

Publication V

M. Heinonen and S. Sillanpää, The effect of density gradients on hydrometers, *Measurement Science and Technology* **14**, 625 - 628 (2003).

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The effect of density gradients on hydrometers

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Received 21 January 2003, accepted for publication 11 March 2003

Published 4 April 2003

Online at stacks.iop.org/MST/14/625

Abstract

Hydrometers are simple but effective instruments for measuring the density of liquids. In this work, we studied the effect of non-uniform density of liquid on a hydrometer reading. The effect induced by vertical temperature gradients was investigated theoretically and experimentally. A method for compensating for the effect mathematically was developed and tested with experimental data obtained with the MIKES hydrometer calibration system.

In the tests, the method was found reliable. However, the reliability depends on the available information on the hydrometer dimensions and density gradients.

Keywords: density, hydrometer, temperature gradients

1. Introduction

Reliable density measurements of liquids are vital to process control in many branches of industry. They also form a solid basis for reliable flow measurements. A hydrometer is one of the simplest yet most accurate instruments for measuring the density of liquids. Because all electrical density measurement instruments of the same accuracy level are significantly more expensive, hydrometers are still widely used in industry and laboratories.

The operation of a hydrometer is based on the Archimedes principle: while in equilibrium, the mass of liquid displaced by the hydrometer is equal to the mass of the hydrometer if the surface tension is negligible [1–3]. The liquid surface shows the density reading on the hydrometer scale. In a case of non-uniform density, the hydrometer reading depends on the density distribution and the shape of the hydrometer.

Density gradients in liquid samples are often due to non-uniform temperature. Despite careful stirring before measurements, evaporation and conductive heat transfer to and from the ambient maintain temperature differences and cause density gradients in the sample. In a common practice for carrying out measurements at a controlled temperature, the liquid surface is slightly raised in a glass tube to a level above the top of some thermal insulation or a thermally controlled jacket. Poor thermal insulation at the top of the tube intensifies the heat transfer between the liquid and the ambient. As a

result, the vertical density gradient in the liquid close to the surface may be significant.

In this work, we studied the effect of vertical density gradients on hydrometer readings theoretically and experimentally. Experiments were carried out using a hydrometer calibration system at the Centre for Metrology and Accreditation (MIKES).

2. Theory

When a hydrometer is partly immersed in a liquid, the net force (F_c) acting on the instrument is a combination of the gravitational force (F_g) and buoyant force (F_b) acting in opposite directions (see figure 1):

$$F_c = F_b + F_g = g(\rho_{l0}V_2 + \rho_a V_1 + \delta F_b) - (m_h g + \pi d_{h0} \gamma_l) \quad (1)$$

where ρ_a , ρ_{l0} , m_h , g , d_{h0} and γ_l are the air density, bulk density of the liquid, mass of the hydrometer, acceleration due to gravity, stem diameter of the hydrometer at meniscus level and surface tension of the liquid, respectively. V_1 and V_2 are the volumes displacing air and liquid, respectively. The correction δF_b due to the non-uniform density is defined as

$$\delta F_b = F_b - g(\rho_{l0}V_2 + \rho_a V_1). \quad (2)$$

We assume that the effect of the density gradients in air can be omitted. When the hydrometer is in equilibrium, $F_c = 0$.

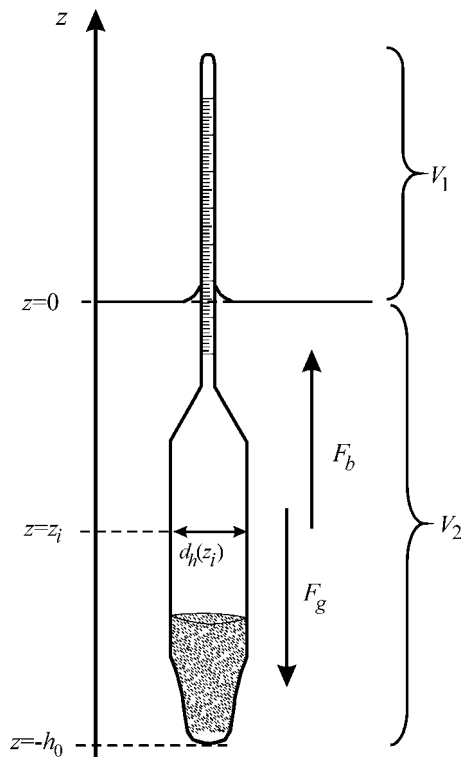


Figure 1. Forces acting on a hydrometer.

If there is only a vertical density gradient in the liquid and the hydrometer is symmetrical with respect to the z -axis, we can write

$$\begin{aligned} \delta F_b &= \frac{\pi g}{4} \int_{-h_0}^0 [\rho_l(z) - \rho_{l0}] d_h^2(z) dz \\ &\approx \frac{\pi g \Delta z}{4} \sum_{i=1}^n [\rho_l(z_i) - \rho_{l0}] d_h^2(z_i) \end{aligned} \quad (3)$$

where $d_h(z_i)$ and $\rho_l(z_i)$ are the diameter of the hydrometer and the actual liquid density at a point z_i , respectively. The integral was approximated by a sum of results calculated at discrete points $i \Delta z$ ($h_0 = n \Delta z$). If the vertical density gradient is due to the temperature distribution $t(z_i)$ along the z -axis, $\rho_l(z_i) = \rho_l(t(z_i))$.

Because $\delta F_b \propto d_h^2$, the gradient effect depends significantly on the position of the hydrometer. The density distribution is often exponential near the liquid surface but almost constant far from the surface. In such a case, the correction may be negligible if the observed reading is at the top of the hydrometer scale. If the reading is at low end of the scale (i.e. near the hydrometer body), however, the correction may be significant.

3. Experiments

3.1. Measurements

We studied the effect induced by a vertical temperature gradient experimentally. Measurements were carried out using the MIKES hydrometer calibration system [4, 5] and a hydrometer with a scale covering the range 1.950–2.000 g cm⁻³, i.e. 1950–2000 kg m⁻³. The scale division is 0.0005 g cm⁻³ =

0.5 kg m⁻³ and the maximum diameter of the stem and body are 4 and 42 mm, respectively. The length of the hydrometer scale is 128 mm and the total length is 335 mm.

The operation of the calibration system is based on Cuckow's method [1, 6], i.e. a hydrometer is weighed in air and while submerged in calibration liquid to the calibration point. The hydrometer reading is then compared with a reference value (ρ_x) calculated from the difference in the weighing results (M_a, M_l). If the correction δF_b is taken into account, the equation for ρ_x is

$$\rho_x = \rho_a + \frac{(\rho_l - \rho_a) [M_a (1 - \frac{\rho_a}{\rho_R}) + \frac{\pi d \gamma_x}{g}] [1 + \beta(t - t_R)]}{M_a (1 - \frac{\rho_a}{\rho_R}) - M_l (1 - \frac{\rho'_a}{\rho_R}) - \frac{\delta F_b}{g} + \frac{\pi d \gamma_l}{g}} \quad (4)$$

where

ρ_a, ρ'_a	= density of air during weighing in air and in liquid, respectively
ρ_R	= reference density for weights (8000 kg m ⁻³)
ρ_l	= density of the calibration liquid
γ_l, γ_x	= surface tension of the calibration liquid and the liquid in which the hydrometer is normally used, respectively
M_a, M_l	= reading of the balance when weighing in air and in the liquid immersed up to the measurement point, respectively
g	= acceleration due to gravity
d	= stem diameter of the hydrometer at meniscus level
t	= temperature of the liquid
t_R	= reference temperature
β	= thermal cubic expansion coefficient of the hydrometer material.

During the experiments, the ethanol (as the calibration liquid) was in a jacketed glass vessel. A thermostatic bath controlled the jacket temperature. A miniature video camera was used for observing the position of the liquid surface on the hydrometer scale. The immersion depth of the hydrometer was changed by lifting or lowering the jacketed glass vessel.

Differences in thermal conditions were obtained between two sets of calibrations by changing the liquid level with respect to the top of the jacket. After both sets of calibrations, the vertical temperature distribution in the liquid was measured using two small Pt 100 probes of a digital thermometer. In the experiments, the mean liquid temperature was +15 °C while the ambient temperature was +25 °C. As shown in figure 2, the liquid level was at first about 60 mm above the liquid surface in the jacket. In the second set of calibrations, the liquid level was about 7 mm below the jacket liquid surface. Due to the different liquid levels, the temperature gradients in the calibration liquid were significantly different in the two sets of measurements (see figure 3).

In both sets of calibrations, five nominal points covering the range above 1960 kg m⁻³ were measured twice. The density of the ethanol at a point below the hydrometer was determined by means of hydrostatic weighing of a silicon sphere.

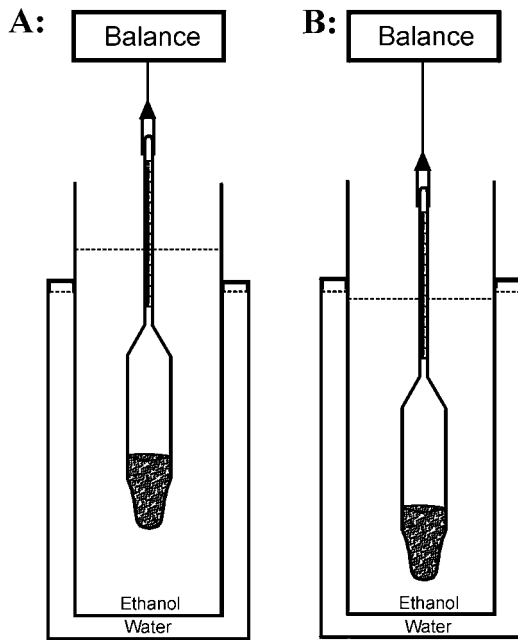


Figure 2. The measurement set-up in the first (A) and second (B) set of calibrations.

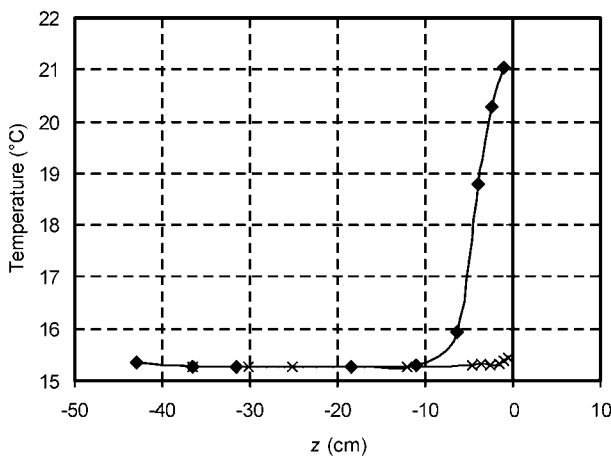


Figure 3. The measured vertical temperature distribution (◆: first calibration set; ×: second calibration set).

3.2. Results

Figure 4 shows all the results calculated omitting the density gradient effect (i.e. assuming $\delta F_b = 0$). The ethanol density, at different temperatures, was calculated using the formula developed by Bettin and Spieweck [7]. A clear systematic difference can be identified in the figure. The difference at high density values (i.e. at a point on the hydrometer scale close to the hydrometer body) is equal to the scale division multiplied by six! This is assumed to be due to the effect of the significant temperature gradient close to the liquid surface on the hydrometer body.

Because the measurement set-ups in the two calibration sets were essentially the same (except the difference in thermal conditions), it is assumed that the difference between the results was entirely caused by the difference in vertical temperature distribution. Therefore, we can test the theory by

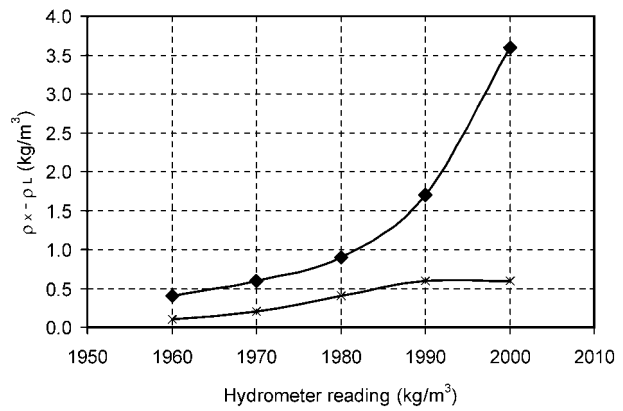


Figure 4. Calibration results without density gradient correction (ρ_x = reference density calculated using equation (4) with $\delta F_b = 0$; ρ_L = hydrometer reading; ◆: first calibration set; ×: second calibration set).

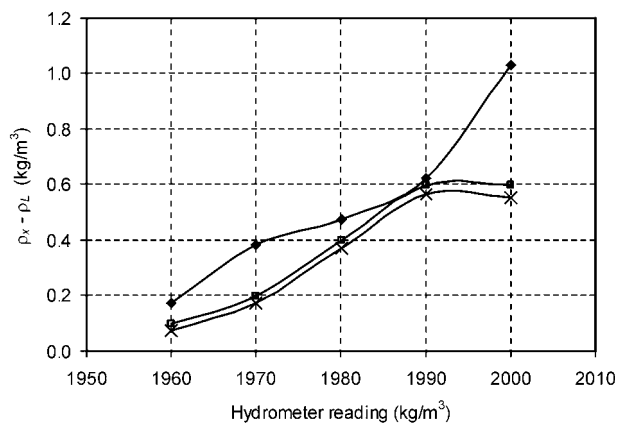


Figure 5. Re-calculated calibration results (ρ_x = reference density calculated using equations (3) and (4); ρ_L = hydrometer reading; ◆: first calibration set; ×: second calibration set) compared with the original results for the second set (□).

calculating the density gradient correction in the way described and by re-calculating the results.

For the purpose, the diameter of the hydrometer was measured at intervals of 3 mm using two methods: digital photograph processing software and a calliper rule with a digital display. Because of the limited resolution of the photograph and the shape of the hydrometer, the measurement uncertainty was 0.16 mm at worst. Using the dimensional and temperature distribution data, we calculated new results according to equations (3) and (4).

As shown in figure 5, the difference between the two calibration sets in the re-calculated results is less than 0.5 kg m^{-3} (equal to the scale division). Furthermore, the difference is mostly smaller than the estimated reading accuracy (equal to a fifth of the scale division). According to our estimation, the expanded uncertainty of a single calibration (excluding the thermal gradient effect) was 0.5 kg m^{-3} . The effect of thermal expansion was found to be insignificant compared with the density gradient effect. The calculated correction for the results of the second set of calibrations was less than 0.05 kg m^{-3} at all points.

4. Discussion and conclusions

Our experiments showed that the density gradients due to non-uniform temperature might have a significant effect on hydrometer measurement results even in a temperature-controlled set-up. The effect should be considered especially if the hydrometer body is near the liquid surface. This is also the most probable explanation for the discrepancies observed in the results of the comparison between findings from MIKES and the Istituto di Metrologia 'G. Colonnetti' (IMGC) in 1998–1999 [5] because the measurements were carried out in a jacketed glass tube with a slightly raised liquid level.

Unfortunately, significant temperature gradients cannot always be avoided for practical reasons, e.g. when measuring liquid density at a temperature considerably higher or lower than the ambient temperature. For such cases, we developed a mathematical method for compensating for the effect of a density gradient on hydrometer measurement results. In the experimental work presented in this paper, errors equivalent to several scale divisions were reduced below the level of one scale division by applying the mathematical method. At the point of highest density, the error could probably be reduced further by increasing and improving the data on the hydrometer dimensions and temperature gradient. On the other hand, our results show that useful results can be obtained with moderate-dimension data.

The method presented is a useful tool for improving the quality of hydrometer measurements and calibrations. It can also be used for estimating the uncertainty related to temperature gradients and density gradients in general.

References

- [1] Gupta S V 2002 *Practical Density Measurement and Hydrometry* (Bristol: Institute of Physics Publishing)
- [2] Wagenbreth H, Gorski W and Kozdon A 1985 Ein verbessertes Prüfverfahren für Normalräometer *PTB-Mitteilungen* **95** 322–6
- [3] Bowman H A and Gallagher W H 1969 An improved high-precision calibration procedure for reference standard hydrometers *J. Res. Natl Bur. Stand. C* **73** 57–65
- [4] Heinonen M 1999 Intercomparison of the hydrometer calibration systems at the IMGC and the MIKES *MIKES Publication J9/1999*
- [5] Lorefice S, Heinonen M and Madec T 2000 Bilateral comparisons of hydrometer calibrations between the IMGC-LNE and the IMGC-MIKES *Metrologia* **37** 141–7
- [6] Cuckow F W 1949 A new method of high accuracy for the calibration of reference standard hydrometers *J. Chem. Ind.* **68** 44–9
- [7] Bettin H and Spieweck F 1990 A revised formula for the calculation of alcoholometric tables *PTB-Mitteilungen* **100** 457–60