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Cramér-Rao Bound for Angular Propagation Parameter Estimation in MIMO Systems

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***Abstract*— Realistic channel models are crucial in developing future wireless systems. Channel sounding and consequently propagation parameter estimation are key tasks in creating such models. In this paper we present an estimator for the angular distribution of the diffuse scattering component that is frequently observed in channel sounding measurements. The large sample performance of the estimator is studied by finding the Cramér-Rao lower bound and comparing the variances. The simulations show that the variance of the proposed estimation technique closely reaches the Cramér-Rao lower bound for small sample size and for any number of antennas.**

I. Introduction

Channel sounding and extensive channel measurement campaigns are necessary in order to build new powerful multidimensional channel models for the development of beyond 3G communications systems. Such models are an important tool in developing transceiver structures and designing networks with high spectral efficiency.

Most of the current development of parameter estimation techniques for channel sounding are based on estimation of specular components, that are considered as deterministic signals with unknown parameters. However, in measurement campaigns available in the literature, it has been acknowledged that part of the signal is actually transmitted by diffuse scattering. For example, in [1] an scheme has been proposed for estimation of time-domain behavior of this diffuse scattering component. However, no information about the angular properties of this component are estimated in [1].

An estimation method for the parameters of the angular distribution of the diffuse scattering component was derived in [2]. Since the diffuse scattering is a random process, the signal model is stochastic. The angular distribution is modeled by the von Mises distribution, which is suitable for directional data. This approach leads to lower complexity and faster convergence in comparison to deterministic model based on Space Alternating Generalized Expectation-maximization (SAGE) algorithm [3], [4]. These benefits are due to the smoother likelihood function and the reduced number of unknows.

In this paper we derive the Cramér-Rao Lower Bound (CRLB) for estimation of the model parameters for the estimator presented in [2]. We also study the small sample performance of the estimator by comparing the variance of the estimates to the CRLB and show that the bound is attained for relatively small sample size.

This paper is organized as follows: in Section II we describe the signal model used in this article. The channel sounding technique employed is described in Section III. In Section IV the technique for parameter estimation is described. In Section V the Cramér-Rao lower bound is established. Finally, in Section VI we present some simulation results and compare the large sample performance of the estimation technique to the CRLB.

II. Signal model

The transmitter is assumed to be elevated and therefore not obstructed by local scatterers, while the receiver is surrounded by a large number of local scatterers. No line-of-sight is assumed between the transmitter and the receiver. We consider that the waves are planar (far-field) and only single scattering occurs. This is called the "one-ring" model [5].

Let us define the output of the each antenna at the receiver by the $N \times 1$ vector $\mathbf{y}(k)$, the transmitted sequences by the $M \times 1$ vector $\mathbf{u}(k)$, the $N \times M$ MIMO channel matrix $\mathbf{H}(k)$, and the $N \times 1$ vector $\mathbf{n}(k)$ with random noise. The signals defined above are discrete-time versions of continuous-time signals sampled at time instants kT_s, where T_s is the sampling period. Using these definitions we model the received signal as

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{u}(k) + \mathbf{n}(k). \tag{1}$$

The defined channel matrix represents the combination of all waves that impinge the receiver array after being reflected by the surrounding scatterers. In [3], [4] the channel is modeled using a different approach, where the received signal is described as a combination of several waves with unknown parameters together with random noise. The received signal is then written as a function of the wave parameters, and estimation techniques such as SAGE can be used to yield estimates of the parameters. However, as observed in [6], the resulting parameters will be randomly distributed with a distribution that is independent of the actual propagation environment and is only an artifact of the estimation procedure.

If this method is employed for estimation of the diffuse scattering component, the parameters from several waves must be estimated, leading to a search in a very high dimensional space. Therefore, the algorithms often experience convergence problems due to large number of local minima in the likelihood function.

III. Channel sounding technique

We consider the same channel sounding technique used in [3], [4], [7], where the radio channel sounder is assumed to be equipped with switches at both transmitter and receiver ends. The sounding signal is applied by means of a switch during a period T_t at the input of each element of the transmit array. The sounding signal $u(k)$ with power P_u is periodic with period T_t. Each period of the sounding signal can be described as a periodically repeated burst $u_s(k)$ of duration T_u and a cyclic prefix of duration T_g, i.e.,

$$u(k) = \sum_{p=-\infty}^{\infty} u_s(k - pN_u), \quad -N_g \le k < N_t - N_g, \quad (2)$$

where $N_u = T_u/T_s$, $N_g = T_g/T_s$, and $N_t = T_t/T_s$. The cyclic prefix is transmitted during the guard time T_g, which must be greater than the maximum propagation delay in order to ensure that the signals from different antennas in Array 1 do not interfere with each other in the receiver.

At the receiver side the outputs from each element of Array 2 are successively scanned during a period T_r. For each period T_t, corresponding to the transmission of the sounding signal by one element of Array 1, one sensing cycle is performed, during which all elements in Array 2 are scanned once.

For a more detailed description of channel sounding arrangements, method, technique, and setup see [2]–[4], [7].

IV. Parameter estimation

For the derivation of the estimation method we will consider the transmission from one transmitter antenna at a time, as implemented in the channel sounder described in Section III. However, the estimation method applies to MIMO systems in general, since after one cycle all transmit antennas have been switched, leading to an M-transmit, N-receive MIMO system. If one transmit antenna is considered at the time, the channel matrix $\mathbf{H}(k)$ in (1) is a $N \times 1$ vector denoted by lowercase $\mathbf{h}(k)$ and the transmitted signal $\mathbf{u}(k)$ is a scalar. We will assume that the channel is constant during one measurement cycle, in which case the signal model of equation (1) can be simplified to

$$\mathbf{y}(k) = \mathbf{h}u(k) + \mathbf{n}(k). \quad (3)$$

We will assume that $\mathbf{y}(k)$ are i.i.d. zero-mean complex Gaussian. After removing the constant terms not dependent on the signal, the log-likelihood function can be rewritten as

$$-\log|\mathbf{C}_y| - \mathrm{tr}\{\mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y\}, \quad (4)$$

where $\mathbf{C}_y$ is the covariance matrix of the received signal, $\mathbf{C}_y = E[\mathbf{y}(k)\mathbf{y}^H(k)]$, $\widehat{\mathbf{C}}_y$ is the sample covariance matrix, defined as $\widehat{\mathbf{C}}_y = \frac{1}{N_s}\sum_{k=0}^{N_s-1}\mathbf{y}(k)\mathbf{y}^H(k)$, $N_s = T_r/T_s$, $|\cdot|$ denotes the determinant, and $\mathrm{tr}\{\cdot\}$ denotes the trace.

As described in Section III, the sounding sequence is preceded by a time-guard interval whose duration is greater than or equal to the channel delay spread. Hence, we can write $\mathbf{C}_y = E[\mathbf{y}\mathbf{y}^H]$ as

$$\mathbf{C}_y = P_u E[\mathbf{h}\mathbf{h}^H] + \mathbf{C}_n, \quad (5)$$

where P_u is the power of the transmitted sequence and $\mathbf{C}_n = E[\mathbf{n}\mathbf{n}^H]$. See [2].

Using the channel model in [5] we can write $E[\mathbf{h}\mathbf{h}^H]$ as a function of the channel parameters. It is assumed that the receiver is surrounded by a large number of local scatterers, and that the waves reflected by different scatterers arrive at the array with the same power. This situation can be represented as a ring of scatterers around the receiver, as depicted in Figure 1 for any 2 antennas at the transmitter and receiver [5].

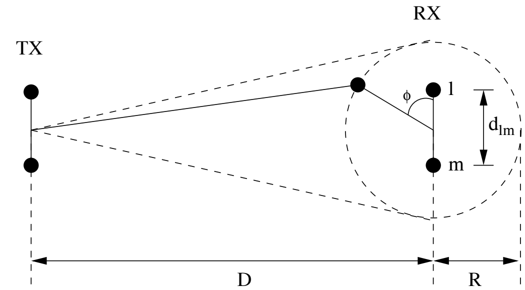

Fig. 1. Illustration of the geometrical configuration of a 2x2 channel with local scatterers at the receiver, where D is the distance between the transmitter and receiver arrays, R is the radius of the ring of scatterers around the receiver, and d_{lm} is the distance between elements l and m in the receive array.

Assuming that the angle spread at the transmitter is small and $D \gg R \gg d_{lm}$, it is possible to show that the cross-correlation between any two sub-channels l and m is given by

$$\rho_{lm} = \frac{1}{\Omega}E[h_l h_m^*] = \int_{-\pi}^{\pi} \exp(b_{lm}\cos(\phi))f(\phi)d\phi, \quad (6)$$

where $f(\phi)$ is any angular PDF of ϕ, $b_{lm} = j2\pi d_{lm}/\lambda$, h_l is the l-th element of $\mathbf{h}$, and Ω is the variance of each MIMO subchannel, $\Omega = E[|h_l|^2] = E[|h_m|^2]$, assumed known. See [5].

It is clear from equation (6) that $E[h_l h_m^*]$ depends on the angular distribution. A Gaussian distribution is not appropriate, since it assumes the data is in the range $(-\infty, \infty)$, while angular data is by definition in the range $[0, 2\pi)$. A suitable PDF, that is defined for angular data and is analytically tractable, is the Von Mises [8], defined as

$$f(\phi) = \frac{1}{2\pi I_0(\kappa)}\exp(\kappa\cos(\phi - \mu)), \quad (7)$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero, μ is the mean angle and κ is a scatter type of parameter, which can be chosen between 0 (isotropic scattering) and ∞ (extremely concentrated). Using the Von Mises PDF, the cross correlation between elements l and m of $\mathbf{h}$ may be written as [2], [5]

$$\rho_{lm} = \frac{1}{I_0(\kappa)} I_0(\{\kappa^2 + b_{lm}^2 + 2\kappa b_{lm}\cos(\mu)\}^{\frac{1}{2}}). \quad (8)$$

We can now find the values for μ and κ that maximize the log-likelihood function in (4), i.e.,

$$\{\widehat{\mu}, \widehat{\kappa}\} = \mathrm{argmax}_{\mu,\kappa}\left\{-\log|\mathbf{C}_y| - \mathrm{tr}\{\mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y\}\right\}, \quad (9)$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace. In order to optimize (9), we will use the Nelder-Mead simplex algorithm [9], as implemented by the `fminsearch` function of Matlab, but any maximization procedure can be used.

V. CRAMÉR-RAO BOUND

In this section we derive the Cramer-Rao bound for the estimation of the parameters of the angular distribution.

The log-likelihood equation is given by

$$\mathcal{L}\{\mu,\kappa\} = -N_s \log \pi^N - N_s \log |\mathbf{C}_y| - N_s \mathrm{tr}\{\mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y\}. \quad (10)$$

In order to compute the derivatives of $\mathcal{L}\{\mu,\kappa\}$ we will use the following matrix differentiation rules [10]:

$$\mathbf{C}(\mathbf{X}) = \mathrm{tr}\{\mathbf{A}\mathbf{X}\} \quad \Rightarrow \quad d\mathbf{C}(\mathbf{X}) = \mathrm{tr}\{\mathbf{A}d\mathbf{X}\} \quad (11)$$

$$\mathbf{C}(\mathbf{X}) = \log|\mathbf{X}| \quad \Rightarrow \quad d\mathbf{C}(\mathbf{X}) = \mathrm{tr}\{\mathbf{X}^{-1}d\mathbf{X}\} \quad (12)$$

$$\mathbf{C}(\mathbf{X}) = \mathbf{X}^{-1} \quad \Rightarrow \quad d\mathbf{C}(\mathbf{X}) = -\mathbf{X}^{-1}(d\mathbf{X})\mathbf{X}^{-1}, \quad (13)$$

where $d(\cdot)$ denotes the differential. Using the expressions above we can calculate

$$\begin{aligned} d\mathcal{L}\{\mu,\kappa\} &= -N_s d\log|\mathbf{C}_y| - N_s \mathrm{tr}\{d\mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y\} \\ &= -N_s \mathrm{tr}\{\mathbf{C}_y^{-1} d\mathbf{C}_y\} + \\ &\quad + N_s \mathrm{tr}\{\mathbf{C}_y^{-1}(d\mathbf{C}_y)\mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y\} \\ &= -N_s \mathrm{tr}\{(\mathbf{I}_N - \mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y)\mathbf{C}_y^{-1}(d\mathbf{C}_y)\}, \end{aligned} \quad (14)$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix.

Using equation (8), the elements of the derivatives of $\mathbf{C}_y$ with respect to μ and κ are given by

$$\begin{aligned} \{\mathbf{D}_\mu\}_{l,m} &= \frac{\partial\{\mathbf{C}_y\}_{l,m}}{\partial\mu} \\ &= -\Omega P_u I_0^{-1}(\kappa) I_1(\beta)\beta^{-1}\kappa b_{lm}\sin\mu \end{aligned} \quad (15)$$

$$\begin{aligned} \{\mathbf{D}_\kappa\}_{l,m} &= \frac{\partial\{\mathbf{C}_y\}_{l,m}}{\partial\kappa} \\ &= \Omega P_u \left[-I_0^{-2}(\kappa)I_1(\kappa)I_0(\beta) \right. \\ &\quad \left. + I_0^{-1}(\kappa)I_1(\beta)\beta^{-1}(\kappa + b_{lm}\cos\mu)\right], \end{aligned} \quad (16)$$

where $\beta = (\kappa^2 + b_{lm}^2 + 2\kappa b_{lm}\cos\mu)^{1/2}$, $I_0(\cdot)$ and $I_1(\cdot)$ are the modified Bessel function of the first kind of order zero and one, respectively.

Hence, using equations (14), (15) and (16), we can write

$$\frac{\partial\mathcal{L}\{\mu,\kappa\}}{\partial\mu} = -N_s \mathrm{tr}\{(\mathbf{I}_N - \mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y)\mathbf{C}_y^{-1}\mathbf{D}_\mu\} \quad (17)$$

$$\frac{\partial\mathcal{L}\{\mu,\kappa\}}{\partial\kappa} = -N_s \mathrm{tr}\{(\mathbf{I}_N - \mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y)\mathbf{C}_y^{-1}\mathbf{D}_\kappa\} \quad (18)$$

We obtain the second derivatives by differentiating (17) and (18). We will first find $\partial^2\mathcal{L}\{\mu,\kappa\}/\partial\mu\partial\kappa$

$$\begin{aligned} \frac{\partial^2\mathcal{L}\{\mu,\kappa\}}{\partial\mu\partial\kappa} &= -N_s \mathrm{tr}\{[\partial(\mathbf{I}_N - \mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y)/\partial\kappa]\mathbf{C}_y^{-1}\mathbf{D}_\mu + \\ &\quad + (\mathbf{I}_N - \mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y)[\partial(\mathbf{C}_y^{-1}\mathbf{D}_\mu)/\partial\kappa]\} \\ &= -N_s \mathrm{tr}\{\mathbf{C}_y^{-1}\mathbf{D}_\kappa\mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y\mathbf{C}_y^{-1}\mathbf{D}_\mu + \\ &\quad + (\mathbf{I}_N - \mathbf{C}_y^{-1}\widehat{\mathbf{C}}_y)[\partial(\mathbf{C}_y^{-1}\mathbf{D}_\mu)/\partial\kappa]\}, \end{aligned} \quad (19)$$

We can then express $E[\partial^2\mathcal{L}\{\mu,\kappa\}/\partial\mu\partial\kappa]$ as

$$E\left[\frac{\partial^2\mathcal{L}\{\mu,\kappa\}}{\partial\mu\partial\kappa}\right] = -N_s \mathrm{tr}\{\mathbf{C}_y^{-1}\mathbf{D}_\kappa\mathbf{C}_y^{-1}\mathbf{D}_\mu\}, \quad (20)$$

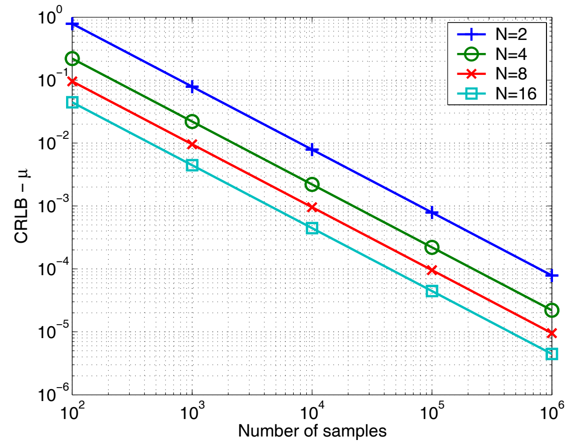

Fig. 2. CRLB for the mean angle μ for several array sizes. The SNR is fixed at 20 dB per antenna.

where the fact that $E[\widehat{\mathbf{C}}_y] = (1/N_s)\sum_{k=0}^{N_s-1} E[\mathbf{y}(k)\mathbf{y}(k)^H] = \mathbf{C}_y$ is used. Similarly, we can write

$$E\left[\frac{\partial^2\mathcal{L}\{\mu,\kappa\}}{\partial\mu^2}\right] = -N_s \mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\mu)^2\} \quad (21)$$

$$E\left[\frac{\partial^2\mathcal{L}\{\mu,\kappa\}}{\partial\kappa^2}\right] = -N_s \mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\kappa)^2\} \quad (22)$$

The Fisher information matrix can then be written as

$$\mathbf{I} = N_s \begin{bmatrix} \mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\mu)^2\} & \mathrm{tr}\{\mathbf{C}_y^{-1}\mathbf{D}_\kappa\mathbf{C}_y^{-1}\mathbf{D}_\mu\} \\ \mathrm{tr}\{\mathbf{C}_y^{-1}\mathbf{D}_\kappa\mathbf{C}_y^{-1}\mathbf{D}_\mu\} & \mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\kappa)^2\} \end{bmatrix} \quad (23)$$

The inverse Fisher information matrix can be easily calculated from (23) as

$$\mathbf{I}^{-1} = \frac{N_s}{|\mathbf{I}|} \begin{bmatrix} \mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\kappa)^2\} & -\mathrm{tr}\{\mathbf{C}_y^{-1}\mathbf{D}_\kappa\mathbf{C}_y^{-1}\mathbf{D}_\mu\} \\ -\mathrm{tr}\{\mathbf{C}_y^{-1}\mathbf{D}_\kappa\mathbf{C}_y^{-1}\mathbf{D}_\mu\} & \mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\mu)^2\} \end{bmatrix}, \quad (24)$$

where the determinant of $\mathbf{I}$ is given by

$$\begin{aligned} |\mathbf{I}| = N_s^2(&\mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\mu)^2\}\mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\kappa)^2\} - \\ &- \mathrm{tr}^2\{\mathbf{C}_y^{-1}\mathbf{D}_\kappa\mathbf{C}_y^{-1}\mathbf{D}_\mu\}) \end{aligned} \quad (25)$$

The Cramer-Rao bounds for μ and κ can then be obtained directly from (24) as

$$\mathrm{CRLB}_\mu = \frac{N_s \mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\kappa)^2\}}{|\mathbf{I}|} \quad (26)$$

$$\mathrm{CRLB}_\kappa = \frac{N_s \mathrm{tr}\{(\mathbf{C}_y^{-1}\mathbf{D}_\mu)^2\}}{|\mathbf{I}|} \quad (27)$$

Figures 2 and 3 shows the behavior of the derived bounds with respect to the number of samples and the number of antennas in the receiver, for parameters μ and κ, respectively. The noise is assumed to be temporally and spatially white, i.e., $\mathbf{C}_n = N_0\mathbf{I}_N$, and $N_0 = 100\Omega P_u$.

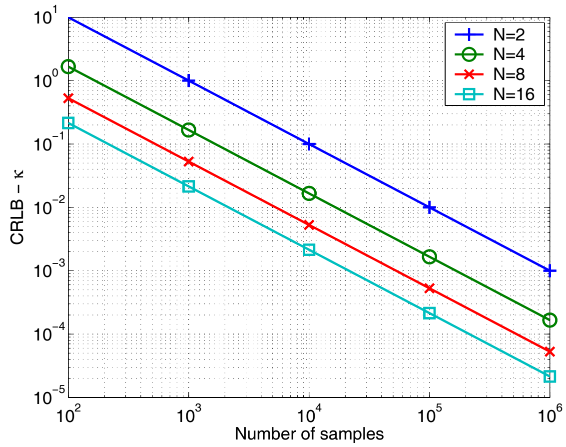

Fig. 3. CRLB for the scatter parameter κ for several array sizes. The SNR is fixed at 20 dB per antenna.

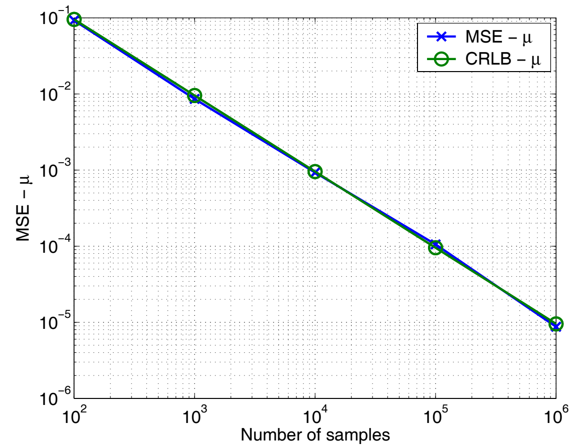

Fig. 4. Comparison of the MSE of the mean angle, μ, to the CRLB as a function of the number of samples. The SNR is fixed at 20 dB per antenna.

VI. Simulation Results

In this section we present some results obtained in simulation in order to compare the performance of the estimator to the CRLB. In all simulations the receiver has an uniform linear array (ULA) with $N = 8$ antennas. The angle of arrival ϕ follows a von Mises distribution with parameters $\mu = 75^\circ$ and $\kappa = 26.93$, corresponding to a angular spread of approximately 8°. The results are averaged over 200–500 runs.

We compare the performance first by generating the signals as a circular complex white Gaussian process with covariance matrix as in (5). This is accomplished by generating the received signal as

$$\mathbf{y}(k) = \mathbf{R}^{1/2}\mathbf{w}(k) + \mathbf{n}(k), \tag{28}$$

where $\mathbf{R}^{1/2}$ is obtained by Cholesky decomposition of $E[\mathbf{h}\mathbf{h}^H]$, i.e., $E[\mathbf{h}\mathbf{h}^H] = \mathbf{R}^{1/2}(\mathbf{R}^{1/2})^H$, and $\mathbf{w}(k)$ is a circular complex white Gaussian process. In order to assess the performance for a frequency-selective scenario, we also generate the signal substituting $\mathbf{w}(k)$ in (28) by $\mathbf{w}'(k) = \mathbf{w}(k) * \alpha(k)$, where $\alpha(k)$ is an exponential sequence with decaying factor chosen as to correspond to a delay spread of $2\mu s$, i.e.,

$$\mathbf{y}_{\text{FS}}(k) = \mathbf{R}^{1/2}(\mathbf{w}(k) * \alpha(k)) + \mathbf{n}(k), \tag{29}$$

In Figures 4 and 5 we compare the MSE for the narrowband model in equation (28) to the CRLB with respect to the number of samples. The SNR is fixed at 20 dB per antenna, and $\Omega = 1$. Figures 6 and 7 compare the MSE to the CRLB with respect to the signal-to-noise ratio (SNR) per antenna, where the number of samples is $N_s = 10^3$. In Figures 8 and 9 the comparison is done with respect to the number of antennas in the receiver. In all cases, it can be noted that the MSE for both parameters closely match the CRLB for all values of sample size, SNR, and number of antennas.

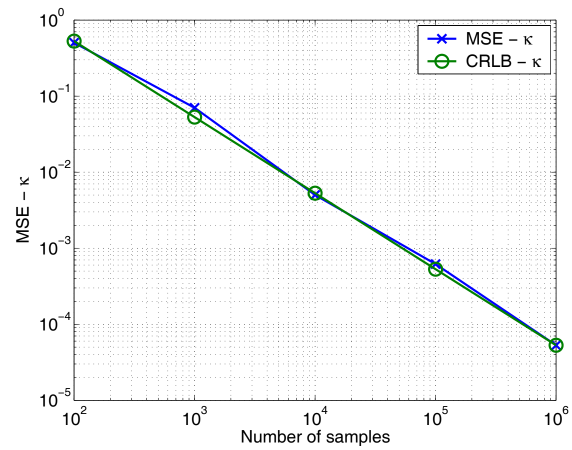

Fig. 5. Comparison of the MSE of the scatter, κ, to the CRLB as a function of the number of samples. The SNR is fixed at 20 dB per antenna.

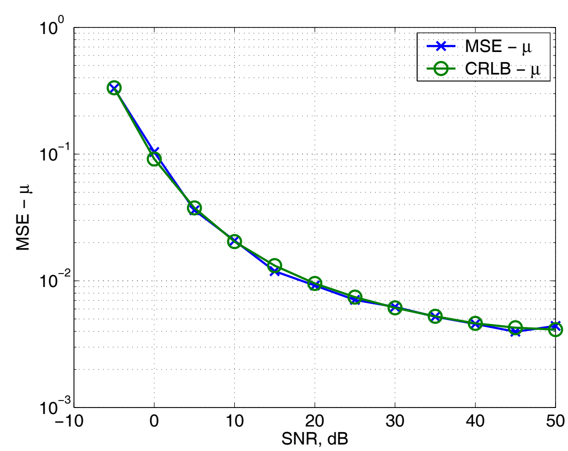

Fig. 6. Comparison of the MSE of the mean angle, μ, to the CRLB as a function of the SNR for 10^3 samples.

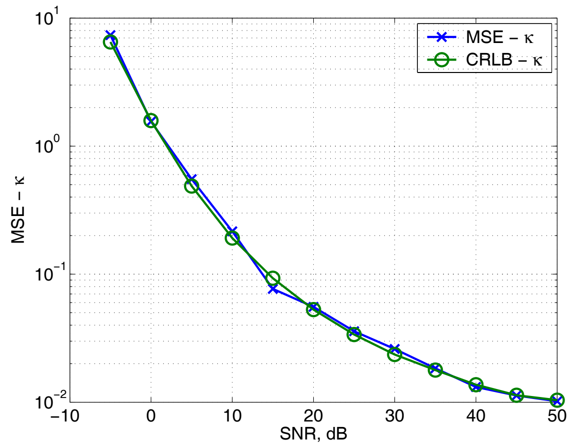

Fig. 7. Comparison of the MSE of the scatter, κ, to the CRLB as a function of the SNR for 10^3 samples.

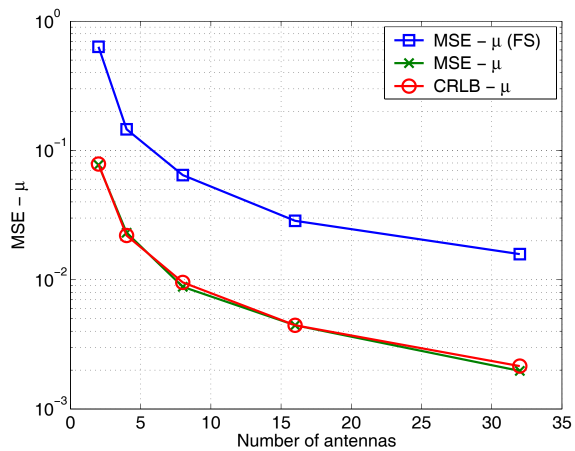

Fig. 8. Comparison of the MSE of the mean angle, μ, to the CRLB as a function of the number of receive antennas, for SNR=20 dB per antenna and 10^3 samples. Performance is shown for both narrowband and frequency-selective (FS) models.

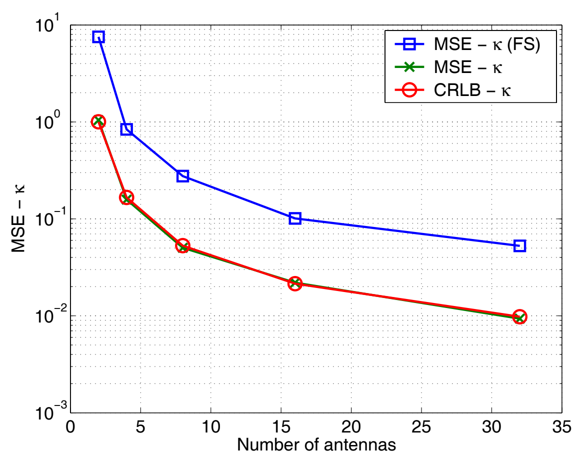

Fig. 9. Comparison of the MSE of the scatter, κ, to the CRLB as a function of the number of receive antennas, for SNR=20 dB per antenna and 10^3 samples. Performance is shown for both narrowband and frequency-selective (FS) models.

VII. Conclusion

In this paper we derived the Cramér-Rao Lower Bound (CRLB) for a technique first presented in [2] for estimation of the parameters of the angular distribution of the diffuse scattering in channel sounding applications. Angular von Mises distribution is used as a model. We also evaluated the small sample performance of the estimator by comparing the variances to the CRLB.

The results show that the variance for the proposed estimation technique attains the CRLB for both parameters with relatively small sample size. Albeit the technique is optimal for a narrowband channel, for the simulated frequency selective channel there is some degradation in the performance. However, high-precision estimates may still be obtained.

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