Modeling of bioreactors

Elina Nauha
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Abstract

Chemicals and biofuel components can be produced with the help of microorganisms in bioreactors. Microorganisms have very specific demands for optimal growth which makes the design and especially scale up of bioreactors challenging. Bioreactors are multiphase systems with complex hydrodynamics. Mathematical modeling can provide information on the functioning of bioreactors without the need for experimentation.

Computational fluid dynamic (CFD) calculations are combined with a compartmental modeling approach for the calculation of 1) bubble column photobioreactors for the cultivation of algal cells and 2) large aerated stirred tanks for the cultivation of aerobic bacteria. Inclusion of dispersed phase population balances, mass transfer and reaction models makes the compartmental model very comprehensive. Local values of variables such as oxygen concentrations are attained. Simulation of bubble columns and heterogeneous conditions is enhanced with a new model for the calculation of dispersed phase flows that allows for bubble induced flow.

New models for the consideration of light distribution and algal growth kinetics in the compartmental model facilitate the comprehensive simulation of algal growth. The results correspond well with measurements taken from literature and provide new insights to the design and run strategies of photobioreactors.

It is shown that large industrial-sized stirred tank bioreactors operate at heterogeneous conditions due to the high oxygen demand of microorganisms. A new simple model estimates gas holdup and mass transfer rates at these conditions. Furthermore, a large stirred tank bioreactor is modeled at homogeneous and heterogeneous conditions with the compartmental modeling approach. Models accounting for the increase in coalescence at high volume fractions are required, without them the mass transfer results are too optimistic. A new reaction model for the uptake of oxygen enables the calculation of local reactor oxygen mass transfer capability at different hydrodynamic conditions. Dead spaces in the reactor are found and suggestions made for reactor design.

The importance of considering local conditions in bioreactor modeling is shown by comparison to model runs where local differences are neglected. Omission of local detail leads to different growth dynamics in photobioreactors and overestimated mass transfer in stirred tank bioreactors. The study of outside cultivation in photobioreactors shows that the existence of the night, rather than the light extinction due to biomass during the day, decreases the maximum achievable cell density. Therefore, further modeling studies to the feasibility and optimum run strategy of outside cultivation are warranted. The study of large stirred tank bioreactors presents several issues concerning the use of these reactors in large scale. Designs of vessels and mixers can be altered, but eventually, a change to bubble driven systems such as airlift reactors should be considered.

Keywords Bioreactors, computational fluid dynamics, compartmental modeling, mass transfer, scale-up
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### Tiivistelmä


Käytetty mallinmusemenetelmä yhdistää virtauslaskennan (CDF) lohkomallinnukseen. Työssä tutkitaan 1) levän kasvatusta kuplakolonnifotobioreaktorissa ja 2) aerobisesti kasvavan bakterin kasvatusta suuren mittakaavan ilmastetuissa sekoitusseudiöissä. Dispergoidun faasin populatiotaseiden, aineensirron ja reaktiomallien yhdistäminen lohkomallissa mahdollistaa hyvin kattavan mallinnuksen. Tuloksena saadaan paikallisia arvoja, kuten liuennon hapen pitoisuus. Usi dispergoidun faasin virtausmalli, joka kehitettiin laskemaan kuplien aiheuttamia virtauksia, parantaa tuloksia kuplakolonnien sekä heterogeenisten virtausolosuhteiden mallinnuksessa.

Lohkomalliin lisätty valon jakautumista ja levän kasvaa kuvaavat mallit mahdollistavat kattavan leväkasvatuksen mallinnuksen. Tulokset vastaavat hyvin kirjallisuudesta haettuja mittauksia ja antavat uutta näkemystä bioreaktoreiden suunnittelusta ja ajostrategioihin.


Kun verrataan lohkomallin tuloksia tulokiin, joissa paikallinen vaihtelun on jätetty pois, on selvä, että paikallisten olosuhteiden huomionvirta on tärkeää. Mallinnuksen yksinkertaistus johtaa muun muassa erilaiseen kasvunäkymään leväkasvatuksissa ja yliarvoinnit aineensiirtonopeuksiin sekoitusseudiöissä.

Levän ulkokasvatuksen mallintaminen osoittaa, että yllämainitun aikaa rajoittaa levän maksimisulupitoisuutta enemmän kuin solujen aiheuttama varjostus päivällä. Tämän vuoksi ulkokasvatuksen mallintamista ja ajostrategian kehitämistä on syytä jatkaa. Suurten teollisten mittakaavan sekoitusseudiöbioreaktoreiden mallinnus osoittaa, että tämän tyyppin reaktoreiden käyttö suuressu suittaa mittakaavassa on haasteellista. Sääliöiden geometriaa ja sekoitintyyppejä voidaan vaihtaa, mutta lopulta kannattaa harkita reaktorityyppin vaihtoa esimerkiksä ilmanostereaktoriin.

### Avainsanat
- bioreaktori, virtauslaskenta, lohkomallinnus, aineensiirto, skaalaus

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I am grateful for all the people I met along the way. Firstly, Professor Ville Alopaeus, in addition to the expertise and support, was fun to travel with. Whale watching in Canada, scuba diving in Mexico, as well as touring in Poland and the Netherlands are worth remembering.

I want to thank all the coworkers I had a pleasure of hanging out with through the years, Friday salmon was always something to look forwards to. I would love to acknowledge you all but would run out of space. So to name a few: Olli Visuri, my academic big brother, without whom I would never even have started all this. Raisa Vermasvuori saved me from being secretary at all meetings and turned into a great boss. Jama Mohamed Ali, who had to endure my slight perfectionism but was always fun to talk to. Zbyněk Kálal for showing me why people have cowriters on their articles. Susanna Kuitunen, without whom working at the lab would have been a drag.

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From my family, I want to especially mention my mom, who always told me to keep going at it and dad for giving me the chemical engineer genes and of course, my twin sister Elisa, who I could not let be the only doctor in the family. I want to thank my kids for having the sense to be born on the proper due dates, so I could finish sending in my articles. Above all others, Jaakko, thanks for allowing me to waste all our holidays on this for the last years. This year we’re going to have a proper holiday!

Espoo, 3 January 2019
Elina Katrīna Nauha
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List of Abbreviations and Symbols

Abbreviations

- CFD: Computational Fluid Dynamics
- UDF: User-defined function
- UDM: User-defined memory
- NB: total number of compartments
- NP: total number of particle size categories
- PSF: Photosynthetic factory

Symbols

- $A$: area between the compartments or at the liquid surface (m²)
- $a$: specific mass transfer area (m² m⁻³)
- $b$: bubble growth rate (m s⁻¹)
- $c$: concentration (mol m⁻³)
- $D_L$: diffusion rate (m² s⁻¹)
- $d$: bubble diameter (m)
- $d_{comp}$: compartment average distance from the side of column (m)
- $F$: flow rate (m³ s⁻¹)
- $g$: breakage rate (s⁻¹)
- $H$: height of reactor (m)
- $h$: coalescence rate (m³ s⁻¹)
- $I$: light intensity (μE m⁻² s⁻¹)
- $K$: power law consistency index (kg s⁻² m⁻¹)
- $k$: turbulence kinetic energy (J kg⁻¹)
- $k_{angle}$: change quotient for zenith angle (rad s⁻¹)
- $k_L$: mass transfer coefficient (m s⁻¹)
\( l \) light path length (m)
\( m \) mass (kg)
\( \bar{M}_{i_f} \) Term accounting for interfacial forces in momentum equation
\( Me \) Maintenance term of algal growth equation (h\(^{-1}\))
\( N \) gas-liquid mass transfer flux (mol m\(^{-2}\)s\(^{-1}\))
\( n \) power law index (-)
\( n \) amount of substance (mol)
\( P \) collision efficiency (-)
\( p \) pressure (Pa)
\( P/V \) power dissipation per volume (W m\(^{-3}\))
\( Q \) gas feed (m\(^3\) s\(^{-1}\))
\( r \) chemical reaction rate (mol m\(^{-3}\) s\(^{-1}\))
\( T \) diameter of reactor (m)
\( t \) time (s)
\( U \) gas velocity at the compartment boundary (ms\(^{-1}\))
\( U_T \) terminal velocity of bubbles (m s\(^{-1}\))
\( u \) velocity (ms\(^{-1}\))
\( V \) volume of the liquid phase (m\(^3\))
\( v \) particle volume (m\(^3\))
\( v_s \) superficial gas velocity (m s\(^{-1}\))
\( X \) biomass concentration (g L\(^{-1}\))
\( X_{atm} \) air mass (-)
\( x_1 \) fraction of PSF in inactive state (-)
\( x_2 \) fraction of PSF in active state (-)
\( x_3 \) fraction of PSF in inhibited state (-)
\( Y \) particle number density (# m\(^{-3}\))
\( z \) zenith angle of the sun (rad)

**Greek letters**
\( \alpha \) void fraction (-)
\( \alpha \) rate constant of photon utilization to transfer \( x_i \) to \( x_2 \) ((\mu E m^{-2})^{-1})
\( \alpha_g \)  
gas volume fraction (-)

\( \alpha_{\text{max}} \)  
maximum packing limit (-)

\( \beta \)  
rate constant of photon utilization to transfer \( x_2 \) to \( x_3 \) \((\mu \text{E m}^{-2})^{-1}\)

\( \beta(d_n,d_p) \)  
probability that a bubble sized \( d_n \) is formed when a bubble sized \( d_p \) breaks (m\(^{-1}\))

\( \delta \)  
rate constant of transfer \( x_3 \) to \( x_1 \) (s\(^{-1}\))

\( \varepsilon \)  
turbulence dissipation (W kg\(^{-1}\))

\( \varepsilon_{\text{atm}} \)  
extinction coefficient of the atmosphere (-)

\( \varepsilon_{\text{biomass}} \)  
extinction coefficient for biomass (g\(^{-1}\) L m\(^{-1}\))

\( \varepsilon_{\text{water}} \)  
extinction coefficient for water (m\(^{-1}\))

\( \gamma \)  
rate constant of transfer \( x_2 \) to \( x_1 \) (s\(^{-1}\))

\( \gamma \)  
correction term for collision frequency (-)

\( \mu \)  
specific growth rate (h\(^{-1}\))

\( \mu \)  
vviscosity (Pas)

\( \rho \)  
density (kg m\(^{-3}\))

\( \phi \)  
gas holdup

\( \sigma \)  
surface tension (kg s\(^{-2}\))

**Subscripts**

\( \alpha \)  
light intensity outside a reactor

\( a \)  
apparent viscosity

bubbles  
referring to velocity of bubbles

CFD  
referring to CFD results

day  
referring to duration of the day

G  
gas

\( i \)  
index of computational cell or compartment

\( j \)  
index of computational cell or compartment

if  
interfacial forces

L  
liquid

\( n \)  
index of particle size category

\( p \)  
index of particle size category

slip  
bubble slip velocity
*slugs* referring to velocity of larger bubbles, slugs

*solar* referring to light intensity of sun outside atmosphere

*sun* referring to the time since sunrise

*surface* liquid surface of reactor

*trans* transition superficial gas velocity

**Superscripts**

' mass concentration

* equilibrium concentration
This doctoral dissertation consists of a summary and of the following publications which are referred to in the text by their numerals


2. Nauha, Elina K.; Alopaeus, Ville; 2015, Modeling outdoors algal cultivation with compartmental approach, Chemical Engineering Journal, Volume 259, Pages 945-960. ISSN 1385-8947. DOI: 10.1016/j.cej.2014.08.073

3. Nauha, Elina K.; Visuri, Olli; Vermaavori, Raisa; Alopaeus, Ville; 2015, A new simple approach for the scale-up of aerated stirred tanks, Chemical Engineering Research and Design, Volume 95, Pages 150-161. ISSN 0263-8762. DOI: 10.1016/j.cherd.2014.10.015

Author’s Contribution

**Publication 1:** Modeling method for combining fluid dynamics and algal growth in a bubble column photobioreactor

Elina Nauha developed, solved and verified the models, analysed the results and wrote the paper. Ville Alopeaus provided guidance and assisted in writing the paper.

**Publication 2:** Modeling outdoors algal cultivation with compartmental approach

Elina Nauha developed, solved and verified the models, fitted the model parameters, analysed the results and wrote the paper. Ville Alopeaus provided guidance and assisted in writing the paper.

**Publication 3:** A new simple approach for the scale-up of aerated stirred tanks

Elina Nauha developed, solved and verified the models, analysed the results and wrote the paper. Ville Alopeaus, Raisa Vermasvuori and Olli Visuri provided guidance and assisted in writing the paper.

**Publication 4:** Compartmental modeling of large stirred tank bioreactors with high gas volume fractions

Elina Nauha performed the CFD runs, developed and performed the compartmentalization, developed and performed the compartmental model runs, verified the models, analysed the results and wrote the paper. Zbyněk Kálal researched the new coalescence models, implemented them into the compartmental model and assisted in writing the paper. Jama Mohamed Ali built the CFD mesh, performed initial CFD calculations and assisted in writing the paper. Ville Alopeaus provided guidance and assisted in writing the paper.
1. Introduction

Bioreactors are common in modern industry. They are closed reactors where microorganisms are grown under controlled conditions. Chemicals can be produced with the help of microorganisms in less extreme conditions than through traditional chemical pathways. Several types of microorganisms, such as yeasts, bacteria and algae, can be utilized in the industry. Each type of microorganism has specific requirements for growth and different production capabilities. Yeasts are traditionally used for alcohol fermentation in anaerobic conditions. Aerobic bacteria are grown in aerated bioreactors. They can produce many types of complex chemicals including pharmaceuticals. Furthermore, algal cells are grown in transparent photobioreactors. Algal cells can produce dyes, be used for carbon capture (Concas et al. 2012) or produce fats as precursors of biofuels (Wijffels & Barbosa 2010). The use of bioreactors has potential to lessen the use of fossil fuels and increase the use of renewable resources. This is why bioreactors are of great interest. However, the complex biology of microorganisms poses challenges to the engineering of these systems. High concentrations of biomass are optimal for extraction of the product, but to achieve this, efficient mixing and mass transfer are required.

Photobioreactors are of interest because algal cells are capable of utilizing light as the source of energy through photosynthesis. The overall equation of photosynthesis is \(6CO_2 + 6H_2O + light \rightarrow C_6H_{12}O_6 + 6O_2\). In photosynthesis, carbon dioxide and water are converted into sugar and oxygen. The energy of light is bound into the sugar, which is further utilized by the algal cells for growth. Oxygen is a product that is excreted out of the cell and needs to be removed as it restricts growth at high concentrations. Especially for the production of biofuels, optimization of photobioreactor systems is required to make them commercially viable (Chisti 2013). Bubble columns, such as the ones depicted in Figure 1, are an example of a possible reactor configuration for photobioreactors. Air that is fed from the bottom of the reactor pushes liquid up at the center of the reactor. Liquid then flows down at the sides of the reactor and creates a circulation that is responsible for the liquid phase mixing.
Inside cultivation with artificial light offers a controlled system, but the sun offers a possibly lucrative, yet constantly variable, free source of energy for outside cultivations. The availability of light restricts growth at high cell concentrations because light penetrates only a short distance into the reactor. Efficient mixing ensures that algal cells circulate between the dark and lit areas of the reactor. Mixing also enhances gas-liquid mass transfer of carbon dioxide and oxygen. Therefore, mixing and fluid flow in photobioreactors is important to consider (Fernández, I. et al. 2012, Luo & Al-Dahhan. 2004, Merchuk et al. 2007) in addition to the light availability.

Stirred tank bioreactors have been studied extensively due to the complexity of the system. In stirred tanks, the gas phase is dispersed through mechanical agitation by one or more impellers. Figure 2 depicts a possible geometry of a stirred tank reactor agitated by a single radial flow impeller, such as a Rushton turbine.
Stirred tank bioreactors can be used for the cultivation of aerobic microorganisms for the production of chemicals and more recently biofuel precursors (Blanch. 2012). Aerobic microorganisms require oxygen for growth and therefore aeration and efficient dispersion is required. Oxygen needs to be transferred from the gas bubbles to the liquid phase through a process called mass transfer in order to be utilizable by the microorganisms. The main cause for research in the area is mass transfer limitations which easily occur and limit the growth of aerobic microorganisms (Garcia-Ochoa & Gomez. 2009). Experimental studies are mostly in very small scale, but commercial viability often requires very large reactors at a scale of hundreds of cubic meters to be used. Small and large-scale reactors perform differently due to differences in the flow regime (Gezork et al. 2000, Gezork et al. 2001). Mixing in small scale stirred tanks is often homogeneous, such that the stirrers define the fluid dynamics of the system. Large scale systems experience heterogeneous conditions, because of the extensive need for aeration. In heterogeneous conditions, the flow dynamics are no longer defined by the stirrers, but also the flow of gas.

The flow dynamics of bioreactors are important to consider. Mixing in reactors, whether stirred tank, bubble column or any other type, is hardly ever ideal. Knowledge of the flow dynamics of the system gives the possibility to define the mass transfer capability of the system. Flow dynamics also reveals the movement of the microbial cells through the reactor from optimal to less optimal locations. Microbial cells experience changing conditions with high to low mass transfer, light availability, oxygen concentration or nutrient concentrations. Computational fluid dynamics (CFD) is a tool that can be used to define the flow dynamics of a system. In CFD, the reactor is split into very
small computational cells, often hundreds of thousands are required. Navier-Stokes momentum equations in these computational cells are calculated to define the flow field. The approach is computationally demanding because of the large number of equations that need to be solved simultaneously. Especially for systems with two phases and possibly heterogeneous conditions, where phase interaction models need to be solved as well, the calculation times limit the usability of CFD.

Compartmental modelling is a common methodology for the simulation of multiphase reactors (Bezzo et al. 2003, Bezzo & Macchietto. 2004, Delafosse et al. 2010, Gresch et al. 2009, Le Moullec et al. 2010, Pigou & Morchain. 2015, Vrábel, P. et al. 1999, Zhao et al. 2017). Most often the reactor that has been solved with CFD is split into compartments, usually from ten to a hundred. The CFD results are averaged for compartments and fluid flows from compartments calculated. With this simplified system, reactions, population balances of bubbles, mass transfer, microbial growth and light distribution models can be added. Computational times are kept reasonable, while the accuracy of the modelling is preserved.

This work confirms the importance of comprehensive modeling of bioreactor systems. In paper [1], the compartmental modeling approach is developed in order to model photobioreactor systems in inside cultivation. Paper [2] expands the model by the addition of new light distribution models for outside cultivation as well as new models for the calculation of gas phase movement in the reactor for bubble driven systems with high gas volume fraction. Results achieved by the model are also compared to measurements and show a very good fit. Paper [3] moves on to aerated stirred tanks as bioreactors. The paper introduces the hydrodynamics of large stirred tanks and proposes a simple predictive model to help in the scale-up of these reactors. Paper [4] continues with large aerated stirred tanks and introduces the use of the compartmental model for comprehensive modeling of these systems at different flow regimes.
2. Compartmental modeling approach

Compartmental models have previously been utilized for the study of multiphase reactors by several authors (Alopaeus et al. 2009, Patterson. 1975, Vrábel et al. 1999). They are used in order to include the effect of fluid dynamics in the modeling of reactions and mass transfer. The compartments are formed based on measurements (Vrábel et al. 1999, Vrábel, Peter et al. 2001) or fluid dynamic calculations (Bezzo et al. 2003, Bezzo & Macchietto. 2004, Delafosse et al. 2010, Delafosse et al. 2014, Rigopoulus & Jones. 2003) (viitteitä bezzo), which removes the need for experiments. The division is based on similar characteristics of flow or some other parameter in the compartment. Submodels for the calculation of mass transfer, reactions and bubble size distributions can be solved in a matter of minutes. The modeling approach gives information on the local conditions in the reactor much faster than the alternative of adding the submodels into full CFD calculations.

In this thesis, a previously described compartmental modeling approach (Laakkonen. 2006) is further developed for the modeling of algal cultivation in bubble column photobioreactors [1-2] and for large-scale heterogeneous stirred tank bioreactors [4]. The use of the model for such different systems is possible because the method is based on phenomenological models, which are scale and geometry independent. Separate closure models, for instance chemical reactions, can be added for these different systems.

Modeling with the compartmental model has three stages: 1. CFD, 2. compartmentalization and 3. compartmental modeling. CFD is used to calculate the flow field and the local values of power dissipation, $\varepsilon$, as the basis of further calculations. Figure 3 shows how the next step of the approach, the compartmentalization, divides the computational domain into compartments. For each compartment, the power dissipation and other cell values are volume averaged and the flows from compartment to compartment summed up at the compartment interfaces. The compartmental model makes use of these initial values for the calculation of several different submodels. Next, each stage is looked at in more detail.
2.1 Computational Fluid Dynamics

Computational fluid dynamics is a tool for the numerical calculation of fluid flow fields. However, as with any tool, the user requires training and experience for its correct use. Although in this work, CFD was used only as a starting point for compartmental modeling, considerable care was taken to make sure the calculations were performed correctly according to general criteria. The CFD is carried out with the commercial software Fluent. Table 1 lists the version of the software used and the main properties of the performed calculations. The modeled systems range in size from a small laboratory photobioreactor to a large commercial size stirred tank bioreactor. CFD was used in papers [1], [2] and [4].

Table 1. Properties of CFD calculations

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<td>Fluent version</td>
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<td>15.1</td>
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<td>Modeled system</td>
<td>1.9 L bubble column photobioreactor</td>
<td>0.06 m³ bubble column photobioreactor</td>
<td>100 m³ stirred tank</td>
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<td>Mesh size (cells)</td>
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<td>164804</td>
<td>682720</td>
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<td>Turbulence model</td>
<td>Realizable k-ε Standard wall functions</td>
<td>Realizable k-ε Standard wall functions</td>
<td>Realizable k-ε Scalable wall functions</td>
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<td>Mesh movement</td>
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<td>Bubble size (mm)</td>
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<td>Phase interaction</td>
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<td>Drag: Tomiyama</td>
<td>Drag: Tomiyama</td>
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<td>Lift: Tomiyama</td>
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<td>Timestep (s)</td>
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<tr>
<td>Results</td>
<td>Single time step</td>
<td>Time-averaged over 100 s</td>
<td>Time averaged over 20 s</td>
</tr>
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</table>

Single phase CFD simulations have previously been used as a starting point for the compartmental model when mechanical agitation has been used and the dispersed phase holdups have been small (Laakkonen et al. 2007). The considered systems in this work have been bubble driven or have exhibited high volume fractions, so single phase CFD as a starting point was not an
option. While single phase simulations with CFD are quite straightforward and reliable with proper computational grids, two phase simulation brings with it phase interaction models which complicate the system causing longer calculations times and uncertainty in the results. All calculations in this work were performed with the dispersed Eulerian multiphase model, where the phases are assumed to be interpenetrating continua. The liquid is considered the continuous phase and the gas dispersed. The mass and momentum equations for the phases are presented in the following equations (1-2). $\bar{M}_{if}$ is the term accounting for the interfacial forces, which can include drag, lift, virtual mass and turbulent dispersion. The drag force is usually considered dominant over the other interfacial forces and thus the others may be neglected (Laborde-Boutet et al. 2009, Tabib et al. 2008). This was done in this work with the exception of paper [2] where it was noted that the calculations produced lower volume fractions than measured when the lift force was neglected. For the calculation of bubble driven flow, as is the case in paper [2], careful scrutiny to the exclusion of interfacial forces needs to be taken. In stirred tanks where mechanical agitation is responsible for most of the flow, drag can more safely be considered dominant.

$$\frac{\partial (\alpha_i \rho_i)}{\partial t} + \nabla (\alpha_i \rho_i \vec{u}_i) = 0 \quad (1)$$

$$\frac{\partial (\alpha_i \rho_i \vec{u}_i)}{\partial t} + \nabla (\alpha_i \rho_i \vec{u}_i \vec{u}_i) = -\alpha_i \nabla p + \alpha_i \mu_i \nabla^2 \vec{u} + \alpha_i \rho_i \vec{g} + \bar{M}_{if} \quad (2)$$

In all CFD simulations, a single representative bubble size was considered sufficient for the prediction of fluid flow fields. Rigorous methods can then be implemented in the compartmental model for the estimation gas holdups.

Mesh size is known to affect the results of CFD calculations. For this reason, grid dependence tests have been performed. The changes in turbulence dissipation, flows fields and gas holdups with different sized meshes usually show an evening out of the results. Mesh sizes where additional refinement does not affect the results any longer are considered optimal.

The underestimation of turbulence dissipation in comparison to the expected values was noted in all calculations. In each paper, the underestimation was corrected in the compartmental model as will be discussed later in this text.

The studied systems were all inherently time dependent, the bubble columns due to the oscillating nature of the gas plumes and the large mixing tanks for the same reason, when high gas volume fractions were modeled. For this reason, steady state solutions were not possible. Transient calculations were performed. In paper [1], a single time step was chosen as a starting point for the compartmental modeling, but in later simulations in papers [2] and [4], a time-averaging scheme was developed.
2.2 Compartmentalization

Compartmentalization is the link between CFD results and compartmental model starting values. The compartmentalization algorithm is based on previous work (Seppälä. 2008, Seppälä et al. 2008). The algorithm is built as a set of user defined functions (UDF) for Fluent with access to cell values and user defined memory locations (UDM). The algorithm first marks each computational cell as part of a compartment and stores the value in an UDM. Secondly, it calculates volume averages of compartment properties as well as compartment boundary values of flows, areas and accelerations and prints these out into a file that can be read into the compartmental model. Figure 3 shows a simple compartmentalization of a stirred tank. The arrows present CFD flows that are summed at the boundaries to attain the compartment to compartment flows. The coloring presents the compartment values that are averaged over the volume of a compartment.

Successful compartmentalization plays a large part for the reliability of the compartmental model results and the speed at which calculations take place. Therefore, it should not be bypassed as a simple averaging tool, but an important stage in the modelling approach. Developments to the compartmentalization method were made during the study.

In paper [1] the automatic compartmentalization cell-by-cell algorithm developed by Maiju Seppälä (Seppälä. 2008), was tested. The algorithm starts out with a single CFD cell and cell by cell, rates the neighboring cells for likeness according to the compartmentalization criteria. The cell rated the highest by the criteria is chosen and added to the compartment. This is performed for all neighboring cells until no cells that adhere to the criteria are found or the compartment reaches a maximum size. A new cell is then chosen to start the next compartment. And the loop goes on, until all cells have an assigned compartment. It is also possible to manually define compartments for introduction of gas feed, gas outlets etc.

The likeness criteria included in the original algorithm were turbulence dissipation and pressure. Because light intensity plays such an important role in algal cultivation, light intensity was added as the main criterion that defined which cell was added to a compartment. Essentially, any cell value criterion could be added or emphasized, so the algorithm is quite versatile. While the algorithm performed quite well and produced compartments that maintained the contours of the original CFD solution, this methodology leaves some room for improvement. A structured grid, in this case, assured that the compartments were nicely shaped and the number of neighbors was kept to a minimum. However, it left some room for improvement while in some places the split was not completely optimal. Because CFD results are available and therefore flow structures available for scrutiny, it is quite possible to manually decide where the compartment boundaries should be. Therefore, in the later papers [2] and [4], manual construction of the compartments was preferred. The manual construction was programmed into the algorithm separately for each case. This way a minimum number of compartments with less compartment boundaries and a clear numbering order could be created. All of
the above affect the speed of calculations in the compartmental model. The manual construction was mainly performed according to geometry, but in paper [4] some compartments were created according to the value of turbulence dissipation.

Originally, the compartmentalization algorithm had been developed for steady state cases. While it was possible to compartmentalize a single time step, in order to truly make use of the transient calculations, the algorithm needed to be expanded to include compartmentalization of time-averaged results. This required another set of UDF. While the time averages of cell values such as turbulence dissipation and pressure, could be calculated and stored into UDM locations for use in the next time step, Fluent did not contain user definable memory locations where cell face flows could be stored. For this reason, it was necessary to define the compartments before time averaging calculations. This also created need to define the compartments through manual construction, because the cell-by-cell algorithm could only handle a single time step, which would not be representative of the time-averaged solution. The compartment boundary values were calculated at each time step and stored externally for averaging. The development of the time average solution and compartmentalization could be followed during the calculations through printouts of the compartmentalization. Eventually the change in the flows and compartment values was so small that further time averaging was deemed unnecessary.

Because the compartmentalization had originally been developed for small gas volume fractions and single-phase cases, some additional developments needed to be performed in order to account for the high gas volume fractions present in the studied systems in papers [2] and [4]. Gas flows at compartment boundaries were also saved and included in the compartmentalization file. Also, the volume fractions of the dispersed phase in compartments were saved to be used as initial values in the compartmental model. The additions made it possible to make further changes in the compartmental model for the calculation of high gas volume fraction cases.

2.3 Compartmental model

The compartmental model takes as starting values the compartmentalization printout. Another file is used to manage the used models and give additional starting values. The compartmental model solves dynamic mass, scalar and population balances for each compartment. Results are printed intermittently to make it possible to follow the development of the solution. The original version of the model has previously been discussed in detail (Laakkonen. 2006). For this reason, this text only describes the parts necessary to allow understanding of the new developments that will be explained in the next chapters. A simplified schematic of the calculations of each compartment is presented in Figure 4. The gas phase flows marked GAS EXIT and GAS FEED are flows out of the system that are only present in specific compartments. All other flows are compartment to compartment flows. Therefore, several such
flows exist for each compartment depending on the number of neighbour compartments.

![Diagram](image)

Figure 4. Simplified schematic of the calculations of a compartment.

### 2.3.1 Mass balance

The initial mass balance is based on the values calculated from the CFD results by the compartmentalization program. To remove small numerical errors, these values are reconciled so that each compartment balance is consistent. In the compartmental model, the mass balance of compartment \( i \) is calculated separately for each chemical species in each assigned phase, namely liquid and gas in this case. For liquid, since there are no flows in or out of the system, the balance considers 1) internal flows in, 2) internal flows out, 3) mass transfer rates and 4) reaction rates as presented by the terms of the following equation:

\[
\frac{dn_{L,i}}{dt} = \sum_{j=1}^{NB} c_{L,j} F_{L,i} - c_{L,i} \sum_{j=1}^{NB} F_{L,j} + N_{GL,i} a_{GL,i} V_{L,i} + r_i V_{L,i} 
\]

where \( i \) and \( j \) are compartment indexes, \( NB \) is the total number of compartments, \( c \) is concentration, \( F \) is the flow rate, \( N \) is the gas-liquid mass transfer flux, \( a \) is the specific mass transfer area, \( r \) is a chemical reaction rate and \( V \) is the volume of the liquid phase. In this work, \( \text{H}_2\text{O}, \text{O}_2, \) and \( \text{N}_2 \) are included in the mass balances. For algal simulations \( \text{CO}_2 \) is also calculated.

For the gas phase balance, 1) the feed of gas and 2) the removal at the liquid surface need to be considered in addition to the 3) internal flow in, 4) internal flow out and 5) the gas-liquid mass transfer. The gas flows are considered separately for each bubble (or particle) size category, because the velocities of different bubble sizes are allowed to differ. The balance becomes:
\[
\frac{d n_{G,i}}{dt} = c_{G,i,in} Q_i - c_{G,i} A_{i,surface} \sum_{n=1}^{NP} v_n U_{n,slip,i} Y_{n,i} + \\
\sum_{j=1}^{NB} [c_{G,i} A_{j} \sum_{n=1}^{NP} v_n U_{n,j,i} Y_{n,j}] - c_{G,i} \sum_{j=1}^{NB} [A_{j} \sum_{n=1}^{NP} v_n U_{n,j,i} Y_{n,j}] - N_{GL} a_{GL,i} V_{L,i} \tag{4}
\]

Where \( Q \) is the gas feed, \( n \) is the index of the particle size category, \( NP \) is the total number of particle size categories, \( v \) is the particle volume, \( A \) is the area between the compartments or at the liquid surface, \( Y \) is the particle number density, \( U \) is the gas velocity at the compartment boundary.

### 2.3.2 Scalar balance

The inclusion of a scalar balance allows for the flow of chemical components in the liquid phase that cannot be described as conventional chemical species, in this case mainly biomass. The mass transfer term is neglected from the liquid balance and the equation becomes:

\[
\frac{d n_{L,i}}{dt} = \sum_{j=1}^{NB} c^{'L,j} F_{L,j,i} - c^{'L,i} \sum_{j=1}^{NB} F_{L,j,i} + r^{'L,i} V_{L,i} \tag{5}
\]

The scalar balance, unlike the chemical components, is followed in terms of mass instead of moles, which is denoted by the apostrophe in the concentrations, \( c^{'} \), and reaction rates, \( r^{'}. \)

### 2.3.3 Population balance

The population balance follows discretized bubbles of diameter \( d \) in a set number of categories (\( NP \)) through movement from compartment to compartment and particle events.

\[
\frac{d n_{L,i}}{dt} = \frac{1}{v_i} Q_i Y_{n,i,in} - \frac{1}{v_i} U_{slip,n,i} A_{i,surface} Y_{n,i} + \frac{1}{v_i} \sum_{j=1}^{NP} U_{n,j,i} A_{j} Y_{n,j} - \\
\frac{Y_{n,i} \sum_{j=1}^{NB} Y_{n,j} A_{ij} + \sum_{p=n}^{NP} \beta(d_n, d_p) \Delta d_n g(d_p) Y_{p,i} + Y_{n,i} \sum_{p=1}^{NP} \frac{g(d_n, d_p) \Delta d}{2} h\left(\left(\frac{d_n^3 - d_p^3}{2}\right)^{\frac{1}{3}}, d_p\right) Y_{p,i}}{v_i} - \\
g(d_n) Y_{n,i} - Y_{n,i} \sum_{p=1}^{NP} \frac{g(d_n, d_p) \Delta d}{2} h\left(\left(\frac{d_n^3 - d_p^3}{2}\right)^{\frac{1}{3}}, d_p\right) Y_{p,i} + \frac{\Delta b(d_n) Y_{n,i}}{\Delta d} \tag{6}
\]

Where the terms in the equation stand for 1) gas feed, 2) gas exiting system, 3) flow out of compartment, 4) flow into compartment, birth of bubbles by 5) breakage and 6) coalescence, death of bubbles due to 7) breakage and 8) coalescence and 9) growth due to mass transfer or varying pressure. \( \beta(d_n, d_p) \) is the probability that a bubble sized \( d_n \) is formed when a bubble sized \( d_p \) breaks, \( g \) is the breakage rate, \( h \) is the coalescence rate, \( b \) is the bubble growth rate.
The population balances are solved utilizing the method of classes. The discretization of the bubble population is performed such that more categories exist in the lower end of the scale. The number of classes can affect results. While more classes tend to provide more accurate solutions, the calculation speed is greatly affected by the number of categories and therefore a minimum number of categories is beneficial for the calculations.

In addition to the balances, the compartmental model relies on a vast set of submodels. These allow the calculation of interfacial forces, mass transfer, physical properties as well as closure models for the population balances. The structure of the compartmental model makes it possible to simultaneously calculate a large set of separate phenomena. For instance, the mass balances link the models for bubble populations with reactions and therefore the effects of bubble size on the reactions can be judged.

The recalculation of gas volume fractions at each time step is a representative example of how different submodels combine to create the results. A flow chart of the calculations is presented in Figure 5.

Gas volume fractions in the compartmental model are calculated based on the results of the gas phase mass balances, equation (4). Local hydrostatic pressure affects the density and, therefore, the volume of the gas phase. Liquid phase volume is based on the liquid phase mass balances, equation (5). Gas volume fractions could be calculated based on either the gas phase mass balances, or the bubble population balances, equation (6). These should be equal at every time step of simulations. However, small discrepancies exist due to pressure, temperature, mass transfer, reaction and other phenomena that affect the gas volume. These discrepancies are reduced by the addition of models for the growth of bubbles due to mass transfer and pressure differences in the population balance. These phenomena account for most of the changes in gas volume since isothermal operation is considered and no gas phase reactions are calculated. Remaining differences and numerical error are avoided by scaling the bubble populations such that the gas volume equals the
value calculated by the mass balance. This calculation methodology assures an important function of the compartmental model, where the volume fraction of the gas phase is allowed to change from the initial values calculated by the CFD.

2.4 Developments to submodels

In order to model systems for the cultivation of algae and large heterogeneous bioreactors, several developments to the compartmental model were required. These include the calculation of time dependent light distribution and algal growth kinetics. Also, the calculation of gas phase movement in the compartmental model needed to be changed for the bubble induced flow systems. Different models for coalescence of very dense dispersions were added. Finally, a simple reaction model for oxygen uptake was constructed. The basic structure of the new models will be explained next.

2.4.1 Light distribution

The basic equation used for the calculation of light intensity is the Lambert-Beer equation

\[ I = I_0 \exp\left(- (\varepsilon_{\text{biomass}} X + \varepsilon_{\text{water}})l \right) \]  

where \( I \) is the light intensity after a given light path length \( l \), \( I_0 \) is the intensity of the light outside the column, \( \varepsilon_{\text{water}} \) is the extinction coefficient for water and \( \varepsilon_{\text{biomass}} \) is the extinction coefficient for the biomass \( X \).

As explained in section 2.2, in algal simulations with the modelling approach, it is important to take into consideration the light intensity even before the compartmental model, in the compartmentalization. Cell values for the light intensity, were calculated within CFD for compartmentalization purposes at a biomass concentration that roughly depicts a halfway point in the cultivation.

Paper [1] considers a reactor that is evenly lit from all sides. The compartmentalization algorithm calculated the average distance from the sides of the column \( d_{\text{ave}} \) for all compartments. Equation (7) was used within the compartmental model to calculate the time variant light intensity at each time step. In order to simplify the calculations, a vessel average biomass concentration was used in these calculations. The biomass concentrations in different parts of the reactor did not vary much due to efficient mixing, so this was a reasonable simplification.

Paper [2] considers an outside cultivation where the light intensity outside the column is a complex function of the 1) time of day in reference to the length of the day, 2) the zenith angle of the sun and 3) the air mass of the atmosphere. Additionally, the light intensity inside the column was considered a function of 4) the division of light into direct and diffuse components and the 5) directionality of the light with regards to the column. Furthermore, the light distribution model included 6) a dark period during the night. The models contained parameters that were fitted against performed measurements.
Figure 6. Measured (Sánchez Mirón et al. 2002) and modeled irradiance in the center of the column as a function of time of day. Also included is light intensity outside the column [2].

The models were programmed into the compartmental model such that the light intensity in compartments could be calculated at each time step. The length of the day, $t_{day}$, which varied from 13.5 to 14 h during the cultivation, was considered a linear function of the number of days since the start of cultivation. The zenith angle, $z$, which defined the direction of the direct light coming from, was calculated as a function of the time since sunrise, $t_{sun}$.

$$
Z = \begin{cases} 
\frac{\pi}{2} \cdot (1 - k_{angle} \cdot t_{sun}) & \text{when } t_{sun} < \frac{t_{day}}{2} \\
\frac{\pi}{2} \cdot k_{angle} \left( t_{sun} - \frac{t_{day}}{2} \right) & \text{when } t_{sun} > \frac{t_{day}}{2} 
\end{cases} \tag{8} 
$$

where the change of the angle, $k_{angle}$, can be calculated simply as $k_{angle} = 2/t_{day}$.

The zenith angle affects the intensity of light that reaches the column through the air mass, $X_{atm}$, which is the thickness of the atmosphere the light needs to pass through. The Lambert-Beer law can be used to calculate the light intensity outside the column

$$
I = I_{solar} e^{-X_{atm} \varepsilon_{atm}} \tag{9} 
$$

where $I_{solar}$ is the light intensity outside the atmosphere and $\varepsilon_{atm}$ is the extinction coefficient of the atmosphere. These two variables were considered model parameters and fitted to measurements. It was assumed that 30 % of this light intensity was diffuse light and the rest direct light. Diffuse light illuminates the column evenly from all sides, but direct light only from a single direction which is dependent on the time of day.
The consideration of light directionality affected the compartmentalization. The column needed to be divided into a western and eastern side to account for the zenith angle of direct light. The eastern side received direct light in the mornings and the western side in the afternoons. The light path to a compartment inside the column at a distance of $d_{\text{comp}}$ from the sides was calculated according to $l = d_{\text{comp}} / \sin z$ on the side toward the sun and $l = (T - d_{\text{comp}}) / \sin z$ on the other, where $T$ is the diameter of the column. Equation (7) was then used to calculate the direct and diffuse components of light intensity and added up to attain the total light intensity of a compartment.

The methods described here and in more detail in paper [2] for the calculation of the local light intensities in the column provide an essential basis for the modelling of outside algal cultivation.

### 2.4.2 Algal kinetics

Calculation of algal cultivation required a model for the growth of the algae. The algal kinetics model was built upon the concept of photosynthetic factory (PSF) (Eilers & Peeters. 1988, Wu, X. & Merchuk. 2002). In the model, biomass is divided into three states: 1) resting, 2) active and 3) inhibited. Within the compartmental model, the three states were each defined as separate scalars and the movement of algal biomass from compartment to compartment was therefore followed by the scalar balance. This allowed the local concentration of biomass in each state to change not only due to growth, but due to the movement of cells from compartment to compartment. Due to this functionality, the effect of mixing on the growth of algae was considered in the modeling. The dynamic structure of the PSF model is shown in Figure 7.

![Figure 7. Structure of the PSF model [1].](image)

The model depicts the growth of algae and light inhibition as a function of light intensity and time as depicted by the following equations.
\[
\frac{dx_1}{dt} = -\alpha I x_1 + \gamma x_2 + \delta x_3 \tag{10}
\]
\[
\frac{dx_2}{dt} = \alpha I x_1 - \gamma x_2 - \beta I x_2 \tag{11}
\]
\[
\frac{dx_3}{dt} = \beta I x_2 - \delta x_3 \tag{12}
\]

where \(x\) is the local fraction of biomass in each state, \(I\) is the light intensity the biomass experiences. \(\alpha\), \(\beta\), \(\gamma\) and \(\delta\) are model parameters that determine the rate of transfer of the biomass from state to state. In the model, biomass starts in the resting state \((x_1)\). A first order reaction with regards to light intensity describes the transition to the active state \((x_2)\). Another first order reaction describes the transition from active to the inhibited state \((x_3)\). The recovery from the inhibited state has a constant rate, \(\delta\). Another constant rate, \(\gamma\), depicts the return to the resting state. This transition is when the production of biomass takes place. Therefore, the specific growth rate of algae, \(\mu\), can be calculated from the following equation.

\[
\mu = k\gamma x_2 \tag{13}
\]

Where \(k\) is the yield of photosynthetic production in the transition from \(x_2\) to \(x_1\). The specific growth rate is related to the total biomass, so the total biomass, \(X\), is calculated as a sum of the different states at each time step. The growth rate of biomass is then calculated from \(r_X' = \mu X\).

All new biomass is added to the resting state. The scalar reaction rates of the three states of biomass, can then be calculated from the following equations

\[
r_1' = \frac{dx_1}{dt} X + k\gamma x_2 X \tag{14}
\]
\[
r_2' = \frac{dx_2}{dt} X \tag{15}
\]
\[
r_3' = \frac{dx_3}{dt} X \tag{16}
\]

These are the reaction rates in the scalar balance, equation (5). The scalar reactions therefore include the transition of biomass from state to state as well as the growth term in equation (14). Separate terms in the scalar balance account for the movement from compartment to compartment, thus the mixing.

In order to judge the effect of mass transfer on the growth of the algae, the reaction model was enhanced to include the production of oxygen and the consumption of carbon dioxide. This made it possible to judge possible locations of poor mass transfer and possible inhibition. The reaction rates were assumed to be proportional to the growth rate of biomass. In paper [1] these rates were linked by the assumed biomass dry weight and elemental composition. The effects of chemical inhibition were included in the specific growth rate equation by Monod-type equations. In paper [2] the approach was
refined somewhat to include a measured respiratory quotient. However, the effects of inhibition were neglected. This was done because, in the measurements the paper was based on, the concentration of CO₂ and O₂ were measured and controlled.

In addition to the daytime growth kinetics, in paper [2], the night time needed to be considered. At night, a decrease in the algal biomass is seen. A maintenance term, \( Me \), was added to the growth model such that equation (13) became

\[
\mu = k \gamma x_2 - Me
\]  

(17)

This term links the decrease in biomass to the calculated amount of biomass such that a proportion of the achieved biomass is lost each night. As the growth rate turns negative, the linked oxygen production turns to oxygen consumption during the night. The maintenance term is active during the day as well accounting for the maintenance metabolism of the algal cells. A constant value was assumed for the simulations, even though research suggests that the value may be dependent on the hydrodynamic conditions (Wu & Merchuk. 2002).

2.4.3 Bubble induced flow to change the flow calculations

Previously, the compartmental model has been used for the simulation of systems with low gas volume fractions and systems where the agitation controls the flow field. Previously, only the liquid flows were imported from CFD. Bidirectional movement at the compartment boundaries was included to improve the accuracy of the solution. The gas phase was assumed to follow the liquid flow field with the addition of the consideration of slip velocity such that the bubble velocities were calculated from convective and slip velocities

\[
U_{n,ij} = \frac{F_{ij}}{A_{ij}} + U_{n,slip,ij}
\]  

(18)

Where \( A_{ij} \) is the area between the subregions where flow is from compartment \( i \) to \( j \).

The rise of bubbles due to slip was assumed to cause a backward liquid flow of equal volume in order to assure that the relative volumes of subregions remained unchanged.

While this approach seemed to work fine in paper [1], in papers [2] and [4], the bubble-driven and heterogeneous systems could not be modeled this way. This is because the underlying assumption of the original model, that gas follows the liquid phase, is incorrect in bubble-driven and heterogeneous flow where, indeed, the flow of liquid is caused by the flow of gas. The gas flows from CFD calculation were therefore used as starting values for a new method of flow calculation in the compartmental model. As explained previously, additional values of gas phase flows were brought into the compartmental model through the compartmentalization results file.
Firstly, the backward liquid flow due to the rise of bubbles was not included. As the compartmentalization was created from 2-phase CFD calculations, the effect of the gas on the liquid flow field had already been accounted for. No changes to the liquid flow fields were made in the compartmental model calculations. Changes in the liquid flow field were assumed to be minimal and were therefore excluded. Momentum balances are not calculated in the system and therefore the changes to the flow fields derived from CFD should be kept minimal.

Since CFD was performed with a single bubble size and an incompressible model for the gas phase, it was clear that the inclusion of bubble size categories and bubble growth due to pressure differences in the compartmental model would cause more changes to the gas flow field. The initial value for all subsequent calculations of gas flows was the velocity of the gas flow from compartment to compartment. This velocity, calculated by the following equation, was used as the velocity exhibited by the bubble size used in the CFD calculations.

\[ U_{\text{CFD,bub},ij} = \frac{F_{\text{gas},ij}}{A_{ij} \Phi_{\text{CFD},i}} \]  

(19)

Velocities of bubbles in all size categories included the compartmental model are calculated from this velocity but changed according to the differences in slip velocities, \( U_{\text{slip}} \), of the two bubble sizes. The slip velocity of bubbles increases with increasing bubble size. There are three possible cases for the calculation of the velocity of a bubble size group at the compartment interface: 1) the slip velocity of the bubble size used in CFD is larger than the calculated gas velocity, 2) slip and gas velocity are in the same direction and 3) slip and gas velocity are in opposite directions.

Case 2) prevails over the other options, as bubbles tend to move in the direction of the slip velocity, most often upwards. Bubbles that are smaller than the size defined in CFD (bubble size category \( n_{\text{CFD}} \)) are slowed down and bubbles that are larger are sped up according to the following equation

\[ U_{n,ij} = U_{\text{CFD,bub},ij} + U_{n,\text{slip},ij} - U_{n,\text{CFD,slip},ij} \]  

(20)

If the slip velocity is large in comparison to the liquid velocity, a situation is possible where the bubbles are slowed down below the liquid velocity. In this case, the velocity of the bubble is set to the liquid velocity. This removes an unphysical scenario where small bubbles move against the liquid flow.

Case 3) happens mostly in horizontal flow where slip velocities are very small compared to vertical flow. Bubbles that are smaller than the size defined in CFD are sped up and bubbles that are larger are slowed down according to the following equation

\[ U_{n,ij} = U_{\text{CFD,bub},ij} - U_{n,\text{slip},ij} + U_{n,\text{CFD,slip},ij} \]  

(21)
If large bubbles are slowed down to negative values, a velocity of zero is used instead. Case 1) takes place very seldom when the gas phase is slowly moving downwards against the slip velocity. The flow is so slow that it cannot be scaled as in the other two cases. Therefore in these cases, a uniform velocity of $U_{n,ij}$ is used for all bubble size categories.

In mechanically agitated systems the flow of liquid is not completely due to the introduction of gas to the system like in bubble columns. Therefore, while Case 3) and Case 1) are more common in stirred tank reactor flow fields in paper [4] than the bubble column flow field in paper [2]. The use of the new calculation method allows calculation of bubble driven and heterogeneous systems greatly increasing the usability of the compartmental model.

The compartmental model allows pressure differences to affect gas phase densities, unlike in incompressible CFD. The last term in the population balance equation is important in this respect. It is worth noting that in large reactors the changes to pressure within the system are large and changes to the gas flow fields may also become large in comparison to the original incompressible CFD solution. Physically, it would be meaningful to consider the effects of this change on the liquid flow field as well. Corrections to the liquid flow were not included because they would have brought further complexities to the system. The consideration of the gas phase density as such brings the simulation of large reactors closer to the physical in comparison to the incompressible CFD solution.

2.4.4 Bubble coalescence in dense dispersions

Earlier work with the compartmental model had been mostly with dilute dispersions. In dense dispersions, as reported in paper [4], the free space for bubble movement is reduced due to the presence of other bubbles. In such conditions, it can be assumed that the collision frequency, $\omega$, between bubbles is increased. A correction term, $\gamma$, can be used to multiply the collision frequency in order to increase the coalescence frequency, $B_{\text{coalescence}} = \gamma \cdot \omega P$, where $P$ is the collision efficiency. Four such models for coalescence correction due to the reduction in free space were implemented to the compartmental model. The models are referenced according to the original authors. The model of Wu (Wu, Q. et al. 1998) takes the form

$$\gamma \rightarrow \frac{1}{\alpha_{\text{max}}^{1/3}(\alpha_{\text{max}}^{1/3} - \alpha_{g}^{1/3})}$$

(22)

Where $\alpha_{\text{max}}$=0.8 is the maximum packing limit.

Hibiki and Ishii (Hibiki & Ishii. 2000) proposed the following equation

$$\gamma \rightarrow \frac{1}{\alpha_{\text{max}} - \alpha_{g}}$$

(23)

Where $\alpha_{\text{max}}$=0.741, which corresponds to the maximum packing of equal-sized spheres.
The equation of Wang (Wang et al. 2005) has a similar form

\[ \gamma = \frac{\alpha_{\text{max}}}{\alpha_{\text{max}} - \alpha_b} \]  

(24)

Where \( \alpha_{\text{max}} = 0.8 \) is slightly higher than the previous due to the consideration that a bubble size distribution allows for denser packing of bubbles.

Lehr proposed a model that functions very differently in comparison to the others, as it receives only values from 0 to 1. Therefore, used as such, it demotes coalescence in small volume fractions and cannot be utilized for coalescence frequencies fitted for dilute dispersions.

\[ \gamma = \exp \left[ - \left( \frac{\alpha_{\text{max}}^{1/3}}{\alpha_b^{1/3}} - 1 \right)^2 \right] \]  

(25)

All of the other models behave similarly to each other, as is depicted in Figure 8. These correction terms grow rapidly close to the packing limit promoting coalescence very heavily. Interestingly, the correction factors of the Wu as well as Hibiki and Ishii do not start out from 1, but rather from 1.16 and 1.35, respectively. This means that these models already affect the collision frequency at very low volume fractions.

![Figure 8. Dependence of factor \( \gamma \) on the volume fraction of dispersed phase [4].](image)

### 2.4.5 Oxygen uptake in bioreactors

In order to model aerobic bioreactor performance in paper [4] and gain an understanding of the locally achievable \( k_{l,a} \) and oxygen transfer rates, a simple reaction model was added to the compartmental model. The reaction
considers only the formation of CO₂ and the uptake of O₂. It neglects all other chemical species and nutrient as well as biomass, as these have little effect on the mass transfer. The oxygen uptake rate (OUR) of a supposed microbial biomass is calculated from a reaction rate coefficient, \( k \), which is assumed equal in all compartments. First order kinetics are assumed, and the reaction equation takes the form \( \text{OUR} = k \cdot C_{O_2} \), where \( C_{O_2} \) is the concentration of oxygen in the liquid phase. Different values of \( k \) can be given to represent different biomass concentrations or microbial species that have different maximum oxygen uptake rates.

This simple reaction model is important because without oxygen consumption, the dissolved oxygen concentration reaches an equilibrium value in the whole reactor. With the addition of oxygen uptake, local values of dissolved oxygen will drop and show the locations of possible dead spots where the oxygen concentration is lowest and detrimental for microbial growth. Local oxygen concentrations are not simply a function of the local \( k_{L,a} \), but also the mixing from compartment to compartment. Also, with the consumption of oxygen, realistic values for the driving force of mass transfer, the difference between the equilibrium and current concentration, can be calculated. The approach leads to more realistic local values of OTR, oxygen transfer rates. This simple reaction scheme can be used also outside the bounds of this modelling method.
3. Algal modeling

The compartmental modeling approach was used for the simulation of algal cultivation in two separate papers [1] and [2]. Paper [1] introduced the modeling method and paper [2] expanded the modeling approach to include outside cultivation and model verification to measurements.

3.1 Modeled systems

The values used for the CFD calculations of the modeled systems are presented in Table 1. Further values that define the system are listed in Table 2.

<table>
<thead>
<tr>
<th>Table 2. Growth properties of algal cultivation models.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>Column diameter (m)</td>
</tr>
<tr>
<td>Liquid height (m)</td>
</tr>
<tr>
<td>Superficial gas velocity (m/s)</td>
</tr>
<tr>
<td>Gas feed location</td>
</tr>
<tr>
<td>Light availability</td>
</tr>
<tr>
<td>Algal species</td>
</tr>
</tbody>
</table>

The system considered in paper [1] is considerably less complex that the one in paper [2]. The column is smaller and the superficial gas velocity lower. The turbulence energy dissipation in CFD cells and compartments of both cases are shown in Figure 9. Comparison of the cell values of the left hand side pictures shows that the flows in case 1 are less heterogeneous in nature than for case 2. This is because of the lower aeration in this case. Therefore, it was possible to use a single time step as the starting point of compartmentalization in paper [1] but the time averaging was necessary for paper [2]. The compartmentalization of the CFD cells was performed with the automated algorithm in paper [1], whereas in paper [2] a manual approach was considered more adequate. Case 1 seems to show more detail in the compartmentalized values as the spreading out of the gas plume is quite nicely depicted. Case 1 included 82 compartments, while case 2 had 53.
compartments. However, while division into 101 compartments was tested in case 2, the overall results were not affected and, therefore, 53 were considered adequate.

The time scales of calculation in both papers are estimated in Table 3. The table emphasizes the speed of the compartmental approach in comparison to CFD. While the CFD is simplified with a minimum number of closure models, the calculation times in comparison to the compartmental model are still over 1000 times longer. Even with efficient parallelization of the CFD calculations, the differences are tremendous. The ideal mixing cases are, of course, extremely fast. The different time scales of calculation show how a compromise between detail and calculation time is necessary. Full scale CFD calculation with the addition of population balances, mass transfer and algal kinetics would increase the computational load of CFD unreasonably. By the simplification of the flow field and exclusion of the momentum balances, a huge save in computational load is possible, which makes it realistic to include the additional models to the calculation. This way, a comprehensive model of the system is possible.

Figure 9. Values of turbulence energy dissipation in CFD cells and compartments in papers [1] and [2].
When comparing the calculation times of the compartmental model in papers [1] and [2], the difference is shocking. The difference in the number of compartments does not explain the differences, but the automated vs. manual compartmentalization, as was hinted earlier in section 2.2. The automated compartmentalization creates compartments only according to the local cell values. The starting cell for a new compartment is chosen to differ most from the previous compartment. Therefore, the algorithm goes through the reactor in a kind of random order. The numbering of the compartments does not follow the geometry of the vessel causing any number of connections for each compartment. These connections may be to any compartment number. New compartmentalization criteria could be added in order to fix this issue. However, the manual compartmentalization can easily be designed so that compartments have as few as possible connections and only to neighboring numbers. This creates sparse matrixes which are solved much faster in the compartmental model.

Table 3. Time scales of calculations, real time (s) taken to calculate a second of simulation time.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CFD</td>
<td>8000</td>
<td>1800</td>
</tr>
<tr>
<td>Compartmental model</td>
<td>1.3</td>
<td>1/240</td>
</tr>
<tr>
<td>Ideal mixing cases</td>
<td>1/2000</td>
<td>-</td>
</tr>
</tbody>
</table>

3.2 CFD vs. compartmental modeling

One important difference between the calculations by CFD to the calculations in the compartmental model is the consideration of the gas phase density. Calculations in CFD with a compressible gas phase were tested but abandoned due to difficulties. Inclusion of the gas phase compressibility seemed to crash the calculations immediately in Fluent 12.1. This lead to the conclusion that while the models exist, the solver of the software was not able to calculate the system. In the 2.05 m tall bubble column, the density of gas decreases by about 20% when moving from bottom to the top. In order to model the reactor as correctly as possible in CFD, the density was set to the value of the middle of the reactor. The local values of volume fraction in the bubble column are presented in Figure 10.
Besides further verification of the successfulness of the time averaging and compartmentalization as seen by comparison of (A)-(C), the figure shows the effect of the density differences calculated by the compartmental model (D). The gas volume fractions at the top of reactor are higher than the ones
calculated by CFD. This is mainly due to the density difference as reported in paper [2] and partly due to the consideration of the bubble size distribution. The compartmental model calculated values are considered more reliable due to the inclusion of these physical models.

3.3 Model parameters

In papers [1] and [2], the submodels for mass transfer, population balance closures and slip velocities were chosen from already existing correlations implemented in the compartmental model. The models were chosen according to likeness to the modeled systems and are further elaborated in the corresponding papers. The new models for light attenuation and algal growth through the PSF model, however, needed new parameters. In paper [1] the parameters were derived from literature or estimated according to best knowledge for an arbitrary algal species. In paper [2], a comprehensive set of measurements reported in the literature (Camacho et al. 2001, Fernández, F. G. A. et al. 1997, Mirón et al. 2003, Sánchez Mirón et al. 1999, Sánchez Mirón et al. 2000, Sánchez Mirón et al. 2002) was available and the parameters were fitted specifically for Phaeodactylum tricornutum UTEX 640 based on these measurements. Because the measurements were not made for parameter fitting of this kind, some parameters needed to be estimated otherwise, such as the term $\delta$, which is the rate of recovery from the inhibited state. The value was taken from a fit performed previously for another algal species (Wu, X. & Merchuk. 2001). The parameter fitting is explained in detail in paper [2].

Figure 11 represents the measured and modeled algal biomass amount as a function of time. The model represents the dynamics of the algal growth very well. Not all the available measurements were used for parameter fitting so this left some room for verification as will be explained later when discussing mass transfer and oxygen concentration in the reactor in section 3.5. The fitted parameters should be system independent and therefore later use in modeling of the specified algal species is possible with the PSF model parameters fitted here.
With the compartmental modeling approach, the combined effect of different phenomena on the cultivation of algae can be assessed. The results of the algal cultivation in both cases clearly show the importance of considering mixing and light availability simultaneously. This can be deduced by taking a look at the changes in the fractions of PSF states during the calculation in different cases. The results for papers [1] and [2] are presented in Figure 12 and Figure 13, respectively. In paper [1], two ideal runs were performed of which the other was calculated with the vessel average irradiance and the other with the maximum irradiance. Note the different scales for the growth rates in each figure. The comparison of ideal runs to the 82 compartment version showcases the results clearly. The dynamics of growth in the different calculations are different. The ideal runs experience a sharp increase in the growth rate until the intensity of light starts limiting growth, very soon in the average case and later in the maximum intensity case. Once the limitation starts, the growth rate drops quickly. This starts the lag phase in the cultivation due to light limitation. In the 82 compartment case, the drop to the growth rate is much slower. The mixing that takes place in the reactor ensures that the algal cells rotate to the sides of the column, where light is still plentiful. This ensures that the fraction of active biomass stays high longer than in the ideal cases. The activity of the biomass is therefore a function of the mixing rate in the reactor. Increased circulation makes it possible to delay the start of the lag phase.

In paper [2], the underlying dynamics are the same, except the lack of light during the night ceases growth and brings all the biomass to the resting state each night. In the outside cultivation, due to the high light intensity and efficient mixing, the fraction of active biomass is quite high even on the last
days of the cultivation, when the batch cultivation was seized. A look at the biomass concentrations (Figure 11) in comparison to the PSF state fractions (Figure 13) reveals the reason. In the outside cultivation, each night brings with it, a decrease in the biomass amount. Once the daily growth rate can no longer surpass this loss in biomass, the cultivation should be ended. Therefore, the end of the cultivation is not light limited because of shading by biomass but because of the lack of light during the night. Efficient mixing is seen to keep up the growth during the day. The addition of artificial light on the last day of cultivation could help to increase the biomass concentration in the reactor. A longer day would increase the biomass concentration. This means seasonal changes to outside cultivations.

While in the cases studied in paper [1], the number of inhibited cells was very small, the beginning of the outside cultivation in paper [2] is slowed down due to inhibition. This is seen from the high fraction of inhibited state, about 0.3, during the first few days. The long lag before the exponential growth period also shows the effects of the inhibition. In the measurements and the model, the column was shaded for the first days to decrease the amount of inhibition. In inside conditions, where the column is artificially lit, this is not necessary, and the light intensity can be chosen such that inhibition does not occur.
Figure 12. Growth rates and fractions of active (PSF2) and resting (PSF1) cells in paper [1].
The effect of mass transfer rates on the cultivation can be studied from the results of the compartmental model. Local and vessel average values of the mass transfer rates as well as oxygen and carbon dioxide concentrations are available. Global values give a representation of the overall functioning of the reactor. Results for paper [1] and paper [2] are presented in Figure 14 and Figure 15, respectively. In Figure 14, the concentrations start at the equilibrium values and begin to change as the growth of algae starts. At the highest growth rates at about 40 hours of cultivation, the concentration of oxygen is almost twice the equilibrium concentration. Mass transfer from liquid to gas phase is unable to keep up with the increase in oxygen due to its production by the algae. At these concentrations, oxygen will start to limit growth. The concentration on CO₂ in the liquid phase is quite high during the cultivation and substrate inhibition is unlikely. CO₂ is fed in excess.

**Figure 13.** Fractions of biomass in the different photosynthetic factory model states as a function of cultivation time. (PSF1) inactive, (PSF2) active and (PSF3) inhibited [2].

**3.5 Mass transfer**

The effect of mass transfer rates on the cultivation can be studied from the results of the compartmental model. Local and vessel average values of the mass transfer rates as well as oxygen and carbon dioxide concentrations are available. Global values give a representation of the overall functioning of the reactor. Results for paper [1] and paper [2] are presented in Figure 14 and Figure 15, respectively. In Figure 14, the concentrations start at the equilibrium values and begin to change as the growth of algae starts. At the highest growth rates at about 40 hours of cultivation, the concentration of oxygen is almost twice the equilibrium concentration. Mass transfer from liquid to gas phase is unable to keep up with the increase in oxygen due to its production by the algae. At these concentrations, oxygen will start to limit growth. The concentration on CO₂ in the liquid phase is quite high during the cultivation and substrate inhibition is unlikely. CO₂ is fed in excess.

**Figure 14.** Oxygen and carbon dioxide concentrations in reactor as a function of cultivation time in paper [1].
The oxygen saturation in the reactor calculated by the model in paper [2] can be compared to measured values. The dynamics are presented correctly with the increase in saturation during the day and decrease during the night when the algal cells utilize oxygen in cellular respiration. In the measurements, gaseous CO$_2$ was added to the reactor at the high growth rates, but the amounts not reported. The increase of CO$_2$ partial pressure in the gas phase and the resulting decrease of oxygen partial pressure which causes changes to the equilibrium conditions for mass transfer is likely the reason for the misrepresentation of oxygen saturation during the last days of cultivation. The increase in the CO$_2$ feed could easily be added to the model and thus the oxygen concentration correctly calculated. However, the amounts were not reported (Sánchez Mirón et al. 2002). The variation in the oxygen saturation however is instrumental in the verification of the model. There are very few points measured for the biomass concentration on the last days of cultivation in Figure 11, however, the oxygen concentrations are reported. The varying oxygen concentrations verify the modelled growth curves. Growth of biomass must be taking place, since production of oxygen is also reported. Therefore, the dynamics of the last days of cultivation are also correctly presented by the model.

From the night time concentrations of oxygen, and how well they are presented by the model, it can be deduced that the maintenance term, $M_e$, is correctly estimated.

Local values of mass transfer rates were reported in paper [2] along with local sauter mean diameters. They are presented here in Figure 16.
Figure 16 shows the mass transfer is concentrated to the center of the column where the bubble plume moves up. The sides where light is plentiful have very small mass transfer rates. This is another reason that the consideration of mass transfer and bubble movement along with local light intensities is important to consider. Activation of biomass and mass transfer happen in
different parts of the reactor. Efficient mixing ensures that biomass experiences steady surroundings where neither light nor oxygen limit growth. Figure 16 also shows how the very bottom of the reactor below the sparger is essentially dead space for mass transfer. Optimization of the reactor would warrant the removal or minimization of this area for improved mass transfer. In this way, the compartmental model results can also be used for the optimization of reactor geometries.
The design of a process involving aerobic bioreactors starts at the scale of the microorganism. The conditions the microorganism needs for growth and high product yields are optimized in the laboratory. Scale-up starts by increasing the size of cultivation from a few hundred millilitres to a few litres. At this bench scale, stirred tank bioreactors are utilized and their performance optimized. Scale-up of stirred tank bioreactors continues to pilot scale (50-300 L) and finally to production scale, where the scale may be hundreds of cubic meters. This is the scale of the large bioreactors referenced in this text.

Four different approaches to scale-up are generally recognized (Garcia-Ochoa and Gomez, 2009): fundamental methods, semi-fundamental methods, dimensional analysis and rules of thumb. Fundamental methods consist of the physics based modeling of bioreactor systems. Paper [3] proposes a new simple physics-based approach to scale-up that can be used much like a correlation. Paper [4] experiments with CFD and the compartmental modelling approach, which can be classified as a fundamental method for scale-up. The use of the compartmental modelling approach, presented in chapter 2 and utilized for bubble columns in chapter 3 of this text, is now presented for use in large stirred tank bioreactors.

### 4.1 Hydrodynamics

Hydrodynamics of large stirred tank reactors are very complex. Paper [3] discusses the hydrodynamics in detail. The basics and most important factors that affect the modelling and design of large bioreactors are summarized here.

#### 4.1.1 Flow regime

When stirred tanks are utilized for mixing of a single phase, the system hydrodynamics are somewhat simple. The mechanical agitation defines the flow field. These systems can quite reliably be modelled by CFD. With the addition of a second phase, in this case gas, the hydrodynamics change. When aeration is small, the agitation still defines the flow field and the gas bubbles move mainly with the liquid with the addition of the slip velocity upwards, of course. With the increase in aeration, the flow field structures start to change as gas changes the flow field. At a point when the flow is no longer mostly defined by the stirring but indeed the gas flow, a transition occurs, and the flow becomes heterogeneous. (Gezork et al. 2000, Gezork et al. 2001)
Heterogeneous systems exhibit complex ever-changing flow fields and often flooding of impellers. Superficial gas velocity can be used to estimate the flow regime in a system. A commonly used transition velocity is 0.3 m/s, above which the flow becomes heterogeneous. Commonly reactor design specifications state that the regime should be kept homogeneous for optimal performance.

4.1.2 Superficial gas velocity vs. vvm aeration

Micro-organisms that are utilized in bioreactors are usually aerobic and therefore require oxygen for growth. Aeration in laboratory vessels is very often kept at around 1 vvm in order to ensure the oxygen demand of the microorganism. In scale-up, using the same vvm aeration rate seems logical, but as Figure 17 shows, conservation of this aeration rate leads to very high superficial gas velocities and therefore the heterogeneous flow regime. High H/T ratio vessels, which are often favoured start to exhibit heterogeneous flow already at a scale of 1 m³. This leads to a situation where the aeration should be lessened, or the vessel designed for heterogeneous flow.

![Superficial gas velocity vs. vvm aeration](image)

**Figure 17.** Superficial gas velocity at the top of a reactor as a function of size for different geometries. Values are calculated for an aeration rate of 1 vvm(NTP). For clarity the transition superficial gas velocity of 0.03 m/s is shown [3].

4.1.3 Hydrostatic pressure

Hydrostatic pressure affects the functioning of large vessels due to the compressibility and solubility of gas. The compressibility of gas has very little effect on the hydrodynamics of small reactors and can therefore be overlooked in design and modelling. As the height of the vessel increases, so does the difference between the density of gas at the top compared to the bottom of the
reactor. In large reactors, the compressibility of gas should be considered in design and modeling. Different hydrodynamics at top and bottom are caused by the difference in superficial gas velocity. The superficial gas velocity at the bottom of the reactor, when the top of the reactor is at $0.03\text{ms}^{-1}$, is shown in Figure 18 as a function of vessel size. This shows that a reactor may be operating in the homogeneous regime at the bottom of the reactor and at the heterogeneous at the top. Figure 19 presents a possible situation in a H/T=3 reactor. The transition velocity has been proposed to be a function of the density of gas as well (Krisha et al. 1991), so the transition velocity has been given a small slope in the figure.

![Figure 18](image1.png)

**Figure 18.** The superficial gas velocity at the bottom of a reactor as a function of vessel size, when the superficial gas velocity is set to 0.03 ms$^{-1}$ at the top [3].
4.1.4 Mass balance

While reactor design specifications for homogeneous flow lead to a maximum aeration rate, the main reason for aeration, the oxygen demand of the microorganism, leads to a minimum aeration rate. Calculations in paper [2] estimate the minimum aeration rate of a H/T=3 bioreactor for a microorganism with an oxygen uptake rate of 100 mol m⁻³ h⁻¹. The result is 0.38 vvm, while the transition to heterogeneous flow happens at 0.31 vvm with the selected H/T ratio. This result verifies that large reactors cannot be designed for homogeneous flow.

4.2 Simple models

Paper [3] proposes simple methods for the estimation of gas hold-up and mass transfer rates in stirred tank reactors. The models are based on simple physical calculations and require no fitting of parameters, but general knowledge of the system.

4.2.1 Estimation of gas holdup

The gas holdup in a stirred tank reactor can be estimated by the following equation for the homogeneous and heterogeneous regime.
\[
\phi = \begin{cases} 
\frac{v_S}{U_{T,bubbles}} & \text{if } v_S \leq v_{\text{trans}} \\
\frac{v_{\text{trans}}}{U_{T,bubbles}} + \frac{v_S-v_{\text{trans}}}{U_{T,slugs}} & \text{if } v_S > v_{\text{trans}}
\end{cases}
\] (26)

Where \( v_s \) is the gas superficial velocity, \( v_{\text{trans}} \) is the transition superficial velocity, usually assumed 0.03 ms\(^{-1}\), \( U_{T,bubbles} \) the bubble terminal velocity and \( U_{T,slugs} \) the slug terminal velocity. In the equation, it is assumed that in the homogeneous regime, all bubbles can be assumed to follow a single averaged terminal velocity. The terminal velocity of bubbles with diameters ranging 4-10 mm in pure water is quite steady at 0.2 ms\(^{-1}\) (Kulkarni & Joshi. 2005, Tzounakos et al. 2004). This is usually the size range of bubbles that is required for adequate mass transfer, so it is a reasonable assumption. For coalescing systems a lesser value of 0.15 ms\(^{-1}\) is better. The model assumes that as the flow turns heterogeneous, further aeration leads to coalescence and the formation of gas slugs that rise at a faster rate than the bubbles. The bubbles are essentially divided into two size classes, the bubbles and slugs, so a bimodal distribution, as often reported in literature (Gezork et al. 2000, Krishna et al. 1991, Yang et al. 2010). Addition of aeration past the transition velocity increases the amount of slugs, but no longer the amount of bubbles. Therefore, the holdup starts increasing at a slower pace than in the homogeneous regime. This is of course a simplification since, in reality, the transition happens slowly with a steady increase in bubble size until the formation of slugs takes place.

The model was tested against measurements and showed a generally good trend as in Figure 20, where the model is compared against measurements in the homogeneous and heterogeneous regimes.

![Figure 20. Holdup as a function of superficial gas velocity. The line depicts the model, measurement points(Gezork et al. 2001) are for different power consumptions. The effect of surface aeration is subtracted from the originally reported values [3].](image-url)
Deviation of the model from measurements is discussed in paper [3] and accredited mostly to the simplification of the bimodal distribution into only two size classes. For the purposes of scale-up, a model with no adjustable parameters, that only requires an estimate of the terminal velocity of bubbles and slugs is very useful. The model is not scale dependent and can therefore produce estimates of holdup for any geometry and size class of vessel. The simple model calculations were compared against CFD and compartmental model calculations in paper [4]. The model produced the holdup in the homogeneous regime quite well but overestimated the holdup in the heterogeneous regime. This was also the case for comparison to measurements in a larger scale and low power input (Vrábel, Peter et al. 2000) in paper [3]. The underestimation may be due to flooding of the impellers, as was seen in the CFD calculations reported in paper [4]. When the lowest impeller is flooded, the formation of bubbles does not take place at the bottom of the vessel and a large part of the reactor does not contain the bubbles the simplified model assumes. Of course, reactors are not be designed for flooded conditions, so the model withstands.

### 4.2.2 Estimation of volumetric mass transfer coefficient

From the simple model for holdup, it is possible to derive the following equation for mass transfer area in a stirred tank reactor.

\[
\begin{align*}
    a &= \left\{ \begin{array}{ll}
    \frac{6\mu_S}{u_T \cdot \text{bubbles}} & \text{if } v_s \leq v_{\text{trans}} \\
    \frac{6\nu_{\text{trans}}}{u_T \cdot \text{bubbles}} + \frac{18(v_s - v_{\text{trans}})}{d_{\text{slug}} u_T \cdot \text{slugs}} & \text{if } v_s > v_{\text{trans}}
    \end{array} \right.
\end{align*}
\]

(27)

The model continues with the assumption of a bimodal distribution and estimates a size for the bubbles \(d_b\) and slugs \(d_{\text{slug}}\). The bubble size can be estimated, for instance, with the following model (Bhavaraju et al. 1978).

\[
d_b = 0.7 \frac{\sigma^{0.6}}{(\frac{P}{V})^{3/2}} \rho_L^{0.2} \left( \frac{\mu_L}{\mu_G} \right)^{0.1}
\]

(28)

Where \(\sigma\) is the surface tension, \(\rho_L\) the liquid density, and \(\mu_L\) and \(\mu_G\) the Newtonian liquid and gas viscosities, respectively. \(P/V\) should always be the total power dissipated due to the mechanical mixing and aeration.

The shape of the slugs is assumed to be a spherical cap and the surface area is estimated separately in paper [3] through an assumed geometry, which leads to the multiplier of 18 instead of 6, which is for spherical bubbles in equation (27) for the heterogeneous regime.

For bioreactor systems, estimation of \(k_L\) can be performed, for example, with the following equation (Kawase & Moo-Young. 1990)

\[
k_L = 0.301 \cdot \sqrt{D_L} \left( \frac{\nu \rho}{K} \right)^{1/(1+n)}
\]

(29)
Where $\varepsilon$ is the same as $P/V$ in the previous equation for bubble size, $D_L$ is the diffusion rate, $K$ is the power law consistency index and $n$ is the power law index. When the liquid is Newtonian, $n=1$ and $K=\mu_o$, the apparent viscosity of the liquid phase. Since the simplified calculation method is meant for scale-up studies, it is fair to assume that the power dissipated can be set according to design specifications. Different power dissipation values can easily be tested. The liquid properties should also be available for design.

Figure 21 shows how the model behaves in relation to measurements found in literature (Gezork et al. 2001). The model deviates clearly in the homogeneous regime but estimates very well in the heterogeneous regime. The deviation is perhaps due to the large fraction of surface aeration, which is not accounted for in the model. In small vessels, the effect of surface aeration is larger than in high H/T ratio vessels, which are usually favoured in scale-up of bioreactors. Therefore, the fit is assumed to be better in large vessels. However, measurements in large scale are very seldom reported in literature and comparison cannot be made.

![Figure 21](image)

**Figure 21.** Measured (Gezork et al. 2001) versus calculated $k_a$ at different superficial gas velocities [3].
4.3 CFD

While the CFD calculations in paper [4] were mainly performed as a starting point for the compartmental modelling, they do show important aspects of large reactor hydrodynamics and emphasize the importance of modelling. The modelling is discussed in detail in paper [4] and the main properties listed in Table 1.

Three different 2-phase cases were run in order to test the models in homogeneous, transitional and heterogeneous flow regimes. The flow conditions and main time-averaged results are summarized in Table 4. The volume fractions clearly increase with added aeration, but to a lesser extent when the flow regime is heterogeneous, in accordance with the earlier presented simple model. The decrease in specific power input is expected when the volume fraction of gas phase increases. The drastic decline is, however, a sign that the impellers may be flooded.

<table>
<thead>
<tr>
<th>Case</th>
<th>Flow regime</th>
<th>VVM</th>
<th>Superficial gas velocity (m s⁻¹)</th>
<th>Mixing rate (rpm)</th>
<th>Time-averaged volume fraction (vol-%)</th>
<th>Specific power input from mixing (W kg⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>homogeneous</td>
<td>0.04</td>
<td>0.007</td>
<td>110.7</td>
<td>3.3</td>
<td>2.06</td>
</tr>
<tr>
<td>Case 2</td>
<td>transition</td>
<td>0.17</td>
<td>0.03</td>
<td>110.7</td>
<td>8.0</td>
<td>1.25</td>
</tr>
<tr>
<td>Case 3</td>
<td>heterogeneous</td>
<td>0.29</td>
<td>0.05</td>
<td>110.7</td>
<td>12.7</td>
<td>1.02</td>
</tr>
</tbody>
</table>

A look at the local gas volume fractions in Figure 22 confirms the assumption that the impellers are flooded. While in the homogeneous Case 1 the gas is spread at the lowest impeller, it seems that even in the transition Case 2, the lowest impeller is flooded. Flooding brings forth transition before the generally regarded transition superficial gas velocity of 0.03 ms⁻¹.
While the CFD results show a likely scenario of flow distribution in a large stirred tank reactor, the simulations lack important physics, namely the compressibility of gas. As discussed in earlier sections, the hydrostatic pressure in large vessels causes a situation where the density of gas is much higher at the bottom than the top. Therefore, the superficial velocity increases when moving up in the reactor. This was not included in the modelling here and very seldom in literature, due to issues with modelling of compressible fluids in Fluent. In the CFD calculations, the aeration and gas density in the whole vessel were set to the superficial gas velocity in the midpoint of the reactor. The values are listed in Table 4. In reality, the superficial gas velocity would be less at the bottom and more at the top. This approach hopefully minimizes the errors due to the exclusion of this important physics. Were the bottom superficial gas velocity used, perhaps the impellers would not be flooded. The compressibility of gas can be considered in the compartmental model and the gas volume fraction results are therefore depicted here in Figure 23 for comparison. In the compartmental model results, the gas volume fractions rise when moving up in the reactor, as would be expected. Therefore, the results of the compartmental modelling for gas volume fractions are more realistic than the CFD results.
The compartmental modelling in paper [4] was performed in order to shed light on the functioning of large bioreactors. The new models for bubble coalescence presented in section 2.4.4 were tested. The results were compared to ideal reactor solutions in order to verify the importance of considering local hydrodynamics. The calculations were based on the compartmentalization of time-averaged solutions of the CFD performed as explained earlier. 89 compartments were used as explained in detail in paper [4]. The new model for gas phase calculations reported in section 2.4.3 was utilized because of the high aeration rates and heterogeneous flow regimes. In the model, the liquid flows were derived from the CFD calculations and the velocities of gas bubbles recalculated based on differences in slip velocities of different sized bubbles.

### 4.4 Compartmental modelling

Vessel average results are used for the comparison of the new coalescence models as well as different drag models in paper [4]. The Sauter mean bubble diameter increased with increasing coalescence adjustment in the order of Wang → Hibiki and Ishii → Wu. The effect was lowest in Case 1, because there are only a few compartments where the volume fraction is large enough to grant significant increases of the collision frequency as seen in Figure 23. Depending on the model, the collision frequency is doubled when the volume
fraction reaches 6%, 25% or 40% for Wu, Hibiki and Ishii and Wang, respectively. In Case 1, the volume fraction is mostly below these values.

Increased Sauter mean diameters that were due to the new models, led to lowered gas phase holdups. Larger bubbles escape the system faster because of their higher slip velocity. The $k_La$ was also lowered when the new models were used because of the decrease in holdup. This shows that the reduction in free space that the coalescence correction models are based on, is important to consider, when omission may lead to results that are too optimistic.

### 4.4.2 Oxygen mass transfer

The oxygen reaction scheme explained in section 2.4.5 was used to test reactor performance in the different cases. With the model, different values for oxygen uptake rates (OUR) were simulated by increasing the reaction rate coefficient, $k$. The oxygen transfer rate (OTR) was simultaneously calculated with the mass transfer models. By changing the reaction rate coefficient to ever larger values, the concentration of oxygen in the liquid phase was decreased, the driving force for mass transfer maximized and the oxygen transfer rate reached a maximum. Therefore, it was possible to asymptotically estimate the maximum possible OTR of each case. The values were 25, 70 and 110 mol m$^{-3}$ h$^{-1}$ for Case 1, 2 and 3, respectively. As discussed in paper [3] and in section 4.1.4 of this text, an oxygen uptake rate of 100 mol m$^{-3}$ h$^{-1}$ can be considered satisfactory for bioreactor performance. Clearly Case 1 and Case 2 are not satisfactory. This verifies the earlier stated fact that large reactors cannot be operated in the homogeneous regime as only the heterogenous regime Case 3 is satisfactory in terms of OTR.

The same conclusion can be made if the outlet concentrations of oxygen, reported in paper [4] are looked at. The outlet molar fraction of oxygen in Case 1 is diminished to 0.04, when the inlet concentration is 0.21. This means that the availability of oxygen is limiting at the top of the reactor and even though mass transfer area exists, the driving force is diminished.

Local results are discussed next. The local Damköhler numbers are presented in Figure 24. The Damköhler number $Da = k/k_{La}$ defines whether the reactions are limited by the reaction rate or mass transfer. If the value is below 1, the microbial growth can be assumed reaction limited, but above 1, mass transfer limitation is more likely.
The bottom of the reactor below the first impeller in all cases is severely mass transfer limited. In Case 2 and Case 3, the result of the flooding is clear as the area below the second impeller is also mass transfer limited except for in the area where the gas plume exists. Case 1 performs quite well in this area due to adequate dispersion. Because of the high pressure and therefore high driving force for mass transfer, at the bottom of the reactor, gas dispersion at the lowest impeller would be most beneficial. Above the second impeller, Case 1 experiences mass transfer limitation due to the lowered availability of oxygen, while Case 2 and Case 3 are reaction rate limited.

In the homogeneous Case 1, throughout the reactor, a microorganism will experience stable conditions. However, the stable conditions involve oxygen limitation. The homogeneous flow regime is not favourable in this scale. In the transition and heterogeneous regime reactors, Case 2 and Case 3, a microorganism will experience changing conditions. The bottom is oxygen depleted with bad circulation of liquid due to the flooded impeller. The top of the reactor is ideal with high gas volume fractions, good mixing and plentiful oxygen. In changing conditions, stress reactions in microorganisms are possible, which leads to lessened growth. Flooding is clearly very detrimental to the system.

**Figure 24.** Local Damköhler numbers for Case 1, Case 2, and Case 3. Values over 10 are depicted in red [4].
4.4.3 Ideal reactor and correlations

The results obtained from the compartmental model runs were compared to ideal reactor runs. In these runs, the reactor was modelled as a single compartment with no local variation and the assumption of ideal mixing in the reactor. The ideal reactor results were found to give overestimations in reactor performance. The assumption of an ideal reactor ignores the effects of flooding, which is a major contributor to the reactor performance in the transition and heterogeneous regime. As discussed earlier in section 4.2.1, the simple model also neglected flooding and caused overestimations in gas holdups compared to the presented results.

Ideal reactor solutions for mass transfer are inadequate in large scale because the availability of oxygen is different in different parts of the reactor. As gas moves up the reactor, oxygen is consumed, so the maximum equilibrium concentration decreases leaving less driving force for mass transfer. At the same time, the pressure of the system decreases, which also decreases the maximum oxygen transfer rate. Therefore, even the gas phase cannot be modelled without the consideration of local variation.

Comparison to the very often used van’t Riet correlation for $k_{La}$ (Van’t Riet. 1979) in paper [4] showed very poor performance. The constant needed to be increased by over a factor of 10. This confirms that small scale correlations are not satisfactory for scale-up.

4.4.4 Design considerations

Figure 23 shows that the volume fractions of gas phase are very low at the bottom of the reactor even in the homogeneous regime, Case 1. Mass transfer and mixing is poor at the bottom below the impeller. In order to avoid dead spots, rounded bottoms for large reactors should be considered instead of the flat bottom modelled here. Rounded bottoms are very often utilized in industry. The addition of draft tubes is another alternative.

The availability of oxygen in the homogeneous regime could be increased by feeding oxygen enriched air. However, this may be a costly endeavour. The reactor geometry presented in paper [4] is not optimal for performance in the large scale. Flooding of the impellers should be avoided. Increasing the mixing rate is not a viable option, because the impeller tip velocities would be increased to too high values. Also, the calculated P/V of 2 W kg$^{-1}$ for the modelled system is already quite high. Tests with CFD showed that flooding was likely at higher velocities as well. Axial flow impellers tend to handle larger gas loads without flooding, so they could be considered in design.
5. Conclusions

The work summarized in this text shows the continued development of a modelling method for bioreactor systems. The modelling framework was further developed to handle systems whose modelling was not previously possible. These included bubble columns and high volume fraction agitated systems. The studied systems exhibit large variation of flow structures with time, requiring time-averaging of compartmentalization. Bubble induced flow can be modelled in addition to the earlier assumed mixer induced flow. Local light availability and algal growth kinetics can be considered simultaneously with fluid flow and mass transfer. The local availability of oxygen for microbial growth can be estimated. The results of the modelling method are able to shed new light to the functioning of photobioreactors and large stirred tank bioreactors.

Efficient mixing in algal cultivation is important for the growth of microalgae. Algal cells move from well lit to unlit conditions. This so-called flashing light effect lessens inhibition but makes sure all cells are activated. The availability of light limits the size of photobioreactors. Optimization of mixing can increase this size.

It was noted in paper [2] that in outside cultivation, the length of the day in relation to the night, limits the maximum achievable cell densities in cultivation perhaps even more than light availability during the day. This realization warrants further studies into the feasibility of outside cultivation in comparison to inside artificially lit cultivation. An interesting study would be outside cultivation with artificial light during the night and whether this could raise the achieved cell densities in outside cultivation. A modelling study could provide estimates for proper light intensities as well as estimate the cell density that could be achieved.

Papers [3] and [4] are instrumental in showing issues concerning the use of large stirred tank bioreactors. The combination of a simple model and compartmental modelling gives a thorough picture of the physics concerning scale-up and design of these reactors. The work presented should aid in the design and choice of reactor for large bioreactors. Issues with mass transfer and hydrodynamics are intertwined.

The simple model is shown to neglect flooding. However, vessels are never designed for flooded conditions, but to avoid flooding. The simple model could be further developed and used systematically for flooded reactors by assuming larger bubble size in different parts of the reactor thereby verifying flooding of already existing reactors. As such the model gives predictions for initial stages
of design, helps in the choice of vessel size and indicates the possible achievable holdups and mass transfer rates independent of scale. Correlations are not capable of this.

The usability of CFD calculations for large two-phase reactors is limited due to the difficulties associated with calculation of a compressible gas phase in mixing applications. Different methods to circumvent the issue could be possible. The reactor could, for instance, be calculated in parts with different densities and superficial gas velocities in each part. Until a truly compressible gas phase solution is possible, the compartmental modelling approach with the consideration of gas compressibility is a very good option.

Flooding detrimental for bioreactor performance. While aeration needs to be at very high levels, the risk of flooding is imminent. Different axial flow impellers are available that do not flood as easily as the considered radial flow Rushton impellers. However, a different impeller only pushes the onset of flooding a little further but does not solve the issue entirely. A complete change in system should also be considered if designing large bioreactors. Instead of a stirred tank, a bubble column or airlift may function better and provide efficient mixing as well as oxygen mass transfer. In essence, the reasons that mechanically agitated tanks are more problematic in large scale (large holdups and high superficial gas velocities) are the reasons bubble driven systems function better in larger than in smaller scale. Further research efforts for bioreactor scale-up should be directed to these systems. The presented compartmental modelling approach can be used for such studies with the existing submodels.
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