SE modes. The measurements were carried out both under ambient air pressure and in vacuum (pressure below 1 mbar) with an input power of 0 dBm and with a bias voltage of 60 V.

5.1. Out-of-plane vibrations

Figure 7 shows out-of-plane vibrations measured under ambient air pressure for three modes together with the corresponding FEM simulation. The scan in figure 7(a) was measured at the resonance frequency of the SE mode, 13.106 MHz. The resonance frequency was determined based on the electrical measurement and on the optical measurement of the in-plane vibrations. The simulated image in figure 7(b) is a superposition of the out-of-plane components of the SE mode and the mode 7, both shown in figure 6. The modes were superposed using an expression \(u_{\text{sum}} = u_7 + a \times u_{\text{SE}}\), where \(a = -350\) gave the best visual match. The excellent correspondence between the measured and simulated profiles
Figure 8. Optical intensity modulation data yielding the relative amplitude of the in-plane vibrations in the upper-right quarter of the resonator plate. The measurement is carried out under ambient air pressure at the frequency of the SE mode (13.106 MHz).

indicates that the observed amplitude profile in figure 7(a) is a superposition of the out-of-plane component of the SE mode and that of the parasitic vertical mode.

The scan in figure 7(c) is measured at the resonance frequency of the Lamé mode, 12.134 MHz. Comparing this to the simulated amplitude field of mode 4 shown in figure 7(d), we can conclude that the vibration profile at 12.134 MHz is dominated by this parasitic vertical mode. In fact, the out-of-plane component of the Lamé mode is expected to be zero according to simulations. The scan in figure 7(e) shows the out-of-plane component of a parasitic mode (mode 2). The measured amplitude profile matches well with the simulated profile shown in figure 7(f). Table 2 summarizes the measured resonance frequencies.

The measured amplitude profiles show vibrations also outside the resonator plate, i.e., in the anchor regions and around the ends of the feeding electrode. This indicates energy leakage from the resonator causing additional losses.

5.2. In-plane vibrations

In-plane vibrations were measured by detecting the intensity modulation, as explained in section 3.1. Figure 8 shows data measured under ambient air pressure at the resonance frequency of the SE mode. The amplitude of the in-plane vibration is obtained by searching the two maxima of each spot pair. The direction of the vibration is given by a straight line drawn through the maximum points of the two spots around each etch hole. Hence the in-plane vibration component of the SE mode measured at discrete points, i.e., on top of each etch hole, can be obtained. Additional spots outside the regular grid of the etch holes are due to dust particles originating from the sample preparation. Note also that the signal at the edges of the resonator plate and at the T-shaped anchors can be seen. In this case, the intensity modulation is proportional to the vibration component perpendicular to the edge. The scan area in figure 8 corresponds to the area shown in the light-reflection image in figure 1(c).

Figure 9. Vector fields of the in-plane vibrations measured under ambient air pressure for (a) the SE mode (13.106 MHz), and (b) the Lamé mode (12.134 MHz) in the upper-right quarter of the resonator. The center of the resonator plate is indicated with a cross.

Figure 10. Light-reflection images indicating the scan areas used. The small scan areas I, II and III marked with yellow boxes are used for frequency response measurements of the SE and Lamé modes.

The measured in-plane vibrations can be presented as a vector field. The vectors extracted from the intensity-modulation data are drawn on top of the etch holes. The orientation and the length of each vector are calculated by determining positions and maximum values of the two spots around each etch hole.

The in-plane vibrations of the SE and Lamé modes measured in the upper-right quarter of the resonator plate are shown in figures 9(a) and 9(b), respectively. The center of the resonator plate is marked with a cross in both figures. It should be noted that the vector field in figure 9(a) is directly extracted from the intensity-modulation data shown in figure 8. The additional spots due to dust particles were excluded from the data by searching the maximum values only in the area close to the etch holes.

5.3. Extraction of quality factors

In order to determine the mechanical frequency responses and the associated $Q$ values of the Lamé and SE modes, scans with high spatial resolution were carried out at several frequencies. The scan parameters and the measured areas are shown in table 3 and in figure 10(b), respectively. Small scanning areas were chosen to minimize the scanning time in order to eliminate the possible drift of the resonance frequencies, e.g., due to temperature changes. The use of a fixed scanning location is justified since the mode shapes of the vibration fields do not change in the narrow frequency range studied.
Both the out-of-plane and in-plane vibrations were studied. For the out-of-plane vibrations, the amplitude was determined at each frequency by taking an average of the amplitude over the scan area, excluding the area of the etch hole, see figures 11(a) and (b). For the in-plane vibrations, the maximum amplitudes of the four spots were first located, see figure 11(d). The four maxima were then averaged to obtain the in-plane vibration amplitude at each frequency. It should be noted that in figure 11(d) the position of the maximum amplitude coincides with the edge of the etch hole, which is in accordance with the principle of the in-plane detection described in section 3.1. Also the shape of the patterns around each etch hole is similar to the one obtained in the simulation of the in-plane detection method, see figure 4.

Figure 12 shows six of the in-plane scans used to extract the frequency response of the SE mode in vacuum. The distance between the two spots around each etch hole and also the shape of the spots changes with the frequency. Hence the maximum amplitude value of the spot does not coincide with the edge of the etch hole. In this case, the amplitude value for each frequency was obtained by determining the maximum amplitude at the edge of the etch hole. The reason for the anomaly is not clear but it could indicate a presence of an additional local vibration mode bound to the etch hole. The same effect occurs for the in-plane component of the Lamé mode in vacuum.

After extracting the mechanical frequency response from the measurement data, the theoretical response given by (2) was fitted to the experimental data. The equation was transformed to logarithmic scale with an expression $20 \log_{10}(|X|) + A$. The fit parameters are the resonance frequency $f_0 = \omega_0 / 2\pi$, quality factor $Q$, parameter $F/K$ and offset $A$.

Figure 13 depicts the mechanical frequency response measured around the resonance frequency of the SE mode. The theoretical curve fits well to the experimental data for all the four cases: the in-plane and the out-of-plane vibrations measured both under ambient air pressure and in vacuum. When the pressure is lowered from the ambient air pressure to vacuum, the resonance frequencies decrease roughly by 3.4 kHz while the maximum amplitudes increase by 18 dB. Also, the $Q$ values in vacuum are about nine times those measured under ambient air pressure. It should be noted that the amplitudes of the in- and out-of-plane vibrations cannot be compared to each other due to differences in the detection principles.

Figure 14 shows the mechanical frequency response measured around the resonance frequency of the Lamé mode. The fitted curves of the out-of-plane component in figure 14(a)
Figure 13. Mechanical frequency responses measured at the frequency of the SE mode: (a) out-of-plane vibrations and (b) in-plane vibrations.

Figure 14. Mechanical frequency responses measured at the frequency of the Lamé mode: (a) out-of-plane vibrations and (b) in-plane vibrations.

were obtained by fitting a sum of two separate theoretical responses, described by (2). The same applies for the in-plane component measured under ambient air pressure in figure 14(b). For the in-plane component measured under ambient air pressure in figure 14(b), a single theoretical response fits quite well with the experimental data. In the responses, two resonance peaks exist with a small frequency spacing. This becomes evident in the vacuum measurements, where the higher $Q$ value leads to a clear separation of the two resonance peaks. The frequency difference between these two peaks is 3 kHz, which corresponds to that observed in the electrical measurements, see figure 2(c). The amplitude fields of the out-of-plane vibrations measured in vacuum at 12.131 MHz and 12.134 MHz appear identical with the amplitude profile shown in figure 7(c). The corresponding in-plane vibrations are similar to those shown in figure 9(b).

Table 4. $Q$ values determined from the out-of-plane and in-plane vibration measurements, and from the electrical responses.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Out-of-plane</th>
<th>In-plane</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE, air</td>
<td>9500</td>
<td>8900</td>
<td>9900</td>
</tr>
<tr>
<td>SE, vac</td>
<td>89000</td>
<td>(65000)</td>
<td>87000</td>
</tr>
<tr>
<td>Lamé, air</td>
<td>(36000/5300)</td>
<td>5500</td>
<td>6100</td>
</tr>
<tr>
<td>Lamé, vac</td>
<td>36000/31000</td>
<td>34000/26000</td>
<td>37000</td>
</tr>
</tbody>
</table>

$Q$ values of the SE and Lamé modes were obtained from the fits of the theoretical response to the experimental data. Table 4 summarizes the $Q$ values as averages over five measurement sets together with the $Q$ values calculated from the electrical measurements shown in figure 2. The $Q$ values of the in- and out-of-plane vibrations are roughly the same and match well with the $Q$ values obtained from
the electrical measurements. The standard deviation of the resonance frequency was about 0.1 kHz in each case. The standard deviation of the $Q$ value in ambient air pressure and in vacuum was $\sim$1% and $\sim$10%, respectively. There was one exception: for the $Q$ value of the in-plane vibration of the SE mode, the standard deviation was 25%. This could be explained by the combination of finite scan step and small drift of the sample position causing an error in the extracted vibration amplitude. It should also be noted that for the out-of-plane response of the Lamé mode in ambient air pressure, the fit is not as good as for the other cases although the measurement data are consistent which is seen as a small standard deviation.

It is also interesting to compare the simulated and measured resonance frequencies summarized in table 2. According to the optical measurements, the resonances of the SE mode and mode 7 are at the same frequency although FEM simulations predict a difference of 1.35 MHz between the resonance frequencies. A similar case can be seen with the Lamé mode and mode 4. The difference in the simulated resonance frequencies could be explained by the fact that the etch holes and the anchor springs were not included in the simulations.

6. Conclusion

Vibration modes in a micromechanical resonator were studied using laser-probe measurements. The measurement results were compared to FEM simulations and to electrical measurements.

The simulated mode shapes match well with the measured ones. All optically measured eigenmodes could be identified from FEM eigenmode analysis. A large number of elements (200,000) was needed to enable simulation of the vertical modes with a sufficient accuracy, although the etch holes and the anchor springs were not included in the simulations. The match between the measured resonance frequencies and the simulated eigenfrequencies was better than 7%.

The in- and out-of-plane vibration amplitudes were measured as a function of frequency for the SE and the Lamé modes along with several other modes. The measurements were conducted both under ambient air pressure and in vacuum for the SE and Lamé modes. From the measured data, the $Q$ values were determined for the SE and Lamé modes. The in- and out-of-plane vibrations feature similar $Q$ values that also correspond to those determined from the electrical measurements.

The measured amplitude profiles show vibration amplitudes also outside the resonator plate, i.e., in the anchor regions and around the ends of the feeding electrode. These additional vibrations indicate energy leakage from the resonator causing additional losses. In addition, splitting of the Lamé mode was observed, which was seen as two resonance peaks also in the electrical measurements. In the optical measurements, both in-plane and out-of-plane, the amplitude profiles for the two resonance peaks appeared identical.

By comparing the laser-probe measurements and the FEM simulations, we can conclude that the observed out-of-plane vibration patterns are partly caused by a parasitic vertical vibration mode. At the frequency of the SE mode, the measured out-of-plane vibration field consists of a superposition of a vertical vibration mode and the out-of-plane component of the SE mode. At the frequency of the Lamé mode, the measured vertical vibration field is dominated by another vertical vibration mode over the vanishing vertical component of the Lamé mode.

According to the FEM simulations, the resonance frequencies of these vertical modes should differ from the resonance frequencies of the in-plane modes (SE and Lamé modes). Based on the laser-probe measurements, however, it seems that the measured in- and out-of-plane vibrations feature about the same resonance frequencies and roughly the same $Q$ values. This could be explained by assuming that the in- and out-of-plane vibration modes are coupled and the out-of-plane vibration mode is excited by the in-plane mode. On the other hand, the difference between the simulated and measured resonance frequencies could also be explained by the fact that the etch holes and the anchor springs were not included in the simulations.

The optical measurements enable studying directly the vibrations in MEMS devices and hence provide new valuable information on the physics of RF MEMS not obtainable from electrical measurements alone. Moreover, by comparing the results of laser-probe and electrical measurements to the FEM simulations, the operation of MEMS devices can be analyzed thoroughly.

Acknowledgments

O Holmgren thanks the Alfred Kordelin Foundation and the Finnish Cultural Foundation for scholarships. This research has been supported by the Academy of Finland within the MIRA project (TULE Programme, Future Electronics Research). Jyrki Kiihämäki at VTT, Finland, is acknowledged for fabricating the sample. The late Professor Martti M. Salomaa had an important role in the research described in this paper and the authors are indebted to him for his support, encouragement and collaboration over the years.

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