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# Dispersion engineering of photonic crystal waveguides with ring-shaped holes

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**Abstract:** The geometry of photonic crystal waveguides with ring-shaped holes is optimized to minimize dispersion in the slow light regime. We found geometries with a nearly constant group index in excess of 20 over a wavelength range of 8 nm. The origin of the low dispersion is related to the widening of the propagating mode close to the lower band gap edge.

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## 1. Introduction

Photonic crystal waveguides (PhCW's) have very different dispersion properties compared to conventional waveguides due to their periodic boundaries. In particular, PhCW's exhibit slow group velocity near the Brillouin zone edge [1]-[5]. Reduced group velocity increases light-matter interaction and nonlinearities, which can be utilized to realize more compact integrated optics devices. However, the slow modes usually have a very high group velocity dispersion (GVD), which leads to optical signal degradation in telecommunications systems. Therefore it is important to realize slow-light structures with tailored dispersion properties. A number of approaches has been used to achieve this, for example using W2 waveguides [6] or modification of the hole radius [7] or period [8] in the first one or two hole row(s) next to the waveguide channel.

Our approach is to use photonic crystal waveguides with ring-shaped holes (RPhCW's) [9]-[12], for which we have measured a group index up to 20 [11]. Due to an extra free parameter in the lattice design, PhCs with ring-shaped holes allow fine-tuning of the waveguide dispersion properties. In this paper we optimize the ring dimensions so that W1 waveguides show minimal dispersion with a relatively high group index.

## 2. Optimization of the ring geometry

Figure 1 shows a RPhCW fabricated into silicon-on-insulator substrate with a 240 nm thick top silicon layer [11]. The waveguide is a single line defect of missing holes in a triangular lattice of rings with outer and inner radii  $R_{out} = 0.344a$  and  $R_{in} = 0.203a$ , respectively.

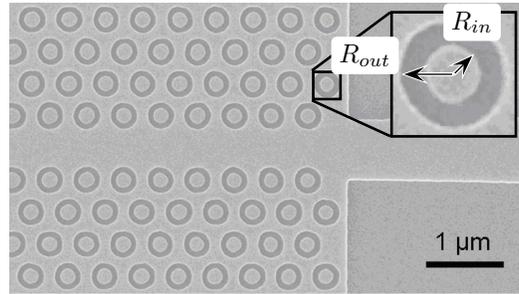


Fig. 1. Scanning electron micrograph of a RPhCW fabricated by electron beam lithography and reactive ion etching [11].

2D simulations with the plane wave method [13] are used. The background refractive index is the effective refractive index of the guided TE polarized mode in the SOI slab. For the 240 nm thick silicon slab on silicon dioxide, which supports only a single TE mode at 1550 nm, we use an effective refractive index of 2.84.

The plane wave simulation gives the dispersion relation  $u(\beta)$ , where  $u$  is the normalized frequency ( $u = \frac{a}{\lambda}$ ) and  $\beta$  is the propagation constant. The group velocity  $v_g$  is defined as

$$v_g = \frac{\partial \omega}{\partial \beta} = \frac{2\pi c}{a} \frac{\partial u}{\partial \beta}. \quad (1)$$

It can be obtained using the built-in function in the MIT Photonic Bands package, where  $v_g$  is calculated for the mode for each value of  $\beta$  by operating the magnetic field using the Hellmann-Feynman theorem [14]. The group index is

$$n_g = \frac{c}{v_g} = \frac{a}{2\pi} \frac{\partial \beta}{\partial u}. \quad (2)$$

Figure 2 shows the calculated dispersion relation of the waveguide in Fig. 1. For comparison, the dispersion relation of the conventional PhC waveguide with the same air fill factor is shown as a scatter plot. The group index of the RPhCW is larger than that of the PhCW between 1610 nm and 1620 nm. The region between 1615 nm and 1620 nm is the most interesting; here the RPhCW has  $n_g > 25$  and a smaller group velocity dispersion parameter  $D$  than the conventional PhCW.  $D$  is defined as

$$D = \frac{1}{c} \cdot \frac{dn_g}{d\lambda}. \quad (3)$$

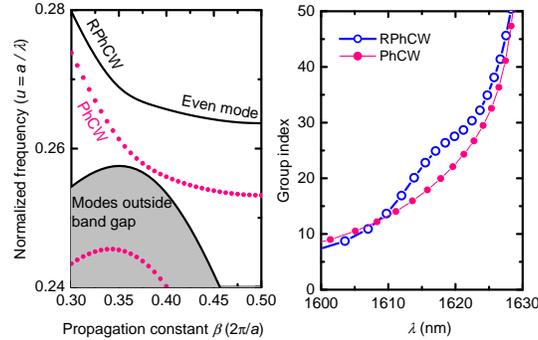


Fig. 2. Dispersion relation and group index of a W1 RPhCW with  $a = 430$  nm,  $R_{out} = 0.344a$  and  $R_{in} = 0.203a$  in comparison with those of a conventional W1 PhCW with  $a = 415$  nm and  $R = 0.278a$ .

Figure 3 shows the shift of the band gap edge and the even waveguide mode frequency as a function of the ring outer radius, while keeping the ring width  $R_{out} - R_{in}$  constant. The cut-off frequency increases with  $R_{out}$ , but not as much as the band gap edge frequency, therefore the band gap edge comes closer to the mode. This affects the shape of the dispersion relation of the waveguide mode, particularly near the transition from the index-guided mode to band gap guided mode around  $\beta = 0.35 \frac{2\pi}{a}$ . The dispersion diagram of the RPhCW with  $R_{out} = 0.38a$  shows a nearly constant slope between  $\beta = 0.35 \frac{2\pi}{a}$  and  $\beta = 0.45 \frac{2\pi}{a}$ .

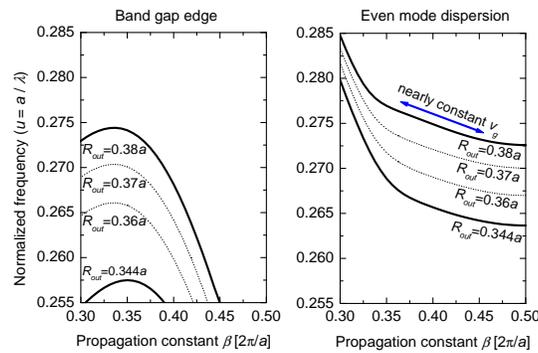


Fig. 3. Change of band gap edge and waveguide mode frequencies with the change of ring radius. The width of the ring  $R_{out} - R_{in}$  is kept constant at  $0.14a$ .

Slow light in the corrugated waveguides is explained as an interaction between the forward and backward propagating modes. The interaction is at its strongest at the edge of the Brillouin

zone, where the group velocity vanishes. When moving away from the zone boundary, the interaction gets weaker due to phase mismatch between the forward and backward propagating modes. However, as the mode of the RPhCW is at its closest to the bandgap edge near  $\beta = 0.35\frac{2\pi}{a}$  it penetrates deeper into the surrounding photonic crystal lattice (Fig. 4). This increases the effect of the periodicity and compensates for the phase mismatch. This explains why the group index is nearly constant over a relatively large wavelength range.

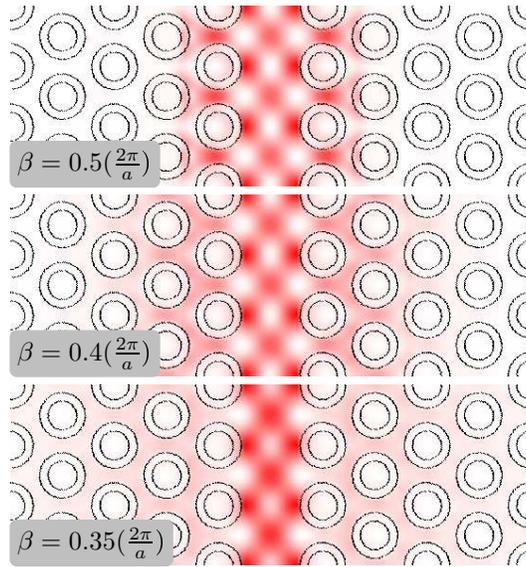


Fig. 4. Electric field energy densities for the slow mode in a RPhCW with  $R_{out}=0.38a$  and  $R_{in}=0.24a$  with three different propagation constants.

Due to the large mode area, a supercell width of  $35a$  is needed in the plane wave simulations in order to prevent the effect of the periodic boundary conditions. In order to verify the results, the group index was also deduced from the Fabry-Pérot oscillations in the RPhCW transmission spectrum simulated by the finite-difference time-domain (FDTD) method [15].

Figure 5 shows the dispersion relation and the group index of a W1 RPhCW with  $R_{out}=0.38a$  and  $R_{in}=0.24a$ . The group index has a quasi constant value of 25 over a wavelength range of 8 nm.

Modes below the  $\text{SiO}_2$  light line are theoretically lossless. Therefore it is desirable to extend the part of the slow mode that is below the light line by decreasing its frequency. This can be realized by increasing the effective refractive index of the silicon slab, i.e., by increasing the silicon layer thickness  $h$ . With  $h = 400$  nm ( $n_{eff}=3.18$ ), we find that the slow light region is almost entirely below the light line. Figure 6 shows the band diagram and group index of a RPhCW with  $R_{out}=0.385a$ ,  $R_{in}=0.235a$  and  $h = 400$  nm. One can see that both the bandgap and the mode frequency have dropped compared to Fig. 5. The group index is  $37 \pm 3$  over a wavelength range of 8 nm.

As can be seen from the definition of the group velocity dispersion parameter  $D$  (Eq. 3), nearly constant group index regions exhibit small dispersion. Figure 7 shows the GVD curves calculated from the group index for the RPhCW's in Figs. 5 and 6. Group velocity dispersion minima can be seen in the nearly constant group index region with  $D < 1\text{ps}/(\text{mm} \cdot \text{nm})$  over 8 nm for the RPhCW of Fig. 5 and  $D < 1\text{ps}/(\text{mm} \cdot \text{nm})$  over 3 nm for the RPhCW of Fig. 6. In a conventional PhCW,  $D$  increases monotonically when coming closer to cut-off wavelength.

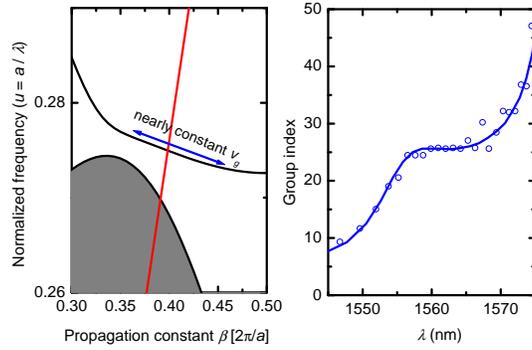


Fig. 5. Dispersion relation and group index of a W1 RPhCW with  $a = 430$  nm,  $R_{out} = 0.38a$  and  $R_{in} = 0.24a$ . The scatter plot shows the results of the FDTD simulations. The light line of SiO<sub>2</sub> is shown in red in the dispersion relation.

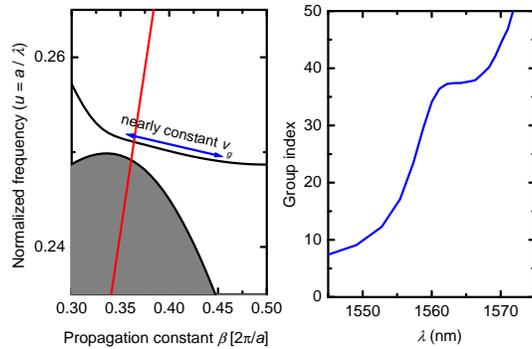


Fig. 6. Dispersion relation and group index of a W1 RPhCW with  $a = 392$  nm,  $R_{out} = 0.385a$  and  $R_{in} = 0.235a$  etched into a 400 nm thick silicon layer on silica. The light line of SiO<sub>2</sub> is shown in red in the dispersion relation.

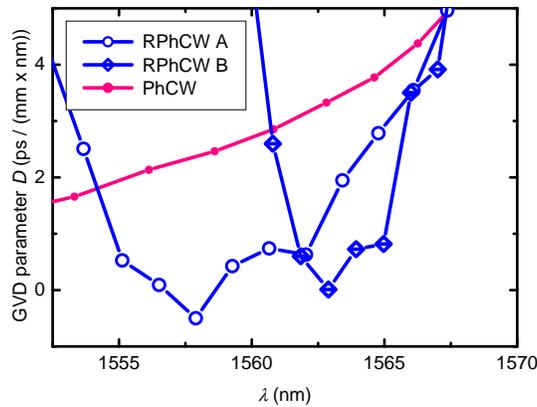


Fig. 7. Group velocity dispersion calculated from the group index spectra of three waveguides. RPhCW A:  $R_{out} = 0.38a$ ,  $R_{in} = 0.24a$ ,  $a = 430$  nm and  $h = 240$  nm, RPhCW B:  $R_{out} = 0.385a$ ,  $R_{in} = 0.235a$ ,  $a = 392$  nm and  $h = 400$  nm. The curve with closed dots corresponds to a conventional PhCW with  $R = 0.278a$ ,  $a = 400$  nm and  $h = 240$  nm. Lattice constants are chosen so that the cut-off wavelength is 1575 nm for all three waveguides.

### 3. Conclusion

We showed that photonic crystal waveguides with ring-shaped holes enable dispersion tailored slow light structures. We show two RPhCW's exhibiting nearly constant group index regimes with  $n_g \approx 25$  and  $n_g \approx 37$ , respectively, both over a wavelength range of 8 nm, with group velocity dispersion parameters  $D < 1\text{ps}/(\text{mm} \cdot \text{nm})$  over a few nanometers. Small dispersion is crucial in high bit rate telecommunications applications. In a wavelength division multiplexed telecommunications system with a channel spacing of 100 GHz ( $\approx 0.8$  nm), the wavelength range of the useful slow light region achieved in this work is equivalent to about 10 channels.

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