Spatial and Spectral Corrections for Integrating Sphere Photometry and Radiometry

Alexander Kokka
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Alexander Kokka

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Abstract

The energy efficiency of lighting is expressed in terms of luminous efficacy. It is the ratio of the visible light emitted by the source to the power consumed in the process. The total amount of useful light produced by a source is described as luminous flux, which is often measured using integrating sphere photometers. These instruments consist of a hollow sphere and a detector whose output signal is proportional to the luminous flux emitted by the device under test. Non-ideal characteristics of photometers and integrating spheres induce measurement uncertainty on the luminous flux measured – and thus the resulting energy efficiency.

With solid-state lighting supplanting the conventional incandescent and energy-saving lamp technologies, the associated measurement techniques need to be revised as well. As integrating sphere photometers are generally calibrated using incandescent light sources, the measurement uncertainty is increased when determining the luminous efficacy of solid-state lighting products such as LEDs. This uncertainty can be reduced by employing correction factors that take into account the imperfections of the measurement system.

In this dissertation, a method based on a fisheye-lens camera was developed to reduce measurement uncertainty due to spatial non-uniformities of integrating spheres. In order to calculate the spatial correction factor, the relative angular intensity distribution of the lamp under test is required. Traditionally, obtaining such a distribution has involved time-consuming and resource-intensive goniophotometric measurements. With the fisheye camera method, the distribution can be measured in seconds using a fisheye camera installed into a port of the integrating sphere.

To reduce the measurement uncertainty due to differences in the spectra of the calibration source and the device under test, a new LED-based reference spectrum was developed for calibrating photometers. The reference spectrum is based on one of the eight LED illuminants that were developed in the same study to be employed in colorimetry. For the LED products and photometers tested, the new reference spectrum reduced the average spectral mismatch errors by a factor of two, when compared with an incandescent calibration source.

Keywords metrology, integrating sphere, camera, photometry, radiometry

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</table>
Valaistuksen energiatehokkuutta kuvataan suurella valotehokkuus, joka on tuotetun näkyvän valon suhde valon tuottamiseen kulutettuun tehoon. Valovirta on lähteen tuottama näkyvän valon määrä. Valovirtaa mitataan yleensä integroivalla pallolla, joka koostuu ontosta pallosta ja fotometrista, jonka tuottama signaali on suhteessa mitattavan lähteen tuottamaan valovirtaan. Fotometrien ja integroivien pallojen epätäydellisyystä aiheuttavat mittausepävarmuutta valovirtamittauksiin ja siten myös mitattuun energiatehokkuuteen.

Puolijohdevalaistuksen syrjäyttäessä perinteiset hehku- ja energiansäästölamput, myös niitä varten kehitetyt mittausmenetelmät vaativat päivitystä. Koska yleisesti integroivien pallojen kalibrointiin käytetään hehkulampputyypissiä lähteitä, mittasepävarmuus on suurempi, kun mitataan puolijohdevalaisimia kuten LED-lamppuja. Tätä epävarmuutta voidaan pienentää käyttäen korjauskertoimia, jotka ottavat huomioon mittausjärjestelmän epätäydellisyystä.


The research for this dissertation was carried out in the scope of European Union project PhotoLED *Future Photometry Based on Solid State Lighting Products* and Academy of Finland project *Quantitative remote sensing by 3D hyperspectral UAVs*. I would like to acknowledge the funded position of Aalto ELEC doctoral school, and the scholarships granted by Walter Ahlström Foundation and the Tutkijat Maailmalle program.

I would like to thank Prof. Erkki Ikonen for supervising this dissertation and for the opportunity to work in the field of metrology. I would also like to thank Drs. Tuomas Poikonen and Tomi Pulli for their advice, almost never-ending expertise, and all the encouragement. This gratitude extends to all my colleagues at the Metrology Research Institute and all the other professionals I had the privilege of working with during these years. I also appreciate the efforts of the preliminary examiners Prof. Kai-Erik Peiponen and Dr. Udo Krüger.

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Espoo, February 21, 2019,

Alexander Kokka
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List of Publications

This dissertation consists of an overview and of the following publications, which are referred to in the text by their Roman numerals.


Author’s Contribution

Publication I: “Fisheye camera method for spatial non-uniformity corrections in luminous flux measurements with integrating spheres”

The author developed the method, performed the measurements and analysis, and wrote the article considering the input of the co-authors and the reviewers.

Publication II: “Validation of the fisheye camera method for spatial non-uniformity corrections in luminous flux measurements with integrating spheres”

The author developed the method, performed the majority of the measurements, carried out the analysis, and wrote the article considering the input of the co-authors and the reviewers.

Publication III: “Development of white LED illuminants for colorimetry and recommendation of white LED reference spectrum for photometry”

The author took part in measuring the LED spectra, carried out the spectral mismatch analysis, and wrote the majority of the article considering the input of the co-authors and the reviewers.

Publication IV: “Flat-field calibration method for hyperspectral frame cameras”

The author co-developed the method, performed the measurements and analysis, and wrote the article considering the input of the co-authors and the reviewers.
Abbreviations

AC Alternating current
ANSI American National Standards Institute
BaSO₄ Barium sulphate
CCPR Consultative Committee for Photometry and Radiometry
CCT Correlated colour temperature
CIE International Commission on Illumination
CMOS Complementary metal oxide semiconductor
CSPD Centroid spectral power distribution
DC Direct current
DUT Device under test
EMPIR European Metrology Programme for Innovation and Research
FPI Fabry-Pérot interferometer
IES Illuminating Engineering Society
IR Infrared
LED Light emitting diode
MAE Mean absolute error
MRI Metrology Research Institute
NIST National Institute of Standards and Technology
NMI National Metrology Institute
PQED Predictable quantum efficient detector
Abbreviations

**PTFE** Polytetrafluoroethylene

**RGB** Red, green, and blue (colour model)

**RSPD** Representative spectral power distribution

**SPD** Spectral power distribution

**SRDF** Spatial responsivity distribution function

**SSL** Solid-state lamp

**TC** Technical committee

**UAV** Unmanned aerial vehicle

**UV** Ultraviolet
Symbols

\( A \) \hspace{1em} \text{Area of precision aperture}

\( C \) \hspace{1em} \text{Closeness score}

\( E_e \) \hspace{1em} \text{Irradiance}

\( E_v \) \hspace{1em} \text{Illuminance}

\( E_v(\theta, \phi) \) \hspace{1em} \text{Illuminance distribution}

\( F \) \hspace{1em} \text{Spectral mismatch correction factor}

\( I(\theta, \phi) \) \hspace{1em} \text{Relative angular intensity distribution}

\( I_e \) \hspace{1em} \text{Radiant intensity}

\( I_v \) \hspace{1em} \text{Luminous intensity}

\( I_v(\theta, \phi) \) \hspace{1em} \text{Luminous intensity distribution}

\( K(\theta, \phi) \) \hspace{1em} \text{Spatial responsivity distribution function (SRDF)}

\( K_m \) \hspace{1em} \text{Photopic normalisation constant}

\( L_e \) \hspace{1em} \text{Radiance}

\( L_v \) \hspace{1em} \text{Luminance}

\( M \) \hspace{1em} \text{Spectral mismatch index}

\( P \) \hspace{1em} \text{Active electrical power}

\( R_a \) \hspace{1em} \text{Colour rendering index}

\( R_f \) \hspace{1em} \text{Colour fidelity index}

\( R_s \) \hspace{1em} \text{Sphere radius}

\( S(\lambda) \) \hspace{1em} \text{Spectral power distribution}
Symbols

$T(\lambda)$ Spectral throughput

$V(\lambda)$ Spectral sensitivity function of human eye

$X$ $x$-coordinate in camera-coordinate system

$Y$ $y$-coordinate in camera-coordinate system

$Z$ $z$-coordinate in camera-coordinate system

$wX$ $x$-coordinate in world-coordinate system

$wY$ $y$-coordinate in world-coordinate system

$wZ$ $z$-coordinate in world-coordinate system

$A$ Angular intensity distribution component

$C$ Camera sensitivity matrix

$D$ Diffuse light level component

$D_G$ Dark signal frame

$F$ Flat-field correction matrix

$G$ One-channel or greyscale image matrix

$K$ Intrinsic parameter matrix

$M$ Projection matrix

$\mathbf{cP}$ Point in camera-coordinate system

$\mathbf{wP}$ Point in world-coordinate system

$R$ Rotation matrix

$S$ Integrating sphere component

$T$ Transformation matrix

$t$ Translation vector

$c_v$ Correction coefficient for total luminous flux measurement

$f$ Focal length

$i$ Photometer signal

$k_a$ Correction factor for non-uniform illuminance at precision aperture

$k_s$ Spatial non-uniformity correction factor

$o_x$ $x$-coordinate of principal point in pixel-coordinate system
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$o_y$</td>
<td>y-coordinate of principal point in pixel-coordinate system</td>
</tr>
<tr>
<td>$p$</td>
<td>Point in image-plane coordinate system</td>
</tr>
<tr>
<td>$p_0$</td>
<td>Principal point in image-plane coordinate system</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Point in pixel-coordinate system</td>
</tr>
<tr>
<td>$p_{i0}$</td>
<td>Principal point in pixel-coordinate system</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial distance to principal point</td>
</tr>
<tr>
<td>$r_d$</td>
<td>Distorted radial distance to principal point</td>
</tr>
<tr>
<td>$r_u$</td>
<td>Undistorted radial distance to principal point</td>
</tr>
<tr>
<td>$s_{rel}$</td>
<td>Relative spectral responsivity of photometer</td>
</tr>
<tr>
<td>$s_x$</td>
<td>Pixel dimension along x-axis</td>
</tr>
<tr>
<td>$s_y$</td>
<td>Pixel dimension along y-axis</td>
</tr>
<tr>
<td>$x$</td>
<td>x-coordinate in image-plane coordinate system</td>
</tr>
<tr>
<td>$x_i$</td>
<td>x-coordinate in pixel-coordinate system</td>
</tr>
<tr>
<td>$x_{ui}$</td>
<td>Undistorted x-coordinate in pixel-coordinate system</td>
</tr>
<tr>
<td>$y$</td>
<td>y-coordinate in image-plane coordinate system</td>
</tr>
<tr>
<td>$y_i$</td>
<td>y-coordinate in pixel-coordinate system</td>
</tr>
<tr>
<td>$y_{ui}$</td>
<td>Undistorted y-coordinate in pixel-coordinate system</td>
</tr>
<tr>
<td>$a_c$</td>
<td>Pitch angle of camera</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Correction factor for incident angle</td>
</tr>
<tr>
<td>$\eta_v$</td>
<td>Luminous efficacy</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Skew coefficient</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>Roll angle of camera</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Azimuth angle in spherical coordinate system</td>
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<tr>
<td>$\phi_{oa}$</td>
<td>Rotation about the optical axis</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Reflectance factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Zenith angle in spherical coordinate system</td>
</tr>
<tr>
<td>$\theta_{oa}$</td>
<td>Deviation from optical axis</td>
</tr>
</tbody>
</table>
Symbols

\( \Phi(\lambda) \)  Relative spectral radiant flux
\( \Phi_e \)  Radiant flux
\( \Phi_\lambda(\lambda) \)  Spectral radiant flux
\( \Phi_v \)  Luminous flux
1. Introduction

1.1 Background

The energy efficiency of lighting is expressed by the ratio of the visible light produced by the source to the power consumed in the process. Artificial light sources have evolved from fire- and gas-based devices to more energy efficient electrical sources. Modern solid-state lighting products, such as light emitting diodes (LEDs), have tremendously increased the power efficiency of electrical lighting when compared with the traditional incandescent and energy-saving lamp technologies, which are phasing out [1–6].

Photometry, the measurement science focused on light, has long relied on standards and measurement methods developed for incandescent sources. For instance, calibrations of detectors and characterisations of complete measurement systems are generally carried out using tungsten-filament lamps [7–11]. However, an increasing share of devices produced, tested, and sold is based on solid-state technologies.

As incandescent lamps will gradually phase out from the consumer market, there is a risk that the availability of tungsten-filament standard lamps used in photometry and radiometry will decline as well. Furthermore, the relatively short operational lifetime and fragility of incandescent lamps pose practical problems for industrial test laboratories, as well as for National Metrology Institutes (NMIs) [12–15].

Many properties of solid-state lamps (SSLs) deviate from those of incandescent sources, affecting the measurement uncertainty [16–18]. For one, compared with energy-saving and tungsten-filament lamps, the wider variety of angular intensity distributions produced by SSLs has increased measurement uncertainty when using integrating sphere photometers to determine the amount of light produced by such products [19–22].

Another source of increased measurement uncertainty is induced by incandescent and solid-state sources having disparate spectral power distributions. When a typical photometer is calibrated using a source with a different type of
spectrum than it is used to measure, the spectral mismatch has to be taken into account, by correcting if possible, or by including the spectral mismatch in the uncertainty budget of the measurement [23]. These uncertainties also propagate to the power efficiency measurements of LED lamps.

1.2 Dissertation outline

Chapter 2 gives an introduction to the concepts of radiometry and photometry. Luminous efficacy, the main concept of the energy efficiency of lighting products, is discussed from the metrology point of view. Chapter 3 focuses on integrating sphere photometry, discussing the measurement instrument widely used to determine luminous efficacy of lighting products. Special attention is given to the correction factors for reducing the measurement uncertainty of luminous efficacy measurements with integrating spheres. That is also the topic of Publications I–III of this dissertation.

Publications I and II introduce a spatial non-uniformity correction method for luminous flux measurements with integrating spheres. The method is based on employing a fisheye-lens camera to determine the angular intensity distribution of the light source under test. Publication III proposes an LED-based reference spectrum for calibrating photometers in order to reduce the uncertainty due to the spectral mismatch when measuring solid-state lighting products.

Chapter 4 presents the fundamentals of imaging and the mathematical camera models that are employed in Publications I, II, and IV. At the core of the fisheye camera method, presented in Publications I and II, is the calibration of the camera with the integrating sphere using a reference light source. This calibration essentially applies a system-wide flat-field correction to the measurement images. Publication IV presents a flat-field calibration method for frame cameras based on scanning the output aperture of an integrating sphere. The method was developed for hyperspectral frame cameras, but it is also applicable to other types of cameras, and is particularly advantageous when characterising wide-angle-lens imaging systems, such as fisheye cameras.

1.3 Scientific contribution

This dissertation contains the following scientific contribution:

Publication I: The publication introduces a method based on using a fisheye camera for determining spatial non-uniformity corrections in luminous flux measurements with integrating spheres. Traditionally, applying the spatial correction has required time-consuming goniophotometric measurements. The fisheye camera method provides a cost and time efficient approach of reducing measurement uncertainty stemming from the spatial non-uniformities of inte-

**Publication II:** The fisheye camera method was validated by measuring six LED lamps using the method in eight integrating spheres, and comparing the relative angular intensity distributions and the spatial non-uniformity correction factors with those obtained using five goniophotometers. The validation measurements showed that the method is applicable to integrating spheres of various diameters, reflectance factors, and port configurations. The method proved to be effortless to retrofit to all eight integrating sphere photometers, as no permanent modifications were made to any of the spheres employed in the study.

**Publication III:** Eight LED-based illuminants were developed, five of which were included in the CIE Technical Report No. 15: Colorimetry, 4th edition [25]. A CIE Technical Committee, TC2-90 *LED reference spectrum for photometer calibration*, was founded to investigate and publish the LED reference spectrum recommended in the publication. By employing the recommended reference spectrum for photometer calibration, the average spectral mismatch errors were reduced by a factor of two for the LED sources and photometers tested in the study, when compared with calibrating the photometers using the widely-used incandescent calibration spectrum.

**Publication IV:** The developed calibration method allows to synthetically create large, uniform radiance and luminance sources for characterising the spatial responsivity of imaging systems, enabling accurate flat-field corrections. The calibration also allows employing relatively small integrating spheres, which are less expensive and require less laboratory room than large integrating spheres, which are often used to create a large uniform radiance source at once.
2. Principles of radiometry and photometry

2.1 Quantities and units

Radiometry is a branch of metrology, which studies measurements of electromagnetic radiation. Methods of photometry, in turn, also take into account interaction of this radiation with the visual system of humans. Any spectral radiometric quantity can be converted into its photometric equivalent by weighting the spectrum of the quantity with the spectral sensitivity function \( V(\lambda) \) of the human eye. For instance, luminous flux \( \Phi_v \) (unit lumen, lm) can be obtained from the respective spectral radiant flux \( \Phi_e(\lambda) \) (unit W/nm) using the equation

\[
\Phi_v = K_m \int_{360\,\text{nm}}^{830\,\text{nm}} V(\lambda) \Phi_e(\lambda) \, d\lambda,
\]

where \( \lambda \) is the wavelength in standard air, and \( K_m \) is the photopic normalisation constant 683 lm/W [26].

Photometric quantities can also be directly measured using a photometer as the detector. The spectral responsivity function of such an instrument is designed to mimic the \( V(\lambda) \) function using optical filters. By employing different filters with a broadband detector, it is also possible to directly measure spectrally weighted data for other applications, such as the ultraviolet (UV) index.

Figure 2.1 shows the spectral responsivity model of the human eye \( V(\lambda) \), as defined by the CIE for the wavelength region of 360–830 nm [26]. The \( V(\lambda) \) function is designed to model the visual sensitivity of humans in well-lit, or photopic, conditions. The maximum value of \( V(\lambda) \) occurs at 555 nm, which perceptually corresponds to the green colour. The figure also shows the spectral radiant flux \( \Phi_e(\lambda) \) of an incandescent lamp. The third curve in the figure, \( V(\lambda) \cdot \Phi_e(\lambda) \), illustrates the amount of radiant power emitted by the tungsten-filament lamp not perceivable by the human eye, leading to excessive energy consumption in electrical lighting when employing incandescent lighting products.
Figure 2.1. Normalised spectral sensitivity model of the human eye, $V(\lambda)$, as defined by the CIE. The colour of the line corresponds to the perceived colour. The $\Phi_e(\lambda)$ is the spectral radiant flux of an incandescent lamp. The third curve shows the effect of the $V(\lambda)$ weighting on the spectrum of the incandescent lamp.

Table 2.1 shows the equivalence of radiometric and photometric quantities, and their respective units [26]. Radiant flux $\Phi_e$ is the total amount of radiant power (unit W) emitted into a particular direction. In the case of the total radiant flux, the emission geometry is $4\pi$ solid angle. Radiant intensity is the power emission of a point source per unit solid angle (steradian, symbol sr). Irradiance is used to describe the incident radiant flux density. Radiance is the emission by a surface per unit solid angle.

Table 2.1. Quantities and units in radiometry and photometry.

<table>
<thead>
<tr>
<th>Radiometry</th>
<th>Unit</th>
<th>Photometry</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Radiant flux</td>
<td>(\Phi_e) W</td>
<td>Luminous flux</td>
<td>(\Phi_v) lm</td>
</tr>
<tr>
<td>Rad. intensity</td>
<td>(I_e) W/sr</td>
<td>Lum. intensity</td>
<td>(I_v) lm/sr, cd</td>
</tr>
<tr>
<td>Irradiance</td>
<td>(E_e) W/m²</td>
<td>Illuminance</td>
<td>(E_v) lm/m², lux</td>
</tr>
<tr>
<td>Radiance</td>
<td>(L_e) W/sr/m²</td>
<td>Luminance</td>
<td>(L_v) lm/sr/m², cd/m²</td>
</tr>
</tbody>
</table>

Subscripts “\(e\)” and “\(v\)” are often used to distinguish between the radiometric and photometric quantities, respectively. Spectral radiometric quantities are expressed as a function of wavelength \(\lambda\), and their units are per nanometre. In this dissertation, radiometric and photometric symbols without a subscript are used to denote the respective relative quantities. Aside from those that are a function of wavelength, quantities without a subscript are related to photometry.
2.2 Luminous efficacy

The power efficiency of electrical lighting products is expressed in terms of luminous efficacy $\eta_v$ [27], which is the ratio of the total luminous flux $\Phi_v$, produced by the source, to the consumed active electrical power $P$

$$\eta_v = \frac{\Phi_v}{P}. \quad (2.2)$$

One way to determine the total luminous flux $\Phi_v$ of a light source is by measuring its luminous intensity distribution $I_v(\theta, \phi)$, and integrating it over $4\pi$ solid angle according to the equation

$$\Phi_v = \int_0^{2\pi} \int_0^\pi I_v(\theta, \phi) \sin(\theta) \, d\theta \, d\phi, \quad (2.3)$$

where $\theta$ and $\phi$ are the zenith and azimuth angles of the spherical coordinate system, respectively [24, 28]. The $\sin(\theta)$ weighting is employed due to the spherical coordinate system.

Figure 2.2 shows an example of a luminous intensity distribution $I_v(\theta, \phi)$. The luminous intensity $I_v$ in any direction is emitted from a point source residing at the origin. The optical axis of the device under test (DUT) is parallel with the $z$-axis. The shape of the distribution is caused by the interference of the four LED filaments of the DUT, which are parallel with the optical axis of the lamp and enclosed by a clear bulb. The total luminous flux $\Phi_v$ of this distribution, calculated using equation (2.3), would be 708.8 lm. With the active power consumption of 5.7 W, that would yield the luminous efficacy of 124.4 lm/W.

![Figure 2.2](Image)

**Figure 2.2.** Luminous intensity distribution $I_v(\theta, \phi)$ of a clear-bulb LED lamp with four filaments. The optical axis of the lamp is parallel with the $z$-axis.

Luminous intensity distributions are traditionally measured using goniophotometers [28, 29]. Figure 2.3 shows the principle of goniometric measurements.
To determine the luminous intensity distribution $I_v(\theta, \phi)$ of the DUT, the luminous intensity $I_v$ of the DUT needs to be measured from all the angles into which the light is emitted using an absolutely calibrated detector residing at a known distance from the DUT. Luminous intensity measurements from different directions can be achieved by rotating the DUT, the detector, or both.

![Figure 2.3. Principle of goniophotometric measurements. The spot-type lamp is rotated about two of its axes to measure all the angles of luminous intensity emission. The aperture array prevents stray light from reaching the detector.](image)

In order to be able to approximate the DUT as a point source, the goniometric measurements need to be carried out in far-field conditions. To satisfy this requirement, the distance between the DUT and the detector needs to be at least five times as large as the largest dimension of the light-emitting surface of the DUT [17, 28]. Figure 2.4 shows a far-field mirror goniophotometer, which combines the rotation of the mirror and the DUT to achieve $4\pi$ measurement geometry. The detector of such an instrument can reside at the distance of tens of meters from the DUT, enabling far-field-condition measurements of large sources, such as street lamps.

![Figure 2.4. Far-field-mirror goniophotometer with a spot lamp under test. The detector of such an instrument can reside at the distance of tens of meters from the lamp under test.](image)
Goniometric measurements are resource intensive, and at high angular resolution also time-consuming, as measuring one DUT from all directions can take several hours. An alternative method for measuring the total luminous flux $\Phi_v$, based on an integrating sphere photometer, is discussed in Section 3.
3. Integrating sphere photometry

3.1 Integrating sphere

Ideally, an integrating sphere is a hollow, empty, and spatially uniform spherical construction, which is built from, or coated with, diffusely reflective and spectrally non-selective material. The operating principle of the integrating sphere is that the total luminous flux $\Phi_v$ produced by the source is proportional to illuminance $E_v$ on any area of the inner-surface of the sphere, not directly illuminated by the light source. In an ideal sphere, the total luminous flux can be obtained using the equation

$$\Phi_v = E_v \cdot \frac{1-\rho}{\rho} \cdot 4\pi R_s^2,$$  (3.1)

where $\rho$ is the reflectance factor of the sphere and $R_s$ is the radius of the sphere [24]. Thus, in theory, by measuring the illuminance at the sphere wall, the total luminous flux can be obtained in one measurement, significantly faster compared with the goniophotometric approach described in the previous chapter.

Figure 3.1 shows a cross-section of an integrating sphere. On the left, there is a photometer, installed into the detector port of the sphere, and the detector port baffle, which prevents direct light from the source reaching the detector. In the figure, the DUT is installed in the centre of the sphere, allowing measurements in $4\pi$ geometry. The lamp holder can also be mounted to the bottom of the sphere when measuring lighting products operated in the base-down orientation, reducing the measurement uncertainty due to the heat distribution within the DUT [17]. On the right is the auxiliary lamp and its baffle. The purposes of the auxiliary lamp and the external-source port, on the left of the DUT, are explained in Sections 3.4 and 3.5, respectively.
Integrating sphere photometers are typically calibrated using a luminous flux standard lamp, with known total luminous flux output $\Phi_{v,\text{std}}$, installed in place of the DUT. The total luminous flux $\Phi_{v,\text{DUT}}$ of the DUT is then obtained from the ratio of the photometer signals $i_{\text{DUT}}$ and $i_{\text{std}}$ produced respectively by the illuminance of the DUT and the standard lamp

$$\Phi_{v,\text{DUT}} = \frac{i_{\text{DUT}}}{i_{\text{std}}} \cdot \Phi_{v,\text{std}} \quad (3.2)$$

The total luminous flux $\Phi_{v,\text{DUT}}$ given by equation (3.2) is accurate if the DUT and the luminous flux standard lamp have the same

- relative angular intensity distribution,
- relative spectral power distribution, and
- dimensions and material. [24]

If the aforementioned properties deviate between the lamps, the measurement uncertainty of the total luminous flux is increased. The coefficient $c_v$ in the equation

$$\Phi_{v,\text{DUT}} = c_v \cdot \frac{i_{\text{DUT}}}{i_{\text{std}}} \cdot \Phi_{v,\text{std}} \quad (3.3)$$

takes into account the non-ideal characteristics of the measurement setup that would otherwise have to be included in the uncertainty budget of the luminous
flux measurement. The correction coefficient $c_v$ consists of

$$c_v = k_s \alpha_v F,$$  \hspace{1cm} (3.4)

where $k_s$ is the spatial non-uniformity correction factor, $F$ is the spectral mismatch correction factor, and $\alpha_v$ is the correction factor for self-absorption [24]. Sections 3.2–3.4 discuss in detail the correction factors that constitute the coefficient $c_v$.

### 3.2 Spatial non-uniformity correction

Integrating spheres are not spatially perfectly uniform [30–35]. Factors such as structural elements of the sphere, uneven coating of the inner surface, and possible contamination cause non-uniform spatial responsivity. Figure 3.2 shows the spatial responsivity distribution function (SRDF), or $K(\theta, \phi)$, of the integrating sphere illustrated in Figure 3.1. The azimuth angle $\phi = 0^\circ$ corresponds to the sphere wall with the auxiliary port, and the angles $\theta = 0^\circ$ and $\theta = 180^\circ$ correspond to the top and the bottom of the sphere, respectively.

The SRDF of an integrating sphere can be measured by scanning the surface of the sphere with a spot-type source, while recording the output of the sphere detector [33,36]. The 1.65-m sphere scanned for Figure 3.2 is coated with barium sulphate ($\text{BaSO}_4$) with 98% nominal reflectance factor $\rho$. The lowered reflectivity of the bottom half of the sphere, due to dust-particle contamination, and certain structural elements of the sphere in Figure 3.1 are distinctly visible in the SRDF of Figure 3.2:

- auxiliary port at $\{\theta = 90^\circ, \phi = 0^\circ\}$,
- detector port at $\{\theta = 90^\circ, \phi = \pm 180^\circ\}$,
Integrating sphere photometry

- external-source port at \( \theta = 90^\circ, \phi = -135^\circ \),
- lamp holder at \( \theta = 0^\circ \),
- base-down lamp-holder socket at \( \theta = 170^\circ, \phi = \pm 180^\circ \), and
- seam of the sphere at \( \phi = \pm 90^\circ \).

Spatial responsivity characteristics of different integrating spheres vary. Figure 2 of Publication II shows the SRDFs of eight integrating spheres with various reflectance factors \( \rho \), diameters, and sphere port/baffle configurations.

The spatial non-uniformity correction takes into account the mismatch between the relative angular intensity distribution of the DUT \( I_{DUT}(\theta, \phi) \) and that of the luminous flux standard lamp \( I_{std}(\theta, \phi) \) employed to calibrate the integrating sphere photometer [31, 32]. Depending on the spatial uniformity of the sphere, and the angular distributions of the DUT and the standard lamp, the spatial correction can be up to a few percent. The spatial correction factor \( k_s \) can be calculated using the equation

\[
k_s = \frac{\int_\phi \int_\theta K(\theta, \phi) I_{DUT}(\theta, \phi) \sin(\theta) \, d\theta \, d\phi}{\int_\phi \int_\theta K(\theta, \phi) I_{DUT}(\theta, \phi) \sin(\theta) \, d\theta \, d\phi} \cdot \frac{\int_\phi \int_\theta I_{DUT}(\theta, \phi) \sin(\theta) \, d\theta \, d\phi}{\int_\phi \int_\theta I_{std}(\theta, \phi) \sin(\theta) \, d\theta \, d\phi},
\]

where \( \theta = 0 \ldots 180^\circ \) and \( \phi = 0 \ldots 360^\circ \). If the integrating sphere were perfectly uniform, i.e. \( K(\theta, \phi) \) constant, the equation would yield \( k_s = 1 \), making the correction unnecessary. That would also be the case if the luminous intensity distributions of the sources were proportional \( I_{\nu,DUT}(\theta, \phi) \propto I_{\nu,std}(\theta, \phi) \) to each other.

Because the angular intensity distribution of the DUT needs to be determined in order to calculate the spatial correction factor, out of the three correction factors of equation (3.4), the spatial correction has traditionally been the most time-consuming one to apply. The fisheye camera method, introduced in Publication I and validated in Publication II, enables to quickly determine the spatial non-uniformity correction factors in luminous flux measurements with integrating spheres. The method is based on determining the relative angular intensity distribution of the DUT using a fisheye-lens camera installed into a port of the integrating sphere.

Figure 3.3 shows a relative angular intensity distribution \( I(\theta, \phi) \) measured using the fisheye camera method. The source of the distribution is the same four-filament lamp whose goniophotometrically measured luminous intensity distribution \( I_\nu(\theta, \phi) \) is presented in Figure 2.2. According to the closeness score discussed in Publication II (equation 2), the similarity of these two angular data sets obtained using the two methods is 95.8 out of 100. Relative angular intensity distributions can also be used together with the total luminous flux \( \Phi_\nu \) of the light source to calculate the respective absolute luminous intensity distribution \( I_\nu(\theta, \phi) \).
3.3 **Spectral mismatch correction**

The spectral responsivities of photometers deviate from the ideal $V(\lambda)$ function. The general $V(\lambda)$ mismatch index $f'_1$ describes how closely the relative spectral responsivity $s_{\text{rel}}(\lambda)$ of the photometer replicates $V(\lambda)$. The $f'_1$ classification of the photometer is defined [23] as

$$f'_1 = \frac{\int_{380\text{nm}}^{780\text{nm}} s^*_{\text{rel}}(\lambda) - V(\lambda) \, d\lambda}{\int_{380\text{nm}}^{780\text{nm}} V(\lambda) \, d\lambda},$$

(3.6)

where $s^*_{\text{rel}}(\lambda)$ is the relative spectral responsivity of the detector normalised according to

$$s^*_{\text{rel}}(\lambda) = s_{\text{rel}}(\lambda) \cdot \frac{\int_{380\text{nm}}^{780\text{nm}} S_A(\lambda) V(\lambda) \, d\lambda}{\int_{380\text{nm}}^{780\text{nm}} S_A(\lambda) s_{\text{rel}}(\lambda) \, d\lambda}. \quad (3.7)$$

The $S_A(\lambda)$ function in equation (3.7) is the spectral power distribution (SPD) of CIE Standard Illuminant A [37], which is the spectrum of a tungsten-filament lamp with the correlated colour temperature (CCT) of 2856 K. The SPD of Standard Illuminant A is included in the normalisation to take into account that photometers have traditionally been calibrated using incandescent lamps set to the CCT of Standard Illuminant A.

Figure 3.4 shows the SPDs of CIE Standard Illuminant A and a blue-pumped white LED with CCT of 4103 K. The peak at approximately 450 nm is the...
emission of the blue LED element, and the broad shape is the light emitted by the phosphor coating. The figure illustrates the discrepancy between a typical LED SPD and Standard Illuminant A, widely used for calibrating photometers.

Figure 3.4. Spectral power distributions of CIE Standard Illuminant A and a white LED of a 4103-K correlated colour temperature.

Figure 3.5 shows the spectral responsivity range of a group of 107 actual photometers. The range shows that especially in the short wavelength region, the responsivities of photometers tend to deviate from the $V(\lambda)$ function. The general $V(\lambda)$ mismatch indices $f'_{1}$ of these photometers range from 0.5% to 8.8%, with the average being 3.2%.

Figure 3.5. $V(\lambda)$ function and the relative spectral responsivity $s_{rel}(\lambda)$ range of 107 photometers.
The spectral responsivity data in Figure 3.5 are the same as used in the spectral mismatch study presented in Publication III. The data set includes the responsivities of devices ranging from handheld illuminance meters to laboratory grade photometer heads.

Because the relative spectral responsivities $s_{\text{rel}}(\lambda)$ of actual photometers do deviate from the ideal $V(\lambda)$ function, the differences in the SPDs of the DUT $S_{\text{DUT}}(\lambda)$ and the photometer calibration source $S_{\text{cal}}(\lambda)$ can cause a spectral mismatch error in the measured photometric quantity. In the case of an integrating sphere photometer, the relative spectral throughput $T_{\text{rel}}(\lambda)$ of the integrating sphere contributes to the spectral responsivity of the system [16, 17, 24]. The spectral mismatch error can be corrected using the spectral mismatch correction factor [16, 23]

$$F = \frac{\int S_{\text{DUT}}(\lambda)V(\lambda) \, d\lambda}{\int S_{\text{cal}}(\lambda)V(\lambda) \, d\lambda} \cdot \frac{\int S_{\text{cal}}(\lambda)s_{\text{rel}}(\lambda)T_{\text{rel}}(\lambda) \, d\lambda}{\int S_{\text{DUT}}(\lambda)s_{\text{rel}}(\lambda)T_{\text{rel}}(\lambda) \, d\lambda}. \quad (3.8)$$

As can be seen from the equation, if $S_{\text{DUT}}(\lambda) = S_{\text{cal}}(\lambda)$, or if $s_{\text{rel}}(\lambda)T_{\text{rel}}(\lambda) = V(\lambda)$, the correction factor is 1, and thus the correction is not required.

If the spectral mismatch correction is not applied, the spectral mismatch needs to be included in the uncertainty budget of the measurement. If the correction is omitted when measuring LED lighting using a photometer calibrated with a Standard Illuminant A spectrum source, it is recommended that the general $V(\lambda)$ mismatch index $f'_1$ of the total relative spectral responsivity of the system should not exceed 3.0% [38].

Employing an LED source to calibrate photometers for measuring LED lighting can significantly reduce measurement uncertainty due to the spectral mismatch [6]. In order to decrease this uncertainty when measuring LED lighting, Publication III proposes an LED-based reference spectrum for calibrating photometers. In that study, by employing the LED-based reference spectrum in LED measurements, the average spectral mismatch errors were reduced by a factor of two, when compared with using Standard Illuminant A as the calibration source of photometers. The proposed reference spectrum is the white LED SPD shown Figure 3.4.

### 3.4 Self-absorption correction

When calibrating an integrating sphere with a luminous flux standard lamp, or when measuring a DUT, some part of the emitted luminous flux $\Phi_\nu$ is absorbed by the source itself. The amount of absorption depends on the shape, size, and material of the device. The error in the measurement due to this self-absorption can be taken into account using the self-absorption correction [17, 24].

To obtain the self-absorption correction factor $\alpha_\nu$ for the DUT, the auxiliary lamp of the integrating sphere is employed. An example of an auxiliary lamp
Integrating sphere photometry configuration is shown on the right-hand side in Figure 3.1. When determining the correction factor, the photometer signal $i_{\text{aux,std}}$ is recorded with the auxiliary lamp turned on while the turned-off standard lamp is installed in the integrating sphere. Then the same procedure is repeated for the DUT to obtain the photometer signal $i_{\text{aux,DUT}}$. The resulting self-absorption correction factor is given by the ratio

$$\alpha_v = \frac{i_{\text{aux,std}}}{i_{\text{aux,DUT}}}.$$ (3.9)

The self-absorption correction also takes into account a possible change of the lamp holder, or any other changes in the mounting system, as long as the near-field absorption effect in the measurement system does not change. The near-field absorption constitutes the portion of the flux getting absorbed into the lamp holder without reflecting first off the integrating sphere surface. To minimise the near-field absorption, the source must be kept as far as possible from other objects inside the sphere. [39]

### 3.5 Absolute integrating sphere method

In the reference-lamp calibration method, described in Section 3.1, the photometer signal $i_{\text{DUT}}$ produced by the DUT is compared with the photometer signal $i_{\text{std}}$ produced by the luminous flux standard lamp. This makes the integrating sphere photometer measurements susceptible to any changes in the standard lamp output. In the absolute integrating sphere method [40], instead of employing a luminous flux standard lamp, the calibration of the sphere is performed by introducing a known amount of reference flux $\Phi_v,\text{ref}$ into the sphere through the external-source port of the sphere [31, 40–44]. The total luminous flux of the DUT is obtained by comparing the photometer signal produced by the DUT to that of the external reference source.

Figure 3.6 shows a schematic of the absolute integrating sphere method measurement system [20] based on the integrating sphere shown in Figure 3.1. The reference flux $\Phi_v,\text{ref}$ is determined by measuring the illuminance $E_v$ over the precision aperture at the end of the aperture array using a standard photometer. Then, by moving the standard photometer out of the way, the reference flux introduced into the sphere equals to

$$\Phi_v,\text{ref} = E_v A,$$ (3.10)

where $A$ is the area of the precision aperture. All the elements in Figure 3.6 reside at the equatorial plane ($\theta = 90^\circ$) of the integrating sphere. Azimuth angles $\phi$ in the figure correspond to those of the spatial responsivity map shown in Figure 3.2.
For an externally calibrated integrating sphere photometer, the correction coefficient \( c_v \) of equation (3.3) requires adjustments to take into account the properties the external-source calibration. The correction coefficient can be calculated as

\[
c_v = \frac{k_s k_a \alpha_v F_{DUT}}{\beta F_{ref}}, \tag{3.11}
\]

where \( k_a \) is the correction factor for the non-uniform illuminance at the precision aperture due to the distance difference between the centre and the edges of the aperture to the reference source. The \( \beta \) factor is the correction for the non-perpendicular incident angle between the reference flux and the sphere surface. The spectral mismatch correction factors have to be determined separately for the DUT and the sphere detector \( F_{DUT} \), and the reference source and the standard photometer \( F_{ref} \). [20, 40, 43]

For the absolute integrating sphere method, the spatial non-uniformity correction factor \( k_s \) of equation (3.5) converges to

\[
k_s = \frac{\int_{\phi} \int_{\theta} K(\theta_{ext}, \phi_{ext}) I_{DUT}(\theta, \phi) \sin(\theta) \, d\theta \, d\phi}{\int_{\phi} \int_{\theta} K(\theta, \phi) I_{DUT}(\theta, \phi) \sin(\theta) \, d\theta \, d\phi}, \tag{3.12}
\]

where \( K(\theta_{ext}, \phi_{ext}) \) is the spatial responsivity of the area on the sphere wall where the reference flux \( \Phi_{v,ref} \) has its first reflection. In the SRDF shown in
Integrating sphere photometry

Figure 3.2, that area of the primary reflection is at $\{\theta = 90^\circ, \phi = -45^\circ\}$ with 0.15\% higher reflectivity compared with the average reflectivity of the sphere.

Self-absorption correction factor $\alpha_v$ is not required if the integrating sphere photometer is calibrated individually for each DUT, by introducing the reference flux with the turned-off DUT already installed inside the sphere. Although, to prolong the lifetime of the reference lamp, the sphere photometer is usually calibrated once with an empty sphere, and then the self-absorption correction is determined using the auxiliary lamp. The principle is the same as for the self-absorption correction when the integrating sphere has been calibrated internally (Section 3.4), but instead of the auxiliary measurement for the luminous flux standard lamp, $i_{aux,\text{std}}$ in equation (3.9), the respective auxiliary measurement is performed with the integrating sphere empty. Thus, the equation

$$\alpha_v = \frac{i_{aux,\text{empty}}}{i_{aux,DUT}}$$  (3.13)

gives the self-absorption correction factor for the absolute integrating sphere method.
4. Imaging and fisheye camera model

4.1 Imaging coordinate systems

Imaging is the coordinate-system transformation of three-dimensional world-coordinate points to the respective image-plane or pixel coordinates of the camera. When forming a digital image of the world object, the coordinate system transformation for each point is carried out in the order $wP \rightarrow cP \rightarrow p \rightarrow p_i$. Table 4.1 shows the point notation for each coordinate system.

**Table 4.1.** Coordinate systems used in geometric image formation.

<table>
<thead>
<tr>
<th>Coordinate system</th>
<th>Point notation</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>World coordinates</td>
<td>$wP$</td>
<td>$wX, wY, wZ$</td>
</tr>
<tr>
<td>Camera coordinates</td>
<td>$cP$</td>
<td>$X, Y, Z$</td>
</tr>
<tr>
<td>Image plane</td>
<td>$p$</td>
<td>$x, y$</td>
</tr>
<tr>
<td>Pixel coordinates</td>
<td>$p_i$</td>
<td>$x_i, y_i$</td>
</tr>
</tbody>
</table>

Figure 4.1 illustrates the relation of the four coordinate systems presented in Table 4.1, when the camera is placed at the detector port of the integrating sphere shown in Figure 3.1, and the origin of the world-coordinate system is set at the centre of the sphere. Additionally, the figure shows the spherical left-hand coordinates corresponding to the SRDF shown in Figure 3.2. The subscripts “0” stand for the origin. The orange square in the figure depicts the camera sensor. The origin of the pixel-coordinate system is in the top left corner of the image, which corresponds to the opposite corner of the camera sensor, because camera-coordinate points get mirrored about the optical axis of the camera when getting projected onto the image plane.
4.2 Rigid transformation

World-coordinate system point \( W^P \) can be transformed to the respective camera-coordinate point \( C^P \) by multiplying it with rotation matrix \( R \), and adding to the product translation vector \( t \) [45]

\[
C^P = R W^P + t = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} wX \\ wY \\ wZ \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}. \tag{4.1}
\]

The translation vector is the world-coordinate system origin \( W^P_0 \) in the camera-coordinate system. For the orthogonal geometry of the camera and the world coordinate systems shown in Figure 4.1, the transformation from the world-coordinate system to the camera coordinates is given by the equation

\[
C^P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} wX \\ wY \\ wZ \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ R_s \end{bmatrix}, \tag{4.2}
\]

where \( R_s \) is the radius of the sphere.

The rotation and translation can be combined into a single multiplication by homogeneous transformation matrix \( T \), consisting of rotation matrix \( R \) and translation vector \( t \). The complete world-to-camera coordinate transformation

\[
V^P = T \begin{bmatrix} wX \\ wY \\ wZ \end{bmatrix}.\tag{4.3}
\]
can thus be expressed as
\[
\begin{pmatrix}
X \\
Y \\
Z \\
1
\end{pmatrix} = \begin{bmatrix}
R_{11} & R_{12} & R_{13} & t_1 \\
R_{21} & R_{22} & R_{23} & t_2 \\
R_{31} & R_{32} & R_{33} & t_3 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{pmatrix}
X' \\
Y' \\
Z' \\
1
\end{pmatrix} = \begin{bmatrix}
R \\
T
\end{bmatrix} \begin{pmatrix}
W X' \\
W Y' \\
W Z' \\
1
\end{pmatrix} = \begin{bmatrix}
R & t \\
0 & 1
\end{bmatrix} \begin{pmatrix}
W P \\
1
\end{pmatrix}.
\] (4.3)

If the camera is not on the equatorial plane of the sphere, or if the camera is rotated about its optical axis, the orthogonal rotation matrix of equation (4.2) needs to be adjusted to take into account the pitch $\alpha_c$ and the roll $\gamma_c$ of the camera, according to
\[
R = \begin{bmatrix}
\cos \gamma_c & -\sin \gamma_c & 0 \\
\sin \gamma_c & \cos \gamma_c & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha_c & -\sin \alpha_c \\
0 & \sin \alpha_c & \cos \alpha_c
\end{bmatrix} \begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}.
\] (4.4)

### 4.3 Perspective projection

Figure 4.2 shows the pinhole camera model, which represents the simplest perspective projection of three-dimensional camera-coordinate point $^C P$ to two-dimensional image-plane coordinate point $p$. The pinhole camera model consists of a distortion-free camera with a small aperture as the centre of projection [46]. A projection is distortion-free when the angle $\theta_{oa}$ between the optical axis and the ray is the same on both sides of the centre of projection. In the figure, $f$ is the focal length of the camera, and $r$ is the distance of point $p$ to the principal point $p_0 = (x_0, y_0)$ of the image plane. Principal point is the point where the optical axis intersects with the image plane.

![Figure 4.2. The pinhole camera model. The imaged object gets projected onto the image plane without any distortions.](chart.png)

The projection distance $r$ is given by the equation
\[
r = f \tan(\theta_{oa}),
\] (4.5)

43
or the equation $r = \sqrt{x^2 + y^2}$. Because of the tangential dependency on the incident angle $\theta_{oa}$, the projection distance rapidly increases on large incident angles.

The $x$ and $y$ coordinates of the projected point in the image-plane coordinate system are given by the equations

$$x = f \frac{X}{Z} \quad \text{and} \quad y = -f \frac{Y}{Z}.$$  

(4.6) 

(4.7)

The minus sign for $y$-coordinate in equation (4.7) is due to the mirroring about the optical axis. The point is also mirrored about the optical axis in the $x$ direction, but due to the opposite directions of the $x$ axes in the camera and the image-plane coordinate systems, the sign remains unchanged.

The respective pixel-coordinate point $p_i = (x_i, y_i)$ can be obtained from the image-plane coordinate point $p = (x, y)$ using the equations

$$x_i = \frac{x - x_0}{s_x} + o_x \quad \text{and} \quad y_i = \frac{y - y_0}{s_y} + o_y,$$  

(4.8) 

(4.9)

where $s_x$ and $s_y$ are the pixel dimensions along the $x$ and $y$ axes, respectively, and $(o_x, o_y)$ is the principal point $p_0$ in the pixel-coordinate system.

All the camera parameters required for calculating the distortion-free perspective projection can also be expressed using the intrinsic parameter matrix

$$K = \begin{bmatrix} f/s_x & \gamma & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix},$$  

(4.10)

where $\gamma$ is the skew coefficient, which deviates from zero if the pixel rows and columns of the sensor are not perpendicular to each other. For modern cameras the assumptions of zero skew and unit aspect ratio of the pixel dimensions are well justified [45]. Intrinsic parameters of a camera can be obtained using various geometric camera calibration, or resectioning, routines [46–51]. For instance, geometric calibration can be performed by capturing a series of images of a known calibration pattern from different positions, and studying the correspondence of the features in the captured images.

Using the intrinsic parameter matrix, the projection of three-dimensional point $^C\mathbf{P}$ to pixel-coordinate point $p_i$, described by equations (4.6)–(4.9), can be combined into a single affine transformation according to the equation

$$Z \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} = \begin{bmatrix} f/s_x & \gamma & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = K^C\mathbf{P}.$$  

(4.11)
The $Z$ component is not preserved in the projected image.

The rigid transformation from the world-coordinate system into the camera-coordinate system, and the subsequent projection to the image plane with the conversion to the pixel coordinates can also be expressed using projection matrix $M$. Projection matrix $M$, also called the camera matrix, is a $3 \times 4$ matrix which combines the intrinsic, rotation, and translation matrices, enabling a single matrix multiplication to obtain pixel coordinates $(x_i, y_i)$ for the respective world-coordinate point $wP$ according to the equation [52]

$$\begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \propto K \begin{bmatrix} R \\ t \end{bmatrix} \begin{bmatrix} wX \\ wY \\ wZ \\ 1 \end{bmatrix} = M \begin{bmatrix} wP \\ 1 \end{bmatrix}. \quad (4.12)$$

### 4.4 Fisheye lens distortion

In the case of a camera with a lens system which affects the angle of the incoming rays, the projection of point $CP$ to the image plane can be calculated by employing an additional operation that gives the distorted distance $r_d$ of the projected point $p$ to the principal point $p_0$. Figure 4.3 shows a camera model where the incident rays are bent towards the optical axis, enabling a larger field of view for the same size image sensor when compared with the pinhole camera model. The larger the angle of incidence $\theta_{oa}$, the more severe is the distortion of the projected image. The subscripts “u” in the figure denote the undistorted, or rectilinear, projection distances $r_u$, equivalent to the pinhole camera model.

Figure 4.3. Camera model with lens distortion. Incident light rays get projected onto smaller distance from the principal point than in the pinhole camera model.

Fisheye-lens cameras present an extreme example of imaging systems with large a field of view [53]. Figure 4.4 shows four common projection functions, or mapping functions, used in fisheye lenses, and the rectilinear projection of
equation (4.5) for comparison [54]. The figure shows, for example, that to capture an image of a 120° field of view along one dimension ($\theta_{oa} = \pm 60^\circ$) using a pinhole camera, the respective dimension of the sensor must be no less than $3.46 \cdot f$.

![Figure 4.4](image)

**Figure 4.4.** Projection distances to the principal point of four common projection functions used in fisheye lenses, and the rectilinear projection for comparison.

One of the most common fisheye lens mapping functions is the equidistant projection function. In the equidistant projection, the distorted projection distance $r_d$ is directly proportional to the incident angle $\theta_{oa}$

$$r_d = f \theta_{oa}.$$  \hspace{1cm} (4.13)

The equidistant mapping function preserves the relative distances between the points along the radial lines joining at the principal point of the image.

Another widely used projection function equisolid, or equal-area, projection maintains the ratios of the areas in the projection. The mapping function of the equisolid projection is

$$r_d = 2f \sin \left( \frac{\theta_{oa}}{2} \right).$$  \hspace{1cm} (4.14)

Stereographic mapping function is designed to preserve the angles in the obtained projection. The stereographic projection is given by the equation

$$r_d = 2f \tan \left( \frac{\theta_{oa}}{2} \right).$$  \hspace{1cm} (4.15)

In orthographic projection, incident rays are deflected to be parallel with the optical axis according to the equation

$$r_d = f \sin(\theta_{oa}).$$  \hspace{1cm} (4.16)

From the equation, and Figure 4.4, can be seen that the maximum field of view for the orthographic mapping function is 180°, as any point from the incident
angle $\theta_{oa} > 90^\circ$ would get projected onto the same point in image plane as the point coming from $180^\circ - \theta_{oa}$.

In addition to the mapping functions of equations (4.13)–(4.16), which fisheye lenses are often designed to conform to, the lens projection can be expressed as the polynomial equation [55]

$$\theta'_{oa} = \theta_{oa} \left( 1 + \kappa_1 \theta_{oa}^2 + \kappa_2 \theta_{oa}^4 + \kappa_3 \theta_{oa}^6 + \kappa_4 \theta_{oa}^8 + \ldots \right), \quad (4.17)$$

where $\kappa_1, \kappa_2, \kappa_3$, and $\kappa_4$ are the distortion coefficients. Typically, the distortion coefficients can be obtained using the same geometric camera calibration routines as for intrinsic parameters matrix $K$.

### 4.5 Undistortion of images

In order to reconstruct the three-dimensional scene from an image by back projecting the image points, the undistorted pixel coordinates need to be calculated to be able to apply inverse linear transformation. For the equidistant projection, undistorted distance $r_u$ from principal point $p_0$ to image point $p$ can be calculated by rearranging equation (4.13) to obtain the undistorted distance equation

$$r_d = f \theta_{oa} = f \tan^{-1} \left( \frac{r_u}{f} \right) \Leftrightarrow r_u = f \tan \left( \frac{r_d}{f} \right). \quad (4.18)$$

Table 4.2 shows the respective undistortion functions for the four fisheye-lens projection functions presented in Figure 4.4 and equations (4.13)–(4.16) [56].

**Table 4.2.** Projection functions and the respective undistortion functions of the four fisheye-lens mapping functions shown in Figure 4.4.

<table>
<thead>
<tr>
<th>Projection function</th>
<th>Projected distance</th>
<th>Undistorted distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equidistant</td>
<td>$r_d = f \theta_{oa}$</td>
<td>$r_u = f \tan \left( \frac{r_d}{f} \right)$</td>
</tr>
<tr>
<td>Equisolid</td>
<td>$r_d = 2f \sin \left( \frac{\theta_{oa}}{2} \right)$</td>
<td>$r_u = f \tan \left( 2 \sin^{-1} \left( \frac{r_d}{2f} \right) \right)$</td>
</tr>
<tr>
<td>Stereographic</td>
<td>$r_d = 2f \tan \left( \frac{\theta_{oa}}{2} \right)$</td>
<td>$r_u = f \tan \left( 2 \tan^{-1} \left( \frac{r_d}{2f} \right) \right)$</td>
</tr>
<tr>
<td>Orthographic</td>
<td>$r_d = f \sin (\theta_{oa})$</td>
<td>$r_u = r_d \left( 1 - \frac{r_d^2}{f^2} \right)^{-1/2}$</td>
</tr>
</tbody>
</table>

In the case of polynomial equation (4.17), the original incident angle $\theta_{oa}$ for every point can be calculated using the equation

$$\theta_{oa} = \theta'_{oa} \left( 1 + \kappa'_1 \theta'_{oa}^2 + \kappa'_2 \theta'_{oa}^4 + \kappa'_3 \theta'_{oa}^6 + \kappa'_4 \theta'_{oa}^8 + \ldots \right), \quad (4.19)$$
where $\kappa_1', \kappa_2', \kappa_3'$, and $\kappa_4'$ are the undistortion coefficients. The first four undistortion coefficients, which are typically sufficient for a fisheye camera lens projection, can be calculated from the respective lens distortion coefficients according to the following group of equations [55]

$$
\kappa_1' = -\kappa_1, \quad (4.20)
$$

$$
\kappa_2' = 3\kappa_1^2 - \kappa_2, \quad (4.21)
$$

$$
\kappa_3' = 8\kappa_1\kappa_2 - 12\kappa_1^3 - \kappa_3, \quad \text{and} \quad (4.22)
$$

$$
\kappa_4' = 55\kappa_4^4 + 10\kappa_1\kappa_3 - 55\kappa_1^2\kappa_2 + 5\kappa_2^2 - \kappa_4. \quad (4.23)
$$

The $x$ and $y$ coordinates of the undistorted distance to the principal point can be calculated using the equations

$$
x_u = \frac{r_u}{r_d} x_d \quad \text{and} \quad (4.24)
$$

$$
y_u = \frac{r_u}{r_d} y_d. \quad (4.25)
$$

From these undistorted image-plane coordinates, the respective undistorted pixel coordinates $x_{ui}$ and $y_{ui}$ can be calculated using equations (4.8) and (4.9).

### 4.6 Back-projection

The back-projection can be used to determine the original world-coordinate points corresponding to the points in the image. In order to back-project the image points to their original three-dimensional coordinates, the coordinate transformation is carried out in the opposite direction than in the image formation: $p_i \Rightarrow p \Rightarrow ^cP \Rightarrow ^wP$. Additionally, for the back-projection, some external information about the $Z$-coordinates is required, because the depth information was lost when projecting points from the camera-coordinate system to the image plane.

Using inverse intrinsic parameters matrix $K^{-1}$, the back-projection of undistorted pixel-coordinates point $p_{ui} = (x_{ui}, y_{ui})$ to three-dimensional point $^cP$ in camera-coordinate system can be calculated using the equation

$$
^cP = \begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = K^{-1} Z \begin{pmatrix}
x_{ui} \\
y_{ui} \\
1
\end{pmatrix}. \quad (4.26)
$$

The depth coordinate $Z$ for the point needs to be calculated from the geometry of the imaged scene. For instance in Publications I and II, where a fisheye-lens camera is used to image the inner surfaces of integrating spheres, $Z$-coordinates were calculated from the sphere geometry using the radius $R_s$ of each sphere according to the equation

$$
Z = 2R_s \cos^2(\theta_{oa}). \quad (4.27)
$$
The original world-coordinate point \( \mathbf{wP} \) is then given by the matrix multiplication of the inverse transformation matrix \( T^{-1} \) and camera-coordinate point \( \mathbf{wP} \):

\[
\begin{pmatrix}
\mathbf{wP} \\
1
\end{pmatrix} = T^{-1} \begin{pmatrix}
\mathbf{cP} \\
1
\end{pmatrix}.
\] (4.28)

Figure 4.5 shows a fisheye camera photograph of the inner surface of the integrating sphere shown in Figure 3.1. The detector port baffle is faintly visible in the centre of the image. Behind the baffle is a spot-type LED lamp illuminating the bottom of the sphere. The rest of the integrating sphere is illuminated by the diffuse light reflected from the sphere walls. On the left is the port for an external reference source for the absolute integrating sphere method. The image was captured using a fisheye lens camera with a focal length \( f \) of 1.98 mm and the equisolid projection function.

![Figure 4.5. A photograph captured using a fisheye camera installed into the detector port of the integrating sphere illustrated in Figure 3.1. The bottom of the sphere is illuminated by a spot-type lamp.](image)

Figure 4.6 shows the sphere reconstructed by back-projecting the fisheye camera photograph shown in Figure 4.5. The bottom lamp holder, the port for an external source, and the seam of the sphere can be easily distinguished from the reconstruction. The area on the left is the port with the camera itself. Because of the absence of the depth data in the image, the \( \mathbf{wZ} \)-coordinates were calculated using equation (4.27). For that reason, the detector port baffle and its holder are projected onto the sphere surface.
4.7 Flat-field correction

In addition to the imaged scene, the pixel intensity values of the image are affected by the responsivity of the camera. For instance, the closer the points are to the detector port in the reconstruction shown in 4.6, i.e. the closer the respective pixels are to the peripheries of the image circle in Figure 4.5, the lower is the average responsivity of the camera hardware in that area. This is mainly due to the light fall-off caused by the camera optics.

The non-uniform spatial responsivity of the imaging hardware can be accounted for by using a flat-field correction [57–61]. In the flat-field correction, the value of each pixel is corrected by taking into account the responsivity of the camera at that point. For a greyscale, or one-channel, image $G$, the correction can be applied using the equation

$$G' = \frac{(G - D_\alpha)_{i,j}}{F_{i,j}},$$

(4.29)

where $G'$ is the corrected image, $D_\alpha$ is its dark signal frame, and $F$ is the flat-field correction matrix. Subscripts $i$ and $j$ are the row and column indices of these matrices, respectively.

In the fisheye camera method, the imperfections of the camera and the structural elements of the sphere are collectively allowed for in the image processing.
stage described by equation (3) of Publication I. In principle, this is equivalent to a system-wide flat-field correction, where, in addition to the camera component $C$, the flat-field matrix $F$ also includes the structural elements of the integrating sphere $S$ and the illuminance distribution $E_v(\theta, \phi)$ produced by the reference lamp on the sphere surface.

To obtain a flat-field correction matrix consisting solely of the camera responsivity $C$, a separate measurement is required. Ideally, the flat-field correction matrix can be obtained by imaging a target of spatially uniform radiance $L_e$ or luminance $L_v$. Because implementing such a target in practice is difficult, the flat-field calibration method developed in Publication IV presents an approach where the flat-field correction matrix is obtained by scanning an opening of an integrating sphere with the camera under test. The procedure is used to synthetically create a uniform radiance source, which fills the entire field of view of the camera.
Photometry has long relied on tungsten-filament lamps for characterising and calibrating measurement instruments. Many properties of solid-state lighting products, such as LEDs, deviate from those of incandescent lamps, increasing the measurement uncertainty of new products. With LEDs superseding energy-saving and tungsten-filament-lamp solutions in lighting, the measurement methods need to be revised as well in order to counter the increased measurement uncertainty.

The power efficiency of electrical lighting is described in terms of luminous efficacy, which is the ratio of the total luminous flux emitted by the source to the consumed active electrical power. The total luminous flux is often measured using integrating sphere photometers. Ideally, such an instrument is a hollow and completely empty sphere with diffuse, spectrally non-selective surface. In reality, integrating spheres have non-ideal characteristics, which need to be taken into account in the measurements.

Compared with incandescent lamps, traditionally employed to calibrate integrating sphere photometers, solid-state lighting products have a wider variety of angular intensity distributions. This increases the measurement uncertainty due to the spatial non-uniformities of integrating spheres. So far, applying spatial corrections has been laborious. Publications I and II present the fisheye camera method for quickly determining spatial non-uniformity corrections when measuring lighting products with integrating sphere photometers.

Another source of increased uncertainty when measuring LEDs is the differences in the spectral power distributions when compared with incandescent lighting. The deviating spectra of the source used to calibrate the photometer and the lamp under test can lead to a spectral mismatch error. If the spectra of both sources and the spectral responsivity of the integrating sphere photometer are known, the error can be corrected. Otherwise, the spectral mismatch contributes to the combined measurement uncertainty of the determined total luminous flux. Compared with the incandescent-based calibration source for photometers, the LED-based reference spectrum developed in Publication III reduced the average spectral mismatch errors by a factor of two when measuring LED products.
Conclusion

In Publication IV, a characterisation method was developed for determining the spatial responsivity of hyperspectral cameras using an opening of an integrating sphere to synthetically create a spatially uniform radiance source. The characterisation method enables applying flat-field corrections to the captured image data. The method is also applicable to other types of cameras, and it is especially advantageous when calibrating imaging systems with a large field of view – for instance fisheye cameras.
References


References


References


Spatial and Spectral Corrections for Integrating Sphere Photometry and Radiometry

Alexander Kokka