Multiaxial Magneto-Mechanical Interactions in Electrical Steel Sheets

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Abstract
This thesis studies multiaxial magneto-mechanical interactions in electrical steel sheets with particular attention to non-oriented electrical steel sheets. Commonly, during the magneto-elastic analysis of electromagnetic applications only uniaxial stress models are used where the effect of multiaxial stress is often neglected.

A new rotational single sheet tester (RSST) that is capable of applying arbitrary in-plane magneto-mechanical loading to steel sheets is designed and manufactured. Experiments on a non-oriented electrical steel sheet are performed for analyzing the effect of multiaxial stress on magnetic properties and iron losses in the material. Performed experiments reveal that the effect of multiaxial stress on magnetic properties and iron losses can be much more significant than that of uniaxial stress.

A simplified multiscale (SM) and a macroscopic Helmholtz energy based (HE) model are used to model the multiaxial magneto-mechanical behavior of non-oriented electrical steel sheets and their prediction capabilities are compared when limited measurement data is available for identification. The models were studied for modeling both anhysteretic and hysteretic behavior of three different materials that were characterized by different measurement setups. Comparison of the modeling results to the measured results shows that the SM model is accurate for certain cases, whereas the HE model is successful for all three materials.

In order to predict the effect of multiaxial stress on hysteresis and excess losses, a stress dependent iron loss model is developed utilizing statistical loss theory. The model is verified with measurements obtained from the manufactured RSST. For validation purposes, both the HE model and the developed loss model is implemented to a 2D finite element model of a transformer that is under mechanical stress. The applicability of the models is proven by comparing the modeling results to the measurements. It is concluded that if mechanical stresses are present in an application, using conventional methods to calculate the losses can lead to inaccurate results.

Keywords Electrical steel sheets, finite element analysis, iron losses, magneto-mechanical effects
Preface

This work was carried out in the Research Group of Electromechanics at Aalto University School of Electrical Engineering, Department of Electrical Engineering and Automation between September 2014 and September 2018. I am grateful for my supervisor Prof. Anouar Belahcen and instructors Asst. Prof. Paavo Rasilo and Dr. Floran Martin for their supervision and guidance during the course of this research. I am also thankful to Prof. Antero Arkkio for his support and constant encouragement during my doctoral studies. I wish to thank M.Sc. Ari Haavisto for his assistance during the experiments. The pre-examination of this thesis was performed by Prof. Xavier Mininger, Prof. Jean-Philippe LeCointe and Prof. Katsumi Yamazaki, whom I am grateful for their time and valuable comments.

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I am grateful for my parents and sister for their support in everything. Finally, I wish to thank my beloved wife, Justina for her support, patience and trust during these years.

Espoo, October 1, 2018,

Ugur Aydin
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List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.


Author’s Contribution

Publication I: “Coupled Magneto-Mechanical Analysis of Iron Sheets Under Biaxial Stress”

In this paper, the concept design of a novel rotational single sheet tester that is capable of applying arbitrary in-plane stress to steel sheets is proposed. In order to analyze the effect of biaxial stress on ferromagnetic sheets as well as to study the applicability of the concept design, a 2D strongly coupled magneto-elastic finite element model of the device was developed using a Helmholtz energy based material model. The simulation results are compared with the results published in the literature for the purpose of validation.

The publication is a joint work among the authors. Ugur Aydin completed the design of the tester geometry, developed a 2D coupled magneto-mechanical finite element model of the setup, and wrote the article. Paavo Rasilo provided the material model and helped with the design. Deepak Singh provided the magneto-mechanical measurements for model identification. Antti Lehikoinen provided an analytical model to verify the sample design. All the co-authors contributed through discussion and commenting on the paper.

Publication II: “Rotational Single Sheet Tester Device for Multiaxial Magneto-Mechanical Effects in Steel Sheets”

In this paper, a detailed design of the rotational single sheet tester device is given. Additionally, measurements on an M400-50A non-oriented electrical steel sheet under uniaxial and biaxial stress configurations with rotating and alternating magnetic field are performed. Results reveal that the effect
of multiaxial stress can be much more significant than that of uniaxial stress.

The publication is a joint work among the authors. Ugur Aydin completed setting up the device, developed the control programs, performed the experiments, analyzed the results, and wrote the article. Floran Martin helped with the flux density control program and sensor placements. Paavo Rasilo helped with analyzing the results. All the co-authors contributed through discussion and commenting on the paper.

**Publication III: “Effect of Multi-axial Stress on Iron Losses of Electrical Steel Sheets”**

In this paper, the effect of multiaxial stress on iron losses is studied. Magneto-mechanical measurements are performed on an M400-50A non-oriented electrical steel sheet using the designed and manufactured rotational single sheet tester. The effect of stress on hysteresis and excess losses are studied. A simple iron loss model to predict the effect of multiaxial stress on iron losses when only uniaxial stress dependent loss data is available is proposed. The modeling results are validated with the measurements.

The publication is a joint work among the authors. Ugur Aydin performed magneto-mechanical measurements, analyzed the results, implemented the model, and wrote the article. Paavo Rasilo helped developing the model. All the co-authors contributed through discussion and commenting on the paper.

**Publication IV: “Magneto-mechanical Modeling of Electrical Steel Sheets”**

In this paper, a simplified multiscale and Helmholtz energy based models for predicting the magneto-mechanical behavior of electrical steel sheets are compared. Two different grades of electrical steel sheets are modeled. The models are identified only by uniaxial stress dependent data. Comparisons show that both approaches are able to model one of the studied materials, whereas for the second material only the Helmholtz energy based model is successful.

The publication is a joint work among the authors. Ugur Aydin identified the models, ran the simulations, analyzed the results, and wrote the
article. The initial models were provided by Paavo Rasilo and Floran Martin. Deepak Singh, Laurent Daniel, Mahmoud Rekik and Olivier Hubert provided the experimental data. All the co-authors contributed through discussion and commenting on the paper.

**Publication V: “Modeling the Effect of Multiaxial Stress on Magnetic Hysteresis of Electrical Steel Sheets: A Comparison”**

In this paper, a simplified multiscale and Helmholtz energy based models that are implemented in the Hauser and Jiles-Atherton hysteresis model for predicting the multiaxial stress dependent hysteresis in electrical steel sheets are compared. The models are identified only from uniaxial stress dependent data. Comparisons between measurements show that both approaches provide reasonably accurate prediction of magnetic hysteresis under multiaxial stress.

The publication is a joint work among the authors. Ugur Aydin identified the models, ran the simulations, analyzed the results, and wrote the article. The initial models were provided by Paavo Rasilo and Floran Martin. Deepak Singh and Laurent Daniel provided the experimental data. All the co-authors contributed through discussion and commenting on the paper.
1. Introduction

1.1 Background

Increasing global energy consumption and global warming effects have led energy policy makers to seek energy efficient technologies. A report published in 2011 stated that rotating electrical machines account for between 43% and 46% of total global electricity consumption releasing 6040 Mt CO$_2$ annually (Waide and Brunner, 2011). In the European Union alone, there are around 5 million distribution transformers, which dissipate between 2.4% and 2.8% of total electricity production as losses, and they are responsible for 30 Mt CO$_2$ emission annually (Yurekten et al., 2013; VITO and BIOIS, 2011). No-load losses that occur in the transformer core account for about 70% of total losses (SEEDT, 2008).

At the same time, with the ongoing full electrification of industrial and transportation systems, authorities such as the International Electrotechnical Commission (IEC) have imposed IE efficiency standards for electrical motors (IEC:60034-34, 2014). An obvious way to increase efficiency is to reduce the losses. Naturally, in order to do that, the sources of the losses should first be clarified. For instance, the losses in rotating electrical machines are commonly divided into load losses, mechanical losses, iron losses and additional losses. Although the sources of additional losses are still unclear, the common belief is that the significant portion is caused by manufacturing processes.

Ferromagnetic steel sheets are widely used in rotating electrical machines, in addition to many other electromagnetic applications such as transformers and energy harvesters as core materials. These materials exhibit magneto-elastic phenomena such as so called magnetostriction and inverse magnetostriction. Briefly, magnetostriction causes deformation in
the material geometry when the material is exposed to a magnetic field. Inverse magnetostriction causes variations in the magnetic properties of the material when external mechanical stresses are present.

Considering the manufacturing processes of rotating electrical machines, the cores are punched, pressed, welded together and shrink fitted to the housing and shaft. Due to these processes, significant mechanical stresses are exerted on the core material. These stresses are multiaxial in nature, and they alter the magnetic properties of the core material. The orientation of the stress and magnetic field in the core may also vary during machine operation. Consequently, the losses in the core material are affected by these complex magneto-mechanical loadings. Usually, during the analysis, the multiaxiality of the magneto-mechanical loadings is neglected.

Magnetostriction and its inverse are not only responsible for the additional losses. For instance, magnetostriction causes stator vibrations in rotating electrical machines. In transformers magnetostriction is responsible for significant audible noise generation. On the other hand, energy harvesters, actuators, and sensors can be designed and manufactured to benefit from these magneto-elastic phenomena of ferromagnetic materials.

Clearly, in order to design more efficient devices and analyze existing ones with higher accuracy, comprehensive multiaxial magneto-mechanical characterization and modeling of ferromagnetic sheets are needed. This thesis aims to contribute to this field by studying the multiaxial magneto-mechanical interactions in electrical steel sheets both experimentally and numerically. A new test setup will be developed for experimental characterization. Then, computationally light models will be studied for simulating multiaxial stress dependent material behavior. Particular attention will be given to the characterization of non-oriented electrical steel sheets.

1.2 Aim and Focus of the Thesis

The aim of this work is to analyze the magneto-mechanical interactions in electrical steel sheets. Particular attention will be given to multiaxial stress effects on the magnetic properties of NO electrical steel sheets. A test setup capable of applying arbitrary in-plane magneto-mechanical loading to steel sheets will be designed, manufactured and used to perform experiments on a certain grade of NO electrical steel sheet. The goal of the experiments is to obtain comprehensive measurement data to analyze the effect of multiaxial stress on the magnetic properties and iron losses in the
Performing multiaxial magneto-mechanical tests to characterize the materials is a practically challenging task. Thus, predictive models are needed that can be identified from measurement data that can be obtained more easily. For instance, uniaxial stress dependent measurements can be performed with considerably simpler test setups. From the modeling point of view, the goal is to predict the effect of multiaxial stress on the magnetic properties and iron losses in the material with models that are identified only from uniaxial stress dependent measurements and that are reasonably accurate. In order to simulate electromagnetic applications using finite element analysis, the models also should be computationally light and convergent. Different approaches will be studied for modeling the effect of multiaxial stress on anhysteretic and hysteretic behavior of NO electrical steel sheets. In addition, a multiaxial stress dependent iron loss model will be developed.

1.3 Scientific Contribution

- A new rotational single sheet tester (RSST) device that is capable of applying arbitrary multiaxial in-plane magnetic and mechanical loading on steel sheets was designed and manufactured.

- Effects of multiaxial stress on the magnetic properties and iron losses of an M400-50A grade NO electrical steel sheet were analyzed by performing experiments using the manufactured RSST. Both rotating and alternating magnetic fields were considered.

- A simplified multiscale (SM) and Helmholtz energy based (HE) models were studied to predict the effect of multiaxial stress on anhysteretic magnetic behavior. The models were identified only with uniaxial stress dependent measurements. The modeling results were compared to measurements performed on three different materials using different test setups. The SM model was shown to be successful for predicting the stress dependent behavior only when the stress affects the material monotonically and only when one of the principal axes of the stress tensor aligns with the magnetization direction. Whereas the HE model proved to be successful for all the studied cases.

- In order to study the effect of stress on magnetic hysteresis, hysteresis
was included in the SM and HE models via two different approaches. Similar to the anhysteretic modeling results the HE model proved to be far more flexible and reasonably accurate for the studied materials.

- A simple iron loss model was developed to predict the multiaxial stress dependent iron losses utilizing statistical loss theory. The model considers the effect of stress on hysteresis and excess losses, and it is identified only from uniaxial stress dependent loss data. The model proved reasonably accurate for the studied M400-50A grade NO electrical steel sheet.

- A small transformer was built, and the iron losses were measured during the application of mechanical stress. The anhysteretic HE model and the developed iron loss model were implemented in a 2D FE model of the transformer. The inclusion of the stress dependent models resulted in more accurate loss calculations when mechanical stresses were present compared to a conventional loss model which does not account for the stress.

1.4 Outline

This thesis is divided into five chapters. The current chapter has presented a brief introduction to the research topic and a summary of the findings. Chapter 2 will review the literature relevant to this thesis. Chapter 3 will describe both experimental and modeling methods that are used in this work. In Chapter 4, the results of the performed studies will be presented. Finally, Chapter 5 will consist of, discussion and conclusions.
2. Review of Relevant Research

In this chapter, a review of the literature in fields related to this thesis is presented. First, the fundamental aspects of the magneto-mechanical interactions in ferromagnetic materials are explained. Next, a review of the macroscopic measurement methods of the magnetic properties of ferromagnetic sheets under mechanical stress is given. Following this, methods to model the magneto-mechanical effects in ferromagnetic materials are discussed. Particular attention is given to modeling the magnetic behavior under multiaxial mechanical stress. Finally, stress dependent iron loss models are reviewed, with the main focus on statistical loss models. It is worth mentioning that throughout the next sections, the focus is kept on Si-Fe non-oriented (NO) electrical steel sheets whenever possible.

2.1 Magneto-Mechanical Interactions in Ferromagnetic Materials

2.1.1 Fundamentals

Majority of the ferromagnetic materials consist of grains, which include magnetic domains that are formed due to the alignment of magnetic dipoles along certain directions over a volume. Therefore, magnetic domains have specific orientation with certain magnetization level so called saturation magnetization. Magnetic domains prefer to align along certain crystallographic axes, so called easy magnetization axes. Despite these easy magnetization axes, at equilibrium domains do not align along the same direction, but the alignment varies from domain to domain, resulting in the material being in a demagnetized state.

The existence of magnetic domains was first proposed by Weiss in the early 20th century (Weiss, 1906, 1907). The first direct confirmation of the existence of magnetic domains was done in 1931 by using fine...
magnetic powder and observing the domain patterns (Bitter, 1931). This was followed by Williams et al. (1949), who used magnetic colloids for their experiments.

The transition layers located between domains with different orientations are called Bloch walls and were suggested by Bloch (1932). Considering a sample with cubic symmetry such as iron, a crystal with several domains is illustrated in Fig. 2.1. The domains are separated with $180^\circ$ and $90^\circ$ walls. There are two mechanisms taking place in the material when an external magnetic field ($H$) is applied. The first is the domain wall movement that increases the volume of domains aligned in the direction of the applied field. As the applied field increases, the second mechanism of domain rotation occurs and results in the disappearance of the $90^\circ$ walls. At this stage, the material is saturated. At low to medium fields the domain wall movement is dominant; the rotation of domains occur mainly when high levels of magnetic field are applied. These mechanisms are illustrated for a domain family under an increasing external field in Fig. 2.1

![Figure 2.1. Illustration of effect of external magnetic field on the structure of a domain family.](image)

These mechanisms that occur in the domain scale cause changes in the macroscopic geometry of the material. This phenomenon is called magnetostriction and was first observed by Joule (1847) in the mid 19th century. Inversely, applied external mechanical deformations affect the magnetization mechanisms, causing variations in the magnetization of the material. This phenomenon was discovered by Villari (1865) and is called the Villari effect, or inverse magnetostriction.

### 2.1.2 Effects of Mechanical Stress on the Magnetic Properties of Ferromagnetic Materials

After the discovery of the magnetostriction and inverse magnetostriction phenomena the interaction between mechanical stress and magnetization in ferromagnetic materials has been of interest since the work of Ewing
The first extensive studies on the magneto-mechanical interactions were conducted by Kittel (1949), Bozorth (1951) and Lee (1955).

Bozorth and Williams (1945) studied the effect of small cyclic stress on iron-nickel samples and observed changes in induction, and they attributed these changes to a constant that is a function of saturation magnetostriction, saturation magnetization and anisotropy parameters of the material. The idea of an equivalent field, a fictitious magnetic field that would affect the material in a manner similar to the application of mechanical stress, was originated by the experiments of Lange and Fink (1943). These experiments led Brown (1945) to derive theoretical formulas for modeling the behavior of soft ferromagnetic materials under a low constant field and low tensile cyclic stresses. He calculated an equivalent field that would act on the domain walls as an application of stress. He also explained that the stress affects the $90^\circ$ domain walls and has no effect on $180^\circ$ domain walls. Brugel and Rimet (1966) continued the studies of Brown and studied the effects of higher stresses, and Schneider and Semcken (1981) extended their work to cover beyond the Rayleigh region. A review of Brown’s and Brugel’s work was given by Birrs (1971).

Brown’s theory predicts that low tension and compression would cause the same effect on the magnetization of a ferromagnetic material that is under constant low magnetic field. This was experimentally confirmed for instance by Craik and Wood (1970), although the magnitude of the changes was different depending on the sign of applied stress (compression or tension).

However, at higher fields, the effect of compression or tension on magnetization depends upon whether the material has a positive or negative magnetostriction coefficient (Bozorth, 1951). For materials with a positive magnetostriction constant such as Terfenol-D, material expands with the application of the magnetic field. For these materials, the application of tension along the magnetization direction increases magnetization, whereas compression reduces it. On the other hand, for materials with a negative magnetostriction constant such as nickel, the effect is the opposite (Bozorth, 1951). This behavior can be described for a single domain based on field-stress energy from Becker and Döring (1939) as

$$E_\sigma = \frac{3}{2} \lambda_s \sigma \sin^2 \theta$$

(2.1)

where $\lambda_s$, $\sigma$ and $\theta$ are the saturation magnetostriction constant, the applied...
stress and the angle between the direction of the saturation magnetization ($M_s$) and the applied stress, respectively. This expression indicates that, for instance, for a positive $\lambda_s\sigma$ the minimum energy is obtained when $\theta = 0^\circ$ or $180^\circ$, which is the condition when $\sigma$ is parallel to $M_s$. Now, let us consider a domain family as shown in Fig. 2.2 (a) at a demagnetized state. When tensile stress is applied to this domain family, as shown in Fig. 2.2 (b), the energies of domains 2 and 4 will increase ($\theta = 90^\circ$), whereas the energies of domains 1 and 3 will not change ($\theta = 0^\circ$). As a result, the domain walls will move in a way to reduce energy, resulting in decreased volume of domains 2 and 4, and this causes the direction of the applied tension to be easier magnetization direction as illustrated in Fig. 2.2 (b).

Figure 2.2. Illustration of effect of tensile stress on the structure of a domain family. (a) Stress free condition, (b) domain family structure under tensile stress.

In fact, (2.1) is an oversimplification with respect to the polycrystalline materials such as iron, since they are anisotropic and consist of many grains, which include domains, domain walls, and impurities. Therefore, the behavior of polycrystalline materials under stress is complicated. As a sidenote, more detailed energy terms including anisotropy and domain wall motions can be found for instance in the works of Kersten (1938) and Chikazumi (1964). Excellent reviews on the subject were written by Chikazumi (1964), de Lacheisserie (1993) and de Lacheisserie et al. (2005).

In Fig. 2.3, anhysteretic $B$-$H$ curves of iron, which is a positive magnetostrictive material under various tensile stresses, applied parallel to the magnetization direction are shown, as reported by de Lacheisserie (1993) after the experiments of Villari (1865). Under low tensile stress, the permeability of iron increases under the knee region of the curve. When high tensile stress is applied, the permeability still improves at low fields, but at higher fields it decreases. This is due to the Villari reversal, which is observed for other alloys as well either for compressive or tensile stress (Langman, 1985; Bieńkowski and Kulikowski, 1991; Devine, 1992; Bieńkowski, 2000; Bieńkowski et al., 2008). This phenomenon is attributed to the contribution of stress to magnetic anisotropy energy (de Lacheisserie,
1993; de Lacheisserie et al., 2005). On the other hand, compression deteriorates the material permeability along the applied direction for positive magnetostrictive materials (Moses and Davies, 1980; Moses, 1981; Devine, 1992; LoBue et al., 2000; Kai et al., 2011a; Rekik, Hubert and Daniel, 2014).

Moreover, the magnetostriction behavior of ferromagnetic materials varies with the application of stress (Banks and Rawlinson, 1967; Atherton and Szpunar, 1986; Yamasaki et al., 1996; Yamamoto and Yamashiro, 2003; Xu et al., 2009). It has been reported that positive magnetostrictive materials start producing negative magnetostriction when externally applied tensile stress is beyond a certain magnitude, which is the Villari reversal point (Banks and Rawlinson, 1967; Moses, 1979; Anderson et al., 2007). Naturally, both of these behaviors are observed in Si-Fe non-oriented electrical steel sheets, which particularly fall under the scope of this work (Pulnikov, 2004; Pulnikov et al., 2005; Yamamoto and Yanase, 2011; Kai et al., 2011a, 2012; Senda et al., 2013; Singh, 2017).

![Figure 2.3](image.png)

**Figure 2.3.** Anhysteretic $B$-$H$ curves of iron under various tensile stresses as reported in de Lacheisserie (1993) after Villari (1865).

### 2.2 Macroscopic Measurements of Magnetic Properties of Ferromagnetic Steel Sheets Under Stress

In order to characterize the stress free magnetic properties of electrical steel sheets, standard methods are often used. Among others, these are the Epstein frame (ASTM:A343/A343M-03, 2003; IEC:60404-2, 2008), ring core (ASTM:A927/A927M-99, 1999), single sheet tester (IEC:60404-3, 2000)
in the case of characterization under alternating flux. In the case of characterization under rotational flux, there are no standardized methods available yet. However, various rotational single sheet tester (RSST) devices have been built previously (Sievert et al., 1992; Zouzou et al., 1992; Hasenzagl et al., 1996; Zurek and Meydan, 2003; de la Barrière et al., 2012). The main aim of these tests is to provide magnetic power losses and $B-H$ loops under certain magnetization frequencies and flux density levels.

Since there are no standard methods available for magnetic property measurements of electrical steel sheets under stress, many different test setups have been developed previously. In this work, these test setups are categorized into two main groups as uniaxial and multiaxial test setups, based on the mechanical loading they are capable of applying. Considering the plane of the sheet, uniaxial test setups are able to apply mechanical stress only along a single axis whereas multiaxial ones can apply stresses along two or more axes. In general, multiaxial test setups can be used for uniaxial testing as well. In this section, general principles of designing these test setups will first be discussed. Then, previously built uniaxial and multiaxial test setups will be reviewed. Particular attention will be paid to the magneto-mechanical measurements of non-oriented electrical steel sheets.

2.2.1 General Principles

One of the most important aspects of designing magneto-mechanical test setups is to ensure uniform magnetic flux density and stress distribution in the measurement area of the sample. In order to ensure these criteria, careful design of the sample geometry is crucial. In addition to sample geometry, the magnetizing system can play a role in the field and stress homogeneity. Any yoke system that is in contact with the sample has the potential to alter the stress distribution. Furthermore, vertical yoke systems, if not designed properly can affect the field distribution due to eddy currents caused by vertical flux penetration to the sample (Sievert, 2011). During the design of the sample and yoke geometries, 2D or 3D electromagnetic and mechanical finite element analysis (FEA) is often used.

A review of the effects of various sample and magnetizing yoke geometries on the field distribution considering the rotational magnetic field was given by Bottauscio et al. (2005) and Sievert (2011). Some of the finite element method (FEM) simulations on different geometries performed by
Bottauscio et al. (2005) are reproduced and shown in Fig. 2.4. The samples have similar dimensions and are placed in the central region of the magnetizers, and in each setup there is a 1 mm airgap between the yokes and the sample. It can be seen that different types of yoke and sample geometries can have a significant effect on the flux density homogeneity in the sample. Among the studied geometries, considering the rotational field, the 3-phase magnetizing setup with a hexagonal sample in Fig. 2.4 (d) provides the best field homogeneity in the sample (Bottauscio et al., 2005). In fact, as Bottauscio et al. (2005) point out, the highest homogeneity would be obtained with a stator-like magnetization system and a round sample. However, with this type of geometry, it is very challenging to design a mechanism to apply mechanical stresses to the sample.

![FEM results of magnetic flux density distributions for different horizontal yoke types and sample geometries. (a) and (b) 2-phase magnetizing setup with square and circular sample geometries, respectively. (c) and (d) 3-phase magnetizing setup with circular and hexagonal sample geometries, respectively. Reproduced after Bottauscio et al. (2005).](image)

In magneto-mechanical setups for single sheet testing, typically search coils or needle probes are used to measure $B$. For magnetic field strength ($H$) measurements, the common method is to use $H$-coils. Reviews of these field measurement techniques were given, for example, by Guo et al. (2008) and Sievert (2011). On the other hand, strain gauges are commonly used to measure magnetostriction due to their simplicity and accuracy (Somkun, 2010).
2.2.2 Measurements Under Uniaxial Stress

Previously, various sample geometries and magnetizing systems have been adopted for uniaxial magneto-mechanical testing. For instance, Banks and Rawlinson (1967) used a test setup with rectangular samples and with direct magnetization of the samples to test the effect of uniaxial tensile and compressive stresses applied along longitudinal and transverse directions on the magnetostriction of 3% Si-Fe grain oriented (GO) electrical steel sheets.

A study on magnetostriction in addition to magnetic losses on a 3% Si-Fe GO electrical sheet was performed by Moses and Davies (1980). They used an apparatus to apply uniaxial tension or compression to an Epstein sample. The sample was magnetized with flux density up to 1.7 T, which was controlled to be sinusoidal using a negative feedback control method. Magnetic flux density, magnetic field strength, and magnetostriction were measured by search coil, $H$-coil, and strain gauge, respectively.

Later, Foster (1984) studied the effect of applied uniaxial tensile stress along magnetization direction in GO and NO materials using a modified Epstein frame. Details about the stress application method were not provided.

A custom made single sheet tester was used by Dąbowski and Zgodziński (1989) to apply uniaxial tension and compression to Si-Fe GO and NO electrical steel sheet samples. The sample geometry was chosen to be 600 mm long and 10 mm wide in order to obtain relatively homogeneous stress and magnetic field distribution across the samples. Details about the application of the stress and flux density control were not provided.

Similarly, LoBue et al. (2000) customized a single sheet tester to hold a spring device for application of uniaxial tension and compression. They performed uniaxial magneto-mechanical measurements on a 3% Si-Fe NO electrical steel under several mechanical stress levels and controlled sinusoidal flux density with various amplitudes and frequencies.

Hasenzagl et al. (1996) designed a measurement setup with a 3-phase excitation system to obtain arbitrary flux density waveform in a hexagonal shaped sample. The magnetization system consisted of six poles with six exciting coils. Later the system was modified to apply uniaxial stress along a certain direction (Krell et al., 2000). In that study, stress was applied by using a clamping device that yielded $\pm 3$ MPa along the rolling direction on a Si-Fe GO electrical steel sheet. Magnetic flux density and field strength
was measured by means of a needle probe and \( H \)-coils. The flux density was controlled to be an elliptical shape.

The effects of uniaxial tensile stress on Si-Fe NO electrical steel sheet were studied by Permiakov et al. (2002), using a custom made single sheet tester device with two excitation systems that adopt a horizontal magnetizing circuit. Magnetic field strength and flux density were measured by \( H \)-coil and needle probes, respectively, and \( B \) was controlled to be sinusoidal in the measurement area. The uniaxial stress was applied along the magnetization direction until the sample was plastically deformed. Later, this setup was also used to investigate the effect of uniaxial stress on a NO Si-Fe electrical steel sheet under rotational magnetization (Permiakov et al., 2004b). Some other measurement setups to test the effect of uniaxial stress on the magnetic behavior of electrical steel sheets were developed for instance by Belahcen (2004); Miyagi et al. (2010); Kanada et al. (2011); Singh (2017); Leuning et al. (2016).

### 2.2.3 Measurements Under Multiaxial Stress

A study to test the effect of biaxial stress on the magnetic behavior of a 3% Si-Fe GO electrical sheet was performed by Moses and Phillips (1978) using an apparatus capable of applying simultaneous tensile or compressive stresses along and perpendicular to the rolling direction, as well as sinusoidal magnetization up to 1.7 T along either direction. The setup is designed to stress a square shaped sample by adding weights to mechanical carriers. A vertical yoke system was adopted for magnetization. Search coils and double \( H \)-coils were used to measure magnetic flux density and field strength, respectively.

Basak and Moses (1978) used a cross shaped sample with magnetization coils wound around the legs of the sample to characterize the magnetic properties of different grades of Si-Fe GO and NO electrical sheets under rotational magnetization. In addition to uniaxial stresses, biaxial loadings were applied by coil springs simultaneously perpendicular to each other. Magnetic flux density was measured by search coils and power losses were obtained from temperature rise measurements.

Pearson et al. (2000) developed a test setup to apply biaxial stresses and alternating magnetization to a 40 mm disc shaped thin sample. The forces were generated by electromagnetic actuators, and a magnetic field was applied using a pair of quasi-Helmholtz coils. The homogeneity of the field and stress in the measurement area were ensured by simulations and
measurements. The surface field strength was measured by a Hall probe, and magnetization was calculated by using an external applied field and estimated demagnetization factor.

A measurement setup with a cross shaped sample was developed by Hubert et al. (2005) for biaxial tests. Later, this setup was improved by Rekik, Hubert and Daniel (2014), and measurements were performed for a 3% Si-Fe NO electrical steel sheet. Stresses up to 100 MPa were applied to each leg of the sample with hydraulic actuators and the sample was magnetized by a U-core vertical magnetization system. Magnetic flux density and field strength were measured by needle probes and \( H \)-coils. A triangle magnetization current waveform was supplied to the coils for magnetizing the sample.

Kai et al. (2011a) built a test setup with a cross shaped sample geometry that is capable of applying biaxial stress and arbitrary magnetization. Search coils and \( H \)-coils were used to measure flux density and field strength, respectively. The waveform of the flux density was controlled to be sinusoidal. They studied the effect of uniaxial stress applied parallel or perpendicular to the magnetization direction on a NO electrical steel sheet using this apparatus (Kai et al., 2011a). The same setup was also used to study the effect of uniaxial stress on the rotational losses in NO electrical steel sheets (Kai et al., 2012).

Another measurement setup was introduced by Kai et al. (2014b) with the ability to apply arbitrary magnetization and in-plane stress tensors to electrical steel sheets. The sample geometry has 8 legs, and stresses were applied to each leg by ball screw guides that were driven manually. The sample was magnetized by two excitation coils and a vertical yoke system. Magnetic flux density and field strength was measured by a needle type V-H sensor, and flux density was controlled to be sinusoidal. Measured maximum flux density amplitude was 0.8 T. The test setup was used to test the effects of uniaxial and shear stress on the magnetic properties of NO electrical steel sheets under alternating and rotational fields (Kai et al., 2014b; Kai and Enokizono, 2017).

The above mentioned studies report results that agree with each other concerning NO electrical steel sheets under alternating magnetization. The application of uniaxial compression parallel to magnetization direction deteriorates the permeability and increases the losses significantly. Application of low uniaxial tensile stress up to about 20 MPa to 30 MPa increases the permeability and reduces the losses whereas, higher tensile
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stress slightly decreases the permeability and increases the losses.

In the case of rotational magnetization, different results were reported by Permiakov et al. (2004b) and Kai et al. (2012). Both studies reported that when the sample is under circular magnetization, the application of uniaxial compression increased the losses. Concerning tensile stress, Permiakov et al. (2004b) reported slight decrease in the losses at low tensile stress levels and increase at high levels. On the other hand, Kai et al. (2012) reported a similar increase in the losses under tensile stress to the case of compressive stress.

The application of uniaxial tensile stress perpendicular to the magnetization direction affects the material in a similar manner when compression along the magnetization direction is applied. Similarly, compression perpendicular to the magnetization direction behaves analogous to a tension along the magnetization direction as shown by (Moses and Phillips, 1978; Kai et al., 2011a). Therefore, with the application of equal biaxial (equibiaxial) stress, one would expect that the material would not be affected significantly. On the other hand, application of compression parallel to magnetization and tension perpendicular to it should have a large effect on magnetic properties. The above mentioned biaxial studies confirm this behavior. In addition, it has been reported that pure shear stress causes significant increase in the losses in NO electrical steel sheets considering both alternating and rotational magnetization (Kai et al., 2014a; Kai and Enokizono, 2017).

2.3 Modeling the Effect of Stress on The Magnetic Properties of Ferromagnetic Materials

As mentioned earlier, many of the electromagnetic device cores are subject to multiaxial magneto-mechanical loadings that affect their performances. In order to characterize different core materials under these conditions, performing multiaxial magneto-mechanical measurements is practically a difficult and time consuming task. Predictive models based on the physical phenomena are therefore needed. Indeed, this could possibly be achieved by creating very comprehensive and complex models at the domain level. However, the computational burden of such models restricts the applicability of these models to large scale simulations such as FEA, which are needed to design more efficient devices. In previous years, there has been interest in developing predictive models that are easy to identify,
This section presents some magneto-mechanical models to take into account the effect of stress on the magnetic properties of ferromagnetic materials, with particular focus on electrical steel sheets. Although magneto-mechanical models that are able to take multiaxial behavior into account fall under the main scope of this thesis, some of the uniaxial models that drew attention in previous years will be reviewed briefly as well.

2.3.1 Uniaxial Models

Earlier, a common approach to model the macroscopic magneto-mechanical behavior of the ferromagnetic materials was to extend already working models that are able to describe the magnetization mechanism in the material. Among these, two models were particularly successful in describing stress free magnetic hysteresis, which were proposed by Preisach (1935) and Jiles and Atherton (1983).

The modifications on the Preisach model were proposed for instance by Bergqvist and Engdahl (1991) by introducing an effective field to account for the effect of uniaxial stress. Sipeky and Ivanyi (2008) introduced stress dependency into the Preisach distribution function to model the effect of uniaxial tensile stress on magnetic hysteresis.

The Jiles-Atherton model was modified by Sablik et al. (1987) by introducing an effective magnetic field to take the effects of uniaxial stress into account. The model was improved later to take into account the magnetostrictive hysteresis (Sablik and Jiles, 1993) and the Villari reversal (Sablik, 1997). Some modifications to the model of Sablik et al. were proposed more recently in (Li and Xu, 2011; Singh et al., 2016).

These models are usually not preferred for modeling electromagnetic devices, especially for optimization purposes, due to their low predictive capabilities, relatively high computational costs, and convergence issues in numerical simulations. For modeling the applications, a common approach is to use single-valued $B$-$H$ curves instead of hysteretic $B$-$H$ loops to describe the magnetization in the machine cores. In fact, it has been shown by Deblecker et al. (2007) and Rasilo et al. (2012) that the errors caused by adopting this approach are small considering loss calculations, and it can be beneficial in terms of computational speed and convergence compared to when hysteresis models are used.

In order to describe the stress dependency of the single-valued $B$-$H$ curves, a study has been performed by Fujisaki and Satoh (2004), where
they parametrized the permeability based on the uniaxial measurements. Belahcen and Arkkio (2008b) developed an analytical formula to express the dependency of permeability on uniaxial stress. The expression includes a few parameters to be identified from measured single-valued $B$-$H$ curves under stress. For example, this model was used in finite element modeling of a surface-mounted permanent magnet synchronous machine by Abdallh and Dupré (2014) to study the effect of stress on the design parameters of the machine. However, these models also lack the ability to predict behavior under magneto-mechanical loadings other than those with which they are identified.

2.3.2 Multiaxial Models

For modeling the multiaxial stress effects on the magnetic properties of electrical steel sheets, several approaches have been used previously. Among these, equivalent stress, multi-scale, and macroscopic energy based approaches will be reviewed.

*Equivalent Stress Models*

The concept of equivalent stress models is based on the assumption that any change caused in magnetic behavior by multiaxial stress can be modeled by an appropriate fictive uniaxial stress (equivalent stress). This allows predicting the multiaxial magneto-mechanical behavior by using only uniaxial models and the measurements under uniaxial stress for required model identification. In past years, several equivalent stress definitions were proposed based on energetic considerations and experimental data. Some of the well known equivalent stress definitions only consider biaxial mechanical loading where the applied field is along one of the principal axes of the applied stress (Schneider and Richardson, 1982; Kashiwaya, 2014; Sablik et al., 1994). Thus, these models are not suitable for modeling many electromagnetic applications, since the relative orientation between the stress and magnetic field usually varies in their core.

In order to overcome this problem, Daniel and Hubert (2009) proposed an equivalent stress definition based on equating the isotropic magneto-elastic energy under multiaxial and uniaxial stresses. Later, Hubert and Daniel (2011) extended this equivalent stress for orthotropic media under disoriented magneto-mechanical loadings, and they proposed a new equivalent stress definition of so called generalized equivalent stress. The equivalent
stress proposed by Daniel and Hubert (2009) has subsequently been used in modeling various applications (Krebs and Daniel, 2012; Yamazaki and Kato, 2014; Yamazaki and Fukushima, 2015; Gueye et al., 2016).

Equivalent stress models provide simple implementation and fast computation. However, accuracy of equivalent stress models is low, especially under certain multiaxial stress states. A review on several equivalent stress definitions was given by Hubert and Daniel (2011) where the lack of accuracy of the studied models under some multiaxial stress states can clearly be observed. In addition, so far the developed equivalent stress models cannot take into account the stress induced anisotropy in the material.

**Multiscale Models**

Another approach to model the multiaxial magneto-mechanical interactions in electrical steel is the multiscale approach, which aims to deduce macroscopic behavior by taking advantage of the physical description of the magneto-mechanical interactions at the domain scale at a reduced cost. The main advantages of multiscale models are that to be identified, they require few physical parameters that can be obtained relatively easily, and they can predict magneto-elastic behavior across a large range of multiaxial loadings. Briefly, the working principle of these models is given by the following:

- Calculating the magneto-mechanical loadings at the domain scale by localization of the externally applied macroscopic loadings.
- Defining a local free energy at the domain scale and describing the magneto-elastic behavior of a single crystal by utilizing this energy.
- Obtaining macroscopic magneto-elastic behavior of the polycrystal by homogenization of the local behavior.

The energy definition at the domain scale is the critical part and is usually given as the sum of several potential energy contributions such as magnetostatic, magneto-elastic, and anisotropy energies. More details about the energy contributions were presented by Armstrong (1997). The local magneto-mechanical behavior can be obtained by minimizing the total potential energy, as suggested by Buiron et al. (1999) or by using a Boltzmann type distribution function, which is a simpler and faster way. This latter approach is commonly used in magneto-elastic multiscale models (Buiron et al., 1999, 2001; Daniel et al., 2008; Vanoost et al., 2016).

The anhysteretic multiscale model of Daniel et al. (2008) is extended
to consider the hysteresis by Daniel et al. (2014) by introducing an irreversible contribution based on the hysteresis model of Hauser (2004). In addition, in the same work, they introduced a configuration field to take into account the non-monotonic behavior under stress (Villari reversal). Later, Vanoost et al. (2017) proposed a correction parameter for calculation of the Boltzmann distribution in order to model non-monotonic behavior under stress with better accuracy. This new parameter needs to be identified using uniaxial stress dependent measurements, and the physical meaning of the introduced parameter is not known. Additionally, the effect of the correction factor on modeling the magnetostriction was not discussed.

Although the computational cost of the multiscale models is reduced compared to the micromagnetic models, due to localization and homogenization schemes it is still too costly to use these models in the numerical simulations of electromagnetic applications. To overcome this issue, Bernard et al. (2011) proposed a simplified version of the anhysteretic full multiscale model of Daniel et al. (2008). In the simplified approach, the material is described as an isotropic fictitious single crystal with properties identified from polycrystal behavior. Similar to the full multiscale model, the simplified model requires only a few physical parameters to be identified. This approach allows skipping the localization and homogenization procedures, allowing an increase in computational speed up to 1000 times greater than the full multiscale model. It was possible to use the simplified multiscale model proposed by Bernard et al. in a 2D finite element model of a switched reluctance motor to study the effect of stress on the machine performance (Bernard et al., 2011).

Later, Daniel et al. (2015) extended the simplified multiscale model to include the hysteresis effects using the Hauser hysteresis model. Similarly, Bernard and Daniel (2015) incorporated the simplified multiscale model to the Jiles-Atherton hysteresis model and implemented it in a 2D finite element model of a switched reluctance motor successfully. The simplification of the full multiscale model comes at a price of reduced accuracy for describing and predicting magneto-mechanical behavior. The accuracy of the simplified multiscale model to predict the multiaxial magneto-mechanical behavior of electrical steel sheets is studied throughout this thesis. Nevertheless, a comparison of full and simplified multiscale models for uniaxial loadings was also given by Vanoost et al. (2017).
Macroscopic Energy Based Models

Macroscopic energy based models are commonly based on determining material behavior using a specific free energy function. Thermodynamic principles are commonly adopted to derive the energy functions in a continuum media with the presence of magnetic fields and mechanical deformations. For describing the nonlinear magneto-elastic interactions, these energy functions are commonly chosen to be polynomials of high order, with several constants to be identified from the experiments. Energy based approaches can provide a suitable description of the material with regards to computational cost and accuracy.

An important study on the topic was performed by Brown (1965) based on the earlier works of Toupin (1956) and himself (Brown, 1951). In this study, Brown derived an energy function depending on deformation gradient and magnetization. He obtained expressions for stress tensor and magnetic field in terms of derivatives of this energy. Brown’s work directly and indirectly influenced many researchers in the field. Extensive and more general studies on the linear and nonlinear continuum magneto-elasticity can be found in the works of Maugin (1988), and Kovetz (2000).

Previous research on developing models for coupled nonlinear magneto-elastic interactions with energy based approaches was mainly focused on modeling the giant magnetostrictive materials. Among these so called standard square (SS) model was developed by Carman and Mitrovic (1995) from a Gibbs energy that is expanded in a Taylor series. Although the SS model is successful predicting the magnetostriction induced by low to mediocre magneto-mechanical loadings, it fails to model the behavior under saturation. In order to model saturation effects, another model was developed by Duenas et al. (1996) based on expressing a Gibbs free energy including hyperbolic tangent functions. In this model the magnetostrictive strain is expressed to be independent of the applied stress, but the effect of stress is modeled indirectly through the stress dependency of the magnetization. Although this approach produces more accurate results than the SS model, it fails to produce good quantitative agreement compared to the experimental results (Duenas et al., 1996). Wan et al. (2003) proposed a so called density domain switching (DDS) model suggesting that magnetostrictive strain appears to be due to the switching of the domains in the material. The DDS model can predict the saturation effects but with large errors compared to experimental results. Later, Jin et al. (2011) proposed a more general 3D magneto-elastic constitutive law based on the Taylor
series expansion of the Gibbs free energy. They used the Langevin function
to express stress free magnetization instead of hyperbolic tangent, as in the
model from Duenas et al. (1996). They also included a thermal expansion
term in the energy function to account for the temperature effects (Jin
et al., 2011). Good agreement between the modeling and measurement
results under uniaxial stress was reported.

Dorfmann and Ogden (2003) and Dorfmann et al. (2004) took a different
approach to model magneto-sensitive elastomers. They determined a set of
invariants as a function of deformation gradient and magnetic flux density
and expressed a Helmholtz energy function using these invariants to model
the magneto-elastic behavior of magneto-sensitive materials. However,
they did not verify the applicability of the models experimentally. Belahcen
et al. (2006) and Belahcen et al. (2008) followed a similar approach to model
the magneto-mechanical behavior of electrical steel sheets, assuming small
deformations. They used invariants expressed as functions of the total
strain and magnetic flux density and derived a Helmholtz energy function
depending on these invariants. Later, Fonteyn (2010) modified the energy
function proposed by Belahcen et al. (2008) and reported that the developed
model was successful in modeling the magnetization and magnetostriction
curves of NO electrical steel sheets under relatively low uniaxial stresses
and including magnetic saturation effects. In addition, the model was
computationally light and was implemented into a finite element model of
an induction machine (Fonteyn, 2010; Fonteyn et al., 2010b,a).

None of the above mentioned studies include hysteresis effects. Pre-
viously, Linnemann et al. (2009) developed a model with hysteresis for
magnetostrictive materials based on the definition of a free energy function
and domain switching criterion. Hysteresis behavior was approximated by
splitting the strains and magnetic fields into reversible and irreversible
components. Another approach to take into account the hysteresis effects is
to implement the anhysteretic models to the stress free hysteresis models
such as the Jiles-Atherton model. For instance, Jin et al. (2012) described
the hysteresis of a Terfanol-D rod and TbDyFe amorphous thin film alloy
incorporating the anhysteretic model from (Jin et al., 2011) to the Jiles-
Atherton hysteresis model. Similarly, Rasilo et al. (2016) modeled the
hysteresis behavior of NO electrical steel sheets under several uniaxial
stress states by implementing the anhysteretic model from (Fonteyn, 2010)
to the Jiles-Atherton model. With this study it was shown that the model
of (Fonteyn, 2010) is also successful at mediocre stress levels from –50 to
80 MPa.

None of the above mentioned studies looked into the predictive abilities of the developed models under multi-axial magneto-mechanical loadings. The accuracy of the slightly modified version of the model from (Fonteyn, 2010) to predict the multiaxial magneto-mechanical behavior of NO electrical steel sheets will be studied throughout this thesis.

2.4 Iron Loss Modeling Under Stress

It is now well established that mechanical stress has significant effects on the magnetic properties of ferromagnetic materials. A conventional way to obtain the stress free losses in electromagnetic devices is to use statistical iron loss models such as those developed by Jordan (1924) or Bertotti (1988) at the post-processing stage of the simulations such as FEA. This way the losses are calculated quickly and easily, since these loss models are just analytical expressions with few coefficients. Therefore, it would be convenient to model the effect of stress on the losses based on these expressions.

In this section, statistical iron loss segregation models are reviewed briefly. First, main focus is given to the models based on the well known Steinmetz loss theory (Steinmetz, 1892). Later, the previous research that reports the effects of mechanical stress on different iron loss components is discussed.

2.4.1 Iron Loss Segregation

Steinmetz (1892) proposed an analytical expression to describe the iron losses in ferromagnetic materials that are under uniaxial sinusoidal excitation. He separated the total iron loss density ($p_{Fe}$) into hysteresis loss ($p_{hy}$) and classical eddy current loss ($p_{cl}$) components and proposed the following empirical equation:

$$p_{Fe} = c_{hy}f B_{p}^{1.6} + c_{cl} f^{2} B_{p}^{2},$$  \hspace{1cm} (2.2)

Here, $B_{p}$ and $f$ are the peak magnetic flux density and frequency, respectively. The hysteresis and eddy current loss coefficients, $c_{hy}$ and $c_{cl}$ are to be determined from iron loss measurements. Various modifications were proposed to improve the Steinmetz equation in previous years. A summary of these modifications was given by (Krings and Soulard, 2010). Among
these, well known models were proposed by Jordan (1924) and Bertotti (1988). The Jordan model is given by

\[ p_{Fe} = c_{hy} f B_p^2 + c_{cl} f^2 B_p^2. \] (2.3)

Bertotti (1988) included an excess loss term \( p_{ex} \) in the Jordan model and proposed a loss model that is given by

\[ p_{Fe} = c_{hy} f B_p^2 + c_{cl} f^2 B_p^2 + c_{ex} f^{1.5} B_p^{1.5}. \] (2.4)

Here, \( c_{ex} \) is the excess loss coefficient to be determined from the experimental data. When a ferromagnetic material is magnetized, hysteresis occurs due to the discontinuous movement of the domain walls, which is caused by impurities in the material. The classical eddy current losses are assumed to be caused by the macroscopic eddy currents induced by time varying magnetization and that they are not affected by the domain structure of the material (Bertotti et al., 1988). The cause of the excess losses was attributed to the microscopic eddy currents that are caused by domain wall motions during the magnetization process. More detailed physical interpretation was given by Stewart (1950), Graham (1892) and Bertotti (1988).

The loss coefficients \( c_{hy} \) and \( c_{ex} \) in (2.4) are usually determined using a set of magnetization measurements under sinusoidally varying flux density at different frequencies and induction amplitudes (Pluta, 2010; Ionel et al., 2006). The instantaneous eddy current losses can be calculated by (Bertotti, 1998)

\[ p_{cl} = \frac{d^2}{12 \rho} \left( \frac{dB}{dt} \right)^2 \] (2.5)

where \( \rho \) and \( d \) are the resistivity and the thickness of the lamination, respectively. Assuming sinusoidal flux density over the thickness of a lamination (2.5) becomes

\[ p_{cl} = \frac{\pi^2 d^2}{6 \rho} B_p^2 f^2 \] (2.6)

resulting in a simple analytical expression for \( c_{cl} \).

Using the above mentioned loss models that assume sinusoidal flux density waveform for loss estimation under non-sinusoidal flux density
could result in significant errors (Fiorillo and Novikov, 1990; Li et al., 2001; Kowal et al., 2015). Therefore, some models to estimate the losses under non-sinusoidal induction waveforms were proposed, see for instance (Fiorillo and Novikov, 1990; Li et al., 2001; Van den Bossche et al., 2004; Roshen, 2007).

In order to model the losses more accurately under rotational fields, several models were developed. Bertotti et al. (1991) proposed a correction factor to calculate the rotational hysteresis losses from the alternating losses. They calculated the eddy current and excess losses as the sum of the losses obtained for two alternating orthogonal components. Later, Zhu and Ramsden (1998) proposed an analytical expression with four fitting parameters for modeling circular hysteresis losses. They modeled the eddy current and excess losses as in (2.4). They also gave an expression to estimate the elliptical losses from alternating and purely circular losses. Belahcen and Arkkio (2008a) proposed an expression with four fitting parameters to take into account the rotational hysteresis losses. The eddy current and excess losses were calculated for two alternating orthogonal components of the flux density waveform. Later, Belahcen et al. (2014) improved this approach and included a 1-D numerical eddy current model to obtain the eddy current loss component.

2.4.2 Modeling the Effect of Stress on Different Iron Loss Components

In previous years, modeling the stress dependency of different iron loss components did not draw very much attention, especially considering the effect of multiaxial stress. In order to study the stress effects on different loss components, a common way is to perform magneto-mechanical experiments at various peak flux density magnitudes and frequencies and obtain different loss components using the Jordan or Bertotti loss models. As discussed previously, the externally applied elastic stress mainly affects the domain wall movement of the ferromagnetic materials. Since it is assumed that the eddy current losses are of a material uniformly magnetized and free of any domain structure (Bertotti et al., 1988), the applied elastic stress does not have any effect on them (Permiakov et al., 2004a). In fact, the permeability of the material is affected by the stress; therefore, stress can alter the skin depth. Depending on the frequency of the applied field, this indirect effect of stress on skin depth could affect the eddy current losses, but skin effect is usually neglected for frequencies below 200 Hz.
Fiorillo et al. (1980) studied the effect of uniaxial stress on different loss components for a GO Si-Fe material. It was reported that the 50 MPa tensile stress reduced the dynamic losses compared to the stress free case. Based on these experimental results, Bertotti (1985) studied the effect of uniaxial tensile stress on the excess losses and explained the effects with the help of the magnetic objects concept, which defines a group of domain walls evolving in a strongly correlated way.

Later, LoBue et al. (1999) showed the evolution of the hysteresis and excess losses with uniaxial compression and tension up to 50 MPa using the statistical loss model of Bertotti for a NO Si-Fe electrical steel sheet. It was shown that the stress affected both the hysteresis and the excess losses in a similar manner, such that the compression increased the losses significantly and tensile slightly reduced them. Similar results were obtained by Permiakov et al. (2004a) who also reported increase in both loss components at tensions higher than 30 MPa. Karthaus et al. (2017) separated uniaxial tensile stress dependent core losses of a NO electrical steel sheet into the hysteresis, excess and non-linear loss components using a modified version of Bertotti loss model from (Eggers et al., 2012). It was shown that the tensile stress affected the hysteresis and non-linear loss components, whereas the effect on the excess loss component was insignificant.

One of the few attempts to model the stress dependency of different loss components was performed by Ali et al. (1997). They segregated the iron losses measured on two different grades of NO electrical steel sheets under uniaxial stress into hysteresis, classical and excess loss components. Then they proposed analytical expressions with fitting parameters to take into account the hysteresis and excess loss evolutions with uniaxial stress. Another study was performed by Singh et al. (2015) with a wide range of data for a non-oriented Si-Fe electrical steel sheet. They proposed two models with a few fitting parameters based on the statistical loss theory of Bertotti to take into account the uniaxial stress effects on the excess loss component.

Although the aforementioned studies can be accurate when describing the losses within the fitted uniaxial stress ranges, none of them were verified to predict the stress dependency of the losses under multiaxial loadings. An attempt was made by Rekik et al. (2014) to predict the biaxial stress dependency of the hysteresis losses when only uniaxial stress dependent measurements are available by adopting an equivalent stress approach. Although they reported satisfactory agreement between the modeled and
the measured results for some cases, the accuracy of the model was limited in general. Recently Yamazaki et al. (2017) adopted a similar approach as Rekik et al. (2014) to model the effect of biaxial stress on the iron losses. They used the Jordan model to separate the losses from magneto-mechanical measurements of (Rekik, Hubert and Daniel, 2014), which was performed under non-sinusoidal flux density waveform with different peak magnetic flux densities and field strengths. As discussed earlier, such an approach can cause significant errors for the loss separation, since the Jordan model assumes sinusoidal flux density (Fiorillo and Novikov, 1990). In addition, the proposed models cannot predict the losses when the principal axis of the mechanical stress is not aligned with the magnetic induction axis.

2.5 Summary and Conclusions

A review of the previous research regarding the magneto-mechanical interactions in ferromagnetic materials was given. Various test setups to measure the effect of stress on the macroscopic magnetic properties of electrical steel sheets and several methods for modeling them were discussed. Special attention was given to magneto-mechanical behavior of NO electrical steel sheets under multiaxial loadings. Considering the macroscopic magneto-mechanical measurements, the effect of uniaxial stress on the magnetic properties of electrical steel sheets have been studied extensively. Although the multiaxial stresses are exerted to the cores of many electromagnetic devices which can cause significant effects, very few measurements are available under these loadings. The situation is similar for modeling the multiaxial stress dependency of the magnetic properties and of the iron losses.

In order to provide comprehensive magneto-mechanical analysis for NO electrical steel sheets under multiaxial stress, a rotational single sheet tester (RSST) device which provides the possibility to apply arbitrary magneto-mechanical loading to a single steel sheet will be introduced. The design of the device will be detailed and the control method for the flux density waveform will be studied. Magneto-mechanical measurements under rotating and alternating flux conditions up to 1.2 T induction level and under up to ±30 MPa multiaxial stress on an M400-50A NO electrical steel sheet will be performed.

The anhysteretic models of Fonteyn et al. (2010b) and Daniel et al. (2015)
will be adopted to test their abilities for predicting the multiaxial magneto-mechanical behavior of two different grades of electrical steel sheets when only uniaxial measurements are available. Hysteresis will be included by using the approaches of Rasilo et al. (2016) and Daniel et al. (2015). In addition they will be tested for modeling the uniaxial stress dependent magnetostriction for another grade NO sheet.

A simple iron loss model to predict the multiaxial stress dependency on the hysteresis and excess loss components will be proposed based on the magneto-elastic invariants from Fonteyn et al. (2010b). The model will be identified only from uniaxial stress dependent loss measurements, and it will be verified for loss modeling under multiaxial stress using the data obtained from the designed RSST.

Applicability of the anhysteretic magneto-elastic model of (Fonteyn et al., 2010b) and the proposed loss model will be tested on a small test transformer that is put under mechanical stress. The models will be implemented into coupled 2D magneto-mechanical FE model of the transformer, and the modeled losses will be compared to the measured ones.
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3. Methods

3.1 Rotational Single Sheet Tester Device

A rotational single sheet tester (RSST) device that provides arbitrary in-plane magneto-mechanical loading possibility was designed and constructed to analyze the effect of multiaxial magneto-mechanical behavior of electrical steel sheets. The concept of the device was given in Publication I, and the design and operation of it was detailed in Publication II, which will be summarized in this section.

3.1.1 Design and Construction

Two main requirements were set for the design of the RSST device. The first requirement was that the device should provide possibility of applying arbitrary magneto-mechanical loading to the sample. The second requirement was to generate uniform stress and magnetic field distribution in a region of the sample that is large enough to perform measurements. This is a common requirement for all the single sheet testers to analyze the magneto-mechanical behavior of steel sheets.

Designing a sample geometry to meet the main requirements is the critical point of the whole design process. The flat geometry of electrical steel sheets allows in-plane study of the geometry. Therefore, the stress tensor can be expressed as $\sigma = [\sigma_{xx} \, \sigma_{yy} \, \tau_{xy}]^T$. In order to control three components of the stress tensor, controlled stress application along three distinct axes is needed. A six leg sample geometry as shown in Fig. 3.1 allows possibility of controlling each stress component of $\sigma$. In order to obtain desired in-plane stress tensor in the measurement area that is located in the central region of the sample, the required mechanical loadings to be applied to each-leg pair can be calculated assuming linear
elasticiy. The calculation steps were given in Publication I and II, and they will not be repeated here.

![Figure 3.1](image)

**Figure 3.1.** The sample geometry for RSST.

In order to ensure homogenous stress distribution in the measurement area, equal mechanical loadings should be applied to each leg pair. In addition, negligible displacement in the central point is preferred. These can be achieved by using independent mechanical actuators for each leg. The tips of the sample legs were designed having T-shapes to ease the clamping for stress application.

Considering the magnetization of the sample, a horizontal yoke system was preferred due to its advantages over vertical yoke systems. For instance, the eddy current formation can be minimized when flux penetrates to the sample along its thickness. In addition, in vertical yoke systems magnetizing yokes are usually in contact with the sample, which means that the yokes have the potential to alter the stress distribution. In horizontal yoke systems, this is easier to avoid by using a appropriate yoke geometry. This also provides significantly easier sample installation possibility, since the magnetization system can be in a fixed position. The magnetization system consists of six magnetization coils wound around magnetizing yokes that are placed between each leg of the sample. This means that while determining the sample diameter, the space for the magnetization system should be taken into account.

Adopting a horizontal yoke system for a six-leg sample geometry would result in a flux path along the legs similar to the case shown in Fig. 2.4 (c) and (d) since the geometry is somewhat similar. In order to concentrate the magnetic flux density on the central area, the leg ends that are close to the central region were designed having narrower widths. With the same applied force to the leg ends, the sample with these narrow regions near the
center provides higher mechanical stress magnitude in the measurement area compared to a sample without narrow regions. To minimize the leakage flux, the yoke geometry was designed to have inclined faces where they reach the sample, as shown in Fig. 3.2.

![Figure 3.2. The sample and yoke geometries. Yokes are shown in red.](image)

In order to ensure the stress and magnetic flux density homogeneity in the measurement area, several sample geometries were modeled using FEM. The initial six-leg sample geometry and the magnetization geometry was adopted inspired by Hasenzagl et al. (1996) since it was shown by Bottauscio et al. (2005) to be effective on obtaining homogeneous field distribution. For the mechanical simulations, 2D plane stress approximation was used, and several stress configurations were simulated. For simulating the magnetic part, 2D approximation which assumes the same thickness for all the geometry, leads to inaccurate field calculation results since in reality, the magnetizing yokes are much thicker than the sample. For this reason the magnetic part was simulated by using 3D vector potential formulation. For both mechanical and magnetic simulations isotropic material is assumed COMSOL Multiphysics® commercial software was used. Magnetic simulations were performed to obtain a rotating field in the central region of the sample. Considering several different sample geometries, the geometry shown in Fig. 3.1 has resulted in a maximum relative standard deviation (RSD) of 4.38% for the stress and 2.74% for the magnetic flux density in the 20 × 20 mm² measurement area. The maximum deviation in $B$ was observed at the instant shown in Fig. 3.3. It is worth mentioning that as the flux density magnitude in the measurement area increases, RSD for magnetic flux density increases slightly as well. It is important to note that using highly anisotropic material such as GO materials might lead higher RSD for the field and stress distributions. Since highly anisotropic materials are out of scope of this thesis, the sample geometry was only designed for NO materials assuming isotropic material.

After the sample geometry was finalized, the components for applying
magneto-mechanical loadings were determined. Mechanical loading was applied by driving a T90 series Thomson ball screw guide using a HIWIN 750W servo motor that is controlled by a HIWIN D2-series servo drive. Between the servo motor and the screw guide, a Girard gearbox with a 60:1 ratio was mounted in order to obtain higher displacement precision. The applied forces were measured using a Tecsis F2301 force sensor. The sample was coupled to the actuators by screwing non-magnetic plates on the T-shaped ends. The sample was placed on a non-magnetic sample holder, and non-magnetic reinforcing plates were added on top of it. In order to avoid buckling under compressive stress, a reinforcing mechanism was used to apply perpendicular force to the sample reinforcing plates. The mechanical loading system and its components are shown in Fig. 3.4. Six of these configurations were used to apply mechanical forces to each leg of the sample.

In order to magnetize the sample, magnetizing coils were supplied with a controlled 3-phase voltage. Each coil has a winding area of $23 \times 89$ mm$^2$, and they contain 2000 turns of enameled copper wire with 0.8 mm diameter. The complete measurement setup is shown in Fig. 3.5.

### 3.1.2 Measurement and Control Procedures

The sensors and the data acquisition modules used to measure mechanical strain $\varepsilon$, magnetic flux density $B$ and magnetic field strength $H$ are summarized in Table 3.1. More detailed information is given in Publication II.

The servo drive control signals to control the displacement of the actuators are provided from an I/O NI-PXI 6224 multifunctional module.
Methods

Figure 3.4. Mechanical loading system, (a) top view and (b) side view.

The magnetizing coils are supplied with 3-phase Elgar SW5250A power amplifier, which is controlled externally. The external control signal is generated from a NI-PXIe 4463 analog output module. All the data acquisition modules are connected to a NI-PXIe 1078 chassis, which includes an

Figure 3.5. Complete RSST measurement setup is shown (a) from top and (b) as a whole.
internal computer. The overall measurement system is illustrated in Fig. 3.6.

The measurement procedure starts with applying static stress prior to magnetization of the sample. The servo drives are programmed to drive the servo motors a pre-determined displacement when they receive a 5 V transistor to transistor logic level. In order to obtain desired stress tensor in the measurement area, the stresses are calculated simultaneously from the measured strains, and servo drives are controlled accordingly with fixed point iteration. After the aimed stress tensor is obtained, the servo motors are locked with electromagnetic brakes, and the flux density control starts in the LabVIEW environment. It is worth noting that when the sample is magnetized, magnetostriction causes small variations on the total strain. Since the sample ends are locked magnetostriction adds additional stress to the sample. For instance, the maximum magnetostrictive stress can be calculated to be up to 1 MPa using the maximum measured stress free magnetostriction for 3% Si-Fe NO alloy from Publication IV. However,

Table 3.1. Sensors and data acquisition devices used for the measurements.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Sensor</th>
<th>Data Acquisition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>Standard rosette strain gauge 120Ω, 2.0 gauge factor.</td>
<td>Bridge amplifier module NI-PXIe 4330/4331.</td>
</tr>
<tr>
<td>$B$</td>
<td>Double search coil with 4 turns each and wound on 20 $\times$ 20 mm$^2$ area. Coil wire diameter is 0.14 mm.</td>
<td>Analog input module NI-PXIe 4464.</td>
</tr>
<tr>
<td>$H$</td>
<td>Double $H$-coil with 17.5 $\times$ 17.5 mm$^2$ area.</td>
<td>Analog input module NI-PXIe 4464.</td>
</tr>
</tbody>
</table>

Figure 3.6. Schematic diagram of the measurement system.
for different alloys this stress might be higher. In these cases magnetostric-
tion should be taken into account during the experiments. This could be
implemented by quantifying the so-called Delta-E effect and making the
corresponding corrections to the applied stress.

The flux density control consists of three main parts, which are phase
angle, amplitude, and waveform corrections of the measured flux density.
The phase angle correction is intended to eliminate the phase difference
between the measured and the reference flux density waveforms. The
amplitude correction brings the amplitude of the measured flux density
close to the reference one. The aim of the waveform control is to generate
the correct voltage waveforms for magnetizing coils to obtain the desired
flux density waveforms along both rolling $B_x(t)$ and transverse $B_y(t)$ direc-
tions in the measurement area. Sinusoidal $B_x(t)$ and $B_y(t)$ are aimed for
throughout this thesis. The control algorithm given in Publication II did
not include the phase angle correction, since in that study the aim was to
obtain only circular $B$-loci and purely sinusoidal flux density waveforms
without phase shifts. Therefore, the possible phase difference between
the reference and measured flux density waveform was small, and the
waveform control alone was able to correct it.

The control procedure starts by sending a balanced 3-phase sinusoidal
voltage $U_{\text{abc}}^0(t)$ with low amplitude to the power amplifier to be amplified
and supplied to the magnetizing coils. Since the aim is to control $B_x(t)$ and
$B_y(t)$, 3-phase coordinate system abc is reduced to an equivalent 2-phase
coordinate system $\alpha\beta$ by applying Clarke transformation to the 3-phase
coil voltages as

\[
\begin{bmatrix}
U_{\alpha}^0(t) \\
U_{\beta}^0(t) \\
U_{\alpha\beta}^0(t)
\end{bmatrix}
= \frac{2}{3}
\begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
U_{a}^0(t) \\
U_{b}^0(t) \\
U_{c}^0(t)
\end{bmatrix}.
\]  

(3.1)

After the Clarke transformation, in order to obtain reference flux density
waveforms along rolling $B_{\text{ref}}^x(t)$ and transverse $B_{\text{ref}}^y(t)$, the respective voltages $U_{\alpha}(t)$ and $U_{\beta}(t)$ are controlled simultaneously in an iterative
manner. At the $i$-th iteration, the reference and the measured flux density
waveform vectors are written as $B_{\text{ref}}^i(t) = [B_{x,\text{ref}}^i(t) B_{y,\text{ref}}^i(t)]^T$ and $B^i(t) = [B_x^i(t) B_y^i(t)]^T$, respectively. Similarly, the coil voltages in the $\alpha\beta$ frame
are expressed as $U_{\alpha\beta}^i(t) = [U_{\alpha}^i(t) U_{\beta}^i(t)]^T$. The flux density control will be
given below step by step starting from the phase angle correction.
1. The phase angle correction is done by calculating a corrected phase angle for the coil supply voltages $U_{\alpha\beta}^i(t)$ by

$$
\phi_{U_{\alpha\beta}}^{i+1} = \phi_{U_{\alpha\beta}}^i - A_{\phi} \left( \phi_{B}^i - \phi_{B_{\text{ref}}}^i \right) / \epsilon_{\phi}^i
$$

(3.2)

where $\phi_{U_{\alpha\beta}}^i$, $\phi_{B}^i$ and $\phi_{B_{\text{ref}}}^i$ are the phase angle vectors of the coil voltages $U_{\alpha\beta}^i(t)$, measured flux density $B^i(t)$ and the reference flux density $B_{\text{ref}}^i(t)$ at the $i$-th iteration, respectively. $A_{\phi}$ is the gain of the phase angle controller, $\epsilon_{\phi}^i$ is the phase angle error vector at the $i$-th iteration and $\phi_{U_{\alpha\beta}}^{i+1}$ is the corrected coil voltage phase angle vector at the next iteration. The phase angles for the waveforms are calculated by performing a fast Fourier transform. The corrected voltage waveforms are obtained by performing an inverse fast Fourier transform with the corrected phase angle.

2. The amplitude correction is applied by using

$$
\epsilon_{\text{amp}}^i = \frac{\max(B^i(t))}{\max(B_{\text{ref}}^i(t))}
$$

(3.3)

$$
U_{\alpha\beta}^{i+1}(t) = U_{\alpha\beta}^i(t) / \epsilon_{\text{amp}}^i.
$$

(3.4)

Here, $\epsilon_{\text{amp}}^i$ is the amplitude ratio vector at the $i$-th iteration.

3. After the phase angle and amplitude correction are performed, the modified coil voltages $U_{\alpha\beta}^{i+1}(t)$ are transformed from the $\alpha\beta$ frame back to the $abc$ frame by using the inverse Clarke transformation. This operation is given by

$$
U_{\alpha\beta}^{i+1}(t) = T^{-1} \cdot \begin{bmatrix}
U_{\alpha}^{i+1}(t) \\
U_{\beta}^{i+1}(t) \\
0
\end{bmatrix}
$$

(3.5)

where $U_{\text{abc}}^{i+1}(t)$ is the corrected 3-phase supply voltage waveform that is supplied to the magnetizing coils.

The above steps are iterated until the errors $\epsilon_{\phi}^i$ and $\epsilon_{\text{amp}}^i$ are below the predetermined tolerances $t_{\phi}$ and $t_{\text{amp}}$. Note that the phase and amplitude correction are performed only at the beginning of the control process to start the waveform control with reasonable initial conditions.

4. The last step of the control process is the waveform control, which starts after the completion of the phase angle and the amplitude corrections. Similar to the previous steps, the 3-phase coil supply voltages are first transformed from the $abc$ frame to the $\alpha\beta$ frame via the Clarke trans-
formation. Then, the voltages in the $\alpha\beta$ frame are modified by comparing the measured flux density waveforms to the reference waveforms. The waveform correction for $U_{\alpha\beta}^i(t)$ is performed by

$$U_{\alpha\beta}^{i+1}(t) = U_{\alpha\beta}^i(t) + A_1 \int (B^i(t) - B_{\text{ref}}^i(t)) dt + A_2 (B^i(t) - B_{\text{ref}}^i(t)).$$  \(3.6\)

Here, $A_1$ and $A_2$ are the constant coefficients of the waveform controller. The values of the coefficients were chosen based on the prior experience. Instead of using constant coefficients, tuning of the parameters can be performed using various methods for instance as proposed by Visioli (2001) or Jinhua et al. (2009). The waveform control expression is (3.6) based on the principles explained by Matsubara et al. (1995) and Zurek et al. (2005). Before supplying the coils with the corrected supply voltages, 30 periods of the previous voltage waveform are supplied. This helps to reduce the transient. The magnetizing coils are supplied with the corrected 3-phase voltage waveforms obtained by applying the inverse Clarke transformation to $U_{\alpha\beta}^{i+1}(t)$ as shown in (3.5). The errors between the measured and reference flux density waveforms are calculated at each iteration by

$$\begin{bmatrix} \epsilon_{x,\text{wav}}^i \\ \epsilon_{y,\text{wav}}^i \\ \epsilon_{z,\text{wav}}^i \end{bmatrix} = \frac{\|B^i(t) - B_{\text{ref}}^i(t)\|}{\|B_{\text{ref}}^i(t)\|}.$$  \(3.7\)

The waveform control is iterated until both conditions $\epsilon_{x,\text{wav}}^i < 1\%$ and $\epsilon_{y,\text{wav}}^i < 1\%$ are met. The flux density control algorithm is shown in Fig 3.7.

### 3.2 Magneto-Mechanical Models

In order to model the anhysteretic magneto-elastic behavior of NO electrical steel sheets, a simplified multiscale model of Daniel et al. (2015) and a slightly modified version of a Helmholtz energy based model of Fonteyn (2010) were adopted. Both models assume isotropic material. The magnetic hysteresis was included following the approaches given by Daniel et al. (2015) and Rasilo et al. (2016). The main focus was on predicting the multi-axial magneto-elastic behavior when only uniaxial stress dependent measurement data is available. The predictive abilities of the two models were compared for anhysteretic and hysteretic behaviors in Publications IV and V, respectively. In this section, the models will be summarized.
3.2.1 Simplified Multiscale (SM) Model

**Anhysteretic Model**

In the SM model, the material is described as a single crystal that consists of a collection of randomly oriented magnetic domains. Assuming initially isotropic material, the local free energy $W_k$ is expressed in domain scale as the sum of the magneto-static $W_{k}^{mag}$ and the magneto-elastic energy $W_{k}^{me}$ terms, and it is given by

$$W_k = W_{k}^{mag} + W_{k}^{me} = -\mu_0 H \cdot M_k - \sigma : \varepsilon_k^{ii}$$  (3.8)
where $\mu_0$ is the permeability of free space, $H$ and $\sigma$ are the applied magnetic field strength and mechanical stress, and $M_k$ and $\varepsilon^\mu_k$ are the local magnetization vector and magnetostriction strain tensor, respectively. These local quantities are classically given for a domain oriented along $u_k$ as

$$M_k = M_s u_k$$ (3.9)

$$\varepsilon^\mu_k = \lambda_s \left( \frac{3}{2} u_k \otimes u_k - \frac{1}{2} I \right)$$ (3.10)

where $M_s$ and $\lambda_s$ are the macroscopic magnetization and the magnetostriction of the saturated material, respectively. $I$ is the second order identity tensor. The volume fraction $f_k$ for a given set of domains with magnetization orientation $u_k$ is introduced by using the Boltzmann probability function (Buiron et al., 1999)

$$f_k = \frac{\exp (-A_s W_k)}{\int_k \exp (-A_s W_k)}$$ (3.11)

where $A_s$ is a model parameter that is a function of the stress free initial anhysteretic susceptibility $\chi_0$ and is given by

$$A_s = \frac{3 \chi_0}{\mu_0 M_s}.$$ (3.12)

The macroscopic magnetization $M$ and magnetostriction $\varepsilon^\mu$ are obtained as the volume average of the corresponding local quantities by integrating the volume fraction over all possible magnetization directions $u_k$ as

$$M = \langle M_k \rangle = \int_k f_k M_k$$ (3.13)

$$\varepsilon^\mu = \langle \varepsilon^\mu_k \rangle = \int_k f_k \varepsilon^\mu_k$$ (3.14)

which can be computed numerically by discretization of a unit sphere for the possible magnetization orientations $u_k$ (Daniel and Galopin, 2008). Since the applied tension or compression affects the material differently, a configuration field $H_{\text{conf}}$ for accounting this non-monotony is introduced as (Daniel et al., 2014)

$$H_{\text{conf}} = \eta \left( N_\sigma - \frac{1}{3} \right) M$$ (3.15)
where $\eta$ is a material parameter to be identified from magnetic measurements under uniaxial stress. The function $N_\sigma$ is given by

$$N_\sigma = \frac{1}{1 + 2\exp(-K\sigma_{eq})} \quad (3.16)$$

$$K = \frac{3}{2} A_s \lambda_s \quad (3.17)$$

$$\sigma_{eq} = \frac{3}{2} h \cdot \left( \sigma - \frac{1}{3} \text{tr}(\sigma) I \right) \cdot h. \quad (3.18)$$

Equivalent stress $\sigma_{eq}$ is defined as the projection of the deviatoric part of $\sigma$ along the externally applied magnetic field direction $h$ (Daniel and Hubert, 2009). Finally, the reversible effective field $H_{rev}$ is obtained as

$$H_{rev} = H + H_{conf}. \quad (3.19)$$

The anhysteretic model parameters $M_s$, $\lambda_s$, $\chi_0$ and $\eta$ are identified from the uniaxial measurements. The first three parameters can be identified from stress free magnetization measurements, whereas $\eta$ can be identified using a few anhysteretic magnetization curves measured under uniaxial tensile stress, which is applied parallel to magnetization direction. A more detailed parameter identification procedure is given in Publication IV.

**Hysteresis Effects**

Hysteresis effects are included in the SM model by adding an irreversible magnetic field contribution $H_{irr}$ to the reversible field $H_{rev}$. The definition of $H_{irr}$ is based on the model of Hauser (2004). Assuming $H_{irr}$ is parallel to the applied field $H$, $H_{irr}$ is defined as

$$H_{irr} = \delta \left( \frac{k_r}{\mu_0 M_a} + c_r \|H\| \right) \cdot \left( 1 - \kappa \exp \left( \frac{k_a}{\kappa} \|M - M_{inv}\| \right) \right) \quad (3.20)$$

where $\delta = 1$ initially and the sign of it changes at each inversion of the magnetic field loading direction. The model parameters $k_a$ and $c_r$ have constant values. Initially $\kappa = \kappa_{ini}$ where $\kappa_{ini}$ is a material constant. The value of $\kappa$ changes with every change in the magnetic loading direction according to

$$\kappa = 2 - \kappa_0 \exp \left( - \frac{k_a}{\kappa_0} \|M - M_{inv}\| \right) \quad (3.21)$$

where $\kappa_0$ and $M_{inv}$ are the values of $\kappa$ and $M$ at the previous inversion of the magnetic loading direction, respectively. The stress dependency of the coercive field is introduced with $k_r$ which is a function of $N_\sigma$ and it is given
as
\[ k_r = k_r^0 \left(1 - \zeta \left(N\sigma - \frac{1}{3}\right)\right). \] (3.22)

Here, \( k_r^0 \) is a material constant, and \( \zeta \) is an adjustment parameter. After the calculation of the irreversible field \( H_{irr} \) the effective field is obtained by adding up reversible field \( H_{rev} \) and \( H_{irr} \) as
\[ H_{eff} = H_{rev} + H_{irr}. \] (3.23)

The material parameters to describe the hysteresis are \( k_a, c_r, \kappa_{ini}, k_r^0 \) and \( \zeta \) which can be identified by least-squares fitting of the model to a measured stress free major hysteresis loop. Therefore, to identify the SM model, including hysteresis effects, magnetization loops under no applied stress and under few uniaxial tensile stresses that are applied parallel to the magnetization direction are needed.

### 3.2.2 Helmholtz Energy Based (HE) Model

**Anhysteretic Model**

In this approach, the constitutive equations to describe the magneto-elastic behavior of the material are derived from a Helmholtz free energy density \( \psi \). The definitions of \( \psi \) given in Publications I and IV appear in a slightly different form and are based on the earlier works of Belahcen et al. (2006, 2008) and Fonteyn (2010). In the following, \( \psi \) used in Publication IV will be given since it is simpler and was found to be more accurate for predicting multiaxial magneto-elastic behavior.

The energy density \( \psi \) is defined as a function of magnetic flux density \( B \) and total strain \( \varepsilon \), and considering an isotropic material, it can be expressed by the following six scalar invariants
\[ I_1 = \text{tr} (\varepsilon), \quad I_2 = \frac{1}{2} \text{tr} (\varepsilon^2), \quad I_3 = \text{det} (\varepsilon) \]
\[ I_4 = B \cdot B, \quad I_5 = B \cdot (\tilde{\varepsilon}B), \quad I_6 = B \cdot (\tilde{\varepsilon}^2 B). \] (3.24)

The first three invariants describe purely mechanical behavior, whereas the fourth invariant, \( I_4 \), describes anhysteretic magnetic behavior. Invariants \( I_5 \) and \( I_6 \) describe the magneto-elastic coupling. In \( I_5 \) and \( I_6 \) the deviatoric part of the total strain \( \tilde{\varepsilon} \) is used to eliminate the effect of hydrostatic
pressure on magnetic behavior. The expression for $\psi$ is then given as

$$\psi = \frac{1}{2} \lambda I_1^2 + 2GI_2 - \nu_0 \left( \frac{I_1}{2} + \sum_{i=0}^{n-1} \frac{\alpha_i}{i+1} I_{i+1} + \ldots \right)$$

where $\lambda$ and $G$ are the Lamé constants of the material, $\nu_0$ is the reluctivity of free space and $\alpha_i$, $\beta_i$ and $\gamma_i$ are the fitting parameters to be determined from the measurements. Since in this work linear elastic material is assumed, $\psi$ does not depend on $I_3$. Finally, the anhysteretic magnetization vector $M$ and the magneto-elastic stress tensor $\sigma_{me}$ are expressed as

$$M (B, \varepsilon) = -\frac{\partial \psi (B, \varepsilon)}{\partial B} \quad \text{and} \quad \sigma_{me} (B, \varepsilon) = \frac{\partial \psi (B, \varepsilon)}{\partial \varepsilon}. \quad (3.26)$$

The magnetic field strength is then obtained as $H = \nu_0 B - M$. The magneto-elastic stress tensor $\sigma_{me}$ includes elastic and magnetostrictive stress contributions.

The model parameters $\alpha_i$, $\beta_i$ and $\gamma_i$ can be determined by fitting of the modeled single-valued $B$-$H$ curves to the measured ones under uniaxial stress applied along the magnetization direction. Usually total three to four $B$-$H$ curves that are under no stress, compression and tension are sufficient to determine the parameters. The number of model parameters $n_{\alpha}, n_{\beta}, n_{\gamma}$ are material dependent. They are typically determined by fitting the model to uniaxial $B$-$H$ curves using various numbers and choosing the best fitted case. This procedure is necessary as different materials present different strength of the investigated physical phenomena. A more detailed parameter identification procedure was described in Publication IV.

**Hysteresis Effects**

Hysteresis effects are included by incorporating the anhysteretic model into the inverse Jiles-Atherton hysteresis model (Gyselinck et al., 2004). The following set of equations summarizes the hysteresis model

$$H_{\text{eff}} = H + \alpha J_a M \quad (3.27)$$

$$M_{\text{an}} = F (H_{\text{eff}}) \quad (3.28)$$

$$d = M_{\text{an}} - M_{\text{irr}} \quad \text{and} \quad \delta = \frac{dB}{dt} \cdot d \quad (3.29)$$
\[
\frac{dM_{\text{irr}}}{dH_{\text{eff}}} = \begin{cases} 
 k(\tilde{\varepsilon})^{-1} \frac{ddT}{||d||}, & \text{if } ||d|| > 0 \text{ and } \delta > 0 \\
 0, & \text{otherwise}
\end{cases}
\]  
(3.30)

\[
\frac{dM}{dH_{\text{eff}}} = c \frac{dM_{\text{an}}}{dH_{\text{eff}}} + (1 - c) \frac{dM_{\text{irr}}}{dH_{\text{eff}}}
\]  
(3.31)

Here, \(\alpha_{ja}\) and \(c\) are constant model parameters. The magneto-elastic effects are included in the hysteresis model by replacing (3.30) with the anhysteretic magnetization \(M\) from HE model given in (3.26). Anhysteretic magnetization \(M_{\text{an}}\) in the Jiles-Atherton model is a function of the effective field \(H_{\text{eff}}\) whereas the input of HE model for obtaining \(M\) is \(B\). Therefore, an equivalent flux density \(B_{\text{an}} = \mu_0 (H_{\text{eff}} + M_{\text{an}})\) is calculated by Newton-Raphson iteration to satisfy \(M(B_{\text{an}}, \varepsilon) = M_{\text{an}}\) for a given total strain \(\varepsilon\). The residual and the Jacobian of this operation are given as

\[
r(B_{\text{an}}) = M(B_{\text{an}}, \varepsilon) - (\nu_0 B_{\text{an}} - H_{\text{eff}})
\]
(3.32)

\[
\text{Jac}(B_{\text{an}}) = \frac{dr(B_{\text{an}})}{dB_{\text{an}}}
\]
(3.33)

Typically Newton-Raphson method takes 2-3 iterations until convergence is obtained. Finally, the stress dependency of the coercive field is introduced by expressing the pinning parameter \(k\) as an isotropic function of \(\tilde{\varepsilon}\) as

\[
k(\tilde{\varepsilon}) = k_0 \left( I + a \tilde{\varepsilon} + b \tilde{\varepsilon}^2 \right)
\]
(3.34)

where \(k_0, a\) and \(b\) are constant model parameters. The model parameters \(\alpha_{ja}, c, k_0, a\) and \(b\) can be determined by least-squares fitting of the modeled major hysteresis loops to the measured ones under various uniaxial stresses, which are applied parallel to the magnetization direction.

### 3.3 Effect of Stress on Iron Losses

A model to predict the multiaxial stress dependency of the hysteresis and excess losses in NO electrical steel sheets was proposed. The model is based on the magneto-elastic invariants given in the previous section, and it is identified only from the loss data under uniaxial stress. The loss data obtained from an M400-50A NO electrical steel sheet using the previously described RSST was used to identify and verify the model as described in Publication III. In addition, iron loss measurements on a small test transformer that is under mechanical stress were performed.
Methods

The applicability of the loss model was tested by modeling the transformer using FEM and comparing the modeling results to the measured ones. The details of the model and the measurements will be given in this section.

### 3.3.1 Stress Dependent Iron Loss Model

Although the iron loss segregation was reviewed in Chapter 1, for the sake of clarity it will be repeated here briefly. Under sinusoidally alternating flux density, the total iron losses in ferromagnetic laminations can be separated into hysteresis losses ($p_{hy}$), classical eddy current losses ($p_{cl}$), and excess losses ($p_{ex}$) (Bertotti, 1988). Assuming that the skin effect is negligible, and that the flux density over the thickness of the lamination is homogeneous $p_{cl}$ can be determined by

$$p_{cl} = \frac{\pi^2 d^2 B_p^2 f^2}{6\rho}$$

(3.35)

where $d$, $B_p$, $f$ and $\rho$ are the thickness of the lamination, peak flux density, frequency of the field and the material resistivity, respectively. Then, total power loss density ($p_{fe}$) per cycle is given by

$$p_{fe} = c_{hy} B_p^2 f + p_{cl} + c_{ex} B_p^1.5 f^{1.5}.$$  

(3.36)

Here, $c_{hy}$ and $c_{ex}$ are the hysteresis and excess loss coefficients, respectively. Assuming that the skin effect is negligible under the studied frequency, and that the studied stress levels are within elastic limits, it is assumed that $p_{cl}$ is independent of the stress state of the material.

Magneto-elastic invariants $I_5$ and $I_6$ given in Subsection 3.2.2 are modified to be dependent on the applied stress and the direction of $B$, and they are given by

$$I'_5 = b \cdot (\tilde{\sigma} b), \quad I'_6 = b \cdot (\tilde{\sigma}^2 b)$$

(3.37)

where $b$ is the direction vector of the flux density, and $\tilde{\sigma}$ is the deviatoric part of the applied stress $\sigma$. Stress dependency is introduced to the loss coefficients $c_{hy}$ and $c_{ex}$ by using the invariants $I'_5$ and $I'_6$ as

$$c_{hy}(I'_5, I'_6) = c_{0,hy} + \beta_h I'_5 + \gamma_h I'_6$$

$$c_{ex}(I'_5, I'_6) = c_{0,ex} + \beta_e I'_5 + \gamma_e I'_6$$

(3.38)

where $c_{0,hy}$ and $c_{0,ex}$ are the stress free Bertotti hysteresis and excess loss coefficients, and $\beta_h$, $\beta_e$, $\gamma_h$ and $\gamma_e$ are the fitting parameters. These
parameters are determined by using the loss data only for the cases when uniaxial stress is applied parallel to the magnetization direction.

### 3.3.2 Test Transformer

An E-I core test transformer was built to analyze the effect of external stress on the iron losses. The primary and secondary windings of the transformer have 305 and 198 turns, respectively. The core of the transformer was constructed by laminating of 30 sheets of M400-50A NO electrical steel sheet that was also used in the RSST measurements. During the magneto-mechanical experiment, the primary of the transformer was supplied with sinusoidal voltage, while the secondary was kept open. The supply frequency was 50 Hz, and the amplitude of the voltage varied from 2 V to 16 V. The power losses were measured by a power analyzer that was connected to the primary side.

Static mechanical compressive stress was applied parallel to the airgap between the I and E cores of the transformer by a hydraulic press. The applied force was measured by a force sensor placed under the transformer. The applied stress magnitude varied from 8 to 16 MPa, after which the laminations started plastically deforming. A 2D cross section of the transformer is illustrated in Fig. 3.8 where the force application direction is shown with arrows. It is worth mentioning that the mechanical stress is applied in this way only to analyze the applicability of the HE and developed loss models in a practical application and with relation to the FE analysis of these devices. Considering the practical use of small transformers, the presence of these stresses are unlikely. On the other hand, considering large transformers, their own mass might lead to significant stress levels in the core.

A 2D coupled magneto-mechanical model of the transformer is developed using the anhysteretic model presented in Subsection 3.2.2. Assuming zero conductivity, the partial differential equations to be solved in the iron are

\[ \nabla \times H (B, \varepsilon) = 0 \]  
\[ \nabla \cdot B = 0 \]  
\[ \nabla \cdot \sigma (B, \varepsilon) = \nabla \cdot \sigma_0 - f. \]

Here, \( \sigma_0 \) and \( f \) are the initial stress and the body force vector, respectively. Vector potential formulation is adopted for the magnetic part of the model...
such that $A = Az$ in $xyz$ Cartesian coordinate system. For the mechanical part, plane strain approximation is used where displacement is expressed in the plane of the sheet as $U = U_xx + U_yy$. Then, the magnetic flux density vector and the strain tensor in Voigt notation becomes

$$B = \begin{bmatrix} \frac{\partial A}{\partial y} - \frac{\partial A}{\partial x} \end{bmatrix}^T$$  \quad (3.42)

$$\varepsilon = \begin{bmatrix} \frac{\partial U_x}{\partial x} & \frac{\partial U_y}{\partial y} & \frac{\partial U_x}{\partial y} + \frac{\partial U_y}{\partial x} \end{bmatrix}^T.$$  \quad (3.43)

Using nodal shape functions $N = [N_1, N_2, ..., N_N]$ and discrete derivative operators $D_m^T$ and $D_s^T$, the flux density $B$ and the strain $\varepsilon$ can be expressed as

$$B = D_m^T a \quad \text{and} \quad \varepsilon = D_s^T u$$ \quad (3.44)

where $a$ and $u$ are the nodal value vectors for the vector potential and displacements, respectively. The discrete derivative operators $D_m^T$ and $D_s^T$ are expressed as

$$D_m^T = \begin{bmatrix} \frac{\partial N}{\partial y} & -\frac{\partial N}{\partial x} \end{bmatrix}$$ \quad (3.45)

$$D_s^T = \begin{bmatrix} \frac{\partial N}{\partial x} & 0 & \frac{\partial N}{\partial y} \\ 0 & \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} \end{bmatrix}.$$ \quad (3.46)

Equations (3.39) and (3.41) are discretized over the iron domain $\Omega$ and expressed in a weak form using the Galerkin method by

$$\int_{\Omega} D_{m}^{T} H (B, \varepsilon) \, d\Omega = 0$$ \quad (3.47)
\[
\int_{\Omega} D_s^T (\sigma (B, \varepsilon) - \sigma_0) \, d\Omega = - \int_{\Omega} \left[ N^T \right] \, f
\] (3.48)

The system is solved iteratively with the Newton-Raphson (NR) method in time domain. The Jacobian matrix for NR iterations is given by

\[
\begin{bmatrix}
\int_{\Omega} D_m^T \frac{\partial H}{\partial B} D_m \, d\Omega & \int_{\Omega} D_s^T \frac{\partial H}{\partial \varepsilon} D_s \, d\Omega \\
\int_{\Omega} D_m^T \frac{\partial \sigma}{\partial B} D_m \, d\Omega & \int_{\Omega} D_s^T \frac{\partial \sigma}{\partial \varepsilon} D_s \, d\Omega
\end{bmatrix}.
\] (3.49)

The primary winding is supplied from a voltage source, and the field equations are coupled with the following voltage equations to solve for primary current \( I_p \) assuming a negligible skin effect in the coils.

\[
U_1 = \left( \int_{C_p} \frac{l N_1}{S_c} \frac{d\alpha}{dt} \, dC_p - \int_{C_n} \frac{l N_1}{S_c} \frac{d\alpha}{dt} \, dC_n \right) + R I_p
\] (3.50)

Here \( l, N_1, S_c \) and \( R \) are the length, number of turns, cross section area and the resistance of the primary winding. \( C_p \) and \( C_n \) are the domains for the positive and negative side of the primary coil, respectively. The time derivative in (3.50) is handled via the backward Euler method.

The nodal displacements of the upper boundary elements are fixed such that the average stress over the boundary results in the measured stress magnitude. In order to obtain higher accuracy in the field solution, for the I piece, transverse direction material properties are used (Gyselinck and Melkebeek, 2001).

After the field solution is obtained, the iron losses per unit volume are calculated in the post processing stage. Hysteresis, classical and excess losses are obtained by Fourier decomposition of \( B \) and summing up the losses for each harmonic. Then the total power loss density is obtained as

\[
p_{Fe} = \sum_n \left( c_{hy} + c_{cl} f_n + c_{ex} f_n^{0.5} \| B_n \|^{-0.5} \right) f_n \| B_n \|^2
\] (3.51)

where \( f_n \) and \( B_n \) are the frequency and the flux density vector of the \( n \)th harmonic. The classical loss coefficient \( c_{cl} \) is obtained from the analytical formula given in (3.35) which was \( c_{cl} = \pi^2 d^2 / 6 \rho \). It is worth mentioning that instead of summing up the losses of each harmonic as in (3.51) the instantaneous classical and excess losses can also be calculated as (Fiorillo and Novikov, 1990)

\[
p_{cl} = \frac{d^2}{12 \rho T} \int_0^T \left( \frac{dB}{dt} \right)^2 \, dt
\] (3.52)
\[ p_{\text{ex}} = \frac{1}{T} \int_{0}^{T} \frac{1}{8.67} c_{\text{ex}} \left| \frac{dB}{dt} \right|^{1.5} \, dt. \]  

(3.53)

This approach could provide more accurate description of the losses in some cases depending on the harmonic content of the flux density waveforms (Geisinger and Knight, 2014; Kowal et al., 2015). In this work, (3.51) is used for the loss calculation since the flux density waveforms in the transformer were mainly sinusoidal under the studied conditions. The hysteresis and excess loss coefficients \( c_{\text{hy}} \) and \( c_{\text{ex}} \) can be modeled to be stress dependent using (3.38). Once the total iron loss density \( p_{\text{Fe}} \) is obtained, the total iron losses are calculated by integrating \( p_{\text{Fe}} \) over the total core volume of the transformer.
4. Application and Results

4.1 Magneto-Mechanical Measurements

In this section the results of the magneto-mechanical experiments performed with RSST will be given. First, the rotational field measurement results will be reviewed; then, the focus will shift to the alternating measurements and iron loss segregation.

All measurements were performed on a 3% Si-Fe M400-50 A grade NO electrical steel sheet. The studied stress configurations include uniaxial stress along rolling (x) and transverse (y) directions, equibiaxial stress, and two cases of pure shear stresses, which are denoted as shear-I and shear-II for brevity. Using the Voigt notation $\sigma = [\sigma_{xx} \sigma_{yy} \tau_{xy}]^T$ (Belytschko et al., 2014), the studied multiaxial stress configurations are listed below.

- Equibiaxial stress: $\sigma = [\sigma \sigma 0]^T$
- Shear-I stress: $\sigma = [\sigma -\sigma 0]^T$
- Shear-II stress: $\sigma = [0 0 \tau]^T$

The magnitudes of $\sigma$ and $\tau$ vary from $-30$ MPa (compression) to $30$ MPa (tension) at $10$ MPa intervals.

4.1.1 Rotational Field Measurements

Rotational field measurements were studied in Publication II. The measurements were performed under circular flux density with peak flux inductions of 0.25 T, 0.5 T, 0.75 T, 1.0 T, and 1.2 T. In order to minimize the dynamic effects on the $B-H$ loops ideally frequencies lower than 1 Hz are preferred. However, at these frequencies the convergence of the flux density waveform control is poor. Thus, the measurement frequency was chosen to be 10 Hz for all the cases as a compromise between low dynamic
effects and better convergence of the waveform control. The studied stress states include uniaxial, equibiaxial, and shear-I cases. The flux density control was performed per the algorithm explained in Subsection 3.1.2. The convergence criteria for the waveform control was set to 1% error for both flux density waveforms along both rolling (x) and transverse (y) directions.

It is known that the application of compressive stress deteriorates the susceptibility along the direction it is applied while improving it along the direction perpendicular to that of the stress application. On the other hand, low tensile stress improves the susceptibility along the stress application direction and deteriorates it along the direction perpendicular to the stress application direction. It would be natural to expect that application of shear-I stress case, which is a linear combination of both uniaxial compression and tension, would enhance the effect of tension as well as that of compression. Similarly, in the case of equibiaxial stress, the effects of both tension and compression would be reduced. Therefore, it is expected that the effect of the shear-I and equibiaxial cases would be the highest and the lowest, respectively on the material. Indeed, this was observed with the experiments in Publication II. In Fig. 4.1 $B$-loci and $H$-loci under the studied stress states where $\sigma = \pm 30$ MPa and at 1 T peak induction level are shown. Clearly, equibiaxial stress affects the material the least, whereas shear-I stress configuration has the largest effect on the $H$-loci.

Analyzing the effect of multi-axial stress on the rotational losses revealed similar conclusions. In order to study the rotational loss evolution with stress, the relative percentage variation of the losses with respect to the stress free case was calculated by

$$\Delta p = \frac{p(\sigma_{xx}, \sigma_{yy}, \tau_{xy}) - p(0, 0, 0)}{p(0, 0, 0)}$$  \hspace{1cm} (4.1)$$

where $p(\sigma_{xx}, \sigma_{yy}, \tau_{xy})$ and $p(0, 0, 0)$ are the iron loss densities per cycle for the stressed and the stress free cases, respectively. In Fig. 4.2, $\Delta p$ is shown for all the studied cases. It is seen that the shear-I stress configuration increases the losses considerably more than the other cases, whereas the equibiaxial stress affects the losses only slightly. Uniaxial stress applied along the rolling direction affects the losses in a slightly different way compared to its application along the transverse direction. This is associated with the initial anisotropy of the material that is mainly related to crystallographic texture (Hubert et al., 2003). Similar results were also
Figure 4.1. Measured $B$-loci and $H$-loci under 1 T peak induction and several stress configurations. (a) $B$-loci for each case. (b) $H$-loci under zero and uniaxial stress along rolling direction. (c) $H$-loci under zero and uniaxial stress along transverse direction. (d) $H$-loci under zero and equibiaxial stress. (e) $H$-loci under zero and shear-I stress.

observed by Kai et al. (2011b) and Kai et al. (2014b) under the rotational field. The effect of stress on the rotational losses is more distinct at low peak flux densities. It is worth noting that for the studied material and the studied cases, the rotational losses can be slightly reduced by the application of uniaxial or biaxial stress states with low tensile along the transverse direction.

4.1.2 Alternating Field Measurements and Loss Segregation

Alternating field measurements and the effect of stress on iron loss components were studied in Publication III. The measurements were taken by magnetizing the sample with sinusoidal flux density along rolling or transverse directions at 1 T peak induction level and at 10 Hz, 30 Hz, 70 Hz, 110 Hz, and 150 Hz frequencies. The studied stress configurations were uniaxial, equibiaxial, shear-I, and shear-II cases. The algorithm explained in Subsection 3.1.2 was used to control the flux density waveform where the convergence criteria was set to 1.5% error due to poorer convergence at high frequencies.

$B$-$H$ loops under magnetization along the rolling direction at 10 Hz frequency and under uniaxial, biaxial, and shear-II stress states are shown.
in Fig. 4.3 (a), (b) and (c), respectively. Similar to the rotational field results, compressive stress along the magneization direction and tension perpendicular to it deteriorates the susceptibility, whereas low tensile stress along the magnetization direction and compression perpendicular to it improves the susceptibility. Equibiaxial and shear-I cases have the least and the most effect on the magnetic behavior, respectively.

Application of shear-II stress case with both $\tau = 30$ MPa and $\tau = -30$ MPa reduces the susceptibility in a very similar way, which ideally should be the same. The difference is associated with the magneto-elastic anisotropy that was also observed for the rotational field measurements.

The iron loss variations with respect to the stress free case are calculated by (4.1) for all the stress configurations and for magnetization along the rolling or transverse directions at 10 Hz and 150 Hz frequencies. The results are shown in Fig. 4.4 for uniaxial and biaxial stress cases and Fig. 4.5 for the shear-II stress case, respectively. For all the cases in Fig. 4.4, significant increase in the losses is observed when uniaxial compressive stress along the magnetization direction and uniaxial tensile along perpendicular to the magnetization direction is applied. When shear-I stress that combines these stress components is applied, the highest increase in the losses (up to 90%) is observed.
Application and Results

Figure 4.3. Measured $B$-$H$ loops at 10 Hz under (a) uniaxial stress, (b) biaxial stress and (c) shear-II stress configurations. Magnetization is along the rolling direction.

Figure 4.4. Measured alternating relative percentage variations of the loss densities ($\Delta p$) for uniaxial and biaxial stress cases for magnetization along rolling (x) direction at frequencies (a) 10 Hz, (b) 150 Hz and magnetization along transverse (y) direction at frequencies (c) 10 Hz, (d) 150 Hz. Note the scale difference in the colormaps.
The losses can be reduced by around 25% when combinations of tension along the magnetization direction and compression perpendicular to it are applied. When the sample is magnetized along the rolling direction, the losses vary more distinctly with stress compared to when magnetization is applied along the transverse direction. Application of shear-II stress configuration with $\tau > 0$ or $\tau < 0$ increases the losses similarly as observed in Fig. 4.5. The rate of increase is similar to when uniaxial compression is applied along the magnetization direction.

Comparison of the results in Fig. 4.4 (a) and (b) with (c) and (d), indicate that when the sample is magnetized along the rolling direction, the loss variations with stress are higher, since stress affects the properties along the rolling and transverse directions in different ways. Similar results are observed also for the shear-II case in Fig. 4.5. For instance, Kai et al. (2011b) and Kai and Enokizono (2017) reported similar experimental results.

Another important result is that variation of the losses is higher at 10 Hz than at 150 Hz. In order to study this further, the losses were segregated into hysteresis, classical, and excess loss components as described in Subsection 3.3.1. It was assumed that the classical loss component is

![Figure 4.5](image_url)

**Figure 4.5.** Measured alternating percentage variations of the loss densities ($\Delta p$) for shear-II stress configuration for magnetization along the rolling (x) direction at frequencies (a) 10 Hz, (b) 150 Hz and magnetization along transverse (y) direction at frequencies (c) 10 Hz, (d) 150 Hz.
independent of stress, since the studied stress states are within elastic limits and the skin effect was neglected. The hysteresis loss coefficient \( c_{hy} \) and excess loss coefficient \( c_{ex} \) were obtained by linear least squares fitting of the total loss density from (3.36) to the measured loss data at each stress state. In Fig. 4.6 (a) and (b), percentage shares of the loss density components in total loss density are shown under uniaxial stress applied parallel to the magnetization for 10 Hz and 150 Hz frequency, respectively. The sample was magnetized along the rolling (x) direction. At low frequency, hysteresis losses are dominant. As the frequency increases, the classical and excess losses start becoming more prominent, since they are higher order function of frequency. At low frequency the losses seem to vary more distinctly under stress because the contribution of \( P_{cl} \) to the total losses is the least, which does not vary with stress.

In Publication III it was shown that stress affects \( c_{hy} \) and \( c_{ex} \) in a similar way. For the sake of compactness, in Fig. 4.7, evolutions of \( c_{hy} \) and \( c_{ex} \)
only under uniaxial and biaxial stress for magnetization along the rolling direction are shown. Although the behaviors of $c_{hy}$ and $c_{ex}$ under stress are similar, the maximum variation in the excess loss coefficient is $+145\%$, which is higher than $+83\%$ of maximum variation in the hysteresis loss coefficient.

4.1.3 Repeatability of the Measurements

Repeatability of the measurements was studied in Publication II. The studied cases include both rotating and alternating field measurements under zero stress and few uniaxial and biaxial stress states. The studied stress states are shown in Fig. 4.8. The magnetizing frequency was 10 Hz and the peak induction levels were 1 T and 1.2 T for rotational and alternating fields, respectively. The sample was dismantled and assembled again several times before performing the second measurements.

Considering the stress free case the maximum differences between the newer and the older measurements for the loss and field strength were 7.44% and 8.44% under rotating and 3.70% and 5.83% under alternating field, respectively. As stated by Zurek (2005) usually the accuracy of a rotational single sheet tester without stress application is not better than 7%. Including the stressing mechanism to the system increased the maximum differences for the loss and field strength to 11.75% and 14.4% under rotating and 4.19% and 8.39% under alternating field, respectively.

![Figure 4.8. Stress cases on which the repeatability of the measurements is studied.](image)

Some of the sources that can affect the repeatability are alignment of the force actuators with respect to the sample, orientation of the rolling and transverse directions of the specimen with respect to the force actuators, ambient temperature, friction between the sample, sample holder and reinforcing plates, the amount of torque applied to the bolts of the coupling...
plates between the sample and actuators, wear and tear of the components.

4.2 Magneto-Mechanical Modeling

This section presents the modeling results of the effects of stress on the magnetization behavior of NO electrical steel sheets. For modeling, the presented simplified multiscale (SM) model and Helmholtz energy based (HE) model were used. Three different Si-Fe NO electrical steel sheets from different suppliers were studied. These include a grade M330-50A (Material I), M400-50A (Material II) and another M400-50A (Material III).

The magneto-mechanical experimental data for Material I was obtained from Rekik, Hubert and Daniel (2014) and the experiments include uniaxial and biaxial stress dependent magnetization measurements. The measurements for Material II were obtained from Singh (2017) and they include uniaxial stress dependent magnetization and magnetostriction data. The measurement data for Material I and II are only for magnetization applied along the rolling (x) direction. Magnetization loops for Material III were measured using the presented RSST under multiaxial stress configurations that were presented in Section 4.1. It is important to note that both models were identified by using only uniaxial stress dependent measurement data for all the materials. Thus, the modeling results under multiaxial stress configurations reveal the predictive abilities of the models.

4.2.1 Anhysteretic Modeling

The parameter identification procedures for anhysteretic SM and HE models were detailed in Publication IV, and the important aspects will be summarized here. Considering Si-Fe alloys, the SM model parameters $M_s$ and $\chi_0$ can be identified using a stress free anhysteretic magnetization curve. To identify the parameter $\eta$ which describes the non-monotonic behavior under stress, an anhysteretic magnetization curve under uniaxial tensile stress is sufficient. The parameter $\lambda_s$ can be identified from stress free magnetostriction measurement, or it can be estimated as long as the Si content of the material is known (Bozorth, 1951). However, this approach might result in inaccurate description of the material behavior. In these cases, the parameter $\lambda_s$ can be adjusted using a single stress dependent magnetization curve.
On the other hand, HE model parameters can be identified from several stress dependent magnetization measurements. The identification can be performed by least-squares fitting of the modeled anhysteretic results to the measured ones. Usually, in addition to a stress free magnetization curve, magnetization curves under few uniaxial stresses are sufficient. It is worth mentioning that the required number of material parameters for HE model is material dependent.

The anhysteretic modeling results of Materials I and II were presented in Publication IV and will be summarized here. In addition, the modeling results for Material III will be presented. In the following, for the stress tensor, the notation $\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{yy} & \tau_{xy} \end{bmatrix}^T$ is used.

**Material I**
The modeled magnetization curves by SM and HE models are compared to the measured ones in Fig. 4.9 (a) and (b), respectively. The HE model being more accurate both models describe the magneto-mechanical behavior of this material under uniaxial stress successfully. Note that the HE model parameters were identified using the $B-H$ curves, which are given in 4.9 (b). The parameter $\lambda_s$ of the SM model was estimated from (Daniel et al., 2004) for 3% Si-Fe alloy. Detailed parameter identification procedures for the models were presented in Publication IV. In Fig. 4.9 (c) and (d) modeled and measured permeability evolutions under uniaxial, equibiaxial and shear-I stress configurations at a magnetic field strength of 500 A/m along the rolling direction are shown for the SM and HE models, respectively. Both models predict the permeability evolutions between -50 MPa and 50 MPa for all the stress cases with reasonable accuracy. At higher stress values, both models overestimate the effect of shear-I case on the permeability. Under equibiaxial stress above 50 MPa, the SM model underestimates the permeability to a considerable degree. Under hydrostatic pressure, both models produce constant permeability that equals that of the stress free case, which is consistent with magneto-elastic theories.

**Material II**
The magneto-mechanical behavior of Material II differs from that of Material I. The permeability of Material II evolves non-monotonically under uniaxial stress, such that both compressive and high tensile stress reduce the permeability. In addition, magnetostriction reversal from positive to negative is observed when more than a certain tensile stress magnitude is applied. A configuration field term $H_{conf}$ was added to SM model to
Figure 4.9. Comparison of measured uniaxial stress dependent anhysteretic magnetization results with modeled results from (a) SM model and, (b) HE model for Material I. Comparison of measured permeability evolution under multiaxial magneto-mechanical loadings with modeling results obtained from (c) SM model and (d) HE model.

take into account this non-monotonic effect of stress on the permeability (Daniel et al., 2015). However, as shown in Publication IV, despite $H_{conf}$ the SM model cannot accurately model this non-monotonic behavior. On the other hand, the HE model can take the variation of both permeability and magnetostriction under uniaxial stress into account. In order to demonstrate the comparison of the models, $B-H$ curve modeling results from the SM and HE models under uniaxial stress compared to the measured ones in Fig. 4.10 (a) and (b). The HE model was identified using $B-H$ curves shown in 4.10 (b). More detailed information on the parameter identification procedure can be found in Publication IV. In Fig. 4.10 (c) and (d), the magnetostriction modeling results from the SM and HE models are compared with measurements under the same stress values. Clearly, the HE model produces a more accurate description of the magneto-mechanical behavior of the Material II. In fact, it was shown in Publication IV that by removing $H_{conf}$ from the SM model, more accurate results for modeling the $B-H$ characteristics under low tensile and compressive stresses could be obtained.

In order to analyze the modeling ability of HE model further, in Fig. 4.11 (a), the relative permeability evolutions under uniaxial and biaxial stresses
are shown under an applied field of 300 A/m. Measured permeabilities under uniaxial stress are also shown. Under the studied uniaxial stress levels, the permeability modeling results from the HE model match well with the measured ones. The modeling results under biaxial stress indicate that the equibiaxial and shear-I stress configurations, respectively, have the smallest and the biggest effects on the material. Although the biaxial measurements for Material II are not available, the modeled behavior under biaxial stress is consistent with the measurements of Material I and Material III.

In 4.11 (b), a comparison of modeled and measured magnetostriction values at certain flux density and uniaxial stress levels is shown. The modeling results are in reasonable agreement with the measurements for the studied cases. However, despite the fact that the modeling magnetostriction curves are close to the measured ones, the rotation mechanism at high flux density levels is not well taken into account.

Material III
Magneto-mechanical measurements for Material III were obtained from the presented RSST device in Publication II. The measurements were per-
formed under sinusoidal magnetic flux density along the rolling direction at 10 Hz frequency and 1.2 T peak induction level for uniaxial and biaxial stress configurations. In addition, measurements under shear-II case at 10 Hz with 1 T induction level were performed. The magnitude of the stress $\sigma$ varied from –30 to 30 MPa at 10 MPa intervals.

Since Material III was not studied in Publication IV, the parameter identification will be detailed here. SM model parameters $M_s$ and $\chi_0$ are identified from stress free magnetization curves. Since Material III is a 3% Si-Fe alloy, $\lambda_s$ can be estimated from Daniel et al. (2004) as in the case of Material I. However, this approach resulted in an inaccurate description of the material behavior under uniaxial stress. Therefore, $\lambda_s$ was adjusted using the measured magnetization curve under 10 MPa compression. Parameter $\eta$ was obtained by fitting the modeled $B$-$H$ curve under 30 MPa tensile stress to the measured one. The determined SM model parameters for Material III are given in Table 4.1. The modeled anhysteretic magnetization curves by SM model are compared to the measurements in Fig. 4.12 (a). Although SM model describes the material behavior under zero stress and compressive stress, under tensile stress the accuracy is poor.

Table 4.1. SM model parameters for Material III.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_s$</td>
<td>$1.25 \cdot 10^6$ A/m</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>$15.5 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>6500</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$7 \cdot 10^{-5}$</td>
</tr>
</tbody>
</table>
The HE model requires ten material parameters to describe the magneto-mechanical behavior of Material III. The parameters were identified by fitting the modeled anhysteretic \( B-H \) curves to the measured ones under uniaxial stresses of \(-20\) MPa, \(-10\) MPa, \(0\) MPa, and \(30\) MPa, which are applied along the magnetization direction. The determined HE model parameters for Material III are given in Table 4.2. Fitting results are compared to the measurements in Fig. 4.12 (b). The anhysteretic magnetization results from HE model fit reasonably well with those of the studied cases.

**Table 4.2.** HE model parameters for Material III.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Value</th>
<th>Param.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>( 8.441 \cdot 10^{-5} ) J/m(^3)T(^2)</td>
<td>( \alpha_5 )</td>
<td>( -6.984 \cdot 10^{-6} ) J/m(^3)T(^2)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>( -8.691 \cdot 10^{-5} ) J/m(^3)T(^4)</td>
<td>( \alpha_6 )</td>
<td>( 1.255 \cdot 10^{-6} ) J/m(^3)T(^4)</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>( 3.282 \cdot 10^{-4} ) J/m(^3)T(^5)</td>
<td>( \alpha_7 )</td>
<td>( 3.770 \cdot 10^{-8} ) J/m(^3)T(^6)</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>( -2.782 \cdot 10^{-4} ) J/m(^3)T(^8)</td>
<td>( \beta_0 )</td>
<td>( -1.017 ) J/m(^3)T(^2)</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>( 9.268 \cdot 10^{-5} ) J/m(^3)T(^10)</td>
<td>( \gamma_0 )</td>
<td>( 4.164 \cdot 10^{3} ) J/m(^3)T(^2)</td>
</tr>
</tbody>
</table>

In order to analyze the modeling abilities of the SM and HE models further, the modeled and measured permeability variations under uniaxial and biaxial stress states are calculated for a 225 A/m applied magnetic field magnitude. In addition, permeabilities under hydrostatic pressure are modeled to verify that models produce constant permeability for this stress state. The results are shown in Fig. 4.12 (c) and (d) for SM and HE models, respectively. Both models produce constant permeability under hydrostatic pressure, as expected. The SM model is successful in modeling the permeability variation when \( \sigma < 0 \) MPa under uniaxial stress and when \( \sigma > -10 \) MPa under equibiaxial stress. For all the other cases, the accuracy of the modeling results compared to the measurements is poor. Although more accurate than the SM model, the HE model overestimates the effect of equibiaxial and shear-I stresses on the permeability when \( \sigma < 0 \) MPa. When \( \sigma > 0 \) MPa, the HE model predicts the permeability variation successfully for all the studied stress configurations.

Since for Material III, measurements under shear-II stress configuration at 1 T peak induction are available, the predictive abilities of the SM and HE models under this stress case are tested. The modeling results from SM and HE model are compared to the measurements in Fig. 4.13 (a) and (b), respectively. The results reveal that the SM model is unsuccessful in modeling the effect of shear-II stress. On the other hand, the HE model produces fairly accurate results. It is worth noticing that since the HE
model is isotropic, it produces identical results when \( \tau < 0 \) and \( \tau > 0 \). However, as discussed in Subsection 4.1.2, the measured \( B-H \) curves are slightly different for these cases due to the magneto-elastic anisotropy.

![Graph](image1)

**Figure 4.12.** Comparison of measured uniaxial stress dependent anhysteretic magnetization results with modeled results from (a) SM model, (b) HE model for Material III. Comparison of measured permeability evolution under multi-axial magneto-mechanical loadings with modeling results obtained from (c) SM model and, (d) HE model.

![Graph](image2)

**Figure 4.13.** Comparison of measured anhysteretic magnetization curves under shear-II stress with modeled results from (a) SM model and, (b) HE model for Material III.

### 4.2.2 Hysteresis Modeling

The inclusion of the magnetic hysteresis to the anhysteretic models was described in Section 3.2. Publication V presents the comparison of SM and HE models to predict the multiaxial stress dependent magnetization be-
havior of Material I, including hysteresis. The results of Publication V will be summarized here for Material I. The hysteresis modeling for Material II was not studied, since no multiaxial measurements are available for this material. Uniaxial stress dependent hysteresis modeling of Material II was performed previously by Rasilo et al. (2016).

Since the SM model was inaccurate in modeling the anhysteretic behavior of Material III under the tensile stress regime, the hysteresis modeling of this material was studied by using only the HE model. The measurements used for this study were performed under uniaxial and biaxial stress states, at a 1.2 T peak induction level and at 10 Hz magnetization frequency.

The hysteresis model parameters of the SM model were identified using a single stress free magnetization loop. On the other hand, the HE model parameters for hysteresis modeling were identified by using various uniaxial stress dependent magnetization loops which were also used to identify anhysteretic parameters.

**Material I**

The predicted hysteresis loops by the SM and HE models are compared to the measurements under various stress states in Fig. 4.14. Modeled hysteresis loops show reasonable accuracy compared to the measured ones for the studied cases. However, accuracy decreases as stress level increases. This is mainly due to the inaccurate prediction of the anhysteretic behavior under stress, which was studied above.

In Figure 4.15 (a) the measured hysteresis energy loss densities are shown. Comparing the loss evolution under multi-axial stress of Material I with that of Material III (Fig. 4.4) reveals contradictory results. This is caused by the fact that the measurement amplitudes of $B$ and $H$ vary with the applied stress for Material I. The relative errors between the measured and modeled hysteresis energy loss densities by the SM and HE models are shown in 4.15 (b) and (c), respectively. The HE model being more accurate, both models predict the stress dependent loss behavior with acceptable accuracy.

**Material III**

The hysteresis model parameters $\alpha_{ja}$, $c$, $k_0$, $a$ and $b$ are determined using the procedure explained in Subsection 3.2.2 and the parameter values are given in Table 4.3. The predicted hysteresis loops by HE model are compared to the measured ones in Fig. 4.16 for several stress states. The modeling results agree with the measurements with acceptable accuracy.
Accuracy could be improved if better anhysteretic fitting of the HE model to the measurements is obtained. Under compressive stress the bowing of the hysteresis loops at low induction, which can be seen in Fig. 4.3 is not taken into account by the adopted Jiles-Atherton hysteresis model. Some modifications to the hysteresis model could also be realized for modeling this behavior. In Fig. 4.17 (a), the measured hysteresis energy loss density variations under uniaxial and biaxial stress states are shown. The relative error between measured and modeled hysteresis losses by the HE model is given in Fig. 4.17 (b). The highest errors are observed under uniaxial compressive stress and shear-I stress case, with compressive stress along...
the magnetization direction (second quadrant) where $|\sigma| > 20$ MPa. Except for these cases the loss modeling results match reasonably well with the measurements.

Figure 4.16. Comparison of the measured and the modeled hysteresis loops with HE model under several stress states for Material III.

Figure 4.17. (a) Measured hysteresis losses of Material III. (b) Errors between measurements and HE model results.

Table 4.3. JA model parameters for Material III.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Value</th>
<th>Param.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{ja}$</td>
<td>$6.660 \cdot 10^5$ A/m</td>
<td>$a$</td>
<td>$-3.322 \cdot 10^3$</td>
</tr>
<tr>
<td>$c$</td>
<td>$1.546 \cdot 10^{-1}$</td>
<td>$b$</td>
<td>$1.012 \cdot 10^7$</td>
</tr>
<tr>
<td>$k_0$</td>
<td>$5.924 \cdot 10^1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Considering the modeling results under shear-II stress configuration, although the anhysteretic model describes the material behavior with reasonable accuracy under this stress state, the hysteresis modeling results are inaccurate compared to the measurements due to the significant bowing of the measured hysteresis loops at low induction levels.

### 4.3 Iron Loss Modeling

Publication III presents the multiaxial stress dependent iron loss modeling results for Material III. The stress dependent loss model was presented in Section 3.3. The model parameters $\beta_h$, $\gamma_h$, $\beta_e$, and $\gamma_e$ were obtained by using the loss data of those cases when uniaxial stress is applied parallel to the magnetization direction. The modeled total energy loss densities for each studied stress case and frequency are compared to measurements in Fig. 4.18 (a) and (b) when magnetization is applied along the rolling (x) and transverse (y) directions, respectively. In Fig. 4.18 losses are plotted by sorting them as ascending with respect to the studied magneto-mechanical cases. The peak induction level was 1 T for all the cases. The predicted losses are in reasonable agreement with the measurements. The errors between measured and modeled losses were 5.6% and 9.9% for magnetization applied along the rolling and transverse directions, respectively. The highest errors are observed when the effect of stress on the losses is significant.

![Figure 4.18](image-url)

**Figure 4.18.** Total energy loss densities for each stress and magnetization state for Material III. Measurements and modeling results from the presented stress dependent loss model. (a) Magnetization along the rolling direction, (b) Magnetization along the transverse direction. The losses are sorted in ascending order.

The stress dependent loss model was used to calculate the iron losses in the test transformer that is under stress which was presented in Sub-
section 3.3.2. To analyze the applicability of the HE model and the stress dependent loss model, three cases were studied. Case I does not include stress dependency, either in the loss nor in the field computation. Case II includes stress dependency only in the loss calculation. Case III includes stress dependency for both the loss and field computation. Stress dependency on the field is taken into account by implementing the HE model into FE model as explained in Subsection 3.3.2. The studied cases are summarized in Table 4.4.

Table 4.4. Studied cases for modeling the transformer.

<table>
<thead>
<tr>
<th>Case number</th>
<th>HE model is implemented into FE model</th>
<th>Stress dependent loss model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Case II</td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>Case III</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

The relative errors between the measured losses and the modeled ones by each case under the studied stresses are given in Fig. 4.19. Under no applied stress, the error between the modeled and the measured losses are 7.3%. This relatively high error between the modeled and measured losses is due to the material degradation by laser cutting of the E and I pieces, which was not taken into account by the FE model. In addition, in reality small airgap between E and I pieces is present and it varies between the laminations. Since the widths of those airgaps are unknown no airgap is considered in the simulation and this might also have an effect on the accuracy of the FE model. As the applied stress increases, clearly Cases II and III give better results compared to Case I. Including the coupled model increases the accuracy for modeling the losses slightly compared to Case II, which is when only stress dependent iron model is used.

Figure 4.19. Errors between measured and modeled iron losses in the transformer.
The percentage loss increase between the stress free case and under 16 MPa applied stress at 18 V primary voltage is calculated, and the distribution in the transformer is shown in Fig. 4.20 (a). The highest increase is observed at the I and back of the E piece, since these parts experience the highest compressive stress that aligns with the magnetization direction. When the material is magnetized along the rolling direction, stress affects the losses more distinctly compared to when the sample is magnetized along the transverse direction. This was observed in Fig. 4.4. Thus, the loss increase for the I piece is lower than the back of the E piece, since the I piece was characterized using transverse direction material properties. At the middle part of the transformer, the magnetization and compressive stress are perpendicular to each other. Therefore, at this location decrease in losses is observed. In Fig. 4.20 (b), norm of the flux density difference distribution between the stressed and stress free case is shown. Similar to the loss variations, the largest flux density differences are observed at the back of the E piece.
Application and Results
5. Discussion and Conclusions

This thesis has focused on the multi-axial magneto-mechanical interactions in NO electrical steel sheets. In this chapter, summary and discussion of the methods and findings are presented. Finally, some suggestions for further research are also given, and the thesis is concluded.

5.1 Discussion of the Methods and Results

5.1.1 Summary of the Findings

Magneto-Mechanical Measurements

A rotational single sheet tester (RSST) device that has ability to apply arbitrary in-plane magneto-mechanical loading to steel sheets was designed and manufactured. The RSST device was used to perform the magneto-mechanical measurements for an M400-50A NO electrical steel sheet in Publications II and III. The developed flux density control algorithm was successful in controlling the 3-phase voltages to obtain rotational and alternating flux density waveforms. The maximum flux density and mechanical stress for the measurements were 1.2 T and $\pm 30$ MPa. Above 1.2 T, the convergence of the waveform control algorithm was poor. The main cause is the saturation of the narrow areas near the central region in the sample geometry. These narrow parts help in obtaining higher stress magnitude in the central area, with lower force application to the legs compared to the sample without these narrow parts. The applied mechanical stress magnitude was limited by the buckling of the sample under compressive loading. Although the sample was reinforced by the retention plates, the force applied by the clamping mechanism was not sufficient to prevent the buckling above these limits.
The magneto-mechanical measurement results revealed that the effect of multi-axial stress can be much more significant than that of uniaxial stress, both on permeability and iron losses. In Publication III, the effect of multiaxial stress on hysteresis and excess loss components was studied utilizing the statistical loss theory of Bertotti et al. (1988). It was assumed that stress has no effect on the classical eddy current losses for the studied frequency and the stress amplitudes, since the macroscopic eddy currents are assumed to be sourced from a domain free structure. This is a strong assumption into which further investigation is needed, especially for frequencies above 100 Hz. Under this assumption, it was shown that the hysteresis and excess loss coefficients behave similarly under stress. However, the maximum variation in the excess loss coefficient was found to be significantly higher than that of the hysteresis loss coefficient. It was also observed that stress affects the material differently depending on the applied magnetization direction, which is attributed to the magneto-elastic anisotropy. This effect may be significant in electromagnetic applications where mechanical stresses are present.

Magneto-Mechanical Modeling
A simplified multiscale (SM) and a Helmholtz energy based (HE) models were studied to predict the anhysteretic multiaxial magneto-mechanical behavior of three different NO electrical steel sheets. The measurements for Materials I and II were obtained from the literature, whereas those for Material III were gathered using the presented RSST. The objective was to use only the uniaxial stress dependent data to identify the models and test their predictive abilities for multiaxial magneto-mechanical behavior.

It was found that although the SM model was successful for Material I, it was inaccurate for the other two materials. Considering uniaxial stress applied along the magnetization direction, the SM model is mostly unsuccessful for predicting the effect of tensile stress, especially when the permeability varies non-monotonically under tension. On the other hand, the HE model was proven to be much more flexible, since it was successful in predicting the multiaxial behaviors of all three materials with reasonable accuracy. A limitation of the HE model is that its predictive ability outside the range of identified stress levels can be poor.

The SM model provides very simple identification procedures and can be useful when limited experimental data is available. However, it should be used with caution. If more comprehensive experimental data is available
Discussion and Conclusions

for identification, the HE model can provide a more accurate description of the magneto-mechanical behavior. In addition, since the input variables of the HE model are flux density and strain, implementation of the model to the general vector potential and displacement field formulations is relatively easy. This provides the advantage of implementing the HE model directly in many readily available magnetic and mechanical FE tools.

Considering the inclusion of magnetic hysteresis, the first requirement to obtain a good description of the hysteresis with the studied approaches is that anhysteretic behavior should be accurately modeled. Thus, the SM model implemented in the Hauser hysteresis model was only accurate for Material I. On the other hand, the HE model incorporated with the Jiles-Atherton hysteresis model was proven to predict the magnetic hysteresis of Materials I and III under uniaxial and biaxial stress states with reasonable accuracy. In the shear-II stress case the HE model was not accurate to describe the hysteresis. The main cause of the inaccuracy comes from the fact that under shear-II stress configuration at low induction levels, significant bowing of the hysteresis loop occurs, and the adopted hysteresis model cannot take this into account.

Iron Loss Modeling

A loss model was developed to predict the effect of multiaxial stress on the iron losses utilizing the statistical loss theory of Bertotti et al. (1988). The model is based on the magneto-elastic invariants that were also used in the HE model and can be identified only from uniaxial stress dependent loss data. In Publication III, the model was verified to predict the multiaxial stress dependency of the iron losses in Material III with reasonable accuracy. The model is simple and computationally light, and it can easily be implemented in FE models to calculate the stress dependent iron losses in electromagnetic applications. It is worth mentioning that although the loss model was successful for the studied material, it should still be tested and verified for other materials and preferably for higher flux density levels.

A small transformer was constructed using Material III to test the applicability of the loss model. Compressive stresses with various magnitudes were applied to the whole transformer up to its destruction, and the iron losses were measured. The loss variations under the studied stress levels before the buckling were rather small. A 2D FE model of the transformer was developed, and the presented stress dependent iron loss model was im-
Discussion and Conclusions

implemented for loss calculation. It was found that the inclusion of the stress dependent loss model provides more accurate loss calculation results when mechanical stresses are present, compared to a conventional loss model without stress. In addition, in order to take into account the effect of stress on the field solution, the HE model was also implemented to the FE analysis of the transformer. Including HE model slightly improved the accuracy of the loss calculation. However, the effect of stress on the field solution strongly depends on the studied geometry and the magneto-mechanical loadings.

5.1.2 Significance of the Work

A new RSST device to characterize multiaxial magneto-mechanical behavior of ferromagnetic steel sheets was developed. The developed RSST and control programs allow for applying arbitrary in-plane magneto-mechanical loading to the steel sheets in an automated manner. In a similar study, one device that could be used to apply arbitrary in-plane mechanical loadings manually was presented that could provide up to 0.8 T maximum induction level in the sample (Kai et al., 2015). Other than this study, to the author’s knowledge, there is no other experimental setup in the published literature that could perform such multiaxial experiments.

It was shown in the experiments that multiaxial stresses that are often ignored can have significant effects on the magnetic properties and therefore on the iron losses in electrical steel sheets. The stress dependent iron losses were segregated, and the effects of multiaxial stress on different loss components were studied. As mentioned in Chapter 2, this topic has received little attention in the previous literature. In fact, considering the effect of multiaxial stress, only one study was performed by segregating the losses using the measurements with non-sinusoidal flux density and uniaxial and biaxial stresses (Yamazaki et al., 2017).

A simplified multiscale (SM) model of Daniel et al. (2015) and a slightly modified version of the Helmholtz energy based (HE) model of Fonteyn (2010) were adopted to predict the effect of multiaxial stress on anhysteretic magnetic behaviors of three different NO electrical steel sheets. Although these models were developed to be fully multiaxial, they have not been verified to be successful under these loadings. The magnetic hysteresis was also included to the SM and HE models following the approaches of Daniel et al. (2015) and Rasilo et al. (2016), respectively. Contrary to expectations, the SM model was successful only for some particular cases,
whereas the HE model described the multiaxial magneto-mechanical behaviors of the studied materials with reasonable accuracy. Both models have limitations, and these were previously discussed.

Modeling the stress dependency of the iron losses has not been studied very much in the literature. Few previous studies have been performed to model the uniaxial stress dependent iron losses, as mentioned in Chapter 2. Considering the multiaxial stress dependency of the iron losses, only one study was performed (Yamazaki et al., 2017). A new model was developed and verified to predict the effect of multiaxial stress on the iron losses when only uniaxial stress dependent loss data is available. Using a transformer that is put under stress, it was shown that if mechanical stresses are present in an application, using the conventional methods to calculate the iron losses could yield significant inaccuracies. The developed stress dependent iron loss model, on the other hand, yielded more accurate loss calculation results.

5.2 Suggestions for Future Work

5.2.1 Magneto-Mechanical Measurements

In this work, the effect of static mechanical stress on the magnetic properties of electrical steel sheets was studied. However, in some applications, mechanical stress can have a dynamic nature. In order to be able to model these devices accurately, characterization of the materials under dynamic stress would be needed. The RSST device designed in this study is capable of applying dynamic multiaxial loadings, and the setup can easily be adapted to test different geometries. Thus, comprehensive characterization of different materials under dynamic stress can be made. In order to do this, one would need to develop an efficient control method for the servo drives to apply the desired dynamic mechanical stresses.

In addition, the present device to test the magneto-mechanical behavior of electrical steel sheets can be modified to obtain higher peak induction levels and mechanical stresses for the experiments. Higher peak induction levels could be achieved by redesigning the sample geometry. Possibly, the sample geometry can be optimized by running an optimization algorithm. Since 3D field simulations are very costly for this purpose, a proper reluctance network could be developed. However, one must be careful here,
especially for the yoke parts and the airgaps between the yokes and the sample. Flux density waveform control can also be improved to achieve better and faster convergence. For instance, the control coefficients can be adapted for certain loadings using various methods (Visioli, 2001; Jinhua et al., 2009).

In order to increase the mechanical stress limits the clamping mechanism for retention plates should be redesigned to avoid the buckling of the sample under compressive loading. Possibly, the clamping for retention plates could be fixed to the sample holder rather than to the outer frame. This would allow applying higher forces to the retention plates without disturbing the alignment of the actuators.

5.2.2 Magneto-Mechanical Models

It was shown that the SM model was unsuccessful for instance, under uniaxial tensile stress coaxial with magnetization for two of the studied materials. The SM model was defined for a single crystal that is free of domain walls, and in the model, stress directly affects these domains. However, it is known that external stress mainly affects the domain walls. A modification to the SM model can be proposed by considering the missing energy terms to take the effect of stress on the domain walls into account. This could indeed be challenging and could cause significant increase in computation costs. Another simpler way could be to assume the stress dependency of the saturation magnetostriction parameter. This would indeed result in more material parameters to be identified. Nevertheless, a more comprehensive study on modification of the SM model can be performed to make it more accurate. If the SM model provides more accurate results, then obtaining accurate magneto-mechanical behavior of many core materials with very limited experiments would be possible.

On the other hand, the HE model with the Jiles-Atherton hysteresis model was lacking accuracy in terms of the bowing of the magnetization loops under low induction and compressive stress. Earlier, a modification was proposed to take this into account by Singh et al. (2016). This could also be implemented in the model with the cost of more material parameters to be identified. In addition, instead of describing the free energy density with polynomials, splines can be adopted for this purpose. Such an approach could result in better accuracy for the model, not only for the identified magneto-mechanical loading ranges, but also for larger loadings.
5.2.3 Stress Dependent Iron Losses

In this work, the effect of stress on the classical loss components was neglected. This assumption was made by neglecting the skin effect and assuming that the studied stresses were within elastic limits. A study performed by Permiakov et al. (2004a) reached the conclusion that the classical eddy current losses do not depend on elastic stress. However, this assumption should be verified by modeling the thickness of the lamination with the stress application. Since the application of stress affects the permeability, the skin depth will be affected and thus the classical eddy current losses might vary under stress.

The developed iron loss model was proven to be accurate for the studied material. However, the applicability of the model should be tested with other materials as well. In addition, the applicability of both the HE model and the stress dependent iron loss model should be tested with other applications such as rotating electrical machines. Indeed, this can be practically challenging, since the core material of the machine should be available for magneto-mechanical testing to identify the models.

5.3 Conclusions

A new RSST test setup was designed and manufactured to test the effect of multiaxial mechanical stress on the magnetic properties and iron losses of ferromagnetic sheets. Experiments on a non-oriented (NO) electrical steel sheet were performed and the results proved the significance of multiaxial stress effects on both the magnetic properties and the iron losses. The capabilities of a Helmholtz energy based model and a stress dependent iron loss model were proven for taking into account the effect of multiaxial magneto-mechanical loadings on NO electrical steel sheets that are often used as core materials in many electromagnetic applications. The studied models in this study, provide useful tools to include the multiaxial stress effects for simulating these devices, when limited measurement data are available for identification purposes.


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References


Considering the manufacturing processes of rotating electrical machines, the cores are punched, pressed, welded together and shrink fitted to the housing and shaft. Due to these processes, significant mechanical stresses are exerted on the core material. These stresses are multiaxial in nature, and they alter the magnetic properties of the core material, affecting the iron losses. Clearly, in order to design more efficient devices and analyze existing ones with higher accuracy, comprehensive multiaxial magneto-mechanical characterization and modeling of ferromagnetic sheets are needed. Magneto-elastic interactions in ferromagnetic materials are not only responsible for additional losses. For instance, due to these interactions stator vibrations in electrical machines and audible noise generation in transformers occur.

This thesis aims to contribute to this field by studying the multiaxial magneto-mechanical interactions in electrical steel sheets both experimentally and numerically.