Heat Transport in Superconducting Quantum Circuits

Jorden Senior
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Aalto University
School of Science
Applied Physics
PICO Group
Supervising professor
Professor Jukka P. Pekola

Preliminary examiners
Doctor Peter Leek, University of Oxford, United Kingdom
Doctor Nicolas Roch, Institut Néel - CNRS, France

Opponent
Associate Professor Jens Koch, Northwestern University, United States of America

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jorden@physics.org
Abstract

Superconducting microwave circuits are a ubiquitous and important tool for devices that exploit the phenomena of superconductivity and cryogenic temperatures as an environment for achieving the generation, manipulation, and detection of quantum states - required for the ongoing development of quantum technologies.

In particular, superconducting circuits are a promising platform for the universal quantum computer, which will require an unprecedented density of quantum-coherent elements to perform large-scale quantum-enhanced calculations and simulations, using the framework of cavity quantum electrodynamics.

Dissipation and heat in these superconducting circuits is a key source of error and inefficiency, however the thermodynamics in this regime is poorly understood despite its increasing relevancy. This thesis describes the integration of superconducting resonators and artificial atoms derived from superconducting quantum circuits with ultra-sensitive bolometry, for looking at heat transport through superconducting circuits.

We will describe the physics and operation of each of these elements, before combining and utilising them to perform heat transport measurements through a superconducting artificial atom coupled to two resonators, each terminated by a normal-metal mesoscopic resistor, with the resistor temperature measured and temperature gradients across the circuit induced by superconducting tunnel-probes. We will present tunable heat transport through this system, firstly when the resonators are symmetric, allowing us to observe the role of dissipation-limited coupling of the resonators to the artificial atom on the locality of the heat transport, then on an asymmetric system, demonstrating a directional rectification of the heat transport. Additionally, we will discuss how each element of the system can be individually characterised, in particular the quality factor of superconducting resonators in the highly-dissipative limit, by exploiting the superconducting transition to perform a background reference.

It is suggested that this hybrid quantum system, and these initial experiments provide a promising platform in the emergent field of circuit quantum thermodynamics. We believe that the techniques and tools developed during this thesis present key steps towards the understanding of the thermodynamics of quantum circuits, towards the realisation of devices that can explore heat transport in the quantum limit, such as a quantum heat engine.

Keywords superconducting quantum circuit, microwaves, qubit, tunnel junction, quantum heat valve, dissipation, heat bath, NIS thermometry, rectification


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Over the four years from August 2015 to August 2019 that have produced the scientific output reflected and summarised in this book, I have had the great joy to meet, work, and develop wonderful relationships with a large and diverse mix of colourful characters. I now have the opportunity and perhaps even responsibility, given how minutely examined these next pages often are, to make an attempt to acknowledge and thank them, however arduous and potentially futile it may be to generate such a comprehensive list.

The first and greatest acknowledgement must undoubtedly go to Prof. Jukka Pekola. Not only for providing me with the means and opportunity to work and pursue our mutual interests in the PICO Group, but also his in-exhaustive support, guidance, teaching, and encouragement.

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Preface


I have had the wonderful opportunity to collaborate internationally with Dr. Tobias Lindström and Dr. Sebastian de Graaf from the National Physical Laboratory (UK), Prof. Joachim Ankerhold of University of Ulm (Germany), Prof. Hsi-Sheng Goan of National Taiwan University (Republic of China). Particular acknowledgements must be given to the Low Temperature Physics group at Lancaster University, where I not only completed the masters degree that sparked my interest in cryogenics and experimental condensed matter, but where I have continued to find kind hospitality and interesting discussions on frequent visits. I have had the fortuitous opportunity to collaborate with Prof. Yuri Pashkin and Dr. Richard George as part of my doctoral studies, resulting in first publication of this thesis.

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To my colleagues in the CQE and QTF centres of excellence and the Micronova community, of whom there are too many to name individually, I have had remarkable discussions, support, and many enjoyable sauna trips over the years. I have been fortunate to have strongly aligned research interests with the QCD, KVANTTI, QT, TQT, AQE, and NAE groups, as-well as shared measurement facilities with NANO, NEMS, and ROTA groups. I also thank Prof. Tapio Ala-Nissilä of the MSP group for acting as reviewer in the mid-term review of my studies.

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To my family; my mother and father, my sister Chloe, our dog Benji, my grandparents, and my extended family, as well as the close friends who have become like family; to Camilla, Jenni, Dups, and my partner Volha. There exist no sufficient words to express my gratitude, but their resolute love, support, and encouragement, has been my strongest motivation in the completion of this book.

Helsinki, November 18, 2019,

Jorden Senior
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This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.


Author’s Contribution

Publication I: “Multiplexing Superconducting Qubit Circuit for Single Microwave Photon Generation”

The author performed DC and RF simulations towards the geometric design of the device, made the lithographic pattern, and fabricated devices. Parts of the measurement assembly; specifically the sample stage mounting, wiring, and shielding were designed and assembled by the author. The author performed measurements on several devices, and contributed to writing the article.

Publication II: “Utilization of the superconducting transition for characterizing low-quality-factor superconducting resonators”

The author conceived the technique, designed and fabricated several samples, based on simulations performed by the author. The author contributed to the writing of the letter.

Publication III: “Tunable photonic heat transport in a quantum heat valve”

The author designed the device, fabricated, and performed measurements on several samples. The author additionally contributed to the writing of the letter.
Author's Contribution

Publication IV: “Heat rectification via a superconducting artificial atom”

The author designed and fabricated devices, and performed measurements. Additionally, the author was responsible for leading the project, the analysis, and the writing of the manuscript.
1. Introduction

This thesis is presented during a time romantically called the second quantum revolution: a technological paradigm shift from building technologies that merely exhibit quantum-originated phenomena; to devices that actively create, manipulate, and readout quantum states for exploiting phenomena such as superposition, entanglement, and the higher Hilbert dimensionality that comes thereto [1].

The development of this ongoing revolution has generally been given the name of ‘Quantum Engineering’, and over the course of this thesis, the term Quantum Engineer has become a popular title and job description, not only in the academic pursuits of quantum mechanics and condensed matter physics, but particularly in the private sector, with both startup-companies and several of the largest multinational companies heavily investing in these technologies. If fifty years ago it was the ‘Race to Space’, it is now a ‘Mission for Superposition’.

In this push for quantum engineering, largely to build ultra-sensitive sensors [2], to provide novel ways of securing communications [3], and especially quantum computation [4, 5] (discussed in more detail in Chapter 3), as well as engineering the environments to generate, operate, and communicate between these devices. It is perhaps interesting then, that the quantum engineering research on actually building a quantum (heat) engine is relatively lacking, with few experimental realisations [6, 7, 8].

Generally, the thermodynamics of quantum systems is poorly understood, and in many cases, thermodynamic properties are not easily defined [9, 10]. While both are statistical in origin, quantum dynamics deals with coherent, low-loss, probabilistic trajectories for single or few particles, whereas thermodynamics has been derived for the incoherent, highly dissipative, collective dynamics for macroscopic numbers of particles.

Yet, as we build devices with increasing density of quantum elements, particularly the millions of quantum elements required to build a fault-tolerant universal quantum computer [11, 12, 13], the need to probe heat
transport in these quantum circuits, and to understand the thermodynamics of quantum systems, is increasingly relevant, and will be required for achieving this second quantum revolution.

This thesis aims to take steps to rectify this challenging problem, by proposing and developing a platform for interfacing thermodynamics with the most popular platform for quantum computing; superconducting circuits, and in doing so, interface the framework of cavity quantum electrodynamics (cQED) [14, 15, 16, 17] describing these superconducting circuits with the tools of superconducting heat transport experiments to establish some of the first experiments in the new field of circuit quantum thermodynamics (cQTD), and explore heat transport and the thermodynamics of superconducting quantum circuits.

1.1 Thesis Outline

This thesis collects and summarises four research outputs performed over the course of the author’s doctoral studies, listed at the beginning of this thesis.

They cover a broad range of physics, starting first by discussing what we mean by heat transport in superconducting circuits, how we realise a thermal bath, engineer and manipulate the flow of heat in a system, and how we go about reading the temperature in Chapter 2.

We then discuss in Chapter 3 how one may design and measure a quantum circuit involving artificial atoms, using superconductors, the framework of cavity quantum electrodynamics, and the broad developments in recent quantum information processing, resulting in Publication I.

In Chapter 4, we then start to interface these two early chapters, by looking at the effect of dissipative elements added to a superconducting microwave resonator, and present a novel method for characterisation of these extremely low-quality dissipative resonators within a superconducting qubit microwave measurement environment, the subject of Publication II.

Next in Chapter 5, we combine these dissipative resonators with the superconducting thermometry of Chapter 2 with the artificial atom of Chapter 3 to look at heat transport through a resonator-qubit-resonator system in a variety of configurations, probing the roles of different constituent coupling elements on both the locality (Publication III) and directionality (Publication IV) of the heat flow.

Finally, we will provide some conclusive remarks, and suggest an outlook of this promising new field of circuit quantum thermodynamics.
2. Heat Transport in Superconducting Circuits

In this chapter, we will explain the design and operational principles of the various constituent devices that we will later combine to form our superconducting circuit.

2.1 Heat and Temperature

To perform heat transport experiments, naturally one of the primary elements of our superconducting circuit should be something with a temperature dependent energy distribution, to act as a thermal bath.

Perhaps the simplest choice is that a piece of metal. The electrons in the metal have an energy distribution that follow the well-known Fermi-Dirac distribution [18], given by Equation 2.1 with energy $E$, chemical potential $\mu$ (which at zero temperature is the Fermi energy $E_F$), and temperature $T_N$.

$$f(E, \mu) = \frac{1}{1 + e^\frac{E-\mu}{k_B T_N}} \quad (2.1)$$

If we were to heat one side of the metal, and measure the conductance of heat as a function of temperature along the piece of metal $T(x)$, then the heat conductivity $\kappa$ is proportional to the electrical conductivity $\sigma$ by the well-known Wiedemann-Franz law, given by Equation 2.2.

$$\kappa = \frac{\pi^2 k_B^2 T(x)}{3 e^2} \sigma \quad (2.2)$$

2.1.1 Electron-phonon interaction

If we place this piece of metal then in contact with another thermal bath at a different temperature $T_{\text{bath}}$, the two will thermalise with each other.
by the exchange of heat. In the low temperature limit where we intend to use this device, we can implement this metal as a piece of copper placed upon an insulating substrate. In this limit, the electrons in the metal are usually hotter than the phonon temperature of the substrate, and the thermalisation will be dominated by the interaction of the hot electrons with the phonons of the substrate.

In this case, for the piece of metal with volume $V$ and material specific coupling constant $\Sigma$, the power from the hot electrons in the heated metal to the phonon bath is given by Equation 2.3 [19]

$$P_{ep} = \Sigma V(T_N^5 - T_{bath}^5)$$  \hfill (2.3)

With the thermal conductance $G_{ep}$ given by

$$G_{ep} = 5\Sigma VT_{bath}^4$$  \hfill (2.4)

then the power can be linearised in the limit $T_N \approx T_{bath}$ as

$$P_{ep} = G_{ep}(T_N - T_{bath})$$  \hfill (2.5)

### 2.1.2 Joule heating

With our piece of metal, its electron temperature and heat conduction defined, one may next ask how do we change the temperature. Cooling can be done by allowing the metal to thermalise with a colder bath, like the substrate as just discussed. Electronic ‘evaporative’ cooling can also be achieved by superconducting probes. Heating, however, is a comparatively simple matter of running an electronic current $I$ through the metal, or similarly applying a potential difference $U$ across the metal.

As electrons transport charge in the metal, they scatter in a process that is quantified by the resistance $R$ of the metal. This scattering process is dissipative, turning of the energy of moving charges into heat by the relation known as Joule heating, and the power is given by $P = RI^2 = V^2/R$.

### 2.2 Measuring the Temperature

So with our temperature defined, and the principles for thermal relaxation, conduction, and dissipative heating explained, how can we measure the temperature?
Ideally this should be done non-invasively, whilst also allowing us to engineer the temperature with the same tool in order to reduce complexity. Whilst many types of thermometry exist and are being continuously developed, this thesis will describe the use of the SINIS technique (superconducting electrode - insulating barrier - normal metal - insulating barrier - superconducting electrode).

2.2.1 Superconductivity

Superconductors are defined by their abruptly vanishing resistance at temperatures below a superconducting transition despite their increasing thermal resistance, first discovered by Kamerlingh Onnes [20] at a similar time to the founding principles of quantum mechanics were being derived [21, 22, 23, 24]. To do this, Kamerlingh Onnes also first liquified helium to experimentally achieve temperatures below 4.2 Kelvin, a technique that is also still used today. It was later discovered by Meissner [25] that an additional effect of the superconducting transition was perfect diamagnetism, another trait one would not expect to find if instead the superconductor was simply a dissipation-less (perfect) conductor.

Several theoretical models for describing the behaviour of superconductors have been developed since their discovery [26, 27], and the most popular microscopic theory is that of Bardeen, Cooper, and Schrieffer [28] known as BCS theory. BCS theory describes how electrons in a superconductor below the transition temperature form Cooper pairs by the electron-phonon interaction, such that there is a gap in the density of states of the superconductor around the Fermi energy. At zero temperature, states with energy less than the superconducting gap are fully occupied by electrons, and all states above the gap remain empty. At finite temperature, however, statistically the distribution of energies for electrons in the superconductor may allow for electrons with energy in excess of the superconducting gap, causing excitations referred to as quasiparticles. For this reason, whilst dissipation and heat conduction are suppressed, they can still occur.

2.2.2 Normal metal - insulating barrier - superconducting electrode (NIS) junctions

One of the most promising and developed tools in mesoscopic physics for on-chip thermometry is the normal metal - insulating barrier - superconducting electrode (NIS) bolometer [29]. In recent years, they have been experimentally demonstrated to work at temperatures as low as 10 mK [30] and on timescales compatible to the coherence of superconducting qubits [31].

These structures take advantage of the difference in densities of states
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across the constituent elements; where the normal metal acts as a temperature sensitive ‘absorber’ of heat; the distribution function for energies in the metal follows the temperature dependent Fermi-Dirac function, whereas the superconducting probe has an approximately temperature independent energy distribution, exhibiting a gap $\Delta$ while operating in the temperature regime corresponding to less than a third of the superconducting transition temperature. We can write the current through the junction as a function of the voltage across the junction as

$$I(V) = \frac{1}{2eR_T} \int_{-\infty}^{\infty} N_S(E)[f_N(E - eV) - f_N(E + eV)]dE$$

(2.6)

where $R_T$ is the tunnel resistance of the junction, a function of the area and tunnel barrier thickness [32] and $N_S$ is the density of states in the superconductor. With a constant current bias $I = I_0$, one can therefore extract the temperature of the normal metal by measuring the voltage drop across the structure, which can be approximated to follow the relation in Equation 2.7, where $\delta$ is a constant [19].

$$eV = \delta - k_B T \ln \left( \frac{\sqrt{\frac{2}{\pi}} \delta k_B T}{eR_T I_0} \right)$$

(2.7)

These devices are typically realised using superconducting aluminium, insulating aluminium oxide, and the normal metal is often selected as copper, silver, gold, or some alloy such as gold/palladium, but in principle any metal that is not superconducting at the temperature in which you wish to operate is a candidate, however limitations during fabrication, and non-standard densities of states of the material may generally restrict the specific implementations of more exotic materials in favour of more practical materials. We use the well established copper, as this material is both well understood from a physical point of view, and has a metallurgical compatibility with aluminium.

By calibrating the voltage-temperature measurements to a known reference, for example - a well-calibrated ruthenium oxide or Coulomb blockade thermometer in thermal contact with the substrate, the local temperature of the normal metal island can be extracted.

Additionally, by using multiple superconducting probes, simultaneous heating and thermometry of the island can be performed.

2.2.3 Josephson junctions

Another important junction to consider is the superconductor - insulator - superconductor junction, first proposed by Brian Josephson [33, 34, 35]
and hence bearing his name, a Josephson junction, also known as an SIS junction.

This device often finds use as a non-linear inductor, the non-linearity derived from the superconducting phase ($\phi$)-dependent current $I_J(\phi)$ through the junction, that follows the relation $I_J(\phi) = I_C \sin(\phi)$, with $I_C$ corresponding to the so-called critical current, the maximum allowed current through the junction before it undergoes a transition back to the normal-metal state, phenomenologically originating from the overlap in the tails of quantum tunnelling probability of electrons on opposite electrodes, combining to form Cooper pairs.

Similarly, the voltage across the junction can be described by

$$V(\phi) = \frac{\Phi_0}{2\pi} \frac{d\phi}{dt}$$

and by using the relation between voltage, inductance, and the time differential of current, $V = L \frac{dI}{dt}$, we can arrive at Equation 2.9 for the non-linear inductance $L_J$ of a Josephson junction. Here, $\Phi_0 = h/2e$ is known as the flux quantum.

$$L_J = \frac{\Phi_0}{2\pi I_C \cos(\phi)}$$

A particularly useful device can be made by using two Josephson junctions to form a loop, known as a superconducting quantum interferometer (SQUID) - so named because the two phase-dependent currents for either side can be tuned by an magnetic flux ($\Phi$) through the loop, forming an interference effect when the currents are combined, and follows the relation

$$I = 2I_C \cos\left(\frac{\pi \Phi}{\Phi_0}\right) \sin(\phi)$$
Similarly, the now flux-tunable inductance can be expressed by

\[ L_J(\Phi) = \frac{\Phi_0}{2\pi I_C \cos \left( \frac{\pi \Phi}{\Phi_0} \right)} \] (2.11)

And where \( E_{J}^{\text{max}} \) is the Josephson energy at \( \Phi = 0 \), the energy can be expressed by

\[ E_J = E_{J}^{\text{max}} \left| \cos \left( \frac{\pi \Phi}{\Phi_0} \right) \right| \] (2.12)

With the cosine dependence on the magnetic flux taken as a magnitude to avoid negative energy states.

The SIS junctions in this thesis are made from two layers of superconducting aluminium, few tens of nanometres thick, separated by an approximately nanometre-thick aluminium oxide insulating layer.

### 2.3 Photonic Heat Transport

When considering heat transport, we must not only take into account the electron-phonon and electron-electron couplings, but also the electron-photon coupling. Hot electrons in our thermal bath may relax from their excited states by the emission of photons. Photonic heat transport has been studied in superconducting devices over a wide frequency range \([36, 37, 38]\), and for the work in this thesis, is the main transport method of interest, as the flow of photon-mediated heat transport can be manipulated using the framework of circuit quantum electrodynamics, discussed later.

Consider two baths, with temperatures \( T_L \) and \( T_R \), separated by some medium with frequency dependent transmission coefficient \( t(\omega) \). In \([39, 36]\)
it has been shown that the power transmitted from one bath to the other can be written as

\[ P_t(\omega) = \hbar \omega |t(\omega)|^2 \frac{1}{\exp \left( \frac{\hbar \omega}{k_B T_t} \right) - 1} \] (2.13)

Which then taking into account thermalisation in the reverse direction, and integrating over all frequencies, can be written as

\[ P_{\text{total}} = \int_0^\infty \frac{d\omega}{2\pi} \hbar \omega |t(\omega)|^2 \left[ \frac{1}{\exp \left( \frac{\hbar \omega}{k_B T_L} \right) - 1} - \frac{1}{\exp \left( \frac{\hbar \omega}{k_B T_R} \right) - 1} \right] \] (2.14)

When taking the transmission to be unity, this power follows a square law, given by equation 2.15, and can be further linearised by taking an average temperature \( T = (T_L + T_R)/2 \) to equation 2.16 where \( G_0 = \pi k_B^2 T/(6\hbar) \) is the quantum of thermal conductance, with this relation having been shown to hold over macroscopic distances [40].

\[ P_{\text{total}} = \frac{\pi k_B^2}{12\hbar} (T_L^2 - T_R^2) \] (2.15)

\[ P_{\text{total}} = G_0(T_L - T_R) \] (2.16)

### 2.3.1 Superconducting Transmission Lines and Resonators

With the perfect diamagnetism and extremely low dissipation, superconductors are a useful tool for constructing high quality transmission lines, that we can use as a medium between the two heat baths.

These transmission lines are usually realised in the form of co-planar waveguides, with a central electrode separated from a ground plane, with the characteristic impedance given by \( Z_0 = \sqrt{L/C} = V/I \) [41]. By applying boundary conditions to generate standing waves, these transmission lines can become resonators. For the experiments presented in this thesis, all resonators were designed as quarter-wave (\( \lambda/4 \)) resonators in the gigahertz frequency range, with the voltage maximum capacitively coupled to the ground, and the current maximum directly terminated through a clean contact, or through a thermal bath resistor. The quality factor, defined as the energy stored in the resonator divided by the energy dissipated per radian, has been demonstrated to reach (internal) values as high as a million [42]. The current big challenges in improving this quality are in reducing the coupling of superconducting resonators to unwanted two level systems such as oxides on the wafer substrate [43]. High quality
superconducting resonators have been used for studying profound quantum mechanics\cite{16, 44}, such as the Casimir effect\cite{45, 46}.

Heat transport through superconducting resonators, including structures containing multiple resonators have been investigated in parallel experiments\cite{47, 48, 49}. In these experiments, devices with both tunable coupling and tunable dissipation were realised for performing heat transport through multiple resonators, with applications in environment engineering for superconducting quantum circuits.

\textbf{Figure 2.3.1.} Image of the multiplexed qubit device presented in Publication I.
3. Superconducting Qubits: Design and Operation

Following a seminal work by Feynman suggesting that the best way to simulate quantum physics was with some kind of quantum system acting as a quantum computer [50], and further seminal works by Deutsch [51] on how one could formally conceive a model for such a universal quantum computer, with motivations on how exploiting the higher Hilbert dimensionality of the superposition of states of interacting quantum systems [52, 53, 54] may provide some quantum speedup compared to conventional computers (more recently preferring the term quantum advantage, as typically one needs both classical and quantum computers working together to perform a useful operation), the field of quantum information processing has undergone a (quantum) speedup and rapid expansion both to realise these devices in a physical system, and also in developing the algorithms for what to do with these devices when they have been made, i.e., quantum software for quantum hardware.

The natural theoretical choice for a quantum system is the two level system, it generally being the simplest to solve, and draws a convenient parallel to the unit of information used for conventional information processing - the bit. This quantum bit, or qubit for short, is usually assigned the state \( |\phi\rangle = \sqrt{\alpha} |0\rangle + \sqrt{1-\alpha} |1\rangle \), where \( |0\rangle \) and \( |1\rangle \) refer to the two energy states of the two level system, and \( \alpha \) the probability to find the qubit in state \( |0\rangle \), analogous to the ubiquitous binary code used in modern computing, where 0s and 1s refer to currents in a transistor network that many millions of transistors later form the devices that provided the technical revolution of the twentieth century (as a comparison, there are roughly 1.75 Billion transistors in a modern computer processor, at a density of \( \sim 17 \) million transistors per square millimetre).

Using atoms themselves, or atomic-like systems that have non-degenerate two level systems has been now successfully demonstrated in a variety of realisations, from ultracold atomic gases [55], trapped ions [56], atomic (nitrogen) vacancies in diamond lattices [57], to quantum optics, and quan-
tum dots [58], each with their own unique benefits and challenges. Each of these implementations has experienced significant development in the past decades, and likely elements of many different implementations will be used to best exploit the benefits of each in the future large scale hybrid device that satisfies the requirements for a universal quantum computer set out by DiVincenzo at the turn of the century [59].

The difficulty in making a quantum computer from atoms themselves is that they are small, are extremely challenging to control in a reliable way, typically have physical parameters that aren’t ideal, and are not widely adjustable. What if we could emulate an atom using something macroscopic, that is non-dissipative, with physical parameters tunable within a wide bandwidth, such as the frequency of the atom, the coupling to the cavity, the anharmonicity between additional energy levels, and tunable not only in making the device, but also tunable in situ to enter different regimes suitable for different activities - say talking to two different atoms individually, and then quickly coupling to a resonator and resetting using the Purcell limitation. And on top of that, we want it to be scalable, potentially on an industrial scale, with the time for individual manipulations/computational operations comparable to conventional computing.

For each of these, superconducting qubits have become perhaps the most promising, most developed, and most likely go-to choice. First demonstrated in 1999 [60], the past twenty years has shown tremendous progress in these superconducting qubits, with over an order of magnitude in coherence time improvements (from \( \approx 10 \text{ ns} \) to \( \approx 100 \mu\text{s} \)), increasing the number of operations one can perform on an individual quantum state before it statistically loses the information to decoherence. Superconducting qubits have gone from a extremely specialist technique few labs in the world were capable of achieving, to in the course of this thesis, now having noisy-intermediate-scale quantum architectures available for not only academics, but also to the general public to generate quantum software on their mobile devices, communicated and interpreted using a cloud based server. It is then compiled into nano-second pulses sent to a quantum chip such as the IBM Q experience in New York [61].

With these superconducting qubit devices integratable with other superconducting circuits, and providing a well-developed means of simulating an artificial atom using a mesoscopic device where one can control aspects such as coupling, excitation frequency, and coherence, they are an ideal component to study the thermodynamics of quantum systems, so how do we make one?
3.1 Designing a Superconducting Artificial Atom

The basic requirement for a superconducting artificial atom is that the energy spectrum should be non-degenerate (leading to anharmonic energy levels), and ideally non-dissipative.

A superconducting qubit is, essentially, a superconducting LC oscillator with capacitive and inductive elements, where the inductor is replaced by a non-linear inductor, to raise the degeneracy and form anharmonic energy levels and thus forms an often named ‘artificial atom’ energy spectrum. As we have a non-linear inductor in the form of a Josephson junction, this is the basic element in constructing a superconducting artificial atom. In fact, it is possible to make both qubits with the degree of freedom in the superconducting phase or flux using a single junction, however these devices are usually challenging to fabricate and operate, so often qubits have a minimum of two junctions.

The qubits that we consider in this thesis operate in what is known as the transmon regime, first proposed in [62]. These consist of a SQUID, shunted by a large capacitance, reducing the sensitivity to noise originating from charging effects. For Josephson energy $E_J$ and charging energy $E_C$ (inversely proportional to the device tunnel junction resistance $R_N$, and device self-capacitance $C_\Sigma$, respectively, defined by 3.1 and 3.2), they operate in the regime where $E_J >> E_C$. This reduced sensitivity to charge noise however comes at the cost of reduced anharmonicity.

\[
E_J = \frac{R_Q \Delta}{2R_N} \quad (3.1)
\]

\[
E_C = \frac{e^2}{2C_\Sigma} \quad (3.2)
\]

where $R_Q = h/4e^2 \approx 6.46 \text{k}\Omega$ is the quantum resistance, and $\Delta$ is the superconducting gap, which for aluminium $\Delta \approx 200 \text{\mu eV}$. For example, in Publication I, $R_N = 8.24 \text{k}\Omega$ and $C_\Sigma = 35.8 \text{fF}$, the Josephson energy is $E_J = 78.2 \text{\mu eV}$ and the charging energy is $E_C = 2.23 \text{\mu eV}$.

In the transmon regime, the transition frequency of the qubit can be described by

\[
h \nu_{01} = \sqrt{8E_JE_C - E_C} \quad (3.3)
\]

and the effective Hamiltonian that describes the qubit is given by

\[
H = 4E_C(\hat{n} - n_\text{g})^2 - E_J \cos \hat{\phi}, \quad (3.4)
\]
where \( \hat{n} \) is the difference in the number of Cooper pairs across the superconducting electrodes; \( n_g \) is the effective offset charge, and \( \phi \) is the phase difference.

Hence, by tuning \( E_J \) with the magnetic flux through the SQUID, either by a local inductive magnetic field, or by a global solenoid; the transition frequency of the qubit should follow a \( \sqrt{\cos(\Phi)} \) dependence, and be tunable to resonate across a wide bandwidth, bounded at the high frequency limit by the \( E_J^{\text{max}} \), and at low frequencies by asymmetry in the SQUID junctions, and Purcell limited at frequencies matching other resonant modes in the environment, i.e. any superconducting resonator cavities.

The self capacitance, \( C_{\Sigma} \), can be designed geometrically, dominated by the capacitance of a shunting capacitor, which can be simulated using finite element modelling. The tunnelling resistance is chosen by a combination of geometric (to determine the area) and in the fabrication process, where the tunnel barrier thickness can be tuned by the amount of oxide grown forming the barrier.

The frequency range of operation is dictated by two limits, with the typical frequency range used in superconducting cQED experiments 3 - 10 GHz. The high frequency limit is defined by the superconducting gap - which on approach would drive the superconductor through a transition back to its normal, dissipative state. The low frequency limit is dictated by the average temperature of electrons in the cavity, in order to suppress spontaneous excitations due to thermal fluctuations, and as such, is limited by the cooling performance and how well-thermalised the device is - typically 10 - 100 mK is the operating temperature for a superconducting qubit device.

So now we have an artificial atom, how do we connect it to the room temperature electronics, and actually operate it on a single excitation level?

This is achieved by not addressing the qubit directly, but by coupling the qubit to a readout resonator, in our case capacitively, and measuring the dispersive shift of the qubit on the dressed resonant frequency of the resonator.

### 3.1.1 The Jaynes-Cummings Hamiltonian

Cavity Quantum Electrodynamics was first derived to study light-matter interactions, one of the first examples being an atom trapped in an optical cavity. This somewhat simple concept is actually an incredibly powerful tool that continues to enable probing of fundamental quantum mechanics by enabling us to probe individual energy levels of the atom, and by
embedding the atom in an off-resonant cavity of sufficiently high quality to suppress spontaneous emission of the specific energy levels one wishes to probe, one can also probe the coherence dynamics of an atom, and readout individual quantum states.

One of the most prolific results in this field is the Jaynes-Cummings Hamiltonian [63, 64], a simplification of the Rabi interaction [65] under the rotating wave approximation (RWA) for a two level system in a cavity, thereby ignoring the effects of decoherence to the environment.

If we take the dispersive limit where the coupling $g$ of the atom to the cavity through the electromagnetic field interaction is smaller than the detuning $\Delta$ between the cavity frequency $\omega_r$ and the transition frequency $\omega_q$, with the dispersive effect of the atom on the cavity frequency $\chi$, then the Jaynes-Cummings Hamiltonian for the system can be written as Equation 3.5, where $\hbar$ is the reduced Planck constant, $\sigma_z$ the Pauli Z matrix, and $a, a^\dagger$ denoting the excitation creation and annihilation operators respectively.

$$
H = \hbar \omega_r a^\dagger a + \frac{1}{2} \hbar \omega_q \sigma_z + \hbar \chi \sigma_z a^\dagger a
$$

The first term here can be identified as the bare energy of the cavity, the second as the energy of the atom, and the final term as the interaction between them.

It is this interaction that manifests as a dispersive shift of the cavity frequency when spectroscopy of the cavity is performed that is the basis of the majority of current state-of-the-art quantum information readout protocols, sometimes referred to as a quantum non-demolition measurement.

In this dispersive limit, $\chi = g^2/\Delta$, and as $g < \Delta$ by this approximation, this shift is small, but can be resolved in sufficiently high quality cavities. Fortuitously, these high quality cavities can also act as filters for the atom, reducing the number of modes that the atom can decohere into, resulting in an increase in the atomic coherence timescales. If however the detuning is small ($\Delta \approx 0$), in the limit where equation 3.5 is no longer valid, the dispersion takes the form $\chi = g/\pi$, known as the vacuum Rabi frequency. This can be observed as a splitting of the resonance peak of the cavity, and gives rise to the Purcell limitation [14], where the coherent interaction between the atom and the cavity is limited by losses in the cavity.

So using this framework of cavity QED, we have a way of measuring quantum dynamics and generating, manipulating, and reading out individual quantum states - and is the basis for superconducting qubit operation.
3.2 Superconducting Circuit Fabrication

The fabrication of a superconducting circuit chip is a multistage process, with multiple elements requiring operating on different length scales. In fabrication, we first realise the microwave structure of the circuit with feature sizes of order micrometres, which we do by an etching process. We then complete the device with its sub-micrometre Josephson elements, requiring much finer precision, achieved by a physical vapour deposition process. We then conclude with post-processing to prepare the fabricated chip for measurement. All processing is done within the Otanano infrastructure, using the Micronova cleanroom facilities. Images of a chip demonstrating the different stages through the fabrication process are presented in Figure 3.2.1 and explained in detail in the following:

**Patterning of the microwave structure:** The substrate used in Publication I and Publication III is single crystal c-axis oriented sapphire. Alternatively, another suitable option used in Publication II and Publication IV is undoped high resistance ($\rho > 10^4 \Omega \text{cm}$) silicon from MTI Corporation that is devoid of a silicon oxide capping layer. A 200 nm niobium film is first prepared by DC magnetron sputtering on the substrate then a high sensitivity e-beam resist of AR-6200 is spin-coated onto the substrate (at 6000 rpm for 60 seconds) and then a 8 mm x 8 mm pattern of the microwave structure is exposed by electron-beam lithography (EBL), typically consisting of a feedline (connected to measurement electronics), inductively coupled $\lambda/4$ resonators, and capacitively coupled qubit elements, as well as lines for local magnetic flux tuning. After patterning by EBL, we develop the resist in AR600-546 developer and a reflow bake is performed to prepare the patterned resist to act as a mask for reactive ion etching. The latter is run with an SF$_6$+O$_2$ process to transfer the pattern from the resist into the niobium film, removing the exposed niobium. The substrate is then cleaned by removing the resist by immersion in AR600-71 (dioxolane-based) resist stripper.

**Deposition of nanoscale structures:** To deposit the sub-micrometre structures, the substrate is prepared by spin-coating two layers of MMA(8.5)MAA (11% ethyl lactate solution) resist to create a 1 $\mu$m film of MMA(8.5)MAA resist, followed by a single 200 nm layer of 950 PMMA (4% anisole solution). To avoid charging offsets during e-beam exposure on non-conductive substrates, such as sapphire, a final layer of conductive polymer “E-spacer 300z” is added. The patterns are exposed using e-beam lithography, initially exposing the fine structure on the PMMA layer, before an additional lower dose beam to form a favourable undercut in the MMA layer. Following lithography, the conductive polymer layer is removed by rinsing in deionised water, whilst the resist structure is developed by immersion in
Figure 3.2.1. a) Optical micrograph of a $\lambda/4$ resonator inductively coupled to a feedline and capacitively coupled to a transmon qubit. This pattern is exposed by EBL and etched from niobium using reactive ion etching. b) Optical micrograph of the transmon qubit, with both the microwave and nanoscale structures visible. c) Scanning electron micrograph of the SQUID element of the transmon qubit, deposited using two angle physical vapour deposition.

methyl isobutyl ketone:isopropyl alcohol solution to open the PMMA top layer structure, and then by immersion in a methyl glycol: methanol solution to clear the undercut in the MMA layer. The structure is then loaded into an electron beam evaporation chamber, equipped with a tilting sample stage to enable deposition from multiple angles, an oxygen inlet, and argon ion milling capability. To allow favourable electrical contact between the deposited metal and the niobium microwave structure already prepared on the substrate, argon plasma milling is performed for 3 minutes. Following this, a crucible containing the desired material is then heated using an electron beam, typically with a current of 20 mA and acceleration voltage of 10 kV, to cause the target material to evaporate. To ensure a ballistic flow of high purity metal being deposited, this is done under high vacuum ($\approx 10^{-7}$ mbar) such that the mean free path of the evaporating material is larger than the boundaries of the chamber. Where aluminium tunnel barriers are required, after the deposition of the first aluminium layer a suitable ($\approx 10$ mbar) amount of oxygen is injected into the chamber and left to react for some minutes to form a thin aluminium oxide layer. The second layer of metal is then deposited on top. As the material only reaches the surface where the resist stack has been exposed, we then remove the excess material with a lift off process in hot acetone. This process is additive, meaning multiple depositions can be done in subsequent lithographic steps, for example to pattern the NIS junctions, which are also grown by electron-beam evaporation.

Post-processing To characterise the yield, we can electrically probe specifically designed test structures on the chip using 1 micrometre point metal probes. It may be required for a particular measurement setup (such as RF) that the chip is diced to size. The structures are protected by a layer of AZ5214E photoresist, and a dicing saw with a thin diamond-embedded resin blade is used to dice the chip within a precision of tens of micrometres.
3.3 Microwave Characterisation

Once a chip has been fabricated and post-processed (detailed in Chapter 3.2), it is glued to a copper sample stage using Apiezon N cryogenic vacuum grease, which acts as both an adhesive and good thermal coupler, with negligible electrical conductivity. Thin metal wires are ultrasonically bonded to the chip to connect it to a printed circuit board. On-chip wire bonds are also added to provide uniform ground plane potentials. The sample holder is covered with a copper shim, designed to limit the interaction with microwave sample-box eigenmodes, and this assembly is mounted into a cryogen-free dilution refrigerator with a base temperature of around 10 mK. As the chip contains magnetic flux sensitive structures, an assembly consisting of two magnetic shields constructed from high magnetic permeability metal optimised for cryogenic operation is used. A bonded chip, and the full cryogenic measurement assembly are shown in Figure 3.3.1.

SMP plugs are used to address the assembly in transmission RF measurements, with the option to control a global magnetic field with DC lines, or on-chip local magnetic flux using an additional coaxial line.

The signal from the room temperature electronics is passed through a series of impedance matched (to ensure the signal isn’t reflected) cryogenic attenuators situated at different temperatures within the cryostat, to reduce the amount of power dissipated to the sample, in the form of Johnson-Nyquist (thermal) noise.
The phenomenological origin of Johnson-Nyquist noise is in the thermal motion of charge carriers in a conductor, giving rise to fluctuating voltage values independent of an applied power. The voltage noise spectral density $S_V(f)$ is described by equation 3.6 that shows the relation between resistance $R$, temperature $T$, and frequency $f$ [66].

$$S_V(f) = 4R \left( \frac{hf}{\exp(hf/k_BT) - 1} + \frac{1}{2}hf \right)$$

(3.6)

In the low frequency limit ($hf \ll k_BT$), which for room temperature electronics can be defined as below the terahertz band, equation 3.6 can be simplified as $S_V(f) \approx 4k_BT R$. To estimate the power that would then be dissipated to the environment, we integrate the voltage spectral density over the bandwidth $\delta f$ to give equation 3.7.

$$P_{\text{noise}} = \frac{\langle \delta V^2 \rangle}{R} = \int_0^\infty S_V(f) df = 4k_BT \delta f$$

(3.7)

We can now see why it is also necessary to have the attenuation to actually generate the signals required. For example, a typical pulse required for manipulating the qubit state requires a power of $\approx -127$ dBm. The Johnson-Nyquist noise, the theoretical minimum thermal noise, for a device operating at room temperature (290 K), generating a 20 ns pulse, is approximately -91 dBm. This is over an order of magnitude higher than the required power at the sample, and hence the dynamics would be limited by the noise. Providing a high power signal at room temperature and having it go through 94 dB of attenuation, distributed to ensure that the power dissipated at each stage of the cryostat is not more than its thermal budget, and is not limited by the thermal noise at each stage, is a suitable alternative.

To measure the output, the signal is first amplified inside the cryostat using a cryogenic low noise amplifier, and then subsequently amplified by room temperature electronics. A schematic of the full measurement setup is shown in Figure 3.3.2

### 3.3.1 Dispersive shift ‘χ’ measurement

The first measurement that is performed is to ‘locate’ the qubit. As the qubits are coupled capacitively to λ/4 resonators, we first must identify the resonator by sending a tone of the correct frequency through the on-chip feedline, and observing a dip in the $S_{21}$, the scattering matrix coefficient corresponding to the transmission from the input to output ports, at the frequency corresponding to the resonator, designed at a specific frequency in the range 6 - 8 GHz. As shown in Figure 3.3.3(a), a fit to the transmission
dip can be performed using the model presented in [67], yielding a coupling quality factor \( Q_c \approx 5500 \) with an internal quality factor \( Q_i \approx 38600 \). This would correspond to a resonator photon decay rate \( \kappa/2\pi \approx 1.4\text{MHz} \).

Now that the resonator has been found, one can verify that in the low power regime, a dispersive shift of the resonance frequency corresponding to the population of the qubit can be observed. This is done by performing the previous single-tone transmission spectroscopy, but now by additionally sweeping the power of this tone. As shown in Figure 3.3.3(b), at low power the qubit is in the ground state \( |0\rangle \) and the resonator experiences a frequency shift \(-\chi/2\pi\) from the unloaded resonator frequency, whereas if the qubit was in the first excited state, \( |1\rangle \), the dispersive shift would be \(+\chi/2\pi\). At higher input powers, the observed resonator becomes saturated with many photons and the resonance frequency returns to that of the unloaded resonator [68, 69]. Modelling this quantum-to-classical transition was recently investigated in [70]. With a qubit-resonator detuning \( \Delta_0 = 990\text{MHz} \), we observe a dispersive shift \( \chi/2\pi \approx 3.9\text{ MHz} \). We now use this information to define the power regime in which we can address the qubit in its ground state, and all further RF measurements presented will be based upon this dispersive shift technique.
3.3.2 Coupling ‘g’ measurement

We next demonstrate a magnetic flux dependence on the dispersive shift of the resonator in the low-power regime. As the effective Josephson energy of the qubit is flux dependent, as described in Equation 2.12, this indicates the presence of a SQUID element coupled to the resonator.

We use a two-tone spectroscopy technique; measuring the transmission through the resonator whilst sending a second probe tone to excite the qubit, highlighting $\nu_{01}(\Phi)$, and measuring the effect on the resonator. We then measure this scanning the flux domain, by controlling the global magnetic field using a superconducting solenoid. Figure 3.3.4 shows the two branches of interest, corresponding to the resonator and the qubit, with an anti-crossing of the qubit and resonator states at around $\Phi = 0.23\Phi_0$. The frequency spacing between the resonator and qubit at the anti-crossing defines the parameter $g/2\pi = 54.3$ MHz, quantifying the coupling of the two elements.

3.3.3 Rabi oscillations, $T_1$ & $T_2$ measurements

Whereas the previous RF measurements can be performed with continuous tones, we now will discuss characterisation of the qubit that require control and measurements in the time domain. We now need to switch to using pulses, generated using an arbitrary waveform generator that can initiate pulse sequences of a defined amplitude, duration, shape, and phase, that is then mixed with an addressing tone provided by a signal generator. For this measurement, we shape the pulses for driving and manipulating the qubit into a Gaussian profile at the generator, however it should be noted that pre-compensated pulse-shaping corresponding to the signal.
The device can be designed in such a way that the difference between the two lowest energy levels typically corresponds to a frequency in the order of 10 GHz, exactly in the regime of superconducting cavities, with these energy levels representing the $|0\rangle$ and $|1\rangle$ states we use for quantum information processing. The qubit can then be driven between the two states by a tuned microwave signal, whilst any spurious excitations due to thermal noise are suppressed by cooling the system to temperatures of approx 20 - 100 mK, as discussed previously.

at the sample is not performed. Readout is performed by measuring the transmission of a rectangular readout pulse addressing the resonator, to determine the frequency shift and hence the population of the qubit. This is then averaged over many identical protocols to determine the population distribution of the qubit for that particular protocol.

Before we begin with these pulse measurements, we first use the global (DC) magnetic field to tune the qubit excitation frequency to the flux insensitive point; for example the maximum of the data in Figure 3.3.4. This point is often denoted in literature as the ‘sweet spot’. By measuring the dispersive shift of the resonator as we sweep a variable length of pulse
around the qubit excitation frequency, we can measure Rabi oscillations [65] corresponding to the periodic excitations in the qubit. The Rabi oscillations are presented in Figure 3.3.5(a) and Fourier transformed in Figure 3.3.5(b). The oscillations exhibit the typical pattern presented in [71], with the lowest oscillation frequency, $\Omega = 6.17\text{MHz}$, at the drive frequency of 8.512 GHz. From this data, we find that a pulse of $\approx 80 \text{ ns}$ at zero detuning excites the qubit with a probability close to unity. We call this pulse a $\pi$-pulse, corresponding to a $\pi$ rotation from the ground state to the excited state on the Bloch sphere that corresponds to all possible qubit states.

With this $\pi$-pulse, we can measure the energy relaxation time for the qubit. To do this, we first initialise the qubit by allowing it to thermalise with the bath, exciting it with a $\pi$-pulse and, after a variable delay, we measure the residual population of the qubit. Fitting an exponential decay model to the data presented in Figure 3.3.6(a) yields $T_1 = 4.72 \pm 0.06 \mu s$, which compares favourably with the transmon relaxation times measured in earlier experiments [72, 73, 74, 75].

Similarly, we can use the $\pi$-pulse calibration to generate a $\pi/2$-pulse and measure the qubit phase coherence. To do this, we use the Ramsey echo technique, of two $\pi/2$ pulses in the $x$-axis of the Bloch sphere, separated by a variable delay $\tau$ with an intermediate $\pi$ pulse in the $y$-axis. This $\pi$ pulse, whilst not required to measure the phase coherence, is used to eliminate low frequency noise. After running this pulse sequence and then measuring the dispersive shift to infer the population of the qubit, we can fit a sinusoidal decay model to the data in Figure 3.3.6(b) to show a phase coherence time of $T_{2,\text{echo}} = 6.69 \pm 0.18 \mu s$ which is also favourable with previous results in the literature.

Finally, we can measure $T_1$ & $T_2$ estimates for qubit excitation frequencies using these protocols close to the anti-crossing with the resonator to show that the coherence times diminish as the relaxation channels become dominated by the qubit-resonator coupling, predicted by the Purcell effect. $T_1$ and $T_{2,\text{echo}}$ as a function of $\nu_{01}$ are plotted in Figure 3.3.7. This shows that if we tune a qubit close to an unpopulated resonator, we can depopulate the qubit effectively, whilst retaining significant coherence timescales (of order few microseconds) when sufficiently detuned.
Superconducting Qubits: Design and Operation

Figure 3.3.5. a) Rabi oscillations recorded as a function of drive frequency and pulse duration. b) Fourier transform of Rabi oscillation plot against pulse.

Figure 3.3.6. The energy and phase relaxation times of the qubit measured at the ‘sweet spot’ ν₀₁ ≃ 8.5 GHz Insets show the pulse sequence used to record the relaxation rates. (a) Relaxation of the qubit. The fit to the data gives energy relaxation time \( T_1 = 4.72 \pm 0.06 \mu s \). (b) Coherent evolution of the qubit with the echo technique applied. The fit to the data gives dephasing time \( T_{2,\text{echo}} = 6.69 \pm 0.18 \mu s \).

Figure 3.3.7. \( T_1 \) and \( T_{2,\text{echo}} \) measured as a function of qubit transition frequency, showing an enhanced relaxation rate as the qubit transition frequency approaches that of the resonator. (7.51 GHz)
4. Adding Dissipative Elements to Superconducting Resonators

With our ability to design, fabricate, and measure nominally dissipation-less superconducting circuits within the framework of cQED to generate, manipulate, and readout quantum states of an artificial atom demonstrated, the next step towards circuit quantum thermodynamics is to investigate adding dissipative elements to act as thermal baths.

In the limit where photons in a cavity are almost-immediately absorbed by a dissipative thermal bath, i.e., the quality factor is low; characterising and even identifying a cavity can prove challenging. The resonance dip is masked within the noise profile of the microwave measurement chain. A technique we developed for performing such characterisation is presented in Publication II and summarised here.

This low-quality limit is an important limit for the experiments that will be presented in later parts of this thesis, as we wish to excite thermal populations in our system, and measure them bolometrically using NIS bolometers. To date, no examples of efficient broadband single microwave photon detectors have been shown to have been developed.

To add this dissipative element to a resonator, we prepare (etch) a gap at the current maximum of the niobium superconducting resonator, where a thin film copper resistor is subsequently grown by electron-beam evaporation. To ensure good electrical contact between the resistor and the niobium, an intermediate layer of aluminium to improve the metallurgical compatibility is grown immediately prior to the copper, resulting in a niobium-aluminium-copper-aluminium-niobium path the ground. This fabrication protocol follows the same procedure as that listed in Section 3.2, removing the in-situ argon ion milling and replacing the second aluminium layer with copper. Similarly, once processed, the device is diced and loaded to the same assembly used for qubit characterisation. Both the device, and the setup, can be observed in Figure 4.0.1.
Figure 4.0.1. Device and measurement setup from Publication II. (a) Diagram of the chip, consisting of two $\lambda/4$ resonators inductively coupled to a common transmission line, used for sending the probe signal. The left resonator contains no dissipative elements, and is used as a reference. The right resonator is, instead, terminated by an aluminium-copper-aluminium constriction. (b) Scanning-electron micrograph of the dissipative constriction, in contact with the center conductor of the resonator at the right side, and in contact with the common groundplane at the left side. The image is coloured with orange for copper, cyan for aluminium, and purple for niobium. (c) An equivalent lumped circuit for the copper terminated resonator. The coupled-inductor symbol represents the dominant coupling mechanism between the in/output transmission line and the resonator. The resistance $R$ represents the resistance of the copper termination. (d) Microwave measurement setup. The microwave signal is introduced from port 1 by a vector network analyser (VNA) at room temperature and passes through several attenuators distributed at different temperatures to the input of the device at 10 mK. The transmission is measured at the port 2 of VNA, with the signal passed through through two cryogenic isolators and both cryogenic and room temperature amplification stages.
4.1 Characterisation

As shown in Figure 4.0.1(d), the sample is loaded to the mixing chamber of a dilution refrigerator, cooled to approximately 10 mK, and the microwave transmission through the sample is measured by a vector network analyser using a heterodyne measurement over a broad bandwidth. At this temperature, the blue trace shown in Figure 4.1.1(a) is the result. While the non-dissipative resonator is clearly visible at 7.246 GHz, the dip in transmission corresponding to the dissipative resonator is not resolvable within the frequency-dependent background of the measurement chain, and indeed the expected dip has amplitude less than 1 dB.

The technique that we use to measure the resonator, then, is to take advantage of having two different superconducting materials in the device, with different superconducting transition temperatures. Hence, by warming the sample stage to a higher temperature (by heating the dilution refrigerator), it is possible to drive the superconducting materials into the normal dissipative state, and measure the spectra of the resonator terminated by an even larger dissipation, with correspondingly lower quality - effectively a background reference.

In a parallel measurement, the DC electronic resistance of a co-processed, nominally identical termination can be measured as a function of this temperature, and is shown in the inset of Figure 4.1.1(a). We can see clearly that the resistance change occurs at precisely the superconducting transitions of aluminium, and later that of niobium. By measuring at the points labelled by arrows - above and below the aluminium transition temperature, we can take the ratio of the two traces and extract the signal corresponding to the dissipative resonator from the background.

This extracted amplitude and phase of the scattering parameter $S_{21}$, corresponding to the transmission through the system, can be modelled using Eq( 4.1), a standard notched resonator model [76].

\[
S^{notch}_{21} = a e^{i\alpha} e^{-2\pi i \tau} \left[ 1 - \frac{(Q_l/Q_c)|e^{i\phi}}{1 + 2i(Q_l/f_r - 1)} \right]. \tag{4.1}
\]

Where $a$ is the overall amplitude and $\alpha$ is the phase shift. Parameters $f$ and $f_r$ denote the probe frequency, and the resonance frequency of the resonator, respectively. $\tau$ is the electronic delay caused by the length of the cable and the finite speed of light, and $\phi$ quantifies the impedance mismatch. $Q_l$ and $Q_c$ are the loaded quality factor and the coupling quality factor respectively, with $1/Q_l = 1/Q_i + 1/Q_c$, providing a means of extracting the internal quality factor $Q_i$. The $Q_l/Q_c$ ratio determines the depth of the notched transmittance and is maximised at $Q_l \approx Q_c$. 
**Figure 4.1.1.** (a) Transmittance spectra of the device show in Figure 4.0.1. The red and blue curves are measured at 2 K and 10 mK, respectively, as indicated by arrows in the inset, which shows the resistance of a co-processed copper termination as a function of temperature, measured in parallel. The inset highlights three temperatures of interest, corresponding to the superconducting transitions of the aluminium contact, and niobium resonator. The yellow region highlights the frequency range of the resistively terminated superconducting resonator, which can be extracted by the division of the blue low temperature 'signal' trace by the red high temperature 'background' trace (b) The extracted resonance dip of the dissipative resonator, the left inset showing the phase, and the dashed green line showing the fit to both of these, giving $f_r$ and $Q_i$ as 6.71 GHz and 45. The right inset shows a similar fitting to the reference resonator, with quality $2.8 \times 10^4$

This fit is shown by the dashed fitting lines in Figure 4.1.1(b), and using this technique, we have extracted extremely low quality factors in the range of $10 – 67$ for resonators with different resistances (thicknesses) of the copper termination element.
5. Heat Transport Experiments

So with our thermal baths, the temperature is read out by probing of the temperature dependent energy distribution of a piece of copper by superconducting aluminium probes separated by a thin layer of aluminium oxide (explained in Chapter 2). These thermal baths are then placed at the current maxima of two superconducting resonators (characterised in Chapter 4). Finally, interfacing the thermal baths with a tunable superconducting qubit capacitively coupled to both resonators (designed and characterised in Chapter 3), we can start to put everything together to study the heat transport through a superconducting quantum circuit. The general circuit diagram for this hybrid structure is shown in Figure 5.0.1.

This has a rather large parameter space to explore, and as such, we first apply the constraint of making the two sides of the qubit symmetric, and investigate how the interplay between the coupling between the resonator and qubit $g$, and the decay rate of the resonator (primarily through dissipation in the copper resistor) $\gamma$ can affect the locality of heat transport through the system. This is the basis for Publication III, and is explained in Section 5.2.

Then, in Section 5.3, we keep this ratio constant, and instead change the frequency of the resonators to explore how one may engineer the directionality of heat flow, and make a magnetic-flux tunable heat rectifier.

5.1 Measurement Methods

These experiments were both performed in a custom-made plastic dilution refrigerator, designed in [77], able to reach base temperature values $\approx \SI{50}{\text{mK}}$. 
Figure 5.0.1. General circuit for the heat transport measurements performed in this thesis, consisting a qubit central element, coupled to two superconducting resonators of frequency \(1/\sqrt{LC}\), each terminated with a copper thermal bath, that can be readout and prepared by superconducting probes through a tunnel barrier of resistance \(R_T\). A third diagnostic resonator is also included, which may be inductively coupled to a microwave feedline for qubit characterisation. The superconducting qubit is tuned by an external solenoid providing a magnetic flux offset through the superconducting quantum interferometer, tuning the non-linear inductance of the qubit through a \(f_{01} \propto \sqrt{\cos(\Phi)}\) dependence.

A bonded chip, shown in Figure 5.1.2, is loaded to the mixing chamber of this dilution refrigerator, with the measurement lines filtered by a 1m-long Thermocoax wire segment, resulting in an effective signal bandwidth of 0 – 10kHz, for low-impedance loads, filtering out the high frequency noise.

As the devices are very sensitive to magnetic flux, a high-permeability magnetic shield is placed around the vacuum can of the dilution refrigerator, with the magnetic field then controlled by a superconducting solenoid wound around the vacuum can, between the can and the magnetic shield.

Electronic current bias for the thermometers, and signal generation for the thermal bath engineering is performed by room temperature electronics, divided by room-temperature voltage dividers to attain the low levels required. Thermometry is then readout via a FEMTO Messtechnik GmbH DLPVA-100 room-temperature low-noise amplifier, passed into an SRS Lock-In Amplifier that is synchronised to the signal generator in order to extract the first harmonic, reducing sensitivity to signal pickup and low-frequency drifts of the voltage amplifier output by operating on a square-wave modulated (22 – 42Hz) signal.

The power is then extracted from this voltage signal by first a thermal calibration, then by using the electron-phonon coupling as the dominant
5.2 Quantum Heat Valve

Let’s write down a Hamiltonian for a superconducting qubit coupled to two identical non-dissipative resonators. We can approximate this by a system of three harmonic oscillators with energy levels \( \hbar f_q \) corresponding to the qubit, and \( \hbar f_r \) corresponding the resonators, where we have assumed that only the first excitation frequency and first harmonic of the resonators. This is justified by operating in the low temperature limit \( \hbar f_r \gg k_B T \). The Hamiltonian of the system then reads

\[
\hat{H} = \hbar f_r \left[ \left( \frac{\hat{a}_D^\dagger \hat{a}_D + \hat{a}_S^\dagger \hat{a}_S}{f_r} \right) + \frac{f_q}{f_r} \hat{b}_S^\dagger \hat{b}_S + g (\hat{b}_D \hat{a}_D^\dagger + \hat{b}_D^\dagger \hat{a}_D + \hat{b}_S \hat{a}_S^\dagger + \hat{b}_S^\dagger \hat{a}_S) + \tilde{g} (\hat{a}_D \hat{a}_S^\dagger + \hat{a}_S \hat{a}_D^\dagger) \right],
\]

where \( \hat{a}_S^\dagger \) and \( \hat{a}_D^\dagger \) are the creation and annihilation operations for the source (S) and drain (D) resonators, depicted in Fig 5.2.1. \( \hat{b}_S^\dagger \) and \( \hat{b}_S \) are the creation and annihilation operators for the qubit, \( g \) is the coupling of the resonators to the qubit, and \( \tilde{g} \) quantifies direct resonator-to-resonator coupling, bypassing the qubit.

It is possible due to non-perfect fabrication that minor differences \( \Delta f \) in the resonant frequencies of the resonators could occur, and we introduce parameter \( a = \Delta f / f_r \ll 1 \). By choosing the minimal four-level basis of \( \{|000\rangle, |100\rangle, |010\rangle, |001\rangle\} \), referring to the source-resonator, the qubit, and the drain-resonator, respectively, results in the matrix representation.
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Figure 5.1.2. A bonded chip on the sample stage, before being enclosed in two brass Faraday cages and loaded to the mixing chamber stage of the dilution refrigerator.

\[ \hat{H} = \hbar f_r \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 + a/2 & g & \tilde{g} \\ 0 & g & f_r & g \\ 0 & \tilde{g} & g & 1 - a/2 \end{pmatrix}. \]  

The eigenmodes of this Hamiltonian are shown in Figure 5.2.2(b), and experimentally verified by performing a two-tone spectroscopy to map out the qubit interaction with non-dissipative identical resonators using the same method and experimental setup presented in Chapter 3.
Figure 5.2.1. a, b: Concept of the experiment and effective thermal model. A transmon qubit having magnetic flux-tunable plasma frequency \( f_q(\Phi) \) is capacitively embedded between two superconducting co-planar waveguide resonators of identical length \( = 4.6 \text{ mm} \), each terminated by a copper resistive thermal bath, shown in (d). We study the temperature of the drain reservoir \( T_D \) as a function of the temperature of the source reservoir \( T_S \) and of the ratio \( r \equiv f_q/\tilde{f}_r \), where \( \tilde{f}_r \) is the fundamental resonant frequency of the resonators. c: Lumped-element idealisation of the device; capacitors \( C_g \) couple the transmon to each \( L_rC_r \) resonator. d: Coloured scanning electron micrograph of a waveguide termination, including three tunnel electrodes. A copper resistor (pink) is in clean contact with aluminium leads (light blue) connecting to the patterned niobium film (light grey) on sapphire substrate (dark grey). The inset shows a magnified orthogonal view of the area spanned by the normal-metal element; the scale bar corresponds to \( 3 \mu \text{m} \). e: Coloured scanning electron micrograph of the SQUID element in the transmon structure; the scale bar corresponds to \( 10 \mu \text{m} \).
Figure 5.2.2. (a): Two-tone transmission spectroscopy data of the qubit interacting with the eigenmodes of the two symmetrical resonators, with the eigenmodes derived from Equation 5.2 superimposed. (b): Eigenmodes derived from Equation 5.2, with the arrows representing allowed transitions used in the quasi-Hamiltonian (global) model, described by Equation 5.4.

So with the non-dissipative environment now established, we can add the thermal baths to the circuit, and look at heat transport through it, in the form of power transmitted from the source reservoir to the drain reservoir, $P_D$.

In general, we can write the power to each reservoir simply as the sum over all eigenstates $k,l$ of the density matrix $\rho$, the transition energy $E_{kl}$, and the rate $\Gamma_{k\rightarrow l}^D$ determined by the drain reservoir, giving Equation 5.3.

$$P_D = \sum_{k,l} \rho_{kk} E_{kl} \Gamma_{k\rightarrow l}$$  \hspace{1cm} (5.3)

Which we can then expand (derived in the supplementary materials of Publication III) to give us

$$P_D = 2\pi hf_r^2 \gamma \sum_{k,l} \rho_{kk} \frac{|\langle k|a_D - a_D^+|l\rangle|^2 (E_{kl}/hf_r)^2}{1 + \left(\frac{f_{kl}/f_r - f_r/f_{kl}}{f_r}\right)^2} \frac{1}{1 - e^{-\beta_D E_{kl}}}.$$

$$\hspace{1cm} (5.4)$$

where $\gamma$ is the inverse of the quality factor of the resonator with resonant frequency $f_r$, and $\beta_D = 1/k_B T_D$ where $T_D$ is the temperature of the drain reservoir.

We call this model ‘Quasi-Hamiltonian (QH)’, owing to its origins in the Jaynes-Cummings Hamiltonian, coupled to the dissipation, and is experimentally verified in Figure 5.2.3(a). In this figure, different coloured traces
Heat Transport Experiments

correspond to source-bath temperatures indicated by the colour bar, with the unbiased temperature of the drain-bath indicated by an arrow.

Using a quality factor $Q_r = \gamma^{-1} = 20$, we see that equation 5.4 accurately describes the features experimentally observed, which is consistent with similarly fabricated devices presented in 4 and Publication II. Optimised fitting suggests the qubit-resonator coupling $g \approx 0.019$ and the resonator-resonator cross coupling $\tilde{g} \approx -0.020$.

This $Q_r \approx 20$ yields a ratio $g/\gamma = gQ_r \approx 0.4$. This ratio, while far from the fully Hamiltonian limit $g/\gamma \to \infty$, is still sufficient to support the creation of these full-circuit ‘global’ eigenmodes predicted and observed by equation 5.2 and Figure 5.2.2. Here, the resonator loss rate is of a similar order to the qubit-resonator coupling.

We next investigate the limit where $g/\gamma \to 0$, where the dissipation is the dominating element of the resonator, with $Q_r \to 0$, by fabricating a lower quality resonator.

In this regime, eigenmodes for the global system should be suppressed, and only the qubit-resonator coupling $g$ should be the dominating element in the tunable photonic heat transport, with a contribution from the resonator-resonator coupling acting as an effective background contribution.

The power from source-reservoir to the drain-reservoir in this highly dissipative non-Hamiltonian ‘local’ case, described in [7] reads

$$P_D = \pi \hbar g f_r^2 \frac{n(\beta_S h f_q) - n(\beta_D h f_q)}{[1 + Q_r^2(r - 1/r)^2][\coth(\beta_S h f_q/2) + \coth(\beta_D h f_q/2)]} + \pi \hbar \tilde{g} f_r^2 \int_0^\infty \frac{n(x \beta_S h f_q) - n(x \beta_D h f_q)}{[1 + Q_r^2(x - 1/x)^2]^2} x^2 dx,$$

where $n(\beta_S h f) = 1/(\exp(\beta_S h f) - 1)$ is the temperature-dependent population in each resonator, following the Bose-Einstein distribution. The second term describes the direct resonator-to-resonator photon transfer, quantified by $\tilde{g}$.

This ‘non-Hamiltonian’ (NH) model is experimentally verified in Figure 5.2.3(b), with optimal fitting at $Q_r = 3.15 \pm 0.14$ and $g \approx 0.0156$, with $g/\gamma = gQ_r \approx 0.05$.

The optimised fitting parameters for both sets of experimental data are given in Table 5.1. Note, that whilst it is perhaps interesting that these microwave parameters can be extracted from a quasi-DC measurement, each can be independently confirmed by a microwave spectroscopy measurement, leading to the main fitting parameter being the quality factor $Q_r$, which falls within the range reported by Publication II.

This ‘Quantum Heat Valve’ device demonstrates the subtlety in tuning
Figure 5.2.3. Total heating power absorbed by the drain reservoir as a function of the applied magnetic flux $\Phi$. Different traces are colour-coded to the source temperature values $T_S$ shown in the adjacent legend bar. In each plot, experimental data is juxtaposed to the optimal fit of the appropriate theoretical model. Panels a, b correspond to quasi-Hamiltonian (Equation 5.4) and non-Hamiltonian (Equation 5.5) regimes, respectively.
simply two parameters, the qubit-resonator coupling $g$, and the reservoir-
resonator relaxation $\gamma$. The interplay of these two parameters influence
both the transmitted power and the locality of the heat transport. It
interesting to see that while the quasi-Hamiltonian ‘global’ regime opens
extra eigenmodes, resulting in a higher power transmitted through the
system, these extra eigenmodes results in a weaker tuning ratio, as the the
qubit-resonator coupling, and our ability to tune it via the magnetic flux
tuning of the qubit plasma frequency, plays a reduced role in the photonic
heat transport. This tuning ratio is shown in Figure 5.2.4.

5.3 Heat Rectification via an artificial atom

So now that we have an understanding on the role that the qubit-resonator
coupling $g$ and the reservoir-resonator relaxation $\gamma$ play on the heat trans-
port through our symmetric system, we now remove the symmetry to
investigate the role of anharmonic coupling of the qubit asymmetrically
to the two microwave resonators. This details the work performed in
Publication IV.

In a pure two-level system, shown in Figure 5.3.1, we can consider the
power flow from left to right using Equation 5.3, assuming one of the
temperatures is $T = 1/(k_B\beta) > 0$ and the other one to be $= 0$, or vice versa,
and now using left bath-qubit coupling $\gamma_1$ and right bath-qubit coupling $\gamma_2$,
takes the form

$$P = \frac{\gamma_1 \gamma_2}{\gamma_1 \coth(E\beta/2) + \gamma_2} E n(E),$$  \hspace{1cm} (5.6)
where \( n(E) = 1/(e^{\beta E} - 1) \) is the Bose-Einstein distribution. This expression is particularly interesting, as it implies that the power flow is directionally impeded when \( \gamma_1 \neq \gamma_2 \), unlike in the fermionic case where the power flow is symmetric.

Taking the rectification ratio \( \mathcal{R} \) of power \( P_i \) to bath \( i \) in the forwards (+) and backwards (−) direction [78] in the two level approximation for the transmon as

\[
\mathcal{R} = \frac{|P_i^+|}{|P_i^-|} = \frac{\gamma_1 + \gamma_2 \coth(\frac{\beta\hbar\omega_0}{2})}{\gamma_1 \coth(\frac{\beta\hbar\omega_0}{2}) + \gamma_2}.
\] (5.7)

Any value \( \mathcal{R} \neq 1 \) corresponds to heat rectification. By introducing the asymmetry in qubit-bath coupling factors \( \delta = 1 - \gamma_1/\gamma_2 \), this expression can be simplified for \( |\delta| \ll 1 \) to read

\[
\mathcal{R} = 1 + e^{-\beta\hbar\omega_0 \delta}.
\] (5.8)

This, in fact, is the simplest realisation of a spin-boson rectifier proposed by [79], and one of the few experimental realisations of heat rectification [80, 81, 82], despite the ubiquity of its electronic rectification counterpart, the electronic diode, omnipresent in modern day electronic devices, and a fundamental tool for electronic logic.

We can fabricate and measure a device using the same protocol as in Section 5.2, changing the resonators to be at 2.8 GHz and 6.7 GHz respectively. We can similarly perform spectroscopic characterisation of the qubit interacting with the two resonators using two-tone spectroscopy, and show that not only do we couple to these two resonators, but also we can see the qubit interacting with the second harmonic of the 2.75 GHz resonator, at 5.5 GHz.
Figure 5.3.1. Diagram of the sample, consisting of a centrally located transmon type superconducting qubit, coupled to two superconducting co-planar waveguide resonators at 2.8 GHz and 6.7 GHz, respectively. Each resonator is terminated at the current maximum with a thin-film copper microstrip resistor, acting as a mesoscopic thermal bath. One of them is shown in the left inset by a colourised scanning electron micrograph of the copper microstrip resistor (orange), with 4 superconducting aluminium probes (green) (separated from the copper by an insulator, not visible) for temperature control and readout, and two superconducting aluminium contacts to the co-planar waveguide resonator (blue). The superconducting quantum interferometer is shown similarly on the right inset. The topmost diagram represents a simple model of the system, with the resonator-qubit-resonator structure represented as a diode, and the forward and reverse directions drawn in purple and green respectively. A third electrode can be seen on the transmon island, which in non-dissipative variants of the device used for spectroscopy, connects the transmon to a readout resonator.
Figure 5.3.2. Two-tone spectroscopic readout of resonator-qubit-resonator structure performed using a tertiary readout resonator coupled to a third electrode of the qubit. We observe qubit-resonator couplings at 2.78 GHz, corresponding to the low frequency resonator, 5.5 GHz, corresponding to the second mode of this resonator, and at 7.05 GHz, corresponding to the high frequency resonator. The parameters used in the calculated energy spectra, shown in the upper right figure, are: $E_J/h = 45$ GHz and $E_C/h = 0.15$ GHz, which give $\omega_{01}(\Phi = 0)/2\pi = 7.2$ GHz.

Figure 5.3.3. (a) Power transmitted between the two baths at three voltage (heating) bias points, with the subplots corresponding to 420 mK (1000 fW), 400 mK (750 fW), and 380 mK (600 fW) source temperatures (powers), in descending order. The bath temperature is kept fixed at 150 mK. Purple is the forward direction, and green the reverse, as shown in Figure 5.3.1. (b) Rectification ratio of traces from (a), with the non-tunable contribution removed.
Writing down the Hamiltonian for this higher Hilbert-dimensionality device is more challenging than in the quantum heat valve case, as we lose the ability to use the same symmetry considerations. We can write the Hamiltonian as

\[ \hat{H} = \hbar \omega_L \hat{a}_L^\dagger \hat{a}_L + \hbar \omega_q \hat{a}_q^\dagger \hat{a}_q + \hbar \omega_R \hat{a}_R^\dagger \hat{a}_R + g (\hat{a}_q \hat{a}_L^\dagger + \hat{a}_q^\dagger \hat{a}_L + \hat{a}_q \hat{a}_R^\dagger + \hat{a}_q^\dagger \hat{a}_R) + \tilde{g} (\hat{a}_L \hat{a}_R^\dagger + \hat{a}_L^\dagger \hat{a}_R). \]

(5.9)

Where \( \hbar \omega_L \), \( \hbar \omega_q \), and \( \hbar \omega_R \) are the energies of the left resonator, qubit and the right resonator, respectively, \( g \) is the common coupling constant of the qubit to the two resonators, and \( \tilde{g} \) is the cross-coupling between the resonators. In the eleven-level basis of \( |000\rangle, |100\rangle, |010\rangle, |001\rangle, |110\rangle, |101\rangle, |011\rangle, |111\rangle, |200\rangle, |210\rangle, |300\rangle, |310\rangle \), where the entries in each state refer to the left resonator, the qubit, and the right resonator, respectively, the matrix form of the Hamiltonian can be written as shown in the supplementary materials of Publication IV. This basis allows up to the third harmonic of the low frequency left resonator, whilst limiting the qubit and high frequency right resonator to single excitations.

The spectra of these states as a function of the ratio \( r = f_q / f_L \), the analogous plot to that shown in Fig 5.2.2 for the Quantum Heat Valve, is shown alongside the experimental spectroscopic readout of the non-dissipative qubit-resonator interactions in Figure 5.3.2.

We can now turn to look at what the power through the system looks like, in both the forward and reverse directions, plotted in purple and green respectively for a series of heater temperatures in Figure 5.3.3(a), with the rectification ratio plotted in Figure 5.3.3(b), to remove a non-tunable rectification for visual clarity. This non-tunable rectification is believed to be an artefact of uncertainty in heater temperature preparation, resulting in large differences in the overall transmitted power, in addition to the direct coupling between resonators.

We see that there is a strong dependence on the magnetic flux-tuning of the superconducting qubit, with features occurring when the qubit is tuned to resonant modes of the system, with the rectification tunable from 0 to 10%.
6. Conclusions

At the beginning of this thesis, we argued the relative lack of understanding of heat transport in quantum systems, and the increasing need to explore this interface of quantum thermodynamics - not just as a purely academical pursuit to understand a fundamental physical boundary, but also to better understand and develop industrial quantum technologies, in particular as the recently-proposed quantum volume, a metric of number of quantum bits (quantum width) and how coherently coupled they are (quantum depth) appear to be following an equivalent Moore’s law to that of the semiconductor microprocessor.

Sufficiently advanced today is the field of quantum computing that during this thesis, the author has not only been involved in the scientific pursuits presented in this thesis, but also in the development of products for the public, such as a quantum dice, using the IBM Q cloud-based quantum computing platform as a quantum-limited dice roll, and in partnership with IBM on the development of a quantum arcade of video and tabletop games. Yet, despite this, there are still plenty of fundamental physics challenges required to achieve a useful universal quantum computer.

This thesis has taken advantage of the tools developed in the development of superconducting quantum information processing, such as the qubits explained, demonstrated, and characterised in Chapter 3 and Publication I, and interfaced it with dissipative elements designed to look at role of dissipation and heat transport in these superconducting quantum circuits.

This has required interfacing a wide combination of sometimes contrasting techniques and technologies, in particular the high-fidelity extremely low energy microwave engineering technologies in both frequency and time-domain for superconducting circuit QED studies, with the highly dissipative, slow timescale regime required for thermodynamics.

To reconcile these two regimes, we have have explored the effect of strong dissipation on superconducting resonators in Chapter 4 and Publication II,
which involved having to develop a technique for isolating and characterising extremely low quality resonators from a typical quantum information microwave background. We then looked at how dissipation can suppress eigenmodes of a resonator-qubit-resonator system, and how we can tune to the qubit in these systems to enhance or suppress the heat transport based on the Purcell effect, in Section 5.2 and Publication III, which we named the Quantum Heat Valve.

Finally, we demonstrated rectification of heat currents when one uses asymmetric resonators and the non-linear anharmonic energy levels of the artificial atom, and by heating sufficiently to populate several modes of the heated resonator, photonically blockading energy states of the artificial atom, we can engineer a system that favours heat transport in a particular direction - again tunable using the Purcell-limited qubit coupling to the resonators.

These experiments combined give an idea of how dissipation could play a role in more complex superconducting circuits, such as those used in quantum computing and information processing, as well as some prospects on how we might manipulate the flow of heat in these systems to improve their performance.

The outlook is quite clear, we should adapt more of the tools of cQED, especially entangling multiple qubits, and as the technology allows us, with more sensitive bolometry (moving to calorimetry), and look to moving towards single quanta excitations/detection and qubit-coherence timescales.

There are some extremely interesting and important experiments to perform in this space, that would be able to truly explore the interface of quantum dynamics with mesoscopic physics, for example - the energy relaxation of a qubit into a well-characterised bath on single trajectories, to look at the energy distribution would be of fundamental interest, and will require a broad bandwidth ultra-sensitive calorimeter, perhaps realised using a dissipative absorber with tunnel junction based readout. Additionally, looking at using a qubit (or multiple qubits) to make a quantum heat engine is currently a topic of great interest in the quantum thermodynamics community.

We hence hope that the work in this thesis goes towards providing a versatile platform for looking at heat transport, by offering an environment with a high dimensionality that can be tuned to engineer specific physical domains, towards quantum heat transport and circuit quantum thermodynamics.
References


References


References


Heat Transport in Superconducting Quantum Circuits

Jorden Senior