Radar-Communications Systems Coexistence and Agile Multicarrier Radars

Marian Bică

Radar-Communications Systems Coexistence and Agile Multicarrier Radars

Marian Bică

A doctoral dissertation completed for the degree of Doctor of Science (Technology) to be defended, with the permission of the Aalto University School of Electrical Engineering, at a public examination held at the lecture hall AS1, TUAS building of the school on 21 September 2018 at 12:00.

Aalto University
School of Electrical Engineering
Department of Signal Processing and Acoustics

Supervising professor
Prof. Visa Koivunen, Aalto University, Finland

Thesis advisor
Prof. Visa Koivunen, Aalto University, Finland

Preliminary examiners
Prof. Marco Lops, University of Cassino, Italy
Dr. Bhavani Shankar, University of Luxembourg, Luxembourg

Opponents
Prof. Marco Lops, University of Cassino, Italy
Dr. Braham Himed, Air Force Research Laboratory (AFRL), USA

NORDIC ECOLABEL

441   697
Printed matter

Author
Marian Bică

Abstract

Rigid allocation policies can no longer satisfy the growing demand for spectrum. There is a great need for flexible and efficient use of spectral resources. Spectrum sharing between different wireless systems is one approach to solve this problem. This requires that the systems are fully adaptive and cognitive. Consequently, these can take advantage of their operational environment knowledge such as the state of the radio spectrum. For such demanding task, multicarrier waveforms provide a convenient and flexible way of generating and designing waveforms. Thanks to their multiple degrees of freedom in time, frequency and code domain, multicarrier waveforms allow for performance optimization as well. In this thesis, theories, methods and justifications for coexistence and spectrum sharing among radar and wireless communication systems are developed.

A generalized multicarrier radar (GMR) model is proposed to enable the required flexible use of spectrum and adaptation to different target scenarios and operational environments. It allows for conveniently representing most of the known radar waveforms as well as designing and optimizing new ones. Using the developed model novel multicarrier waveforms are proposed. These waveforms relax the subcarrier orthogonality constraint for an improved ambiguity function and consequently better radar performance. Constrained radar waveform optimization using the GMR model and a mutual information (MI) based objective function is proposed. Several examples of waveforms employing fast frequency hopping, which are special cases of the GMR model, are also provided.

The problems of radar waveform optimization and adaptation are addressed in the coexistence context using multicarrier waveforms, in particular OFDM. The optimization problems are formulated for different radar tasks, i.e. target characterization, target detection and target parameter estimation. The optimization is performed using information theoretic criteria such as mutual information (MI) or criteria stemming from detection and estimation theory, for example, Neyman-Pearson (NP) criterion or Cramer-Rao bound (CRB). The optimization problems proposed in this thesis are formulated by imposing constraints on the total transmitted radar power and an interference mask provided by the communication system. This interference mask ensures that a desired rate for the communication users can be achieved. The solutions to the optimization problems represent power allocations over the available subcarriers. The performance of the proposed optimized waveforms in this thesis is evaluated in simulations using receiver operating characteristic or root mean squared error plots, for the detection and estimation tasks respectively. Recommendations on which optimization objective functions produce the best performing waveforms are given. It is also demonstrated that exploiting the communication signal reflected off the target can improve the radar performance for all considered tasks.

Preface

The research work for this dissertation has been carried out mainly at the Signal Processing and Acoustics (SPA) Department, School of Electrical Engineering, Aalto University. The completion of this dissertation, as well as the publications on which it is based, would not have been possible without the constant support and guidance of my supervisor, Prof. Visa Koivunen. I am extremely grateful for his efforts throughout the years I have spent under his supervision and the opportunities he has provided for me. Prof. Koivunen's unparalleled patience, constructive feedback and outstanding technical insight are fundamental to this research work.

I would like to extend my sincere gratitude to the pre–examiners of this dissertation, Prof. Marco Lops and Dr. Bhavani Shankar, who have made valuable remarks and suggestions for improving the quality of this dissertation. I would like to thank Prof. Marco Lops and Dr. Braham Himed for serving as opponents at my defense.

I also wish to thank Prof. Urbashi Mitra for hosting me at University of Southern California (USC), facilitating and supporting my short visit there. I am grateful for her guidance and many lessons I have learned during the time I have spent at USC as well as for being an insightful co-author. I would like to acknowledge also Kuan-Wen Huang at USC for our collaboration and many fruitful discussions.

Special thanks are due to several people who have been of great help throughout my time spent at the SPA department. Particularly I would like to acknowledge Dr. Jussi Salmi and Dr. Mário Costa who have been very supportive, helpful and have provided advice on various topics. Especially helpful in the past few years has been Dr. Hassan Naseri through countless discussions and debates on a wide range of topics. I am extremely grateful for his relentless support and advice which have been very useful and motivating. I would like to acknowledge also Dr. Pramod Mathecken, Dr. Tuomas Aittomäki and Karthik Upadhya for their help and interesting discussions as well as practical advice. I am very grateful to Dr. Traian Abrudan for introducing me to Prof. Koivunen and for the many encouragements to embark on this challenge. Without him my presence in the group wouldn't have been possible. Many other colleagues and ex–colleagues

made the time spent in the SPA department fun and delightful. I would like to acknowledge Adriana Chiş, Janne Aho, Elias Raninen, Dr. Shahab Basiri, Robin Rajamäki, Henri Hentilä, Emad Mozafari, Dr. Yongzhe Li, Dr. Jari Miettinen, Dr. Jan Oksanen, Dr. Jarmo Lundén, Dr. Sachin Chaudhari, Dr. Karol Schober, Dr. Maarit Melvasalo, Dr. Jayprakash Rajasekharan, as well as Prof. Stefan Werner, Prof. Esa Ollila, Prof. Sergiy Vorobyov and Prof. Risto Wichman. I would like to extend my sincere appreciation to Prof. Shannon Blunt, my mentor in the AESS Mentorship Program. Thank you for the encouragement and for the many interesting and valuable discussions.

I gratefully acknowledge the assistance of Mirja Lemetyinen for handling many of the practicalities of me joining the SPA department and her efforts of integrating me to the group. I would also like to recognize the help of Marja Leppäharju and Kati Vuotilainen from the doctoral studies team with matters related to various topics throughout my studies. The research work for this dissertation has been financed through research grants and projects acquired by Prof. Koivunen, as well as the Aalto ELEC Doctoral School in 2016. I would also like to acknowledge the additional financial support provided by Aalto University and Nokia Foundation.

Many thanks to my friends with whom I have shared many joyful moments. I have spent many delightful evenings with Susana, Fahimeh, Varun and Martin. Let's keep this going and have more fun in the future. Many thanks also to all my Romanian friends for their support and precious times together.

Special thanks to my extended family in Helsinki, Miki and Timo, who have always supported, inspired and whose help cannot be overestimated. Their two daughters, Saara and Sofia, never seize to amaze and cheer us up. I am very lucky and proud to have them all in my life. Two special persons in my life, who played a decisive role in who I am today, my mother and my father, deserve my deepest gratitude for their unparalleled support. You have always encouraged and believed in me to great extent and I can only hope to make the time spent away from me worth it. Someone who never wavered in her support and who has been by my side throughout this tough journey, Delia, deserves my deepest gratitude and appreciation. You have been a true partner and I am deeply indebted for your patience and accepting me as I am. I hope this is just the beginning of a long and beautiful journey ahead, thank you.

Espoo, August 19, 2018,

Marian Bică

Contents

List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I Marian Bică and Visa Koivunen. Generalized Multicarrier Radar: Models and Performance. *IEEE Transactions on Signal Processing*, vol. 64, no. 17, pp. 4389-4402, September 2016.

II Marian Bică and Visa Koivunen. Frequency Agile Generalized Multicarrier Radar. In *2014 48th Annual Conference on Information Sciences and Systems (CISS)*, Princeton, NJ, pp. 1-6, March 2014.

III Marian Bică and Visa Koivunen. Waveform Optimization for Target Parameter Estimation in Cooperative Radar-Communication Systems. Submitted to *IEEE Transactions on Aerospace and Electronic Systems*, 2018.

IV Marian Bică, Kuan-Wen Huang, Visa Koivunen and Urbashi Mitra. Mutual information based radar waveform design for joint radar and cellular communication systems. In *2016 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Shanghai, pp. 3671-3675, March 2016.

V Marian Bică, Kuan-Wen Huang, Urbashi Mitra and Visa Koivunen. Opportunistic Radar Waveform Design in Joint Radar and Cellular Communication Systems. In *2015 IEEE Global Communications Conference (GLOBECOM)*, San Diego, CA, pp. 1-7, December 2015.

VI Marian Bică and Visa Koivunen. Delay Estimation Method for Coexisting Radar and Wireless Communication Systems. In *2017 IEEE Radar Conference (RadarConf)*, Seattle, WA, pp. 1557-1561, May 2017.

List of Abbreviations

5G	5th Generation Mobile Networks
ADC	Analog to Digital Conversion
ADSL	Asymmetric Digital Subscriber Lines
AF	Ambiguity Function
BS	Base Station
CBRS	Citizens Broadband Radio Service
CDMA	Code Division Multiple Access
COCS	Consecutive Ordered Cyclic Shift
CP	Cyclic Prefix
CRB	Cramér-Rao Bound
CSI	Channel State Information
DAB	Digital Audio Broadcast
DAC	Digital to Analog Conversion
DMT	Discrete Multitone
DSL	Digital Subscriber Line
DVB	Digital Video Broadcast
DVB-T	Digital Video Broadcast-Terrestrial
FCC	Federal Communications Commission
FDRW	Frequency Diversity Radar Waveform
FFH	Fast Frequency Hopping
FFT	Fast Fourier Transform
FH	Frequency Hopping
FI	Fisher Information
GLRT	Generalized Likelihood Ratio Test
GMR	Generalized Multicarrier Radar
IA	Interference Alignment

ICA	Independent Component Analysis
ICI	Inter-Carrier Interference
IDFT	Inverse Discrete Fourier Transform
IFFT	Inverse Fast Fourier Transform
i.i.d.	independent and identically distributed
IS	Identical Sequence
ISI	Inter-Symbol Interference
KKT	Karush-Kuhn-Tucker
LFM	Linear Frequency Modulated
LTE	Long-Term Evolution
LTI	Linear Time-Invariant
MC-CDMA	Multicarrier CDMA
MC-DS-CDMA	Multicarrier Direct Sequence CDMA
MCPC	Multicarrier Phase Coded
MI	Mutual Information
MIMO	Multiple Input Multiple Output
MLE	Maximum Likelihood Estimator
MMSE	Minimum Mean Square Error
MT-CDMA	Multitone CDMA
NP	Neyman-Pearson
OFDM	Orthogonal Frequency Division Multiplexing
PAPR	Peak to Average Power Ratio
PLC	Power Line Communication
PDF	Probability Density Function
PMEPR	Peak to Mean Envelope Power Ratio
PRT	Pulse Repetition Time
PSLL	Peak Sidelobe Level
RMSE	Root Mean Squared Error
ROC	Receiver Operating Characteristic
SFW	Stepped-Frequency Waveform
SIC	Successive Interference Cancellation
SINR	Signal to Interference and Noise Ratio
SISO	Single Input Single Output
SNR	Signal to Noise Ratio
SPR	Subcarrier Power Ratio
TDRW	Time Diversity Radar Waveform
WiMAX	Worldwide Interoperability for Microwave Access

WLAN   Wireless Local Area Network

List of Symbols

a	Phase shift due to the target delay
$\mathbf{A}$	Diagonal matrix containing the phase shifts of each subcarrier due to target delay
$\mathbf{a}_c$	Vector containing the phase shifts due to the target delay for each subcarrier of the communications signal
$\mathbf{A}_c$	Matrix containing the phase shifts due to the target delay for each subcarrier and channel tap for the communications signal
$\mathbf{a}_r$	Vector containing the phase shifts due to the target delay for each subcarrier of the radar signal
$\mathbf{A}_r$	Matrix containing the phase shifts due to the target delay for each subcarrier and channel tap for the radar signal
$\mathbf{b}_c$	Vector containing the amplitudes of the target impulse response corresponding to the communications signal
$\mathbf{b}_r$	Vector containing the amplitudes of the target impulse response corresponding to the radar signal
$\mathbf{B}_{\text{TX}}$	IDFT matrix unaffected by delay and Doppler effects scaling
$\mathbf{B}_{\text{RX}}$	IDFT matrix affected by delay and Doppler effects scaling
$\mathbf{c}$	Vector of transmitted communications symbols
$\mathbf{C}$	Matrix of transmitted frequency domain chips or symbols
$\mathbf{c}_0$	Vector of transmitted frequency domain chips or symbols
d	Range of the target
$\det(\cdot)$	Determinant
f_c	Carrier frequency
$\mathbf{F}^{-1}$	IDFT matrix
$\mathbf{G}$	Matrix representing the target scattering
h_c	Target impulse response for the communications channel
h_d	Impulse response for the direct path channel between the radar and BS

h_e	Impulse response for the channel between the radar and the communications receiver
h_r	Target impulse response for the radar channel
h_s	Impulse response for the channel between the communications transmitter and receiver
$H(\cdot)$	Differential entropy
$\mathcal{H}_0$	Null hypothesis
$\mathcal{H}_1$	Alternative hypothesis
$I(\cdot,\cdot)$	Mutual Information
$\mathbf{I}$	Identity matrix
$\mathbf{I}_N$	Identity matrix of size $N \times N$
j	Imaginary unit
$\mathcal{J}\{\cdot\}$	Operator reshaping a matrix of size $N \times N$ to a matrix of size $pN \times N/p$
$\mathcal{K}\{\cdot\}$	Operator reshaping a matrix of size $N \times N$ to a matrix of size $N \times pN$
M	Number of chips in a modulating code
$\mathbf{n}$	Vector containing the time domain noise samples
N	Number of subcarriers
$L(\cdot)$	Likelihood ratio test
$p(\cdot)$	Probability Density Function
P_{FA}	Probability of false alarm
P_{D}	Probability of detection
P_{T}	Total transmitted radar power
$\mathbf{r}$	Vector of transmitted time domain radar symbols
s	Speed of light
s_{target}	Speed of the target
$T(\cdot)$	Test statistic
T_c	Chip duration
t_k	Minimum data rate for kth subcarrier
$\mathbf{U}$	Selection matrix for active/inactive subcarriers
$\mathbf{v}_c$	Vector containing the error for the communications measurements
$\mathbf{v}_r$	Vector containing the error for the radar measurements
$\mathbf{W}$	Diagonal matrix containing the weights of the subcarriers
w_n	Weight of the nth subcarrier
$\mathbf{y}$	Vector of received time domain samples
$\mathbf{y}_r$	Vector of received time domain radar samples
α	Maximum allowed probability of false alarm
γ	Doppler shift at passband carrier frequency

$\mathbf{\Gamma}$	Matrix containing the Doppler shift introduced by the target
Δf	Intercarrier spacing
η	Detection threshold
μ	Target complex scattering amplitude
μ_c	Target scattering coefficient for the communications signal
μ_r	Target scattering coefficient for the radar signal
σ_n^2	Noise variance
σ_v^2	Variance of the noise and clutter
$\mathbf{\Sigma}$	Covariance matrix of the received signal
τ	Target time delay
$\hat{\tau}_{ML}$	Maximum Likelihood estimate of the target time delay
ψ	Phase shift due to the distance traveled by the echo
$\mathbf{\Omega}$	Covariance matrix of the received signal under $\mathcal{H}_0$
$\lvert \cdot \rvert$	Absolute value
$(\cdot)^F$	Frequency domain
$(\cdot)^H$	Complex Conjugate Transpose (Hermitian)
$\odot$	Element-wise (Hadamard) product

1. Introduction

Radars can perform various tasks. Target detection and target parameter estimation are the most important ones. Other important radar tasks are target characterization and target recognition. Radars have many applications both for military and civilian use. Radar applications include: air defense, for target detection, recognition or missile guiding, air and sea navigation as well as remote sensing, to mention only a few. New areas for radar applications such as the automotive industry, building security, medical devices, civil engineering or geophysics are emerging as well. Recent applications of high interest are vital signs monitoring as well as hand gesture recognition for computer interaction. As radar systems have become more precise and affordable it is expected that their applications and numbers will keep growing massively in the near future.

An important part of radar research efforts is aimed at waveform design and, more recently, waveform optimization and adaptation. In recent years, the popular multicarrier waveforms, which have known great success in modern communications systems, are proposed for radar use as well [1, 2]. These waveforms become attractive to radar use thanks to their many degrees of freedom in time, frequency or code domains. The available degrees of freedom allow the radar to avoid jammers, unintentional interference, as well as adapt its waveform for optimal performance according to target characteristics, task at hand as well as radar channel and propagation environment.

Nowadays there are new and emerging civilian applications for radars, including automotive radars and health applications. Automotive industry has seen a great impact from radar technology. Radars are a key component in improving the safety of modern vehicles. Millions of devices are currently manufactured and a huge growth is predicted for automotive radars. This, combined with the ever growing demand for spectrum in modern communications systems, contribute to the critical problem of spectral congestion. One important solution is the use of new frequencies, however some higher frequencies may not be as applicable to radar as lower frequencies. Another solution for the spectral congestion is the sharing of spectrum among different wireless systems in an adaptive and smart way. Recent requirements for communications systems of larger bandwidth and the shift towards higher frequency communications

are threatening the traditionally privileged position of the radars in terms of spectral allocation. Proposals for spectrum sharing among radar and wireless communications have been made and authorities in the US for example are releasing 100 MHz in the 3.5 GHz band for shared access of wireless broadband in industrial applications with already existing satellite systems and S-band radars [3, 4].

In this thesis the focus is on two timely radar research topics. Multicarrier radar waveforms are developed for their spectral efficiency, frequency and waveform diversity as well as many degrees of freedom. The problem of spectrum sharing and coexistence among radar and communications systems is addressed as well. Multicarrier radar waveforms allow for the flexibility and adaptability of a radar system which is needed in a coexistence scenario with other wireless systems. As it is shown in this thesis, multicarrier radar waveforms are a great candidate for spectral sharing among radar and communications systems. In part, this is also thanks to the fact that modern communications systems utilize the same multicarrier waveforms. Multicarrier radar and communications systems could also use similar transceiver structures and benefit from the cost-efficient development and evolution of wireless systems. Thus, the two topics of this thesis complement each other and provide a potential foundation for future radar systems.

1.1 Objectives and Scope

This thesis focuses on several signal processing theories and methods in topics such as multicarrier radars and spectrum sharing among communications and radar systems. In multicarrier radar research the goal is to develop a generalized model that would describe most of the commonly used radar waveforms as well as novel ones. It is important to have a unifying model for the description, implementation, adaptation and customization in a very easy and computationally efficient manner of radar waveforms. This is achieved by developing a matrix model for multicarrier radars that can be used to represent most currently used radar waveforms. Such model allows for adapting the waveforms simply by filling in the matrices appropriately and designing new waveforms with desirable properties. The second objective is to develop, using the generalized model, novel multicarrier radar waveforms that would outperform other multicarrier radar waveforms proposed in the open literature. Many multicarrier radar waveforms in the literature provide several benefits such as Doppler sensitivity, spectral efficiency, waveform and frequency diversity and additional degrees of freedom for agile use of waveforms. However these can be further improved, for example, by relaxing the subcarrier orthogonality property in order to have a better ambiguity function (AF). Another objective is to provide examples of interesting special cases of multicarrier waveforms, for example, employing techniques such as fast frequency hopping (FFH) in order to illustrate the agility and flexibility

in terms of waveform design, adaptation and optimization that the proposed generalized model can provide.

This thesis also focuses on radar waveform optimization techniques in scenarios involving spectrum sharing with communications systems. Traditionally only the radar receiver is adaptive. However, there has been a major paradigm shift in radar research since the transmitter parameters and the degrees of freedom of the radar system may be adjusted based on the radar task and radar spectrum and target scenario [5, 6]. Multiple input multiple output (MIMO) radar employing spatial, frequency, polarization and waveform diversity methods are one example in this sense [7–10]. The optimization may be performed using information theoretic criteria such as mutual information (MI) or criteria stemming from detection and estimation theory, for example, Neyman-Pearson (NP) criterion or Cramér-Rao bound (CRB). Such optimization techniques allow adjusting waveforms, for example by allocating power to different subcarriers in an optimal manner depending on the state of the radar spectrum or target scenario. Furthermore, an important objective of this thesis is to demonstrate that both radar and communications systems can benefit from the coexistence to improve their performance. Communications systems benefit from spectrum sharing, coexistence and cooperation with radar systems as these scenarios facilitate interference management and achieving the desired rate despite the spectrum being used by more systems. The radar can exploit the communications signals reflected off the target in a passive manner, with little to no cooperation from the communications system. Such benefits are proven in this thesis for both target detection and parameter estimation tasks in cooperative scenarios. This should be a great incentive for spectrum sharing among the two systems, which aims to solve the problem of spectral congestion.

1.2   Contributions

The contributions presented in this thesis have been included in one published and one submitted journal papers and four peer-reviewed international conference articles. Contributions are made in signal processing theory and methods for waveform design and optimization, target detection and target parameter estimation. The contributions of this thesis can be summarized as follows:

1. A generalized multicarrier radar (GMR) model is first proposed in Publication II and further improved in Publication I, which facilitates highly flexible use of radar waveforms and different diversity methods just by appropriately filling in the matrices in the model. Many widely used radar waveforms can be described and easily generated using the proposed model. A few of these are described in Publications I and II. The model facilitates designing and optimizing waveforms. For example flexible allocation of power per subcarrier and waveform optimization can be employed based on the radar task, as well as target scenario and avoiding both unintentional and intentional

interference, i.e. jamming. Frequency agility is discussed in Publication II, while optimal power allocation per subcarrier is discussed in Publication I. The proposed GMR model is an enabler for future cognitive radars.

2. Novel multicarrier radar waveforms which relax the orthogonality of the subcarriers are proposed in Publication I. The resulting design leads to an improved AF and consequently better radar performance. Subcarrier orthogonality is lost anyway in radar receiver because there are no waveforms that would be orthogonal with all delays and Doppler shifts. These novel multicarrier radar waveforms are described using the proposed GMR model. The first proposed waveform, the Time Diversity Radar Waveform (TDRW) considers spreading over time or frequency, similar to the well known Multi-carrier Direct Sequence CDMA (MC-DS-CDMA) in communications systems. The second proposed waveform, the Frequency Diversity Radar Waveform (FDRW) considers spreading only over frequency, similar to the well known Multicarrier CDMA (MC-CDMA) in communications systems. The multicarrier phase coded (MCPC) waveform already proposed in the radar literature employs an orthogonal design. It is shown that the proposed TDRW can outperform the MCPC waveform in terms of lower peak sidelobe levels (PSLLs) of the AF and even better delay resolution compared to the MCPC waveform. The performance improvement can be obtained as long as the inter-carrier interference (ICI) is kept under control. The ICI is controlled by choosing the length of the spreading codes and the intercarrier spacing appropriately.

3. MI based criteria for radar waveform optimization in coexistence scenarios are proposed and investigated for best achievable detection performance in Publication IV. It is shown, using simulation results, that, as expected, a larger maximized MI between the received signal and the target channel impulse response does not guarantee an optimal detection performance in the NP sense. It is also shown that, in a cooperative radar and wireless communications scenario, it is important to exploit the communications signals reflected off the target. This is especially true for weak radar returns, when the communications signal return can improve the detection performance by providing spatial diversity.

4. A radar waveform optimization based on the NP detector which maximizes the probability of detection given a maximum allowed false alarm probability, a total transmitted radar power constraint and an interference mask is proposed in Publication V. The interference mask is considered in the cooperative spectrum sharing scenario in order to guarantee a minimum data rate for the communications system. The constrained optimization problem is solved numerically and its solution is shown to be a power allocation per subcarrier. It is shown that the NP based solution outperforms the MI based one for the radar detection performance, especially in low signal to noise ratio (SNR) regime and when considering lower maximum allowed false alarm probabili-

ties for the radar detector. Using simulations, it is shown that also in the case of NP based waveform optimization the communications signals reflected off the target can improve the detection performance. It is thus shown that the radar and the communications system can benefit from coexisting, not only through spectral efficiency but also improved radar performance and a minimum guaranteed communications data rate.

5. The problem of estimating target parameters in the context of radar and communications system coexistence is addressed. An estimator for target time delay is proposed in Publication VI. It is shown that the maximum likelihood estimator, which exploits the communications signals reflected off the target, achieves lower root mean squared error (RMSE) compared to the case when the radar operates alone. This is due to the additivity property of the Fisher information (FI). The performance gain is shown to depend on the SNR of the reflected communications signals. For a larger SNR the communications signal contributes to the estimation with better estimates.

6. The CRBs for target time delay estimation in cooperative radar and communications systems scenarios are derived. First the CRB derivation is developed in Publication VI for a point target model. The derivation is extended in Publication III for an extended target model. It is shown that exploiting the communications signals reflected off the target can lower the CRB for target time delay estimation. This is motivated by the additivity property of the FI. The larger the SNR of the communications signals reflected off the target the lower the obtained CRB is.

7. Radar waveform optimization for target parameter estimation task for cooperative radar and communications systems scenarios is proposed in Publication III. The objective function looks for a waveform that minimizes the CRB for target time delay estimation. Novel optimization problems with additional constraints on the subcarrier power ratio (SPR) are proposed. The SPR is a measure of how much the power of one subcarrier deviates from the average subcarrier power. The obtained solutions are shown to reduce ambiguities in the delay domain over the unconstrained ones. Furthermore, the unconstrained solution may not give a realizable waveform.

The author of this thesis is responsible for the derivations and simulations in the publications and this thesis, as well as the writing of the publications and this thesis. The co-authors have provided help by steering and planning the research as well as writing and revising the publications.

1.3 Thesis Structure

This thesis contains four chapters. Two main technical chapters address the topics of multicarrier radar waveforms and of spectral coexistence among radar

and wireless communications systems respectively. The analytical and simulation results presented in this thesis are based on the attached Publications I–VI. Detailed derivations are presented in the attached publications. Chapter 1 is this introduction of the thesis.

Chapter 2 deals with the topic of multicarrier radar waveforms. The state of the art research in the literature is reviewed first. The proposed GMR model is presented next. Many existing waveforms are special cases of the proposed model. The model allows for adapting, optimizing waveforms as well as developing new waveforms using the same compact notation. The two sets of equations for transmitter and receiver that build the proposed model are developed. Using the proposed GMR model the novel TDRW is described and its performance in terms of AF properties is presented. Examples of waveform optimization using the proposed GMR model based on MI are also presented.

Chapter 3 deals with the spectral coexistence among radars and wireless communications systems. Several radar waveform optimization examples using MI based objective function are presented and the detection performance of the optimized waveforms is investigated. The detection performance is compared to the case when radar operates alone. It is shown that benefits can be obtained from the cooperation among the two systems. Radar waveform optimization based on the coexistence model and NP detector is presented. The detection performance of the optimized waveforms is compared to radar waveforms optimized using the MI criterion. The estimation task is also considered in Chapter 3, where an estimator for target time delay in cooperative radar and communications scenarios is proposed. Waveform optimization based on minimizing the derived CRB for target time delay estimation in Publication III is proposed. The performance of the optimized waveforms is studied and compared to waveforms optimized for received SNR maximization. Also, the benefit of exploiting the communications signals reflected off the target is demonstrated.

The conclusions of this thesis are presented in Chapter 4. Future work is also discussed in this chapter.

2. Multicarrier Radar Waveforms

Multicarrier data transmission introduced in the 1960s [11, 12] started to get significant interest in the 1980s [13]. The appeal of multicarrier transmission is increased by the advances in digital circuitry as well as their deployment in most present and emerging wireless and wireline communications systems. Multicarrier transmission methods such as discrete multitone (DMT) are included in the digital subscriber line (DSL) systems implemented over telephone lines [14]. Multicarrier transmission methods used in digital communications such as orthogonal frequency division multiplexing (OFDM) are included nowadays in the digital television and radio systems (DVB [15] and DAB [16]), local area mobile wireless networks (WLAN [17, 18]), wireless communications and cellular communications (WiMAX [19] and LTE [20]) [21]. Multicarrier waveforms will be used also in the emerging 5G wireless networks.

Orthogonal multicarrier systems transmit information in symbols. A block diagram for multicarrier transmitter and receiver, employing OFDM modulation, is presented in Fig. 2.1. The transmitted symbols are serial to parallel transformed and an inverse fast Fourier transform (IFFT) is applied. The obtained symbols are transformed to serial again and a cyclic prefix (CP) is added to the symbol. After digital to analog conversion the symbol is transmitted through the medium where it typically experiences a frequency selective channel and it is subject to additive noise. At the receiver, inverse operations are performed to recover the original data symbols. In a linear time-invariant (LTI) multipath channel intersymbol interference (ISI) is created by combining at the receiver contributions from multiple transmitted symbols. In order to mitigate ISI, the CP is used in multicarrier systems. The CP turns linear convolution to circular one and allows turning frequency selective channel to multiple frequency flat channels. The CP is discarded at the receiver. In order to effectively mitigate multipath effects, the length of the CP has to be longer than the channel length [21]. OFDM is the most common modulating technique nowadays which manages ICI via orthogonal subcarriers. It provides advantages such as high spectral efficiency, robustness against ISI and multipath propagation fading, efficient implementation and many others. The major benefit is that the broadband frequency selective channel is divided into multiple narrowband frequency flat channels,
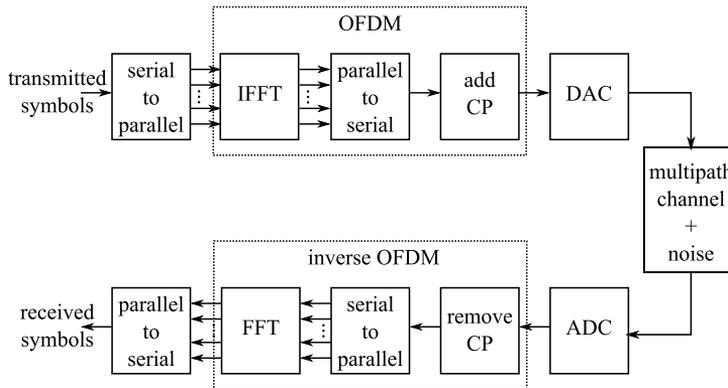
Figure 2.1. Transmitter and receiver block diagram employing OFDM.

which allows for only one tap equalization at the receiver. The use of OFDM also has disadvantages like Doppler sensitivity, efficiency loss due to the CP, which causes lower data rates, and high levels of peak to average power ratio (PAPR) due to the varying envelope. In addition to orthogonal multicarrier systems, multicarrier systems that employ spread spectrum such as Multicarrier CDMA (MC-CDMA), Multicarrier Direct Sequence CDMA (MC-DS-CDMA), Multitone CDMA (MT-CDMA), asymmetric digital subscriber lines (ADSL) [22], wireline systems such as power line communication (PLC) [23] and others [21, 24] are proposed in the literature. After this very brief introduction to OFDM for communications systems, in this thesis OFDM and multicarrier waveforms are considered in the context of radar.

Similar to communications systems, multicarrier waveforms provide several benefits for radar systems as well. Some of the key advantages that multicarrier waveforms bring to radar are:

- Spectral efficiency.
- Waveform and frequency diversity.
- Additional degrees of freedom for agile use of waveforms.
- Robustness against multipath propagation with the use of CP or guard intervals.
- Doppler sensitivity which allows for fast Doppler estimation.
- Easy implementation in digital domain.
- Easily available hardware.

Frequency diversity can be considered the most important advantage. For example, jamming and narrowband interference, as well as attenuation and non-intentional interference can be easily mitigated when frequency diversity is available. The channel state at the receiver that is subject to jamming or interference has to be known prior to adjusting the transmitter appropriately. Subcarriers that are subject to jamming or high attenuation are allocated zero transmit power, for example. When frequency diversity is available, the efficiency of jammers is nonetheless greatly reduced as the power is spread over a large

bandwidth, while narrowband interferers affect only a small portion of the spectrum. High power efficiency can be achieved by adaptive power allocation per subcarrier based on channel gain or attenuation. More power is allocated to subcarriers where the signal is attenuated very little. One drawback of achieving power efficiency is the non-constant envelope and consequently one may have a large PAPR. Also, frequency diversity allows for easy implementation of frequency hopping (FH) techniques and frequency agility in general.

Another important benefit of multicarrier waveforms is waveform diversity. Multiple degrees of freedom are available to design and optimize waveforms in time, frequency or code domains. This allows for optimization based on the radar task at hand, target scenario or channel and propagation conditions. Traditionally only the radar receiver has been adaptive and optimized. Multicarrier signal model facilitates using similar transceiver structures and circuitry as in commercial communications transceivers. Consequently, implementation of multicarrier radars can be made more cost efficient and the transceiver may even be software defined.

Drawbacks to the multicarrier waveforms have been identified as follows:

- Time-varying envelope that results in a high PAPR levels.
- Doppler sensitivity that might spoil the orthogonality of the subcarriers and cause ICI.

Accurate frequency synchronization is required when dealing with Doppler sensitive waveforms. Constant modulus is a desirable property for the radar waveform to ensure efficient use of power amplifiers. Plenty of research on mitigating this problem is available [2, 25–27].

2.1   Generalized Multicarrier Radar Model

Most of the available multicarrier and single carrier radar waveforms can be easily implemented using the GMR model introduced in Publication I. In fact, they are special cases of this GMR model, obtained by choosing the dimensions of the system appropriately. The preliminary idea for a generalized model is presented in Publication II. An improved version is introduced in Publication I. The model is composed of two sets of equations, one for transmitter and one for receiver, both formulated using a matrix notation. The matrix formulation allows for easy implementation and customization of the waveforms as well as facilitates the use in simulation environments such as Matlab for example. The model can also be used for optimizing the transmitter or the receiver as well as agile use of waveforms in future cognitive radars.

Similar to the matrix formulation of the GMR model, the processing of the received MCPC waveform considered in [28] and the OFDM waveform in [29] is described using a matrix notation. Nevertheless, the matrix formulations of [28, 29] are valid for the waveforms considered therein and consequently are not as general as the model in Publication I.

2.1.1 GMR Model Equations

The transmitter equation which describes the baseband transmitted signal is:

$$\mathbf{r} = \mathcal{K}\{\mathbf{U} \odot \mathbf{B}_{\text{TX}}\mathbf{W}\}\mathcal{J}\{\mathbf{C}\}, \tag{2.1}$$

where the matrix of ones and zeros $\mathbf{U}$ selects the active and inactive subcarriers during each subpulse. Matrix $\mathbf{B}_{\text{TX}}$ is the IDFT matrix, matrix $\mathbf{W}$ is a diagonal matrix that contains the weights of each subcarrier on the main diagonal and matrix $\mathbf{C}$ is responsible for the coding operation in frequency, time or both domains. Different coding matrices lead to different time-frequency tilings of the transmitted signal. When coding in time domain is employed, matrix $\mathbf{C}$ is of size $N \times M$, while, when coding in frequency domain is employed, matrix $\mathbf{C}$ may be replaced by vector $\mathbf{c}_0 = [c_0, c_1, \ldots, c_{N-1}]^T$, as $M = 1$ in this case. The operators $\mathcal{K}$ and $\mathcal{J}$ allow for the generation of TDRWs. Operator $\mathcal{K}$ applied to an $N \times N$ size $\mathbf{Z}$ matrix reshapes that matrix to a matrix of size $N \times pN$ by partitioning matrix $\mathbf{Z}$ in blocks containing N/p rows and building a block diagonal matrix using these blocks. Operator $\mathcal{J}$ applied to an $N \times N$ size $\mathbf{Z}$ matrix reshapes that matrix to a matrix of size $pN \times N/p$ by partitioning the matrix $\mathbf{Z}$ in blocks containing p columns and using the columns in each block to create a new column in the resulting matrix. It can be observed that when $p = 1$, reshaping operators $\mathcal{K}$ and $\mathcal{J}$ have no effect on the matrix that they are applied to. Consequently, if the model is used to generate MCPC waveforms or its particular forms like OFDM or even single carrier waveforms, the following simplified version of equation (2.1) can be used:

$$\mathbf{r} = \mathbf{U} \odot \mathbf{B}_{\text{TX}}\mathbf{W}\mathbf{C}. \tag{2.2}$$

The receiver equation which describes the baseband received signal in the absence of noise is:

$$\mathbf{y}_r = \mu\psi\mathbf{\Gamma}_1\mathcal{K}\{(\mathbf{U} \odot \mathbf{B}_{RX})\mathbf{W}\}\mathcal{J}\{\mathbf{AC}\}\mathbf{\Gamma}_2, \tag{2.3}$$

where μ is the complex amplitude introduced by the target (here assumed frequency independent, however it can also be frequency dependent) and $\psi = \exp\left(-j2\pi f_c \frac{2d}{s}\right)$ is the phase shift due to the distance traveled by the echo. Matrices $\mathbf{\Gamma}_1$, of size $N \times N$, and $\mathbf{\Gamma}_2$, of size $M \times M$, are described by $\text{diag}\{\gamma^0, \gamma^1, \ldots, \gamma^{N-1}\}$ and $\text{diag}\{\gamma^0, \gamma^N, \ldots, \gamma^{(M-1)N}\}$ respectively, with $\gamma = \exp\left(-j2\pi f_c \frac{T_c}{N} \frac{2s_{\text{target}}}{s}\right)$ the Doppler shift at the passband carrier frequency due to the radial motion component between the target and the receiver. Matrices $\mathbf{U}$ and $\mathbf{W}$, of size $N \times N$, and matrix $\mathbf{C}$, of size $N \times M$, are the same as for the transmitter equations, while matrix $\mathbf{B}_{\text{RX}}$, of size $N \times N$, has its nth and mth element given by $\left[\beta_{\text{RX}}\right]_{nm} = \left[\exp\left(j2\pi\Delta f \frac{T_c}{N}\left(1 - \frac{2s_{\text{target}}}{s}\right)\right)\right]^{nm}$. When $\Delta f = \frac{1}{T_c}$, matrix $\mathbf{B}_{\text{RX}}$ can be seen as the IDFT matrix affected by the scaling factor due to the Doppler shift. Finally, matrix $\mathbf{A}$ is described by $\text{diag}\{a^0, a^1, \ldots, a^{N-1}\}$, whose elements represent the phase shift of each subcarrier due to the distance traveled by the received echo

$a = \exp\left(-j2\pi\Delta f\frac{2d}{s}\right)$. As in the transmitter case, if the model is used for generating MCPC waveforms or its particular forms like OFDM and single carrier waveforms, equation (2.3) can be simplified to:

$$\mathbf{y}_r = \mu\psi\mathbf{\Gamma}_1(\mathbf{U}\odot\mathbf{B}_{RX})\mathbf{WAC\Gamma}_2, \qquad (2.4)$$

or, when coding only in frequency domain is employed, to:

$$\mathbf{y}_r = \mu\psi\mathbf{\Gamma}_1(\mathbf{U}\odot\mathbf{B}_{RX})\mathbf{WAc}_0. \qquad (2.5)$$

2.1.2 GMR Model Waveform Generation Examples

It is stated in Publication I that a variety of commonly used radar waveforms can be implemented using the proposed model. For example, both OFDM and MCPC waveforms as well as chirp approximations, pseudo and random FH waveforms can be generated using equation (2.2). MCPC and OFDM waveforms have all the subcarriers active at all time instances, thus matrix $\mathbf{U}$ has only elements with value 1. The weighting matrix $\mathbf{W}$ associated with power allocation for different subcarriers is an identity matrix of size N, unless different powers are allocated to different subcarriers. In such case, the values on the main diagonal of $\mathbf{W}$ are chosen accordingly, such that $0 \le w_n \le 1$. Both OFDM and MCPC waveforms require orthogonality among the subcarriers, so the carrier frequencies have to be chosen accordingly. Matrix $\mathbf{B}_{\text{TX}}$ is just the IDFT matrix. Examples of codes that may be used for the generation of MCPC waveforms are the polyphase P3 and P4 codes [25]. Consequently, the coding matrix $\mathbf{C}$ for an MCPC waveform contains on its rows the codes that modulate each subcarrier. For an OFDM signal phase coding could be done in frequency domain and matrix $\mathbf{C}$ becomes a vector $\mathbf{c}_0$ of dimension $N \times 1$. If no coding is employed for OFDM, then vector $\mathbf{c}_0$ is a vector of all ones. It can also be a vector of data symbols, if both communications and radar purposes are desired simultaneously.

Other waveforms that can be easily implemented using the GMR model are approximations of linear frequency modulated (LFM) waveform. For the multicarrier radar, a stepwise approximation of a LFM waveform can be done by exciting a different subcarrier in increasing or decreasing order, depending on whether up-chirp or down-chirp is approximated, at different time instances, like illustrated in Fig. 2.2. This stepwise approximation of an up-chirp or down-chirp pulse can be considered to be a FH technique which excites the bands in monotonically increasing or decreasing order. The generation of such a waveform involves the matrix $\mathbf{U}$ which selects the active/inactive subcarriers at each sampling instance.

Waveforms employing FFH patterns, similar to the ones in Publication II or [30] and [28], where the hopping patterns considered for an MCPC waveform are on a pulse-by-pulse basis, can also be generated. The hopping pattern is implemented using matrix $\mathbf{U}$ while the width of matrix $\mathbf{C}$ controls the number of available pulses/symbols. The number of patterns and waveforms that can

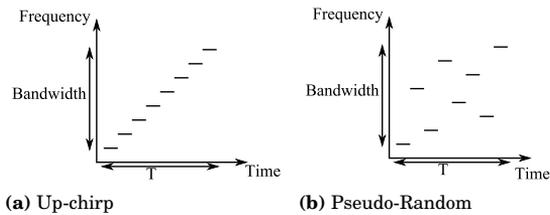**(a)** Up-chirp   **(b)** Pseudo-Random

Figure 2.2. Examples of stepwise approximation of up-chirp pulse and pseudo-random frequency hopping in multicarrier radar. At each sampling instant the next subcarrier is active, creating a frequency hopping in an ascending manner or subcarriers are pseudo-randomly activated over the time duration of the pulse. Copyright 2016 IEEE.

be implemented using the GMR model is not limited to the presented examples, these only show the flexibility and the multitude of available options in designing radar waveforms.

2.2 Review of Multicarrier Radar Waveforms

The use of multicarrier waveforms for radar is proposed by Levanon [1, 25]. The MCPC waveform is introduced, in which each subcarrier is modulated by a code sequence of a specific length. It is a generalization of the OFDM waveform in the sense that after the time domain modulation of each subcarrier is performed, the subcarriers remain orthogonal. Consequently, the intercarrier spacing for an MCPC waveform needs to accommodate the spreading in frequency domain due to the modulation in time domain. The modulation of each subcarrier is done using phase codes, for example, P3 or P4 polyphase codes [1, 25]. The acronym MCPC is used for multifrequency complementary phase coded in [1, 25, 26], whereas it stands for multicarrier phase coded in [2, 27]. The essential difference between the two is that for the former it is assumed that the phase codes modulating each subcarrier form a complementary set, while in the latter case there is no such restriction. A complementary set is built from finite sequences, each called complementary sequence, with the property that the sum of their autocorrelation functions is zero except for the zero shift [2]. The complementary sets are first introduced in [31].

The introduction of the MCPC waveform inspired other researchers to propose the use of OFDM waveforms for radar. It is claimed in [28] that for radar applications the use of CP is not necessary for OFDM waveforms. However a CP is used in [32, 33] for example. Even before using OFDM waveforms for active sensing these waveforms show up in proposals for passive radar [34].

After the MCPC waveform is proposed for radar, the OFDM waveform is also considered for radar use. It is observed that in the literature several sources use the term OFDM when referring to an MCPC waveform. Representative examples include [35, 36] as well as [30, 37–40]. In this thesis it is referred to

an MCPC or OFDM waveform by its mathematical description presented in the respective source. Precisely, an orthogonal waveform which employs time domain coding on each subcarrier is an MCPC waveform, while an orthogonal waveform which does not employ time domain coding on the subcarriers is an OFDM waveform. Both MCPC and OFDM radar waveforms found in the literature are special cases of the proposed GMR model.

Another type of multicarrier radar waveforms is proposed in Publications I and II, which relaxes the orthogonality constraint of the OFDM and MCPC waveforms. The orthogonality constraint can be relaxed as the received waveforms are not orthogonal with all delays and Doppler shifts nonetheless. These proposed waveforms are the FDRW and the TDRW, generalizations of the OFDM and MCPC waveforms respectively. FDRWs and TDRWs relax the orthogonality constraint of OFDM and MCPC waveforms for radar performance improvement, while employing frequency or both frequency and time domain coding respectively. Also these novel radar waveforms are special cases of the proposed GMR model.

Table 2.1 summarizes the characteristics of the multicarrier waveforms considered for radar. The criteria considered for characterization are: orthogonality of the subcarriers and possible coding domain. It can be observed that the MCPC waveform and the TDRW employ coding both in time and frequency domain, while the OFDM waveform and the FDRW perform coding only in frequency domain.

Table 2.1. Characteristics of multicarrier radar waveforms.

Waveform	Orthogonal	Non-Orthogonal	Time Coding	Frequency Coding
MCPC	×		×	×
OFDM	×			×
TDRW		×	×	×
FDRW		×		×

The research on multicarrier radar waveforms can be divided in three main application domains:

- active radar [1, 30, 35],
- passive radar [34, 41, 42],
- joint radar and communications areas [43–48].

When considering multicarrier radar waveforms these can be either continuous [29, 45, 49] or pulsed [30, 37, 50] waveforms. Also, waveforms using the CP and ignoring it have been reported in the literature [28, 45]. It is claimed in [28] that CP is not necessary in radar applications in contrast to communications applications, where it transforms convolution to a circular one and facilitates 1-tap equalization.

Various designs are proposed for multicarrier radar waveforms in the liter-

ature. Pulse-to-pulse frequency agility is introduced in [30]. The waveform contains a burst of MCPC pulses, each of which takes advantage of the available frequency diversity by exciting only certain subcarriers among available ones. A wideband OFDM system for joint radar and communications designed to take advantage of pulse-to-pulse waveform diversity capabilities is described in [47]. Stepped-frequency waveforms (SFWs) are considered in [28] in order to obtain both range and Doppler information from the target. The SFW contains a set of pulses, each pulse being an MCPC pulse with a different set of active frequencies. The subset of active subcarriers changes from pulse to pulse according to a Costas code. Similar to the SFW in [28], Publications I and II introduce multicarrier radar waveforms employing FH patterns. The difference is that the frequency agility pattern is considered for one subcarrier at a time and not for a block of subcarriers as in [28].

The main drawback of multicarrier waveforms in general is the time-varying envelope which affects the achieved peak to mean envelope power ratio (PMEPR) or PAPR. The PMEPR is an approximation for the PAPR (which can be lower), mostly used for baseband signals [2]. Such effects are caused by the fact that a sum of signals is transmitted which can cause great variations depending on their instantaneous amplitude and phase. This causes the power amplifiers to function inefficiently and, especially for radar systems which try to use the amplifiers at almost full power, this is highly detrimental. Consequently, efforts in lowering the PMEPR of the multicarrier waveforms have been reported in [2, 27]. In [35] frequency and phase modulation for MCPC waveforms is introduced in order to produce a constant envelope. MCPC waveforms with constrained PMEPR at 3 dB are proposed in [51] based on Golay codes. In [39] a waveform optimization based on a genetic algorithm is proposed in order to lower the PMEPR. Genetic algorithms require huge amount of computation in general and may not be preferred as optimization algorithms. It is shown in Publication I that the proposed TDRWs can provide a lower PAPR than MCPC waveforms just by relaxing the orthogonality constraint of the subcarriers and spreading the available power more uniformly in the spectrum. It is however noted in Publication I that for multicarrier waveforms employing phase coding, like MCPC and TDRW, the achieved PAPRs depend on the type of used codes [2], thus TDRWs might not provide lower PAPRs for every waveform design. Consequently, analyzing the PAPR for a waveform design is recommended.

The AF plays an important role in the radar performance studies through the insight that it provides on the performance under different delays and Doppler shifts. Also, multicarrier waveforms are analyzed using the AF for their ambiguity properties and the achieved delay and Doppler domains resolution. Such analysis is provided for MCPC waveforms in [1, 2]. Moreover, AF is used for the design and analysis of the MCPC and OFDM radar waveforms [30, 36, 52]. In Publication I, plots of the AF are used to compare the achieved PSLLs and resolution in both delay and Doppler domains for the TDRWs and MCPC waveforms. Similarly in Publication II, the AF is used to compare multicarrier

waveforms employing different types of FFH patterns.

The most basic radar tasks are to estimate target's range and velocity. As multicarrier waveforms are candidates for future radar systems, researchers investigate the ability of such waveforms to extract delay and Doppler information from the target. A Doppler processing technique for pulsed MCPC waveforms is presented in [30] for the case of frequency agile waveforms. In [37] the Doppler processing technique is extended for a burst of pulses in both narrowband and wideband cases. A processing technique that solves the Doppler ambiguity for random phase-coded OFDM signals is proposed in [49] and [29]. A processing technique for both range and Doppler information for wideband OFDM radar waveforms is presented in [45]. A compressed sensing approach to process frequency agile MCPC waveforms is proposed in [53] which eliminates the need for matched filtering and allows for obtaining the range profile. A technique to resolve both range and Doppler information using a stepped MCPC waveform is proposed in [28]. A technique to compress MCPC waveforms with low sample rate is introduced in [54].

Other radar tasks are considered in the literature as well. For example in [50] an approach to characterize scattering centers from their radar return using flat spectra OFDM is introduced. The task of target recognition is considered in [40], where symbols matching known target responses are used for target recognition. The utilized waveform in [40] is an MCPC waveform built from a concatenation of symbols matching different targets, the number of symbols being equal to the number of different targets desired to be recognized. A sparsity based multi-target tracking algorithm using an OFDM radar is proposed in [55]. The target tracking task for a single target in a low-grazing angle scenario is considered in [48] using an OFDM waveform. Target characterization is considered in Publication I by proposing multicarrier waveforms that maximize the amount of information from the target at the radar receiver. Such waveforms are obtained by optimizing an information theoretic based objective function.

The multitude of degrees of freedom brought by multicarrier waveforms allow for the optimization of the transmitted waveform. Various objective functions can be considered for the optimization. For example, optimization based on the desirable AF is considered in [36, 52]. As pointed out earlier, the PMEPR is very important for the radar transmitter and the optimization aimed at minimizing the PMEPR value is proposed in [39]. Waveform optimization is also considered for several radar tasks, including target detection [56, 57], target tracking [48] or target recognition [40]. An information theoretic approach to multicarrier waveform optimization is proposed in Publication I. The goal of the optimization problem is to maximize the MI between the received waveform and the target impulse response, which maximizes the amount of information about the target that is delivered at the receiver.

The multicarrier radar waveforms considered in the literature are very similar to each other and can be considered special cases of the GMR model. The relationship among the multicarrier radar waveforms available in the literature

is illustrated in Fig. 2.3. It can be observed that the FDRW and the MCPC waveform are special cases of the more general TDRW. Also, the OFDM waveform is a special case of both FDRW and MCPC waveform. All these multicarrier
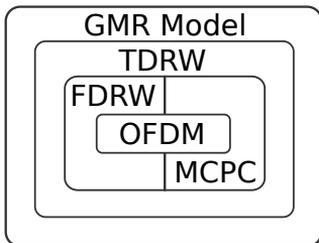

Figure 2.3. Relationship between multicarrier radar waveforms. TDRW is a generalization of both FDRW and MCPC waveform, which in turn are generalizations of the OFDM waveform.

radar waveforms can be obtained from the GMR model by appropriately choosing the dimensions of the matrices and specific parametrization.

2.2.1 Orthogonal Multicarrier Radar Waveforms

The first multicarrier waveform introduced for radar systems is the MCPC waveform [1]. The most general version of an MCPC waveform contains N subcarriers which are modulated using sequences of phase codes of length M. Each element in the modulating sequence corresponds to a chip. The intercarrier spacing is chosen to accommodate the spreading due to the time domain coding and ensure orthogonality among subcarriers. Precisely, the intercarrier spacing has to be equal with the inverse of the chip duration. A block diagram for generating an MCPC pulse containing N subcarriers modulated by code sequences of length M is presented in Fig. 2.4. Although the length of the codes M can be smaller or larger than the number of subcarriers N, it is common in the literature to consider codes of length equal to the number of subcarriers. This way a complementary set can be obtained. For a chip
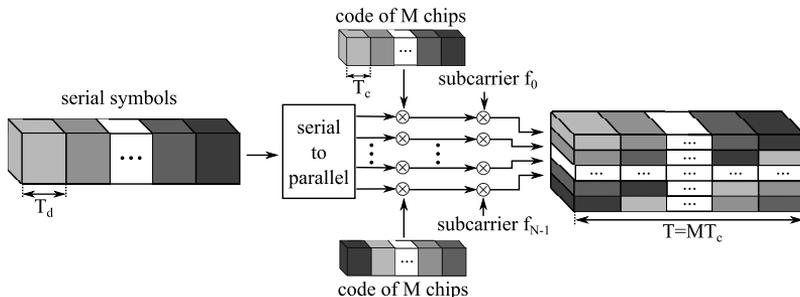

Figure 2.4. Generation of a $N \times M$ MCPC pulse containing N subcarriers modulated by codes of length M. The intercarrier spacing is $1/T_c$ and the overall symbol duration is $T = MT_c$. This MCPC pulse contains M OFDM sub-pulses of length T_c.

duration T_c, a modulating sequence of length M has a total duration of $T = MT_c$. Similarly, the MCPC pulse described in Fig. 2.4 for example contains M symbols, each of duration T_c, with a total duration of $T = MT_c$. Given the chip duration of T_c, the intercarrier spacing is chosen as $\Delta f = 1/T_c$. The overall bandwidth occupied by an MCPC pulse is $B = N\Delta f = N/T_c$. The M symbols contained in the MCPC pulse are basically OFDM symbols. Consequently, an MCPC pulse contains a number of OFDM symbols, the first symbol contains the chip on each subcarrier, the second symbol contains the second chip on each subcarrier and so on. The number of OFDM symbols is equal to the length of the modulating sequence, M for example.

The modulating sequences on each subcarrier are phase codes. Commonly employed phase codes are the P3 and P4 polyphase codes [1, 2]. These code sequences are ideal sequences. An ideal sequence is a sequence that has zero autocorrelation sidelobes. The modulating sequences can be identical on each subcarrier or can be different cyclic shifts of the same sequence. Other codes are proposed in [25] based on Barker codes that also have ideal periodic autocorrelation function. While a Barker code contains phases $0°$ and $180°$, the codes proposed in [25] contain phases $0°$ and $138.59° (= \arccos(-3/4))$. The phases are chosen such that the out of phase autocorrelation is zero. Different ways of finding such codes are presented in [58].

The complex envelope of an $N \times M$ MCPC is given by [2]:

$$g(t) = \sum_{n=1}^{N} \sum_{m=1}^{M} w_n a_{n,m} s[t - (m-1)T_c] \exp\left[2\pi j\left(n - \frac{N+1}{2}\right)\frac{t}{T_c}\right], \quad 0 \le t \le MT_c \tag{2.6}$$

where $a_{n,m}$ is the mth element of the sequence modulating the nth subcarrier, w_n is the complex weight associated with the nth subcarrier and $s(t) = 1$ for $0 \le t \le T_c$ and zero elsewhere. The complex weight w_n can be used for amplitude modulation of the subcarriers, which can be useful in PMEPR reduction or power allocation based on channel gain in each subcarrier for example. The overall duration of such MCPC pulse is MT_c, with the duration of one chip of T_c. The intercarrier spacing is $1/T_c$ and the effective bandwidth is N/T_c (without weighting). The spectrum of an MCPC pulse is relatively flat. The autocorrelation main lobe width of such signal is T_c/N, while the AF main lobe width in Doppler domain is $1/T$. The main lobe width for an MCPC waveform is comparable to the main lobe width of a single-frequency pulse-compression radar waveform, employing for example a P4 polyphase code of length M^2, and the same overall duration [1, 25]. The nulls in the autocorrelation function of an MCPC waveform are present at multiples of phase element duration T_c. These are a result of the orthogonality of the subcarriers with $1/T_c$ spacing and the utilized complementary set.

The major drawback of an MCPC waveform, similarly to OFDM and other multicarrier waveforms in general, is its time-varying envelope. Two solutions proposed to overcome this are based on the choice of the modulating sequences on

each subcarrier. One option is based on the use of identical sequences (ISs) [2,27], which uses the same ideal sequence to modulate all subcarriers. Lowering the PMEPR then becomes a problem of amplitude and phase modulation of each subcarrier using the complex weights w_n. The advantage of this method is that closed-form solutions are available for a varying number of subcarriers. Their common disadvantage is that the resulting PMEPR is low, but not really close to the optimum value achievable [2]. Closed form solutions of carrier initial phase that lower the level of PMEPR are given in [2,27]. An ideal PMEPR value is 1 (0 dB), i.e. constant modulus waveform. A carrier phasing design that takes into account carrier weighting achieves a PMEPR of 1.9 dB in an example presented in [2]. This example is for a $N = 15$ carrier setup with amplitude modulation over the carriers that follow a square root of Hamming window rule. Further reduction of PMEPR is possible through iterative algorithms. An example of such algorithm is the time-frequency switching and clipping approach described in [59]. This approach offers a PMEPR level of 0.86 dB for the example presented in [2]. More such iterative algorithms are presented in [60] which offer even lower PMEPR levels. The advantage of such iterative algorithms is that they provide the best possible phases for the lowest achieved PMEPR level, however the provided solutions might be local ones or their solutions depend on the chosen initialization values for the phases.

Another option is to use the so called consecutive ordered cyclic shifts (COCSs) of an ideal sequence [2,25,26], which means that an ideal sequence based on P3 or P4 codes for example is generated and on each subcarrier a cyclic shifted version of it is used for modulation. When $N = M$ the COCS method creates a complementary set, which guarantees nulls in the autocorrelation function at multiples of chip duration. Also carrier amplitude and phase modulation can be used in conjunction with the COCS method. For large compression ratios, when N takes large values close to M, the PMEPR using COCS of P4 codes is reported in [2] to be close to 2.67 dB. With carrier modulation and optimal selection of N ($N = M$), the COCS method can achieve a PMEPR level of around 1.76 dB [2].

Frequency agile patterns for MCPC waveforms can be generated by applying an IDFT over a matrix containing different phase codes such as the polyphase P3 or P4 codes. Nulls are placed in the matrix of codes where the corresponding subcarriers are inactive. For the waveform proposed in [30], different Doppler processing techniques are investigated in [37]. The processing techniques are based on fast Fourier transform (FFT). If the pulse shift is negligible with respect to the time resolution, the complex samples corresponding to one range gate can be processed using an FFT over the samples spaced at the pulse repetition time (PRT). Another Doppler processing method is proposed in [30]. First, the received MCPC symbols are transformed from time to frequency domain using an FFT and then the change in frequency of the subcarriers is observed over the available symbols. The Doppler frequency is obtained by using another FFT operation over the considered symbols. The signals presented in [30,37] are wideband. This means that each subcarrier may experience a different Doppler

shift. Consequently, Doppler frequency may be estimated from one pulse only.

The OFDM waveform is another orthogonal multicarrier waveform employed in radar applications. OFDM is also common in passive radar systems employing DVB-T or DAB signals, for example. It can be considered a special case of an MCPC waveform, without the coding on each subcarrier. If one considers an MCPC waveform with bandwidth $B = N_{\text{MCPC}}\Delta f_{\text{MCPC}}$, where $\Delta f_{\text{MCPC}} = 1/T_c$, and overall duration $T = MT_c$, an OFDM waveform can be defined with the same bandwidth and duration. The following relationships between the two waveforms exist:

$$N_{\text{OFDM}}\Delta f_{\text{OFDM}} = N_{\text{MCPC}}\Delta f_{\text{MCPC}}$$
$$N_{\text{OFDM}}\frac{1}{MT_c} = N_{\text{MCPC}}\frac{1}{T_c} \qquad\qquad (2.7)$$
$$N_{\text{OFDM}} = MN_{\text{MCPC}}.$$

Thus, the difference between an MCPC and an OFDM waveform with equal bandwidth and duration is in the number of subcarriers and the intercarrier spacing. To be more specific, the MCPC waveform has an M times larger intercarrier spacing and M times less subcarriers. As it also suffers from the time-varying envelope, all the techniques to reduce the PMEPR of the MCPC waveform that use only carrier modulation can be employed for the OFDM waveform as well.

A considerable research effort has been made in the area of coexistence among radar and communications systems. It is of particular interest to design wave-forms for both radar and communications purposes, as well as systems capable of performing both tasks simultaneously. The OFDM waveforms are considered an attractive candidate for such applications. Among the first proposals of an OFDM waveform for simultaneous radar and communications tasks is made in [32]. A technique to resolve both range and radial velocity is also proposed in [32]. The technique is based on FFT and is independent of the phase coding. A similar processing technique which allows for both radar and communications operations simultaneously is proposed in [61] and tested in [62]. The processing method consists of using the transmitted information and received information at the output of the OFDM de-multiplexer before the channel equalization and decoding. This way the distortion from the channel is fully contained in the complex modulation symbols of received data. The frequency domain channel transfer function can be easily obtained through an element-wise division:

$$I_{\text{div}}(n) = \frac{I_{\text{RX}}(n)}{I_{\text{TX}}(n)}, \qquad\qquad (2.8)$$

where $I_{\text{RX}}(n)$ is the received information and $I_{\text{TX}}(n)$ is the transmitted infor-mation at the nth subcarrier. The performance of the method proposed in [61] is evaluated in [33]. It is found that target speed and distance estimation for OFDM radars works very well above an SNR threshold of -36.7 dB and rapidly degrades under this threshold [33]. The SNR threshold is found through simulations that utilize the same target at different distances.

2.2.2 Novel Non-Orthogonal Multicarrier Radar Waveforms

Generalized versions of OFDM and MCPC waveforms are proposed in Publications I and II as FDRW and TDRW respectively. Both relax the orthogonality constraint of OFDM and MCPC waveforms for radar performance improvement. Similar to the OFDM and MCPC waveforms, these are special cases of the GMR model. Resolution as well as PSLLs in both delay and Doppler domains are considered for performance evaluation and are obtained from the AF. The design of such waveforms is justified by the fact that the orthogonality of the subcarriers is lost anyway at the receiver due to the delay and Doppler effects introduced by the target. There is no waveform that would be orthogonal in all delays and Doppler shifts. It is demonstrated in Publication I that the TDRW achieves lower PSLLs and even better delay resolution compared to the MCPC waveform. It is also shown in Publication I that the Doppler shift induces a lower ICI for a TDRW compared to an MCPC waveform. The TDRW waveform allows for the use of longer modulating codes on the subcarriers. It is demonstrated in Publication I that this may lead to lower PAPR levels due to spreading the power on each subcarrier over a larger bandwidth.

The proposed TDRW and FDRW can be presented in relation to the well known MCPC and OFDM waveforms. First, in this thesis when waveforms are compared it is assumed these use the same amount of resources, i.e. have the same total power, occupy the same bandwidth and have the same duration. The FDRW is a generalized version of the OFDM waveform which employs coding in frequency domain over subcarriers and subcarrier orthogonality relaxation. As a consequence of the orthogonality relaxation, in contrast to OFDM waveforms, FDRWs have an intercarrier spacing which is smaller or larger. Consequently, FDRWs have a number of subcarriers which is larger or smaller respectively. The TDRW is a generalization of the MCPC waveform which relaxes the orthogonality of the subcarriers. In terms of orthogonality relaxation the TDRW has two degrees of freedom. This can be achieved by either increasing or decreasing the intercarrier spacing or by increasing or decreasing the length of the codes modulating each subcarrier. The proposed GMR model accounts for all the different ways of generating the FDRWs and TDRWs.

2.2.3 Performance of Novel Multicarrier Radar Waveforms

The novel TDRWs proposed in Publication I using the GMR model relax the orthogonality constraint for improved radar performance. Their performance is compared against the MCPC waveforms. The AF plots are used to evaluate the performance which is established in terms of delay and Doppler resolution as well as PSLLs in both delay and Doppler domains. The delay and Doppler resolutions are determined by the width of the main lobe in the respective domain cut. The width of the main lobe is determined by the first null in the respective plot. Noise free scenarios are considered when comparing the

two waveforms which are constrained to use equal amount of resources, i.e. they occupy the same bandwidth and have the same total power and duration. The performed simulations indicate that a TDRW can offer a better radar performance than the MCPC waveform proposed in [1, 2, 25].

The simulation results in Publication I reveal that a TDRW with more subcarriers or longer modulation codes than a similar MCPC waveform can benefit from lower PSLLs in both delay and Doppler domains of the AF. However the level of ICI needs to be kept under control for the TDRW. Having lower PSLLs reduces the ambiguities when dealing with target detection. In Fig. 2.5 the AF plots for the $N = 8$, $M = 8$ MCPC waveform is compared to an $N = 15$, $M = 32$ TDRW. Lower overall sidelobes are also experienced for TDRWs with longer modulating codes of $M = 16$ or $M = 32$. Simulation results in Publication I reveal that increasing the code length for TDRW decreases the PSLL in delay domain and increases the PSLL in Doppler domain. However, the PSLL in Doppler domain is still smaller than the one for the MCPC waveform with equal use of resources.
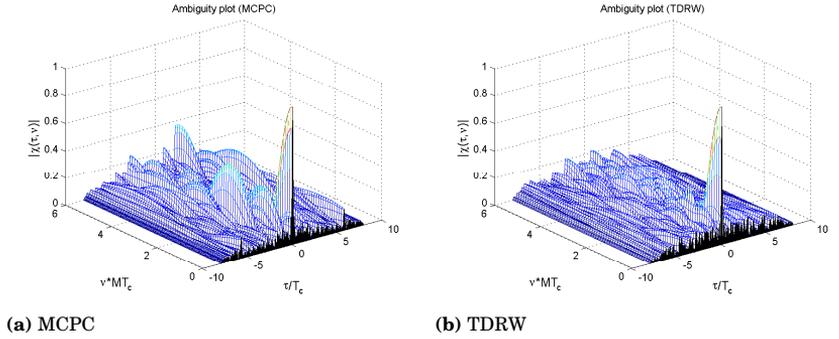

(a) MCPC   **(b)** TDRW

Figure 2.5. AF plots for MCPC waveform ($N = 8$, $M = 8$, P3 code) and TDRW ($N = 15$, $M = 32$, P3 code). The sidelobes in both delay and Doppler domain corresponding to the TDRW are much reduced compared to the sidelobes corresponding to the MCPC waveform. Copyright 2016 IEEE.

A better delay resolution is obtained for the TDRW with lower number of subcarriers than the MCPC waveform. This may be observed for example when comparing an $N = 5$, $M = 5$ MCPC waveform and an $N = 3$, $M = 20$ TDRW. The chip duration T_c for TDRW is 4 times smaller than for the MCPC waveform in this case. Such result can be justified by the difference in the effective bandwidth between the two waveforms, which is larger for the considered TDRW than for the MCPC. It is shown in [63] that the theoretical bound on delay estimation variance depends on the effective bandwidth of the transmitted waveform.

The proposed TDRWs also provide lower PAPR levels compared to the MCPC waveforms. TDRWs employing both P3 and P4 codes are compared with equivalent MCPC waveforms in Publication I and it is shown that the additional spreading for the TDRWs facilitates reducing the PAPR significantly. A TDRW

with a PAPR level as low as 2.32 dB is obtained for codes of length $M = 32$, while a PAPR level of 4.64 dB is obtained for an MCPC waveform with codes of length $M = 8$. It is important to note that the PAPR values depend on the employed codes, the cyclic shifts and initial phase of the ideal sequence for both TDRW and MCPC waveforms [2]. Consequently, low PAPR values may not be always achieved and special consideration is needed when designing low PAPR multicarrier radar signals.

Received radar signals are typically subject to Doppler shifts caused by the radial velocity component of the target. A direct consequence of the Doppler shift experienced by multicarrier radar signals is ICI at the receiver and loss of orthogonality. This ICI affects all multicarrier radar signals and has to be taken into account when analyzing the performance of these special radar waveforms. Another source of ICI is the orthogonality constraint relaxation, which is employed in TDRW design. The ICI can be strong if very long codes or nonstandard subcarrier spacing are considered for the TDRW. It is concluded in Publication I that the TDRWs are less sensitive to Doppler shifts than an MCPC waveform, which makes them the preferred option for certain radar applications. For reasonable code lengths, the longer the spreading codes for the TDRW the better tolerance for Doppler shifts is achieved. The increased ICI of TDRWs does not impact the radar performance in a significant manner as long as it is kept under certain levels through careful waveform design.

Multicarrier waveforms facilitate taking advantage of frequency diversity. In case of narrowband jammers or high attenuation for certain parts of the frequency band occupied by the signal, one can adaptively allocate less or no power to subcarriers affected by such phenomena. This way the power resources are not wasted on subcarriers experiencing severe jamming or challenging propagation conditions. A simulation study performed in Publication II establishes how nulling some of the subcarriers of TDRWs and MCPC waveforms impacts the radar performance in terms of AF properties. It is found that the delay and Doppler resolutions of the respective waveforms are not impacted by the subcarrier nulling. On the other hand the PSLLs in both delay and Doppler domains experience degradations. For example, a TDRW with codes of length $M = 32$ has reduced PSLL in delay domain compared to an MCPC, as it can be observed in Fig. 2.6. Also, in Doppler domain, lower PSLLs are experienced for the TDRW with codes of length $M = 32$ in more cases rather than for the MCPC waveform.

2.3   Waveform Optimization

Waveform optimization is extremely important for radar as it allows for adapting the transmitted waveform to the radio environment, target properties and radar task at hand. Exploiting situational awareness and agile use of waveforms are key concepts in cognitive radars [5]. The awareness may be provided, for example
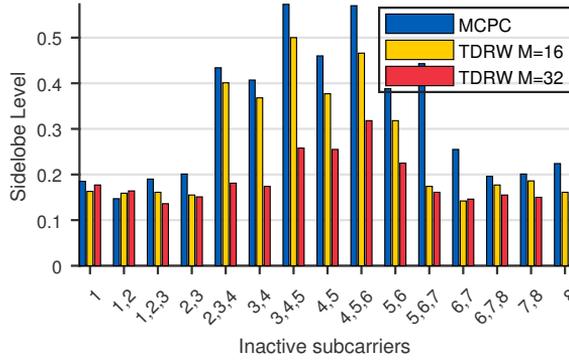
Figure 2.6. Sidelobe levels in the delay domain of the AF plots for agile MCPC waveform ($N = 8$ subcarriers, code length $M = 8$) and TDRWs ($N = 8$ subcarriers, code lengths $M = 16$ and $M = 32$). The horizontal axis contains the index of the inactive subcarriers, chosen from the total of 8 available subcarriers for every signal. It is observed that the inactive subcarriers cause, in general, higher sidelobe levels in delay domain. However, lower sidelobe levels in delay domain are obtained for a TDRW with code length $M = 32$. Copyright 2014 IEEE.

from the receiver back to the transmitter. The transmitted waveform can be dynamically optimized over time provided the scene does not change faster than the time it takes to acquire the awareness and perform the optimization at the transmitter.

It is mentioned in [6] that one of the earliest efforts in optimizing the input signal for a known channel can be tracked to [64]. In late the 1980s and the early 1990s a considerable research effort is made in waveform adaptation for maximizing the response of weak targets [6]. There is plenty of research literature available on the topic of waveform design in radar. The interest in this chapter is on waveform optimization for multicarrier waveforms in order to take advantage of the various degrees of freedom that these waveforms provide.

2.3.1 Waveform Optimization for Multicarrier Radar

The multitude of degrees of freedom brought by multicarrier waveforms facilitate adjusting and optimizing the transmitted waveforms. In [36] for example, a phase-matching iterative method to obtain MCPC waveforms with an AF as close to the ideal one as possible is presented. Also based on the AF, an optimization technique for obtaining the desired spectrum of an OFDM waveform, such that the delay resolution is improved, is presented in [52]. The optimization procedure takes into account the scattering characteristics of the target. An algorithm to optimally design the OFDM waveform for the detection of a moving target in a multipath scenario is proposed in [56]. The detection problem is formulated as a statistical hypothesis test and a generalized likelihood ratio test (GLRT) is employed for deciding the presence of the target at a particular range cell. The design of an OFDM radar waveform using a multiobjective

optimization technique is proposed in [57] for the detection of a moving target. Two objective functions are employed, one that minimizes the upper bound on the estimation error of the sparse vector for the scattering coefficients and one that maximizes the Mahalanobis distance to increase the performance of the underlying detection problem. The sparsity of multiple paths and the knowledge of the environment at a particular range cell are exploited to estimate the paths for the target response. The symbols composing the waveform used for target recognition in [40] are obtained through a genetic algorithm based optimization method to be as close as possible to the true responses of the targets. In [39], two waveform optimization strategies are proposed for an MCPC waveform. One single-objective optimization based on a genetic algorithm considers the reduction of the PMEPR, while a dual-objective optimization considers the reduction of both PMEPR and autocorrelation sidelobes.

The use of information theory to design radar waveforms for the measurement of extended radar targets is considered in [65]. Although considering classical single carrier radar waveforms, [65] introduces the maximization of the MI between the target ensemble and the received radar waveform for the first time. Another information theoretic approach, this time for OFDM waveform optimization for target tracking in a co-located MIMO system, is proposed in [48]. Degrees of freedom in frequency and polarization are used for richer target information and resolving of multipath components respectively. The optimization maximizes the MI between the state and measurement vectors at the next pulse by making use of all the measurement history up to the current pulse. Another information theoretic approach to multicarrier radar waveform optimization is considered in Publication I. The goal of the optimization problem is to maximize the MI between the received multicarrier waveform and the target impulse response. This maximizes the amount of information about the target that is delivered at the receiver.

2.3.2 Information-Theoretic Waveform Optimization Using the GMR Model

The GMR model that is introduced in Publication I is very convenient since it facilitates optimizing the degrees of freedom and waveforms. For example, power allocation per subcarrier can be done in an adaptive manner, based on the quality of the signal in each subcarrier observed at the radar receiver. The quality of the channel may be hampered by properties of target radar cross section, jamming, unintentional interference, propagation conditions or transceiver nonidealities. One way to model the optimization problem is to use a MI based criterion from information theory, which is first proposed in [65].

A simple and intuitive example of multicarrier waveform optimization is provided in Publication I for an OFDM waveform. Using the GMR model particularized for an OFDM signal and a stationary target the equation for the

received baseband signal is given by:

$$\mathbf{y} = \mathbf{y}_r + \mathbf{n} = \mathbf{F}^{-1}(\mu\psi\mathbf{A})\mathbf{c}_0 + \mathbf{n} = \mathbf{F}^{-1}\mathbf{G}\mathbf{c}_0 + \mathbf{n}, \tag{2.9}$$

where $\mathbf{F}^{-1}$ is the IDFT matrix, $\mathbf{G} = \mu\psi\mathbf{A}$ is an $N \times N$ diagonal matrix with $g_i = \mu\psi \left[\exp\left(-j2\pi\Delta f \frac{2d}{s}\right)\right]^i$, $i = 0\ldots N-1$, d is the distance to the target and s is the speed of light. Matrix $\mathbf{G}$ represents the target scattering composed of the complex amplitude introduced by the target μ and the delays introduced by the target, ψ, at passband carrier frequency, and $\mathbf{A}$, at subcarrier level in baseband. The conditional MI maximization between the received samples $\mathbf{y}$ and the radar target scattering, described by impulse response $\mathbf{G}$, given the knowledge of the transmitted waveform $\mathbf{r}$ provides the optimum power allocation among subcarriers. The greater the MI between the received signal at the radar and the target scattering, the better the radar is able to estimate the target parameters. Matrix $\mathbf{G}$ is estimated at the receiver and fed back to the transmitter, where the MI based optimization takes place. The MI can be expressed as:

$$I(\mathbf{y}, \mathbf{G}|\mathbf{r}) = H(\mathbf{y}|\mathbf{r}) - H(\mathbf{y}|\mathbf{G}, \mathbf{r}) = H(\mathbf{y}|\mathbf{r}) - H(\mathbf{n}), \tag{2.10}$$

where $H()$ denotes the differential entropy, and can be further written as:

$$I(\mathbf{y}, \mathbf{G}|\mathbf{r}) = \log_2[\det(\mathbf{\Sigma})] - \log_2[\det(\sigma_n^2\mathbf{I}_N)] = \log_2\left[\frac{\det(\mathbf{\Sigma})}{\det(\sigma_n^2\mathbf{I}_N)}\right]. \tag{2.11}$$

Maximizing the MI is done under a constraint on the total transmitted radar power P_T. This constraint ensures the radar follows regulatory constraints regarding energy emission.

The optimization problem is solved in closed form in Publication I using the method of Lagrange multipliers. In the considered setup, the MI objective is a concave function of the transmitted radar powers on each subcarrier. Adding the linear constraint on the total transmitted radar power results in a convex problem, which can be solved using its Lagrangian form and the Karush-Kuhn-Tucker (KKT) conditions. As the problem is convex, KKT conditions are sufficient for finding the optimal solution [66]. The obtained solution is a water filling solution which allocates power to subcarriers that give higher quality radar returns, i.e. are not subject to large noise, or get strong returns from the target, or is not severely attenuated on its way from the transmitter to the target and to the radar receiver. An example of power allocation per subchannel is presented in Fig. 2.7, where an OFDM radar waveform with $N = 32$ subcarriers is considered. It can be observed that power is allocated only to subcarriers where the receiver is experiencing high channel gain and/or low noise.

A similar optimization problem, this time for an MCPC waveform, is also presented and solved in closed form in Publication I. The solution to this optimization problem is a set of M power allocations per subcarrier, corresponding to each OFDM symbol in the MCPC waveform.

The waveform optimization examples provided in Publication I show that the GMR model can take advantage of the diversity and awareness of the state of the spectrum by using an information theoretic objective function based on MI.
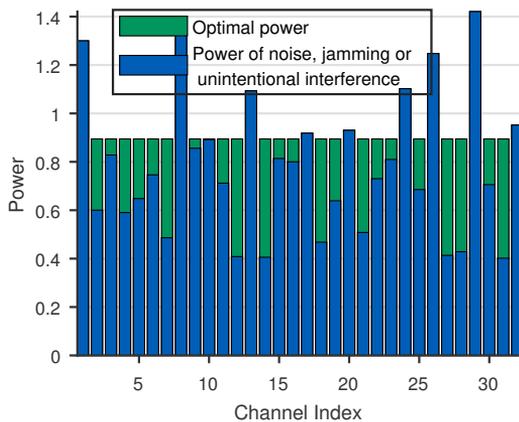
Figure 2.7. Example of power allocation per subchannel based on channel quality for an OFDM type of radar waveform with $N = 32$ subcarriers. The limited power is allocated to subcarriers experiencing strong target returns or low attenuation, whereas subcarriers that have low channel gain and/or large noise are allocated little or no power. Copyright 2016 IEEE.

2.4 Discussion

The success and the wide spread of multicarrier systems for communications in the past forty years have impacted the radar research community as well. Many of the benefits, as well as their drawbacks, are applicable to radar as well, although traditionally the two systems have different goals. Multicarrier waveforms efficiently make use of several degrees of freedom in frequency, code or time domain. This allows for waveform adaptation to the changing state of the spectrum, target scenario and radar task at hand. Drawbacks such as high PAPR levels are easy to overcome as, in contrast to communications systems, the transmitted waveform can be anything as long as it has the desired properties. This way, amplitude modulation and coding over subcarriers is able to reduce the PAPR.

A GMR model is introduced in this thesis. The model can be used to describe and implement most common radar waveforms as well as novel ones such as TDRW or FDRW. Two sets of equations, one for transmitter and one for receiver, are introduced. The model is formulated using matrix notations, which facilitates agile design and generation of waveforms in simulation environment using a simple matrix model. Moreover, it facilitates easy hardware implementation. The flexibility of the proposed GMR model may allow for the design and generation of new waveforms that can outperform the existing ones presented in this thesis. Traditionally only the receiver is optimized, however, the advent of cognitive radar [5,6] brings fully adaptive transmitter and receiver and the use of channel and spectrum knowledge in agile design of both the transmitter and receiver. The GMR model, with its multiple degrees of freedom for optimization,

is an enabler for cognitive radar. Also, for the more recent topic of spectrum sharing/coexistence between radar and communications systems, the GMR model is an attractive choice. It allows for exploiting various degrees of freedom in designing radar or communications waveforms for dual-use, interference-avoidance from one system or the other and even co-design strategies. Examples of information-theoretic based waveform optimization are provided in this thesis, in particular waveform optimization examples for OFDM and MCPC waveforms. Such optimizations provide the best possible power allocation per subcarrier for the given target characteristics and propagation conditions.

The MCPC waveform is the first multicarrier waveform proposed for radar in the literature. It is a special case of the proposed GMR model and a generalized version of the OFDM radar waveform, also proposed later in the literature. The MCPC waveform employs time domain coding over each subcarrier while satisfying the orthogonality condition at the same time. Various designs and processing techniques are proposed in the literature for both waveforms. Due to the mathematical simplicity, subcarrier orthogonality and fast and easy Fourier based implementation, these waveforms allow for very simple processing techniques. The MCPC waveforms benefit from frequency diversity, similarly to the OFDM waveform, as well as code diversity and high compression gains. The high compression gain of the MCPC waveform allows for a thumbtack AF, a desirable property, that offers the MCPC waveform high delay resolution and reduced ambiguities. Both OFDM and MCPC waveforms suffer however from a time-varying envelope due to the inherent properties of the transmitted signal which is a sum of possibly scaled orthogonal sinusoidal signals. Plenty of research is available on reducing the PMEPR of these multicarrier signals. Few of the pros and cons of the MCPC and OFDM waveforms are presented in Table 2.2.

Subcarrier orthogonality is a key property of both OFDM and MCPC waveforms. In this thesis, waveforms that relax the orthogonality constraint are proposed and at the same time are able to improve the radar performance. These are also special cases of the proposed GMR model. However, the ICI caused by the orthogonality loss has to be kept under control in order to ensure the improved radar performance. In essence, the orthogonality relaxation is not a significant issue as anyway at the radar receiver the subcarriers are not orthogonal after the delay and Doppler effects introduced by the target. Benefits of the orthogonality relaxation are presented in this thesis, such as: lower PSLLs in both delay and Doppler domains and even improved delay resolution for some particular designs. The radar performance of the TDRW is compared with the MCPC waveform and it is shown that the advantages of relaxing the orthogonality property allows the TDRW to outperform the MCPC waveform. Other novel multicarrier waveforms employing FFH on a chip-by-chip basis at subcarrier level, different than the SFWs in [28] on a pulse-by-pulse basis at block of subcarriers level, are shown to provide low PSLLs in both delay and Doppler domains. Such FFH technique can also be combined in the future with the

orthogonality relaxation of the TDRW for potential additional gains. The benefit of frequency diversity, which allows for efficient power utilization in cases of attenuation, jamming or interference, is available to all multicarrier waveforms. It is demonstrated by simulations in this thesis that the TDRW provides lower PSLLs compared to the MCPC waveform when some of the subcarriers receive no power due to such unfavorable propagation conditions. A brief comparison of the main pros and cons that TDRWs, OFDM and MCPC waveforms have are presented in Table 2.2.

Table 2.2. Comparison between multicarrier radar waveforms.

	MCPC	OFDM	TDRW
	Flat Spectrum	Flat Spectrum	Flat Spectrum
	Frequency Agility	Frequency Agility	Frequency Agility
			Lower PSLLs
Pros			Improved delay resolution
	Low PAPR		Lower PAPR
			Improved Doppler tolerance
	Thumbtack AF		Thumbtack AF
	High PSLLs	High PSLLs	
	Doppler sensitivity	Doppler sensitivity	
Cons			Higher ICI
		Higher PAPR	
			Out of band emissions for boundary subcarriers[1]

---

[1]This can be handled with proper power allocation to boundary subcarriers, which leaks less power to the other bands.

3. Coexistence and Spectrum Sharing among Wireless Communications and Radar Systems

In this chapter, the problem of coexistence and spectrum sharing among radar and wireless communications systems is treated from the radar point of view. Traditionally, the two systems are designed independently. However the changes in spectrum regulation [67] are forcing a paradigm shift. Traditionally, radars have been allocated a variety of frequencies with exclusive use. New emerging wireless communications systems and the need for higher rates have led to a situation where the systems are required to share the spectrum and coexist so that both systems get their desired performance while managing the interference caused to the other systems. New regulations are such that spectrum is allocated in larger chunks but a number of radio systems are sharing it.

Nowadays a considerable increase in the number of radar devices is experienced, especially in the automotive industry. This trend makes the topic of spectral coexistence among radar and communications systems even more timely. Although the spectral coexistence among radar and communications systems is the topic of this chapter, the need for such methods and techniques extends beyond this particular case to a considerable number of wireless systems that operate in the same spectral band, such as LTE, 5G, WLAN and S-band radars.

The need for spectrum sharing and coexistence among radar and wireless communications systems is pointed out for example in [67] and proposals by US spectrum regulator Federal Communications Commission (FCC) are made already [3]. The coexistence problem has received a lot of attention from the research community in the recent years [68]. In the following chapter key contributions in the existing literature are presented.

3.1 Coexistence Models

A first step in investigating the radar and communications system coexistence problem is to characterize the interference that one system can cause to the other. Some of the recent results are presented in [69–71]. A variety of techniques for mitigating the interference between the two systems, as well as joint design of the two systems or systems capable of both communications and radar tasks are

present throughout the literature [68].

Different system configurations for spectrum sharing among radar and wireless communications systems are available in the literature. It is referred to coexistence when the two systems operate in the same bandwidth. The systems can be in the same or different locations [72]. Also, various modes of operation for the two systems are identified, ranging from interference mitigation to joint design. All of these are enabled through waveform design and optimization or system design. One of the most futuristic approaches to the coexistence problem in general is proposed in [72], where the system elements are dynamic and adapt to the task at hand (communications or radar task, for example). Based on the relative locations of the two systems, two categories emerge from the existing literature: co-located and distributed systems. These two topologies are illustrated in Fig. 3.1. Co-located systems where the same hardware is capable of performing both radar and communications tasks are proposed in [68, 73–75] for example. This category also includes dual-use waveforms described for example in [61, 62, 76]. Distributed systems are considered for example in [77–80] and Publications III–VI. Under this configuration the two systems are located in different locations close enough that mutual interference could be experienced.
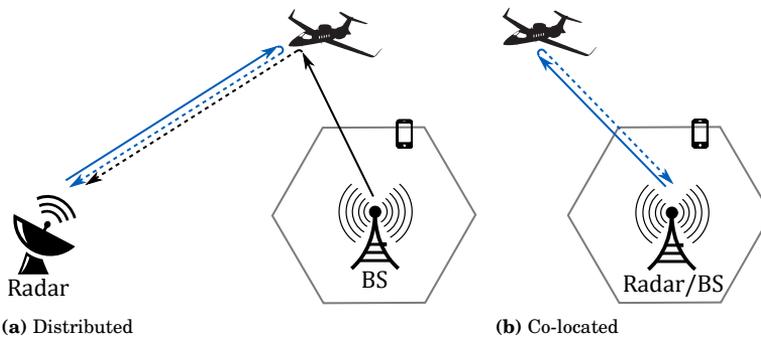

(a) Distributed   **(b)** Co-located

Figure 3.1. Commonly found coexistence configurations in the literature. In the distributed configuration the two systems are separated in space. In the co-located configuration several systems are found: a system capable of both radar and communications tasks using different waveforms, a system with a dual-use waveform, or a passive opportunistic radar system.

In terms of the relationship between the two systems, the following categories are identified: passive, competitive, coexisting, cooperative. The passive class is represented by passive radar systems that exploit other systems' transmissions without transmitting anything themselves. See for example [34, 81–83] for more details. In the passive class the sub-class of opportunistic systems is identified. This sub-class contains radar systems that are an evolution of passive radars, for example the ones considered in [84–86]. Such radar systems contain one or more receivers and make use of the waveform of another system from which certain information is received. Consequently, having access to various information from the communications system differentiates opportunistic systems from typical

passive systems. For example, azimuth, timing or even the transmitted data could be available at the opportunistic passive radar system [85]. Also friendly or adversary radars can be used as illuminators of opportunity whose location can be known or estimated respectively by the opportunistic radar [87]. A competitive relationship between the two systems is observed in legacy systems, designed in isolation and separated in frequency domain or far away from each other in order to minimize the mutual interference. Coexisting radar and communications systems are designed by taking into account the other systems and managing the interference caused to the other systems sharing the spectrum. Few examples are found in [78, 79], where the radar waveform is designed to fill spectral gaps available between the spectrum allocated for the communications systems. Also in [88–96] precoders are designed for the two systems in order to avoid mutual interference. Another coexistence example can be found in [97, 98], where coexistence is possible through dynamic spectrum access, using the concept of Licensed Shared Access/Authorized Shared Access. Cooperative radar and communications systems exchange certain information for their mutual benefit. Few examples of cooperative systems can be found in [72, 77, 96, 99] as well as Publications III–VI. For example, channel state information (CSI) is shared with the radar in [96]. In [99] the MI between the communications receiver and transmitter, which quantifies the amount of conveyed information, is shared with the radar. The purpose is to build a NP based cooperative joint risk that can be used to find the cooperative optimal point of operation for both systems. In [72] information regarding the communications system that would help with forward error correction and spectral shaping is shared with the radar system with the goal of removing the incoming communications signal from the observed waveform. In [77] and Publications III–VI the communications system shares an interference mask with the radar system, which ensures a minimum desired rate for the communications systems can be achieved.

For the distributed configuration illustrated in Fig. 3.1a several situations are encountered. The communications signals arrive at the radar through a direct path as well as indirect path reflected off a target. In a plain coexistence scenario, both communications returns act as interference to the radar. Alternatively, in a cooperative scenario, the communications signal reflected off the target can be exploited for improving the estimation or detection performance as demonstrated in Publications III and V, respectively. At the same time, the direct path between the two systems can be used for decoding the transmitted communications signal, interference cancellation, or for exchanging useful information. Consequently, the communications signal reflected off the target can be considered as interference or useful energy, as in Publication IV.

At the system level various configurations are identified in the literature. Both single input single output (SISO) [77, 78, 80] and MIMO [48, 76, 92, 98] systems are proposed, either for the radar or for the communications systems. Also in Publications III–VI a SISO system is considered for both coexisting systems. The

waveforms utilized by the two systems can be single carrier [78–80], multicarrier [42, 52, 61, 70, 77, 91] or frequency hopping type of waveforms [100, 101]. In particular, in Publications III–VI the multicarrier waveform such as OFDM is employed for both the radar and the communications systems. Proposals for integrated radar and communications systems have been made, where the same hardware is capable of performing both radar and communications tasks [68, 73–75]. Techniques based on dual-use waveforms are proposed as well [61, 62, 76].

3.2 Waveform Optimization in Coexistence/Cooperative Scenarios

Radars have a variety of tasks and different waveforms may be needed for different tasks. Target detection is the most important task. This tasks is performed first, before the parameter estimation/tracking or target characterization tasks can take place. It is common for radars to adapt or optimize the transmitted waveforms in order to improve the performance of the radar task at hand. The optimization or adaptation is done based on acquired awareness, such as channel state information or SINR, obtained through sensing and feedback from the receiver. Such approach is considered also in the coexistence/cooperative scenarios with communications systems addressed in this thesis.

As it can be observed from the existing literature available on coexistence among radar and wireless communications systems, waveform design plays a central role in facilitating the coexistence. When considering different waveforms for the radar and the communications tasks, the optimization of either waveform or both waveforms has been proposed in the literature. The proposals can be categorized as: radar-centric [77, 102, 103] when optimizing the radar waveform, communications-centric [96, 98, 104, 105] when optimizing the communications waveform, or co-design when both waveforms are optimized jointly [106, 107].

Considering the coexistence among radar and communications systems, many propose the optimization of either the transmitted radar waveform or the communications waveform in order to control the interference between the two systems. Another objective is to take advantage of the non-contiguous spectral allocation and non-continuous use of spectrum (e.g. carrier aggregation) by the communications systems. A radar-centric approach to the optimization is taken for example in [78, 102, 103], as well as in Publications III–VI. A communications centric approach is taken in [96–98, 104, 105] for example. Various objective functions are used for the proposed radar-centric optimization problems. Most of these are focused on SNR or signal to interference and noise ratio (SINR) maximization [78, 79, 108], probability of detection [102], interference at the radar [97], or, in a multiobjective optimization, the probability of detection and the communications user throughput [97]. Another interesting objective function used for optimizing the radar waveform for target characterization task is the MI used in [77] and Publications I and IV. In [77] the MI between the

received radar signal and the target channel impulse response is maximized. This ensures that the information about the target at the receiver is maximized. The focus of Publication IV is to evaluate various MI criteria possible in a coexistence scenario and find the one that provides the best possible waveform design. MI as an objective function is introduced for a traditional radar setting by the seminal work in [65]. The paper introduces maximizing the MI between the Gaussian distributed target impulse response and the received signal at the radar for target channel estimation. The intuition behind this is to capture as much information about the target at the radar receiver. The work in [65] is extended in others such as [109–111]. A connection between information theoretic measures and estimation theoretic measures, namely MI and minimum mean square error (MMSE), is established for MIMO radars in [110]. Also in [112] connections between information theory and estimation theory are presented in terms of estimation information and integrated MMSE respectively.

Several other information-theoretic measures have been used for radar waveform design as well [57, 113–116], however not in a coexistence setting. Such measures are the relative entropy, also known as Kullback-Leibler (KL) divergence, and J-divergence [115–117], as well as Bhattacharyya distance [116, 118] and squared Mahalanobis distance [57] between the two densities under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$. According to Stein's lemma [119], for a fixed probability of false alarm or a fixed probability of detection, the maximization of the corresponding KL divergence ($D(f_0 \| f_1)$ or $D(f_1 \| f_0)$ — f_0 and f_1 the densities under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ respectively) leads to an asymptotic maximization of the probability of detection or minimization of the probability of false alarm respectively [116]. The Bhattacharyya distance simultaneously provides an upper bound on the probability of false alarm and a lower bound on the probability of detection [119]. It is claimed in [120] that the Bhattacharyya distance is a better optimization metric than the J-divergence for detection performance. It has been proven in [107] that maximizing the J-divergence (which is in fact the sum of the two KL divergences — $D(f_0 \| f_1)$ and $D(f_1 \| f_0)$) is equivalent to maximizing the SINR. In [121], based on [122], it is mentioned that MI maximization is related both to the ability to estimate unknown characteristics of the target and also to the maximization of the probability of detection for a fixed probability of false alarm. The latter connects MI maximization with KL divergence (precisely with $D(f_0 \| f_1)$). The radar waveform optimization problem that maximizes the MI between the received signal and the target channel impulse response in coexisting radar and communications systems scenario is considered in [77] for the first time. The optimization is done under constraints on total transmitted power for the radar and interference caused to the communications system. Although radars have plenty of power available these have to obey regulatory levels in shared spectrum. Therefore a constraint on the total transmitted radar power is employed. Moreover, they may wish to use low probability of detection waveforms to avoid jamming. As the two systems considered in [77] are in close proximity and share the same frequency band, interference is likely to happen.

It is considered in [77], as well as in this thesis, that a minimum data rate for the communications system is guaranteed and a certain SINR value is needed for that. This can be expressed by using the interference constraint in the optimization problem. The radar is cognitive in the sense that it uses the target and channel information to adjust its transmitted waveform. The cooperation between the two systems enables the formulation of the interference constraint. This can be formulated for example by using knowledge of the second order statistics of the communications signal $c(t)$ and the channel between the radar and the communications receiver $h_e(t)$, as well as the knowledge of the channel between the communications transmitter and the communications receiver $h_s(t)$. The objective function to be maximized is the MI between the received radar signal $y(t)$ and the target impulse response $h_r(t)$ conditioned on the transmitted radar signal $r(t)$ [77]:

$$I(y; h_r | r) = H(y | r) - H(y | h_r, r). \tag{3.1}$$

The optimization problem for a single base station (BS), when both the radar and the communications systems use OFDM waveforms, may be written as [77]:

$$\begin{aligned}
\max_{\{|r^F[k]|^2\}} \quad & \sum_{k=0}^{N-1} \log \left(1 + \frac{\sigma_{h_r}^2[l] |r^F[k]|^2}{\sigma_c^2[k] |h_d^F[k]|^2 + \sigma_v^2} \right) \\
\text{s.t.} \quad & \log \left(1 + \frac{\sigma_c^2[k] |h_s^F[k]|^2}{\sigma_{h_e}^2[k] |r^F[k]|^2 + \sigma_n^2} \right) \geq t_k, \ \forall k \\
& \sum_{k=0}^{N-1} |r^F[k]|^2 \leq P_{\mathrm{T}}.
\end{aligned} \tag{3.2}$$

The second term in the logarithm of the objective function is the SINR at the radar receiver, where the communications signal through channel h_d acts as interference. The first constraint in the optimization problem is a capacity constraint per subcarrier k at the communications receiver. Also here, the second term in the logarithm is the SINR at the communications receiver, where the radar signal through channel h_e acts as interference. The last constraint in the optimization problem is the total transmitted power constraint on the radar signal. It is assumed in [77] that the second order statistics of the channels h_r, h_e, as well as of the communications signal and the noise are known. The radar signals and channels h_d and h_s are assumed to be known and deterministic. The optimization problem in (3.2) is convex and it is solved in closed form using the technique of Lagrange multipliers. The obtained solution is a water-filling type of solution and represents a power allocation per subcarrier of the transmitted radar signal.

Radars have a variety of tasks and different waveforms may be needed for different tasks. Two main tasks are the detection and the target parameter estimation tasks. Target detection is the most important radar task. This was also the main function of the radar in its early times, before evolution and the multitude of purposes of the modern day radars. It is also needed for target tracking in most cases. A common approach to target detection is the hypothesis

testing, which tries to distinguish between two hypotheses:

$$\begin{cases} \mathcal{H}_0, \text{no target present} \\ \mathcal{H}_1, \text{target present}. \end{cases} \tag{3.3}$$

The optimal way, based on the NP theorem, to build such hypothesis testing is through the likelihood ratio test. In such test, the ratio between the likelihoods associated with hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$ respectively is compared against a threshold. If the ratio is larger than the threshold then hypothesis $\mathcal{H}_1$ is decided, otherwise hypothesis $\mathcal{H}_0$ is decided. Based on the received samples, the likelihood ratio test can be defined as:

$$L(\mathbf{y}) = \frac{p(\mathbf{y}; \mathcal{H}_1)}{p(\mathbf{y}; \mathcal{H}_0)} \gtrless \eta, \tag{3.4}$$

where $p(\mathbf{y}; \mathcal{H}_1)$ is the likelihood associated with hypothesis $\mathcal{H}_1$, $p(\mathbf{y}; \mathcal{H}_0)$ is the likelihood associated with hypothesis $\mathcal{H}_0$ and η is the threshold deciding which hypothesis is in place. When one has multiple measurements that are independent and identically distributed (i.i.d.), the likelihoods associated with either hypotheses $\mathcal{H}_i$ ($i = 0, 1$) can be written as:

$$p(\mathbf{y}; \mathcal{H}_i) = \prod_k p(y[k]; \mathcal{H}_i), \tag{3.5}$$

with $p(y[k]; \mathcal{H}_i)$ the probability density function (PDF) of sample $y[k]$ under hypothesis $\mathcal{H}_i$. It is common that instead of the likelihood ratio test $L(\mathbf{y})$ a test statistic $T(\mathbf{y})$, which is a monotonic function of $L(\mathbf{y})$, is used without affecting the optimality of the test.

Besides choosing from different waveforms for the task at hand, waveform optimization can be employed with the aim of exploiting the awareness and the available degrees of freedom for improved performance. Radar waveform optimization in coexistence scenario for improved or best possible detection performance is proposed in several works [74, 79, 97, 98, 102, 108, 123] as well as Publication V. In [102] the problem of optimum radar code design is addressed. The detection probability is maximized under constraints on the amount of interference caused to the communications system, as well as energy and similarity constraints. The role of the similarity constraint is to make sure the optimized code has ambiguity and resolution properties close to a reference code or desired ideal AF. The same problem of radar code design in a similar setup is considered in [79], this time with the radar SINR as a figure of merit. It is assumed in [79, 102] that the radar has access to a radio environment map. The false alarm probability constraint is not considered in the optimization problems proposed in [102]. In [108] the optimal power spectral density of a radar waveform is proposed via radar SINR maximization. It is assumed that the communications system interferes with the radar. The radar tolerates this interference for the sake of coexistence and tries to cause as little interference as possible to the communications system. The detection performance of the designed waveforms

and the performance of the communications system are investigated in [108]. Although it is shown that the optimization method proposed in [108] reduces the interference to the communications system, no interference limitation to the communications system or probability of false alarm constraints are considered in the optimization. In [123] algorithms for radar precoder design are proposed for coexistence with communications systems. The detection performance of the optimized radar waveforms is investigated for different probability of false alarm values. The precoder is designed with coexistence as a goal and does not take into account the false alarm probability. In [97, 98] a beamformer and a precoder respectively are designed for the communications system coexisting with a radar system. While in [97] the probability of detection is considered in the precoder design, in [98] the detection performance is investigated after designing the optimum beamformer that minimizes the interference at the radar. In [124] a precoder-decoder design based on interference alignment (IA) is proposed for a radar coexisting with a communications system. The design in [124] is extended in [125] for MIMO systems and its performance is tested using the derived GLRT for target detection and ML estimator for target direction. Also in [124, 125] the probability of detection or false alarm is not employed in the proposed precoder-decoder designs. The radar waveform optimization proposed in Publication V in a cooperative scenario maximizes the probability of detection under a constraint on maximum allowed false alarm probability and a constraint on the total transmitted radar power and the interference caused to the communications system. It is demonstrated in Publication V that cooperation between the two systems can improve the detection performance.

Not much effort addressing target parameter estimation in coexistence scenarios is found in the literature. Bounds on estimation rates for a radar system in a joint design with a coexisting communications system operating in the same spectral band are introduced in [126]. The available bandwidth is split between the radar and the communications systems, with a fraction of the bandwidth for shared use. As such, a joint system is considered in [126]. A different approach is considered in Publication VI, where, for systems in different locations, target time delay parameter estimation bound as well as a time delay estimator are derived. Radar waveform optimization for target time delay estimation in coexistence scenarios is addressed for the first time in Publication III. Similar to Publication VI, the coexisting systems are assumed to be in different locations and share the same spectral resources. The communications system shares an interference mask with the radar system, which ensures a minimum desired rate for the communications systems can be achieved.

Earlier works in the field of satellite positioning propose waveform optimization based on the CRB minimization, for example [63, 127, 128]. It is observed that without other constraints this is equivalent to maximizing the Gabor bandwidth of the signal, which is defined in [63] as:

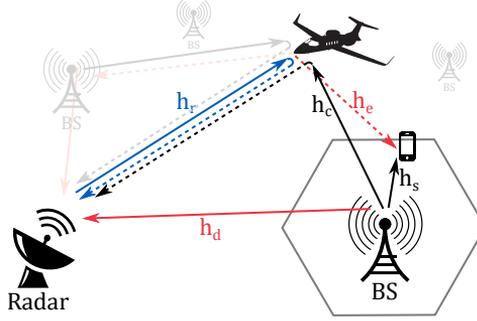$$\int_{-B}^{+B} f^2 |G(f)|^2 df. \tag{3.6}$$

Figure 3.2. System model for spectrum sharing among radar and communications systems. Active sensing done by the radar is illustrated using blue color, passive sensing is illustrated using black, while red is used for interference. Dashed lines symbolize reflections off the target. Signals from the communications BS act as interference, passive illumination as well as communications inside the cell. Multiple communications systems/BSs can be considered and these are illustrated using higher transparency.

Thus, the solution to the optimization problem allocates all the available energy on the edges of the frequency band. Such solution does not correspond to any realizable finite energy pulse [63]. A trade-off between allocating power based on channel gain or towards the edges of the available spectrum is shown for the optimization proposed in Publication III.

3.2.1 Information-Theoretic Optimization

MI based radar waveform optimization in a cooperative setting with wireless communications systems is considered in Publication IV also. This builds upon [77] and looks at various MI criteria to be maximized in a coexistence scenario and proposes radar waveform optimization problems based on such criteria. The considered cooperative scenario is illustrated in Fig. 3.2 and it consists of a monostatic radar and several communications BSs, either from the same system or from different systems. It is assumed both systems use OFDM waveforms occupying the same spectral band. Different than [77], in Publication IV it is considered that the communications signal reflected off the target arrives at the radar receiver. Consequently, the communications signals are received by the radar both reflected off the target and on a direct line of sight. The latter can be considered as interference or a reference copy of the communications signal that is not affected by the target channel. Similarly to [77], the solutions to the optimization problems in Publication IV are power allocations over the available subcarriers.

The reflected communications signal off the target can be considered as useful energy, as interference, or can be ignored altogether. In each of these cases a different MI criterion can be established for the radar waveform optimization. These are illustrated in the Venn diagram of Fig. 3.3, where it is assumed that there is some common information about the target in the channels h_r and h_c.
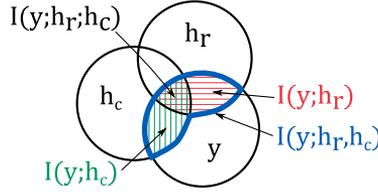
Figure 3.3. Venn diagram of information theoretic measures considered in Publication IV for three random variables $\mathbf{y}$ — the received signal at the radar, $\mathbf{h}_r$ — the target impulse response corresponding to the radar signal, and $\mathbf{h}_c$ — the target impulse response corresponding to the communications signal. The areas with the horizontal red stripes and vertical green stripes correspond to $I(\mathbf{y};\mathbf{h}_r)$ and $I(\mathbf{y};\mathbf{h}_c)$ respectively. The area with only the horizontal red stripes corresponds to $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_c)$, while the area with only the vertical green stripes corresponds to $I(\mathbf{y};\mathbf{h}_c|\mathbf{h}_r)$. The area marked by the thicker blue line corresponds to $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_c)$. Copyright 2016 IEEE.

Communications signal as useful energy. For the case when the reflected communications signal off the target is considered as useful energy, the objective function to be maximized is formulated as:

$$I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_c) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{h}_r,\mathbf{h}_c), \qquad (3.7)$$

where $\mathbf{y}$ is the vector corresponding to the signal at the radar receiver, $\mathbf{h}_r$ is the vector corresponding to the target impulse response experienced by the radar signal and $\mathbf{h}_c$ is the vector corresponding to the target impulse response experienced by the communications signal. In this case the MI between the received signal and jointly the two target impulse responses corresponding to the radar and the communications signals is maximized. The optimization problem is formulated as:

$$\max_{\{|r^F[k]|^2\}} \sum_{k=0}^{N-1} \log\left(1 + \frac{|r^F[k]|^2\sigma_{h_r}^2[k]+|c^F[k]|^2\sigma_{h_c}^2[k]}{|c^F[k]|^2\sigma_{h_d}^2[k]+\sigma_v^2[k]}\right)$$

$$\text{s.t.} \quad \log\left(1 + \frac{|c^F[k]|^2\sigma_{h_s}^2[k]}{r^F[k]|^2\sigma_{h_e}^2[k]+\sigma_v^2}\right) \geq t_k \qquad (3.8)$$

$$\sum_{k=0}^{N-1}|r^F[k]|^2 \leq P_{\text{T}},$$

where $|r^F[k]|^2$ and $|c^F[k]|^2$ are the power of the radar and the communications signals, respectively, for the kth subcarrier and $\sigma_{h_r}^2[k]$, $\sigma_{h_c}^2[k]$, $\sigma_{h_d}^2[k]$, $\sigma_{h_s}^2[k]$, $\sigma_{h_e}^2[k]$ and $\sigma_v^2[k]$ are the power of the corresponding channels and the noise and clutter, respectively, for the kth subcarrier. The SINR term in the objective shows that the communications signal reflected off the target is considered as useful energy, in contrast to (3.2) where it is ignored. This optimization problem is solved using the technique of Lagrange multipliers and a closed form power allocation solution is provided in Publication IV.

Communications signal as interference. For the case when the reflected communications signal off the target is considered as interference, the objective function to be maximized is formulated as:

$$I(\mathbf{y};\mathbf{h}_r) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{h}_r), \qquad (3.9)$$
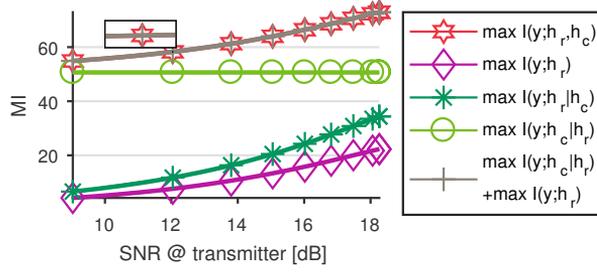
(a) Maximized $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_c)$, $I(\mathbf{y};\mathbf{h}_r)$, $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_c)$, $I(\mathbf{y};\mathbf{h}_c|\mathbf{h}_r)$ and the sum of $\max I(\mathbf{y};\mathbf{h}_c|\mathbf{h}_r)$ and $\max I(\mathbf{y};\mathbf{h}_r)$. It is observed that $\max I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_c) > \max I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_c) > \max I(\mathbf{y};\mathbf{h}_r)$.
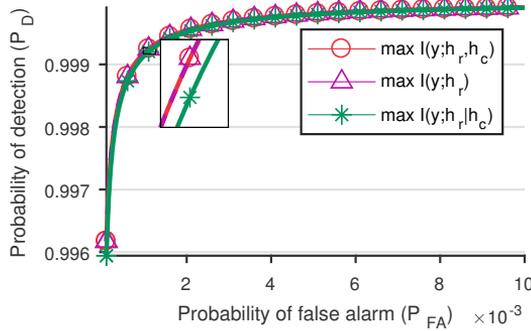
(b) ROC curves achieved with radar waveforms that maximize $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_c)$, $I(\mathbf{y};\mathbf{h}_r)$ or $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_c)$. The radar waveforms that maximize $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_c)$ or $I(\mathbf{y};\mathbf{h}_r)$ provide better detection performance.

Figure 3.4. Although $\max I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_c) > \max I(\mathbf{y};\mathbf{h}_r)$, a better detection performance is obtained when using the maximization of $I(\mathbf{y};\mathbf{h}_r)$ as the objective function. Thus, a larger maximized MI does not guarantee an optimal detection performance. Copyright 2016 IEEE.

which replaces the objective function in the optimization problem in (3.8) with

$$\sum_{k=0}^{N-1} \log\left(1 + \frac{|r^F[k]|^2 \sigma_{h_r}^2[k]}{|c^F[k]|^2 \sigma_{h_c}^2[k] + |c^F[k]|^2 \sigma_{h_d}^2[k] + \sigma_v^2[k]} \right). \tag{3.10}$$

The SINR term in the objective shows that the communications signal reflected off the target is considered as interference. A closed form solution to the optimization problem is provided in Publication IV.

Communications signal is ignored. This case corresponds to the one in (3.2) and presented in [77]. When the reflected communications signal off the target is not considered at the radar receiver (however, it is present), the objective function to be maximized is formulated as:

$$I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_c) = H(\mathbf{y}|\mathbf{h}_c) - H(\mathbf{y}|\mathbf{h}_r,\mathbf{h}_c), \tag{3.11}$$

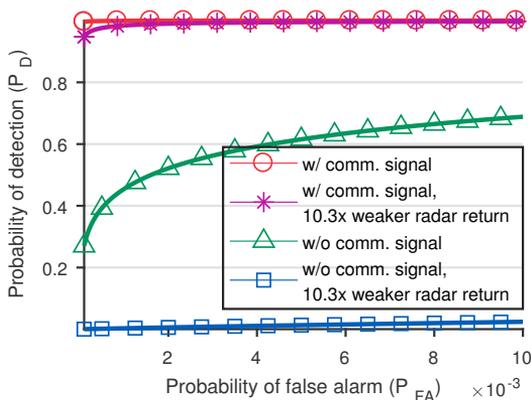which also replaces the objective function in the optimization problem in (3.8)

Figure 3.5. ROC curves achieved with waveforms that maximize $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_c)$ for different cases. The detection capability decreases considerably when the communications signals reflected off the target are not considered and it becomes much lower when the radar is also dealing with weak returns. Copyright 2016 IEEE.

with

$$\sum_{k=0}^{N-1} \log \left(1 + \frac{|r^F[k]|^2 \sigma_{h_r}^2[k]}{|c^F[k]|^2 \sigma_{h_d}^2[k] + \sigma_v^2[k]} \right). \tag{3.12}$$

The SINR term in the objective function shows that the scattering due to the communications signal is ignored. A closed form solution to this optimization problem, which is identical to the one obtained in [77], is provided in Publication IV.

The discrete values of the maximized MI for the considered criteria are shown in Fig. 3.4a. It is observed that $\max I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_c) > \max I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_c) > \max I(\mathbf{y};\mathbf{h}_r)$. This ordering is due to the fact that the maximized MIs are functions of the optimized radar waveform and the SINR. It is observed from the SINR term of the objective functions of (3.8), (3.10) and (3.12) that the reflected communications signal contributes to the signal part in (3.8), to the noise, clutter and interference part in (3.10) and to none of the terms in (3.12). Similar ordering can be established among the SINRs, which justifies the ordering of the maximized MIs.

Exploiting the communications signals reflected off the target at the radar receiver is demonstrated in Publication V to improve target detection in the case of a NP detector. It is also shown that, for example in the case of a low observable target, when the radar direction backscattered energy is weak, considering the communications signal in the optimization of the radar waveform can be beneficial for the detection task since it can take advantage of spatial diversity. The detection performance loss for a low observable target is shown in Fig. 3.5 when considering the communications signals and when not. The detection performance is greatly decreased when the communications signals are not considered in the radar waveform design.

3.2.2 Detection-Theoretic Optimization

One of the first works to consider the detection task in a cooperative setting with a communications system is the one in Publication V. Radar waveform optimization is proposed based on the probability of detection maximization for a maximum allowed false alarm probability. The detection performance of the proposed waveforms is compared against the detection performance of other waveforms optimized using MI. Also, the importance of exploiting the energy of the communications signals in a passive way is emphasized and demonstrated in simulation results.

The same system model presented in Fig. 3.2 is considered in Publication V. The target detector is based on the NP approach by posing a binary hypothesis testing problem using the following hypotheses:

$$
\begin{cases}
\mathcal{H}_0 : \mathbf{y} = \mathbf{y}_d + \mathbf{v} \\
\mathcal{H}_1 : \mathbf{y} = \mathbf{y}_r + \mathbf{y}_c + \mathbf{y}_d + \mathbf{v},
\end{cases}
\tag{3.13}
$$

where $\mathbf{y}_d$ is the vector containing the samples corresponding to the communications signals arriving through direct links to the radar receiver, $\mathbf{y}_r + \mathbf{y}_c$ is the vector containing the samples corresponding to the echoes off the target, due to both radar and communications signals, and $\mathbf{v}$ is the vector of noise and clutter samples.

For a zero mean i.i.d. Gaussian distributed data under both hypotheses, with covariance matrix $\mathbf{\Omega} + \sigma_v^2 \mathbf{I}$ for $\mathcal{H}_0$ and covariance matrix $\mathbf{\Sigma} + \sigma_v^2 \mathbf{I}$ for $\mathcal{H}_1$, the likelihood ratio test is written as:

$$
L(\mathbf{y}) = \frac{\frac{1}{\pi^N \det(\mathbf{\Sigma}+\sigma_v^2\mathbf{I})} \exp\left(-\mathbf{y}^H(\mathbf{\Sigma}+\sigma_v^2\mathbf{I})^{-1}\mathbf{y}\right)}{\frac{1}{\pi^N \det(\mathbf{\Omega}+\sigma_v^2\mathbf{I})} \exp\left(-\mathbf{y}^H(\mathbf{\Omega}+\sigma_v^2\mathbf{I})^{-1}\mathbf{y}\right)} \gtrless \eta.
\tag{3.14}
$$

The test statistic as well as the probability of false alarm P_{FA} and the probability of detection P_{D} can be obtained from (3.14) as in Publication V. The radar waveform optimization problem is stated as:

$$
\begin{aligned}
\max_{\{|r^F[k]|^2, \eta'\}} \quad & P_{\text{D}} \\
\text{s.t.} \quad & P_{\text{FA}} \leq \alpha \\
& \log\left(1 + \frac{\sigma_c^2[k]|h_s^F[k]|^2}{|r^F[k]|^2 \sigma_{h_e}^2[k] + \sigma_v^2}\right) \geq t_k, \ \forall k \\
& \sum_{k=0}^{N-1} |r^F[k]|^2 \leq P_{\text{T}},
\end{aligned}
\tag{3.15}
$$

which provides the radar waveform that maximizes the probability of detection P_{D} given a maximum allowed probability of false alarm α and the same constraints as in Publication IV on the interference to the communications system and maximum allowed transmitted power. The optimization problem in (3.15) is non-convex. However it may be solved numerically by employing an interior point method that finds several local minima and selects the smallest one. The
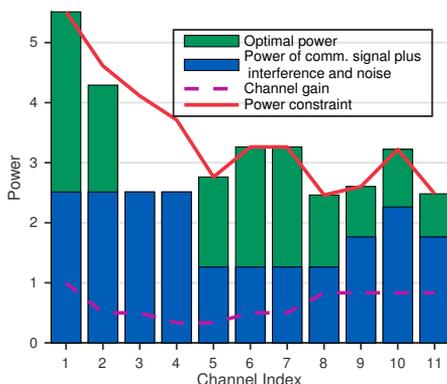
Figure 3.6. Opportunistic radar power allocation example for 11 channels. The radar power is shown with respect to the power of the communications signals reflected off the target plus the interference power of the communications signals arriving through a direct path at the radar and noise power. The radar power is allocated to the channels with a better gain, while satisfying the capacity constraint for the communications signal. Copyright 2015 IEEE.

obtained solution is essentially a power allocation across the different carriers. An example of optimized power allocation is illustrated in Fig. 3.6 for 11 sub-carriers and the maximum allowed P_{FA} of $\alpha = 0.01$. It can be observed that the radar power is allocated to the channels with a larger gain. Also the radar power in each channel satisfies the capacity constraint imposed by the communications system.

The detection performance of radar waveforms optimized based on MI in [77] and on NP detector in Publication V is compared. The power allocation solutions obtained by MI based optimization are employed in the derived NP detector and the probability of detection is obtained. The results in Publication V show that, as expected, the NP based solutions provide better detection probability in low SNR regime and for lower values of maximum allowed P_{FA} in comparison to the MI maximization based solutions.

The benefit of taking also the communications signal into account in target detection is demonstrated in Fig. 3.7. The receiver operating characteristic (ROC) curves for the NP based and MI based optimized radar waveforms respectively, with and without communications signals reflected off the target at the radar receiver are presented in Fig. 3.7. When considering the NP detector, the detection performance is greatly improved if the communications signals reflected off the target are exploited at the radar receiver. The same conclusion is valid when considering MI as a criteria for optimization.

3.2.3 Estimation-Theoretic Optimization

A method of estimating the target time delay in a cooperative radar and communications scenario is investigated in Publication VI. Similar to the detection task
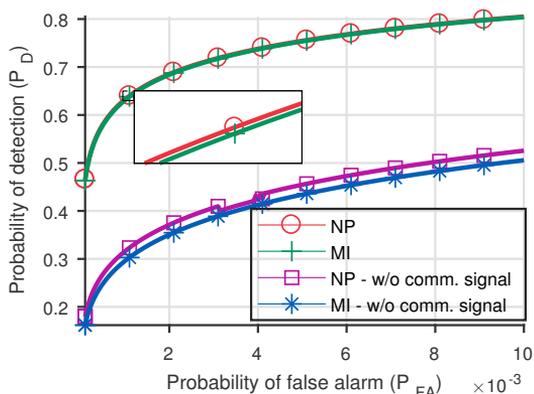
Figure 3.7. Receiver operating characteristic (ROC) curves for the NP based detector and MI maximization based detector for the two cases when the communications signal reflected off the target is considered at the radar receiver and when it is not considered. Only the upper left corner of a typical ROC plot is shown, for a practical P_{FA} range of 10^{-2} to 10^{-4}. The detector performs better when the communications signal reflection is considered, with a 8.5 dB and a 9.3 dB gains for the NP and MI cases respectively. Copyright 2015 IEEE.

considered in Publication V which can benefit from the communications signals reflected off the target, it is shown that the estimation performance can also benefit from the communications signals. Both CRB and RMSE of a proposed estimator are shown to improve in a cooperative scenario in Publication VI. It is desirable for the radar in a cooperative scenario to use an optimal waveform for the estimation task. Thus, in Publication III radar waveform optimization is proposed that minimizes the CRB for the target time delay estimation. The optimization is proposed subject to power constraints due to the interference caused to the communications system and maximum allowed emissions. Publication III is an extension of Publication VI in few regards. First, the target model considered in Publication III is an extended target model, which is a generalization of the point target model considered in Publication VI. As the considered extended target model is similar to a dense multipath channel, two cases are considered in Publication III: one where the channel tap amplitudes are known and another one where they need to be estimated. The channel amplitudes can be assumed known for example when dealing with a cognitive radar that is continuously learning about the target and spectrum environments [5]. Second, the independence assumption on the error terms in (3.17) considered in Publication VI is generalized in Publication III to a dependence assumption. These terms result from the processing steps employed for exploiting the communications signals reflected off the target. As expected, similar results are reported in Publications III and VI.

For the cooperation scenario considered in Publications III and VI the system model is presented in Fig. 3.8, which is based on the model in Fig. 3.2 with certain particularities. It is assumed that the radar is capable of beamforming
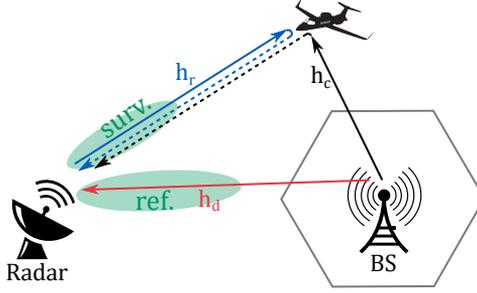
Figure 3.8. System model for spectrum sharing among radar and communications systems as considered in Publications III and VI. The radar is capable of beamforming on two directions or has two directive antennas aimed towards the target and the BS respectively.

in two directions, one required for target surveillance and one for receiving the reference communications signal along the direct path. Beamforming also rejects interferences from other angles. It is assumed that the communications system provides the radar with an interference mask aimed at guaranteeing a minimum data rate for its users. This is one way of cooperating between the radar and the communications system. Consequently, the radar receives the following signals:

$$
\begin{cases}
y_{\text{surv}}(t) = y_{\text{rad}}(t) + y_{\text{com}}(t) + n(t) \\
y_{\text{ref}}(t) = y_d(t) + n(t),
\end{cases}
\tag{3.16}
$$

where $y_{\text{surv}}(t)$ and $y_{\text{ref}}(t)$ are the received signals on the surveillance and reference channels respectively. In addition, $y_{\text{rad}}(t)$ is the radar target return, $y_{\text{com}}(t)$ is the communications signal reflected off the target and $y_d(t)$ is the reference communications signal arriving on the direct path, while $n(t)$ accounts for the receiver noise.

In order to exploit the common information about the target that the two received signals contain it is important to separate them. One way of achieving the separation is to use the technique of successive interference cancellation (SIC), as presented in [126, 129]. This technique involves removing the radar or the communications signal (the strongest one) from the observed signal in order to obtain the communications or the radar return, respectively, free of interference. This approach is applicable if the transmitted communications and radar signals are digital, with the radar symbols drawn from a finite alphabet, for example polyphase P3 and P4 codes. For analog modulated radar and communications signals, source separation techniques, such as blind source separation or independent component analysis (ICA) can be used instead, as long as the signals are statistically independent [130]. In Publications III and VI, it is assumed that SIC is used for separating the radar and communications signals since both of them use digital (discrete-time) waveforms.

Two different approaches are taken in Publications III and VI for the target model, however both consider frequency domain models. The frequency domain

measurement models for the separated radar and communications returns in Publication VI are given by:

$$\begin{cases} \mathbf{y}_r^F = \mu_r \mathbf{a}_r + \mathbf{v}_r^F \\ \mathbf{y}_c^F = \mu_c \mathbf{a}_c + \mathbf{v}_c^F, \end{cases} \tag{3.17}$$

where scalars μ_r and μ_c denote the scattering coefficients introduced by the target for the radar and communications signals respectively, vectors $\mathbf{a}_r$ and $\mathbf{a}_c$ contain the phase shifts due to the target delay for each subcarrier of the radar and the communications signals respectively, while vectors $\mathbf{v}_r^F$ and $\mathbf{v}_c^F$ contain the errors for the radar and communications measurements respectively. The elements of vectors $\mathbf{a}_r$ and $\mathbf{a}_c$ are given by $a_r[k] = \exp(-j2\pi k\Delta f \tau)$ and $a_c[k] = \exp(-j2\pi k\Delta f \tau/2)$ respectively.

The frequency domain measurement models for the separated radar and communications returns in Publication III are given by:

$$\begin{cases} \mathbf{y}_r^F = \mathbf{R}\mathbf{A}_r \mathbf{b}_r + \mathbf{v}_r^F \\ \mathbf{y}_c'^F = \mathbf{C}\mathbf{A}_c \mathbf{b}_c + \mathbf{v}_c'^F, \end{cases} \tag{3.18}$$

where $\mathbf{R} = \mathrm{diag}\{\mathbf{r}^F\}$ and $\mathbf{C} = \mathrm{diag}\{\mathbf{c}^F\}$ are $N \times N$ diagonal matrices containing the frequency domain radar and communications transmitted symbols respectively on the main diagonal. The elements of matrices $\mathbf{A}_r$ and $\mathbf{A}_c$ of size $N \times L$ are given by $[\mathbf{A}_r]_{k,l} = \exp(-j2\pi k\Delta f(\tau + lT_s))$ and $[\mathbf{A}_c]_{k,l} = \exp(-j2\pi k\Delta f(\tau/2 + lT_s))$ respectively, for $k = -N/2,\dots,N/2-1$ and $l = 0,\dots,L-1$. The measurements in both equation (3.17) and (3.18) can be used to build the measurements vector

$$\mathbf{y}^F = \begin{bmatrix} \mathbf{y}_r^F \\ \mathbf{y}_c^F \end{bmatrix}, \tag{3.19}$$

which is used to compute the FI and CRB as presented in Publications III and VI. An example of plots for the square root of CRB, as presented in Publication III, is illustrated in Fig. 3.9. It is observed that in the cooperative case, the CRB is always lower, both when target channel amplitudes are assumed known and when they need to be estimated. The strength of the communications signal reflected off the target impacts the improvement obtained in the cooperative case. The better the communications signal quality, the smaller the CRB is. Nevertheless, the greatest improvement is obtained when the SNR for the radar signal is smaller than the one for the communications signal. Lowering the CRB in the cooperative scenario suggests the fact that exploiting the communications signal reflected off the target can improve the estimation performance. This is shown to be true in both Publications III and VI, for optimized and unoptimized radar waveforms respectively.

A maximum likelihood estimator (MLE) for target time delay in cooperative scenarios is proposed in Publication VI. The proposed estimator exploits also the communications signal received at the radar receiver in a passive way. Given
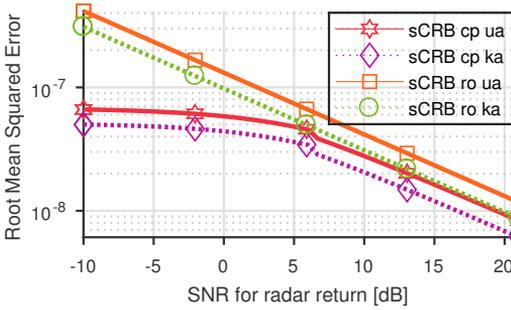
Figure 3.9. Square root of CRB for the cooperative and the radar-only cases with and without known amplitudes. For the cooperative case it is considered the $\text{SNR}_{\text{com}} = 6$ dB, which is visible on the CRB plots for the cooperative case as the point where SIC removes first the communications and then the radar return. As the CRBs are lower, it is always beneficial to use cooperation and the communications signal. Also, when the channel amplitudes are estimated, the estimation performance is degraded compared to the case when the channel amplitudes are known.

the likelihood of the measurements $L(\mathbf{y}^F; \tau)$, the MLE for τ can be obtained as:

$$\hat{\tau}_{ML} = \arg\max_{\tau} L(\mathbf{y}^F; \tau), \tag{3.20}$$

however it is not trivial. Consequently, a simple approach is proposed in Publication VI for finding the value of τ, which essentially is a discrete grid search. A clear disadvantage of such estimation method is that its performance is limited by the search resolution. In practice, finding the parameter of interest τ, as presented in Publication VI, can be done using a bank of two matched filters on the measurements $\mathbf{y}_r^F$ and $\mathbf{y}_c^F$. A similar estimator based on a discretized delay is employed also in Publication III for the evaluation of the estimation performance using the optimized waveforms.

It is demonstrated in Publication VI that by exploiting the communications signal in a passive way at the radar receiver the CRB can be considerably lowered. This happens in a regime where the SNR of the received radar signal is smaller than the SNR of the received communications signal. In this cooperative scenario, however, the CRB is generally found to be lower than the CRB for the case when the radar operates alone. Thus, an estimator that can take advantage of the information available in the communications signal reflected off the target at the radar receiver can outperform one that does not.

Optimizing the radar waveform based on CRB minimization is proposed in Publication III. The optimization is performed with constraints on the total power transmitted by the radar, as well as an interference power mask, which is imposed by the communications system. The interference mask is provided at the radar by the communications system under the cooperative framework. The optimization problems presented in Publication III are non-convex due to the objective function, however a local solution can be obtained by solving the problem numerically using interior point methods. The solutions to the optimization problems are power allocations over the subcarriers. An example
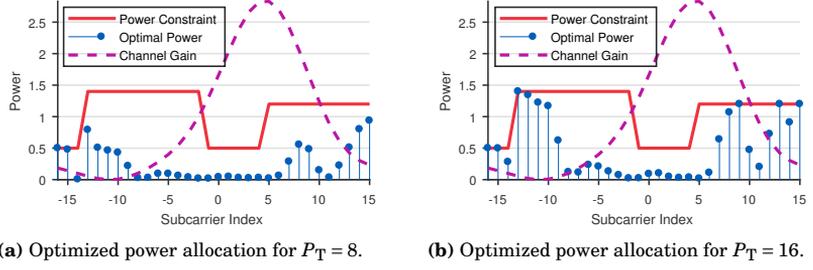
(a) Optimized power allocation for $P_T = 8$.

(b) Optimized power allocation for $P_T = 16$.

Figure 3.10. Optimized power allocations using fmincon for the cooperative case with unknown channel amplitudes. The trade-off between allocating power based on channel gain or subcarrier index is observed.
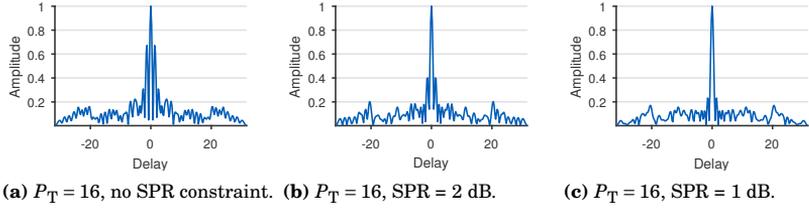

(a) $P_T = 16$, no SPR constraint.

(b) $P_T = 16$, SPR = 2 dB.

(c) $P_T = 16$, SPR = 1 dB.

Figure 3.11. Autocorrelation function for different SPR constraints. It is observed that the SPR constraint can decrease the ambiguities in the delay domain.

of optimized power allocation over subcarriers is illustrated in Fig. 3.10 for different constraints on the total transmitted radar power. The trade-off between allocating power based on channel gain and edge of the available spectrum, or subcarrier index for the considered OFDM waveform, is observed.

It is known that such waveforms, optimized based on the CRB, suffer from high ambiguities. Consequently, a constraint on the SPR is proposed in Publication III, which reduces the ambiguities. The SPR is a measure of how much the power of one subcarrier deviates from the average subcarrier power. Naturally, this constraint forces a more uniform power distribution over the available subcarriers and, in consequence, reduces delay domain ambiguities of the optimized waveforms. This is illustrated in Fig. 3.11 for different SPR values.

The estimation performance of the optimized waveforms is investigated in Publication III and shown to be improved over the case when the radar operates alone as well as over the case when the radar waveform is optimized for receiver SNR maximization. These results are presented in Fig. 3.12. It is observed that the optimized waveform greatly improves the target delay estimation performance, despite the local nature of the solution to the optimization problem. Also, it is once more demonstrated the benefit of exploiting the communications signal reflected off the target for target parameter estimation at the radar receiver.
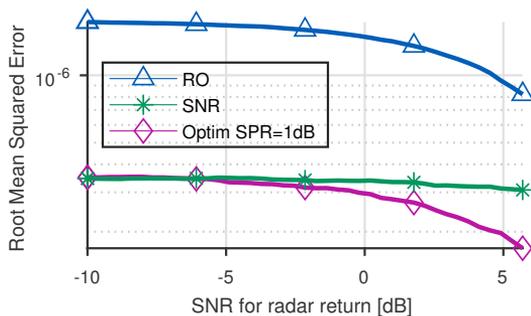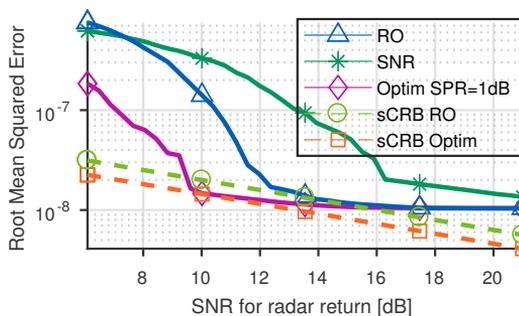
(a) RMSE for $SNR_{rad} < SNR_{com} = 6$ dB.

(b) RMSE for $SNR_{rad} > SNR_{com} = 6$ dB.

Figure 3.12. RMSE of the waveforms optimized in the case of unknown channel amplitudes with an SPR constraint (1 dB). It is observed that the optimization improves the estimation performance in both low and high SNR regime for the radar return. The limitation of the implemented estimator is also shown in the high SNR regime, where the RMSE becomes flat.

3.3 Discussion

The ever growing demand for spectrum has pushed the research community to find ways for meeting such demands. Spectral sharing among radar and communications systems is proposed as a way of solving the spectral congestion in scenarios where different radio systems need to coexist in the same frequency bands. The methods for spectral coexistence among radar and communications systems presented in this thesis are applicable to other systems that are sharing radio spectrum as well.

A key enabling method for the coexistence among radar and communications systems is waveform optimization and adaptation. Three categories are identified:

- radar-centric, when the radar waveform alone is optimized
- communications-centric, when the communications waveform alone is optimized
- co-design, when both waveforms are optimized jointly.

The third category can be considered as a future scenario where the two systems

converge in a single dynamic one that is able to perform multiple tasks. The first two categories include solutions which more or less can be applied to the current wireless systems such as 4G, 5G, WiFi and citizens broadband radio service (CBRS) as well as new emerging systems such as cognitive radios and radars. Within these categories many approaches are identified, depending on the spatial configurations of the two systems, MIMO or SISO configurations, dual-use waveforms, precoder design and waveform optimization. Different radar tasks are also considered in the literature: detection, estimation or target tracking. Another important aspect is the relationship between the two systems. The existing literature can be divided into four categories:

- passive, when the radar system exploits the transmissions of the communications systems
- competitive, when the two systems compete for the available resources
- coexisting, when either system takes into account the existence of the other one
- cooperative, when the two systems exchange certain information for their mutual benefit.

In the cooperative case, the systems know waveforms such as pilot signals or information that can help with forward error correction. Also, these can share CSI, the achieved communications rate or interference masks that tell how much interference can be tolerated for example.

In this thesis, contributions to coexistence among radar and communications systems were presented. The considered configuration is a distributed one, in which the radar and the communications systems are in different locations. It is assumed that both systems utilize SISO configurations and multicarrier waveforms such as OFDM, occupying the same spectral band. Also, it is assumed that the two systems are in a cooperative relationship and share information. For example, the communications system provides the radar with an interference mask, which tells the radar how much power it can use in each subcarrier so that it does not cause too much interference to the communications users. The communications users wish to maintain their minimum desired data rate despite the spectrum sharing. A radar-centric approach is proposed to the coexistence problem, where the radar waveform is optimized using different objectives and constraints on the total transmitted power and the interference caused to the communications system.

It can be observed in the contributions made by this thesis that the radar system can benefit from the cooperation with the communications system and improve its estimation or detection performance. This is conditioned on the radar being cognitive and sensing the environment or cooperation with the communications system. Nevertheless, the cognitive function of the radar plays an important role in the optimization methods presented in this thesis, which allows the radar to optimize and adjust the waveforms for the target and radio spectrum environment in which it operates.

MI between the received signal at the radar and target impulse response

corresponding to the radar or the communications signal is one objective which is proposed in this thesis for the optimization of the radar waveforms. The MI criteria which provide optimal radar waveforms with improved detection performance are introduced. Using simulation results it is demonstrated that a larger maximized MI does not guarantee an optimal detection performance in the NP sense where detection probability is maximized under false alarm level constraint. Also, it is shown that the detection performance can be improved by exploiting the communications signals reflected off the target.

For the target detection task, radar waveform optimization based on the NP detector is proposed. Although the optimization problem is not convex it can be solved numerically and it is shown that the solution provides improved detection performance compared to optimal waveforms in the MI sense. It is observed that the improvement is more significant in low SNR regime and when a lower maximum allowed false alarm probability level is required. Also for the NP based waveform optimization it is shown that the detection performance benefits from the energy provided by the communications system in a passive manner, similarly to the MI based optimization.

Radar waveform design that minimizes the CRB for the target time delay estimation is proposed for cooperative scenarios. This is motivated by the additivity property of the FI. Analytical derivations for the FI and CRB are provided. Such optimization is considered for the first time in this setting and the trade-off between allocating power towards the edge of the available spectrum and based on channel gain is pointed out. It is shown that by exploiting the communications signals in a passive manner the estimation performance can be improved over the radar-only case. Also, the RMSE of the estimation performed with the optimized waveforms is demonstrated to be lower than the one corresponding to a waveform optimized for receiver SNR maximization.

One limitation of the contributions provided in this thesis is the considered radar-centric approach. It is thus assumed that only the radar waveform can be optimized, while the communications waveform remains unaltered, however achieves the desired minimum rate. The potential benefits for the cooperative radar and communications systems scenarios is much larger if a joint design approach is considered. Both theoretical bounds and actual estimation errors could be considerably lowered by optimizing and adapting also the communications signal. The choice of such approach is justified by the difficulty in impacting standardization and introducing changes that may cause marginal capacity losses to a profit driven industry. Regulatory authorities can however impose such requirements eventually, however meanwhile the contributions of this thesis can be considered as new enabling technologies and valuable intermediate steps in spectrum sharing and cognitive radar and radio systems.

4. Conclusions

Future radar systems need to be fully adaptive and cognitive and take advantage of all available degrees of freedom for optimizing their performance. For such demanding task, multicarrier waveforms provide a convenient and flexible way of generating and designing waveforms. Moreover, many of the technical solutions and transceiver structures developed for multicarrier communications systems can be utilized in radars as well. Radio spectrum is becoming more and more scarce and a valuable resource. Proposals for more flexible and efficient use of spectral resources as well as changes in regulation have been made already [3, 4, 67]. Spectral sharing and coexistence among radar and communications systems is one approach for solving the problem of congested spectrum use and inefficiency caused by rigid regulation of spectrum use. This requires that both radar and communications systems are adaptive and can take advantage of their operational environment knowledge such as the state of the radio spectrum. Multicarrier waveforms allow the radar systems to fulfill such requirements by using the available degrees of freedom in frequency and code domains in a flexible manner. Moreover, a convenient generalized multicarrier model that allows for representing most of the commonly known radar waveforms as well as designing and optimizing new waveforms is needed.

4.1 Multicarrier Radar Waveforms

In this thesis, a GMR waveform model is proposed. It is used to optimize radar waveforms based on spectral awareness as well as for designing new waveforms with desirable properties for different radar tasks. The flexibility of the proposed GMR model and multicarrier radar waveforms is first demonstrated and then multicarrier waveforms are employed in the context of coexistence among radars and communications systems. The choice of multicarrier waveforms for the radar systems is justified not only by the available degrees of freedom but also by the fact that modern communications systems utilize similar waveforms. Therefore, similar transceiver structures may be used for both purposes.

This thesis is comprised of six publications which contribute towards future

radar systems that are fully adaptive and cognitive. A GMR model is proposed which is able to describe and generate the most common single and multicarrier radar waveforms in a convenient way. The model provides a unifying framework for multicarrier radars which also allows for designing new multicarrier waveforms as well as adjusting waveforms to different radar spectrum and target scenarios. Based on the well-known OFDM and MCPC waveforms a relaxation of the subcarrier orthogonality constraint is proposed and the novel TDRW and FDRW are introduced. The orthogonality of the subcarriers is anyway lost at the receiver due to the various delays and Doppler shifts that a target can introduce. There exists no radar waveform that would be orthogonal with all delays and Doppler shifts. The relaxation is introduced with the benefit of improved AF properties and consequently better radar performance. It is shown that in general this allows for lower PSLLs in both delay and Doppler domains and also improves delay resolution in comparison to the MCPC waveforms for example. Besides generating and describing radar waveforms, the GMR model is also capable of exploiting the degrees of freedom that the multicarrier waveforms provide in a flexible manner. For example, waveforms employing FFH patterns are generated by filling in the matrices of the GMR model appropriately and their performance is evaluated using the AF. Also, frequency diversity may be conveniently exploited using the GMR model for avoiding and mitigating jamming and unintentional interference. If one or several subcarriers experience such effects these would not be allocated any power. A comparison between TDRWs and MCPC waveforms is performed in such scenario. It is shown that the former performs better in terms of the PSLLs in both delay and Doppler domains for various code lengths.

The proposed model is written using a compact matrix formulation which allows for easy waveform customization and generation just by filling in the matrices of the model appropriately. It is also shown that it can be used for radar waveform optimization and examples for OFDM and MCPC waveforms are provided. The proposed constrained optimization problems employ an information theoretic MI criterion and allow the radar to adapt to target characteristics, propagation environment and state of the spectrum. Consequently, the proposed GMR model can be considered an important enabler for future cognitive and fully adaptive radar systems.

4.2   Coexistence among Radar and Communications Systems

The spectral coexistence among radars and communications systems accounts for a significant portion of this thesis. The multicarrier radar model is particularly suitable for coexistence scenarios that require flexible use of spectrum and adaptation to different target scenarios and operational environments. In this thesis, the problems of waveform optimization and adaptation are addressed using multicarrier waveforms, in particular OFDM. Several radar waveform

optimization criteria are proposed in coexistence scenarios: MI maximization, probability of detection maximization under probability of false alarm constraint (NP sense) and CRB minimization. The waveforms are optimized for different radar tasks, i.e. target characterization and detection and target parameter estimation. The optimization problems proposed in this thesis are formulated by imposing constraints on the total transmitted radar power and an interference mask provided by the communications system. This interference mask tells the radar how much power it can allocate to certain subcarriers such that a tolerable interference level is experienced by the communications system and consequently, a desired rate can be achieved. Also, the coexistence scenarios considered in this thesis assume some degree of cooperation among the two systems, for example through the exchange of an interference mask. In the considered setup, the MI objective is a concave function of the transmitted radar powers on each subcarrier. Adding the linear constraint on the total transmitted radar power results in a convex problem, which can be solved using its Lagrangian form and the KKT conditions [66]. The solutions to the optimization problems represent power allocations over the available subcarriers. Some of these solutions can be obtained in a closed form, while other require numerical optimization.

The performance of the proposed optimized waveforms in this thesis is evaluated in simulations using ROC or RMSE plots, for the detection and estimation tasks respectively. Recommendations on which optimization objective functions produce the best performing waveforms are given. It is demonstrated that exploiting the communications signal reflected off the target can improve the radar performance for all considered tasks.

This thesis provides theory, methods and justifications for spectrum sharing among radar and communications systems. The methods proposed in this thesis are steps towards future fully adaptive radars where both the transmitter and receiver can be adapted and cognitive radars that learn and exploit knowledge of their operational environments. There is also more immediate need for spectrum sharing because many wireless systems such as 5G, WiFi and 4G evolution and CBRS occupy the same frequency bands as S-band radars, for example. As such, spectrum sharing is proven to be possible with little to no change in the existing radar and communications systems, with the extra benefit of improved performance.

4.3 Future Work

Few objective functions have been employed for the radar waveform optimization problems presented in this thesis. The use of other information-theoretic measures can be explored as well. For example, the KL divergence, J-divergence, as well as Bhattacharyya distance between the two densities under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$.

It would be interesting to employ the methods proposed in this thesis for a co-design approach, where also the communications waveform is optimized and adjusted to local channel and interference conditions. With this design approach there is potential for further performance improvement in both systems. Also, different configurations for the distributed configuration considered in this thesis can be examined. For example, the BS is able to receive the radar signals reflected off the target and help the monostatic radar. If digital waveforms are used the BS can employ SIC to separate the radar signal from the communications signals as described in [129] or Publication III.

Another improvement to the efforts presented in this thesis is addressing the performance of the communications system directly. It is assumed in this thesis that a minimum data rate for the communications users is guaranteed, by limiting the interference from the radar and ensuring sufficiently high SINR at the communications receiver. However, the actual performance of the communications users is not investigated in the proposed scenarios.

More realistic scenarios and assumptions can be employed. The idealistic nature of operational environments and scenarios employed in this thesis serves the purpose of demonstrating the potential benefit of spectrum sharing, as well as the possibility for coexistence in itself. However, real world radio and target environments may be significantly more complicated and change rapidly.

A significant future contribution to the research presented in this thesis is to consider a dynamic target and environment. Doppler effect increases the modeling complexity and can also be a parameter of interest for the radar. Another interesting future research topic is addressing the target tracking task. The added complexity due to constantly changing environment and target state make this particularly interesting, also from a cognitive, fully adaptive radar point of view. New cooperation methods need to be developed in such scenarios for the mutual benefit of the systems sharing the spectrum.

The automotive industry will likely be revolutionized by autonomous vehicles [131]. When legislation regarding such vehicles will be in place there will be a significant number of vehicles equipped with one or several radars, communications devices as well as numerous other sensors. These sensors are responsible for example with the collection of information on weather conditions, blind spot detection, traffic light advisories, parking assistance, optimized cruise control, lane change and access to vehicular wireless networks among others [132]. All these sensors will be required to share the spectrum and cooperate for their mutual benefit and avoid the potential high level of interference. More research efforts in accommodating such a large number of active devices and cooperation methods are needed.

The health industry is another field where radars are experiencing an increased use. Few examples are vital signs monitoring [133–135] and detection in emergency situations [136, 137]. As these medical and consumer monitoring devices based on radar technology become prevalent, spectrum sharing techniques for such applications are also needed.

References

[1] N. Levanon, "Multifrequency radar signals," in *Record of the IEEE 2000 International Radar Conference*, 2000, pp. 683–688.

[2] N. Levanon and E. Mozeson, *Radar Signals*. John Wiley & Sons, Inc., 2004, pages 327–372.

[3] Federal Communications Commission (FCC), "FCC Proposes to Enable Innovative Small Cell Use of Spectrum in the 3.5 GHz Band," December 2012. [Online]. Available: https://www.fcc.gov/document/fcc-proposes-innovative-small-cell-use-35-ghz-band

[4] ——, "FCC Puts Final Rules in Place for New Citizens Broadband Radio Service," April 2016. [Online]. Available: https://www.fcc.gov/document/fcc-proposes-innovative-small-cell-use-35-ghz-band

[5] S. Haykin, "Cognitive radar: a way of the future," *IEEE Signal Processing Magazine*, vol. 23, no. 1, pp. 30–40, January 2006.

[6] J. Guerci, *Cognitive Radar: The Knowledge-Aided Fully Adaptive Approach*, ser. Artech House radar library. Artech House Publishers, 2010.

[7] E. Fishler, A. Haimovich, R. Blum, D. Chizhik, L. Cimini, and R. Valenzuela, "MIMO radar: an idea whose time has come," in *Proceedings of the 2004 IEEE Radar Conference*, April 2004, pp. 71–78.

[8] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik, and R. A. Valenzuela, "Spatial diversity in radars-models and detection performance," *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 823–838, March 2006.

[9] C. Y. Chen and P. P. Vaidyanathan, "MIMO Radar Waveform Optimization With Prior Information of the Extended Target and Clutter," *IEEE Transactions on Signal Processing*, vol. 57, no. 9, pp. 3533–3544, September 2009.

[10] H. Godrich, A. P. Petropulu, and H. V. Poor, "Power Allocation Strategies for Target Localization in Distributed Multiple-Radar Architectures," *IEEE Transactions on Signal Processing*, vol. 59, no. 7, pp. 3226–3240, July 2011.

[11] G. Porter, "Error distribution and diversity performance of a frequency-differential PSK HF modem," *IEEE Transactions on Communication Technology*, vol. 16, no. 4, pp. 567–575, August 1968.

[12] M. Zimmerman and A. Kirsch, "The AN/GSC-10 (KATHRYN) variable rate data modem for HF radio," *IEEE Transactions on Communication Technology*, vol. 15, no. 2, pp. 197–204, April 1967.

[13] J. Cimini, L., "Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiplexing," *IEEE Transactions on Communications*, vol. 33, no. 7, pp. 665–675, July 1985.

[14] T. Starr, T. Starr, J. Cioffi, and P. Silverman, *Understanding Digital Subscriber Line Technology*, ser. Prentice Hall communications engineering and emerging technologies series.   Prentice Hall PTR, 1999.

[15] H. Sari and G. Karam, "Orthogonal frequency-division multiple access and its application to CATV networks," *IEEE Transactions on Consumer Electronics*, vol. 9, no. 6, pp. 507–516, November-December 1998.

[16] K. Taura, M. Tsujishita, M. Takeda, H. Kato, M. Ishida, and Y. Ishida, "A digital audio broadcasting (DAB) receiver," *IEEE Transactions on Consumer Electronics*, vol. 42, no. 3, pp. 322–327, August 1996.

[17] B. Crow, I. Widjaja, L. Kim, and P. Sakai, "IEEE 802.11 wireless local area networks," *IEEE Communications Magazine*, vol. 35, no. 9, pp. 116–126, September 1997.

[18] R. van Nee, G. Awater, M. Morikura, H. Takanashi, M. Webster, and K. Halford, "New high-rate wireless LAN standards," *IEEE Communications Magazine*, vol. 37, no. 12, pp. 82–88, December 1999.

[19] A. Ghosh, D. Wolter, J. Andrews, and R. Chen, "Broadband wireless access with WiMax/802.16: current performance benchmarks and future potential," *IEEE Communications Magazine*, vol. 43, no. 2, pp. 129–136, February 2005.

[20] H. Holma and A. Toskala, *LTE for UMTS - OFDMA and SC-FDMA Based Radio Access*.   Wiley, 2009.

[21] Z. Wang and G. Giannakis, "Wireless multicarrier communications," *IEEE Signal Processing Magazine*, vol. 17, no. 3, pp. 29–48, May 2000.

[22] J. S. Chow, J. C. Tu, and J. M. Cioffi, "A discrete multitone transceiver system for HDSL applications," *IEEE Journal on Selected Areas in Communications*, vol. 9, no. 6, pp. 895–908, August 1991.

[23] H. Dai and H. V. Poor, "Advanced signal processing for power line communications," *IEEE Communications Magazine*, vol. 41, no. 5, pp. 100–107, May 2003.

[24] K. Fazel and S. Kaiser, *Multi-Carrier and Spread Spectrum Systems: From OFDM and MC-CDMA to LTE and WiMAX*.   Wiley, 2008, pages 55–100.

[25] N. Levanon, "Multifrequency complementary phase-coded radar signal," *IEE Proceedings - Radar, Sonar and Navigation*, vol. 147, no. 6, pp. 276–284, December 2000.

[26] N. Levanon and E. Mozeson, "Multicarrier radar signal - pulse train and CW," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 38, no. 2, pp. 707–720, April 2002.

[27] E. Mozeson and N. Levanon, "Multicarrier radar signals with low peak-to-mean envelope power ratio," *IEE Proceedings - Radar, Sonar and Navigation*, vol. 150, no. 2, pp. 71–77, April 2003.

[28] G. Lellouch, A. K. Mishra, and M. Inggs, "Stepped OFDM Radar Technique to Resolve Range and Doppler Simultaneously," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 2, pp. 937–950, April 2015.

[29] R. F. Tigrek, W. J. A. D. Heij, and P. V. Genderen, "OFDM Signals as the Radar Waveform to Solve Doppler Ambiguity," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 1, pp. 130–143, January 2012.

[30] G. Lellouch, P. Tran, R. Pribic, and P. van Genderen, "OFDM waveforms for frequency agility and opportunities for Doppler processing in radar," in *Proceedings of IEEE Radar Conference*, May 2008, pp. 1–6.

[31] M. Golay, "Complementary series," *IRE Transactions on Information Theory*, vol. 7, no. 2, pp. 82–87, April 1961.

[32] R. Tigrek, W. de Heij, and P. van Genderen, "Solving Doppler Ambiguity by Doppler Sensitive Pulse Compression Using Multi-Carrier Waveform," in *Proceedings of European Radar Conference*, October 2008, pp. 72–75.

[33] M. Braun, C. Sturm, and F. Jondral, "Maximum likelihood speed and distance estimation for OFDM radar," in *Proceedings of IEEE Radar Conference*, May 2010, pp. 256–261.

[34] D. Poullin, "Passive detection using digital broadcasters (DAB, DVB) with COFDM modulation," *IEE Proceedings - Radar, Sonar and Navigation*, vol. 152, no. 3, pp. 143–152, June 2005.

[35] R. Mohseni, A. Sheikhi, and M. A. M. Shirazi, "Constant envelope OFDM signals for radar applications," in *2008 IEEE Radar Conference (RadarConf)*, May 2008, pp. 1–5.

[36] M. A. Sebt, A. Sheikhi, and M. M. Nayebi, "Orthogonal frequency-division multiplexing radar signal design with optimised ambiguity function and low peak-to-average power ratio," *IET Radar, Sonar Navigation*, vol. 3, no. 2, pp. 122–132, April 2009.

[37] G. Lellouch, R. Pribic, and P. van Genderen, "Wideband OFDM Pulse Burst and its Capabilities for the Doppler Processing in Radar," in *Proceedings of International Conference on Radar*, September 2008, pp. 531–535.

[38] G. Lellouch, A. Mishra, and M. Inggs, "Impact of the Doppler modulation on the range and Doppler processing in OFDM radar," in *2014 IEEE Radar Conference*, May 2014, pp. 0803–0808.

[39] G. Lellouch, A. K. Mishra, and M. Inggs, "Design of OFDM radar pulses using genetic algorithm based techniques," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 52, no. 4, pp. 1953–1966, August 2016.

[40] ——, "Orthogonal frequency division multiplexing phenomenology: radar technique combining genetic algorithm-based pulse design and energy detector for target recognition," *IET Radar, Sonar Navigation*, vol. 10, no. 5, pp. 912–922, 2016.

[41] C. R. Berger, S. Zhou, P. Willett, B. Demissie, and J. Heckenbach, "Compressed sensing for OFDM/MIMO radar," in *2008 42nd Asilomar Conference on Signals, Systems and Computers*, October 2008, pp. 213–217.

[42] C. Berger, B. Demissie, J. Heckenbach, P. Willett, and S. Zhou, "Signal processing for passive radar using OFDM waveforms," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 1, pp. 226–238, February 2010.

[43] G. Lellouch and H. Nikookar, "On the Capability of a Radar Network to Support Communications," in *2007 14th IEEE Symposium on Communications and Vehicular Technology in the Benelux*, November 2007, pp. 1–5.

[44] D. Garmatyuk, J. Schuerger, Y. T. Morton, K. Binns, M. Durbin, and J. Kimani, "Feasibility study of a multi-carrier dual-use imaging radar and communication system," in *2007 European Microwave Conference*, October 2007, pp. 1473–1476.

[45] Tigrek, R.F., "A Processing Technique for OFDM-Modulated Wideband Radar Signals," November 2010. [Online]. Available: http://repository.tudelft.nl/islandora/object/uuid:5127253d-2acf-47b0-b4ec-d23ce5ad8267?collection=research

[46] D. Garmatyuk and J. Schuerger, "Conceptual design of a dual-use radar/communication system based on OFDM," in *MILCOM 2008 - 2008 IEEE Military Communications Conference*, November 2008, pp. 1–7.

[47] D. Garmatyuk, J. Schuerger, K. Kauffman, and S. Spalding, "Wideband OFDM system for radar and communications," in *2009 IEEE Radar Conference (Radar-Conf)*, May 2009, pp. 1–6.

[48] S. Sen and A. Nehorai, "OFDM MIMO radar with mutual-information waveform design for low-grazing angle tracking," *IEEE Transactions on Signal Processing*, vol. 58, no. 6, pp. 3152–3162, June 2010.

[49] R. F. Tigrek and P. V. Genderen, "Compensation of Range Migration for Cyclically Repetitive Doppler-Sensitive Waveform (OFDM)," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 4, pp. 2118–2123, October 2010.

[50] G. Lellouch, A. K. Mishra, and M. Inggs, "Orthogonal frequency division multiplexing phenomenology: recognition of canonical scatterers using flat spectra OFDM pulses," *IET Radar, Sonar Navigation*, vol. 10, no. 4, pp. 647–654, 2016.

[51] Y. Paichard, "Orthogonal multicarrier phased coded signal for netted radar systems," in *2009 International Waveform Diversity and Design Conference*, February 2009, pp. 234–236.

[52] S. Sen and A. Nehorai, "Adaptive design of OFDM radar signal with improved wideband ambiguity function," *IEEE Transactions on Signal Processing*, vol. 58, no. 2, pp. 928–933, February 2010.

[53] G. Lellouch, R. Pribic, and P. van Genderen, "Merging frequency agile OFDM waveforms and Compressive Sensing into a novel radar concept," in *2009 European Radar Conference (EuRAD)*, September 2009, pp. 137–140.

[54] R. Mohseni, A. Sheikhi, and M. Masnadi Shirazi, "Compression of Multicarrier Phase-Coded radar signals with low sampling rate," in *Proceedings of International Conference on Radar*, September 2008, pp. 718–721.

[55] S. Sen and A. Nehorai, "Sparsity-Based Multi-Target Tracking Using OFDM Radar," *IEEE Transactions on Signal Processing*, vol. 59, no. 4, pp. 1902–1906, April 2011.

[56] ——, "Adaptive OFDM Radar for Target Detection in Multipath Scenarios," *IEEE Transactions on Signal Processing*, vol. 59, no. 1, pp. 78–90, January 2011.

[57] S. Sen, G. Tang, and A. Nehorai, "Multiobjective Optimization of OFDM Radar Waveform for Target Detection," *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 639–652, February 2011.

[58] S. Golomb, "Two-valued sequences with perfect periodic autocorrelation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, no. 2, pp. 383–386, April 1992.

[59] E. Van der Ouderaa, J. Schoukens, and J. Renneboog, "Peak factor minimization using a time-frequency domain swapping algorithm," *IEEE Transactions on Instrumentation and Measurement*, vol. 37, no. 1, pp. 145–147, March 1988.

[60] M. Friese, "Multitone signals with low crest factor," *IEEE Transactions on Communications*, vol. 45, no. 10, pp. 1338–1344, October 1997.

[61] C. Sturm, T. Zwick, and W. Wiesbeck, "An OFDM System Concept for Joint Radar and Communications Operations," in *Proceedings of 69th IEEE Vehicular Technology Conference*, April 2009, pp. 1–5.

[62] C. Sturm, T. Zwick, W. Wiesbeck, and M. Braun, "Performance verification of symbol-based OFDM radar processing," in *Proceedings of IEEE Radar Conference*, May 2010, pp. 60–63.

[63] L. Giugno and M. Luise, "Optimum pulse shaping for delay estimation in satellite positioning," in *2005 13th European Signal Processing Conference*, September 2005, pp. 1–6.

[64] J. H. H. Chalk, "The optimum pulse-shape for pulse communication," *Proceedings of the IEE - Part III: Radio and Communication Engineering*, vol. 97, no. 46, pp. 88–92, March 1950.

[65] M. Bell, "Information theory and radar waveform design," *IEEE Transactions on Information Theory*, vol. 39, no. 5, pp. 1578–1597, September 1993.

[66] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge University Press, 2004.

[67] President's Council of Advisors on Science and Technology, "Realizing the Full Potential of Government-Held Spectrum to Spur Economic Growth," July 2012. [Online]. Available: https://obamawhitehouse.archives.gov/sites/default/files/microsites/ostp/pcast_spectrum_report_final_july_20_2012.pdf

[68] B. Paul, A. R. Chiriyath, and D. W. Bliss, "Survey of RF Communications and Sensing Convergence Research," *IEEE Access*, vol. 5, pp. 252–270, December 2017.

[69] S. Heuel, D. McCarthy, and Y. Shavit, "Test and measurement of coexistence between S-Band radar and mobile networks," in *2016 26th International Conference Radioelektronika (RADIOELEKTRONIKA)*, April 2016, pp. 1–4.

[70] N. Nartasilpa, D. Tuninetti, N. Devroye, and D. Erricolo, "Let's share CommRad: Effect of radar interference on an uncoded data communication system," in *2016 IEEE Radar Conference (RadarConf)*, May 2016, pp. 1–5.

[71] M. A. Govoni, "Enhancing spectrum coexistence using radar waveform diversity," in *2016 IEEE Radar Conference (RadarConf)*, May 2016, pp. 1–5.

[72] A. R. Chiriyath, B. Paul, and D. W. Bliss, "Radar-Communications Convergence: Coexistence, Cooperation, and Co-Design," *IEEE Transactions on Cognitive Communications and Networking*, vol. 3, no. 1, pp. 1–12, March 2017.

[73] S. Melo, S. Pinna, A. Bogoni, I. F. da Costa, D. H. Spadoti, F. Laghezza, F. Scotti, and S. A. Cerqueira, "Dual-use system combining simultaneous active radar communication, based on a single photonics-assisted transceiver," in *2016 17th International Radar Symposium (IRS)*, May 2016, pp. 1–4.

[74] A. K. Mishra and M. Inggs, "White space symbiotic radar: A new scheme for coexistence of radio communications and radar," in *2015 IEEE Radar Conference*, October 2015, pp. 56–60.

[75] P. M. McCormick, S. D. Blunt, and J. G. Metcalf, "Simultaneous radar and communications emissions from a common aperture, Part I: Theory," in *2017 IEEE Radar Conference (RadarConf)*, May 2017, pp. 1685–1690.

[76] A. Hassanien, M. G. Amin, Y. D. Zhang, and B. Himed, "A dual-function MIMO radar-communications system using PSK modulation," in *2016 24th European Signal Processing Conference (EUSIPCO)*, August 2016, pp. 1613–1617.

[77] K. W. Huang, M. Bica, U. Mitra, and V. Koivunen, "Radar Waveform Design in Spectrum Sharing Environment: Coexistence and Cognition," in *2015 IEEE Radar Conference (RadarCon)*, May 2015, pp. 1698–1703.

[78] A. Aubry, A. D. Maio, Y. Huang, M. Piezzo, and A. Farina, "A new radar waveform design algorithm with improved feasibility for spectral coexistence," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 2, pp. 1029–1038, April 2015.

[79] A. Aubry, V. Carotenuto, A. D. Maio, A. Farina, and L. Pallotta, "Optimization theory-based radar waveform design for spectrally dense environments," *IEEE Aerospace and Electronic Systems Magazine*, vol. 31, no. 12, pp. 14–25, December 2016.

[80] C. Sahin, J. Jakabosky, P. M. McCormick, J. G. Metcalf, and S. D. Blunt, "A novel approach for embedding communication symbols into physical radar waveforms," in *2017 IEEE Radar Conference (RadarConf)*, May 2017, pp. 1498–1503.

[81] P. Stinco, M. S. Greco, F. Gini, and M. Rangaswamy, "Ambiguity function and Cramer-Rao bounds for universal mobile telecommunications system-based passive coherent location systems," *IET Radar, Sonar Navigation*, vol. 6, no. 7, pp. 668–678, August 2012.

[82] Q. He and R. S. Blum, "The Significant Gains From Optimally Processed Multiple Signals of Opportunity and Multiple Receive Stations in Passive Radar," *IEEE Signal Processing Letters*, vol. 21, no. 2, pp. 180–184, February 2014.

[83] P. Stinco, M. S. Greco, F. Gini, and B. Himed, "IEEE 802.22 passive radars: multistatic detection and velocity profiler," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 52, no. 5, pp. 2298–2313, October 2016.

[84] Y. Wu, Q. He, J. Hu, and R. S. Blum, "Demonstrating significant passive radar performance increase through using known communication signal format," in *2017 51st Asilomar Conference on Signals, Systems, and Computers*, October 2017, pp. 75–79.

[85] E. Grossi, M. Lops, L. Venturino, and A. Zappone, "Opportunistic Radar in IEEE 802.11ad Networks," *IEEE Transactions on Signal Processing*, vol. 66, no. 9, pp. 2441–2454, May 2018.

[86] P. Kumari, J. Choi, N. González-Prelcic, and R. W. Heath, "IEEE 802.11ad-Based Radar: An Approach to Joint Vehicular Communication-Radar System," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 4, pp. 3012–3027, April 2018.

[87] P. Samczynski, P. Krysik, and K. Kulpa, "Passive radars utilizing pulse radars as illuminators of opportunity," in *2015 IEEE Radar Conference (RadarConf)*, October 2015, pp. 168–173.

[88] A. Khawar, A. Abdel-Hadi, and T. C. Clancy, "MIMO radar waveform design for coexistence with cellular systems," in *2014 IEEE International Symposium on Dynamic Spectrum Access Networks (DYSPAN)*, April 2014, pp. 20–26.

[89] B. Li, A. P. Petropulu, and W. Trappe, "Optimum Co-Design for Spectrum Sharing between Matrix Completion Based MIMO Radars and a MIMO Communication System," *IEEE Transactions on Signal Processing*, vol. 64, no. 17, pp. 4562–4575, September 2016.

[90] B. Li, H. Kumar, and A. P. Petropulu, "A joint design approach for spectrum sharing between radar and communication systems," in *2016 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, March 2016, pp. 3306–3310.

[91] S. Kim, J. Choi, and C. B. Dietrich, "Coexistence between OFDM and pulsed radars in the 3.5 GHz band with imperfect sensing," in *2016 IEEE Wireless Communications and Networking Conference*, April 2016, pp. 1–6.

[92] A. Khawar, A. Abdelhadi, and T. C. Clancy, "Coexistence Analysis Between Radar and Cellular System in LoS Channel," *IEEE Antennas and Wireless Propagation Letters*, vol. 15, pp. 972–975, October 2016.

[93] S. Sodagari, A. Khawar, T. C. Clancy, and R. McGwier, "A projection based approach for radar and telecommunication systems coexistence," in *2012 IEEE Global Communications Conference (GLOBECOM)*, December 2012, pp. 5010–5014.

[94] A. Khawar, A. Abdel-Hadi, and T. Clancy, "MIMO radar waveform design for coexistence with cellular systems," in *2014 IEEE International Symposium on Dynamic Spectrum Access Networks (DYSPAN)*, April 2014, pp. 20–26.

[95] J. A. Mahal, A. Khawar, A. Abdelhadi, and T. C. Clancy, "Spectral Coexistence of MIMO Radar and MIMO Cellular System," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 53, no. 2, pp. 655–668, April 2017.

[96] C. Shahriar, A. Abdelhadi, and T. C. Clancy, "Overlapped-MIMO radar waveform design for coexistence with communication systems," in *2015 IEEE Wireless Communications and Networking Conference (WCNC)*, March 2015, pp. 223–228.

[97] E. Yousif, F. Khan, T. Ratnarajah, and M. Sellathurai, "On the spectral coexistence of colocated MIMO radars and wireless communications systems," in *2016 IEEE 17th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, July 2016, pp. 1–5.

[98] E. H. G. Yousif, M. C. Filippou, F. Khan, T. Ratnarajah, and M. Sellathurai, "A new LSA-based approach for spectral coexistence of MIMO radar and wireless communications systems," in *2016 IEEE International Conference on Communications (ICC)*, May 2016, pp. 1–6.

[99] C. D. Richmond, P. Basu, R. E. Learned, J. Vian, A. Worthen, and M. Lockard, "Performance bounds on cooperative radar and communication systems operation," in *2016 IEEE Radar Conference (RadarConf)*, May 2016, pp. 1–6.

[100] B. Ravenscroft, P. M. McCormick, S. D. Blunt, J. Jakabosky, and J. G. Metcalf, "Tandem-hopped OFDM communications in spectral gaps of FM noise radar," in *2017 IEEE Radar Conference (RadarConf)*, May 2017, pp. 1262–1267.

[101] A. Hassanien, B. Himed, and B. D. Rigling, "A dual-function MIMO radar-communications system using frequency-hopping waveforms," in *2017 IEEE Radar Conference (RadarConf)*, May 2017, pp. 1721–1725.

[102] A. Aubry, A. D. Maio, M. Piezzo, and A. Farina, "Radar waveform design in a spectrally crowded environment via nonconvex quadratic optimization," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, no. 2, pp. 1138–1152, April 2014.

[103] K. Shepherd and R. Romero, "Radar waveform design in active communications channel," in *2013 Asilomar Conference on Signals, Systems and Computers*, November 2013, pp. 1515–1519.

[104] S. Kim, J. M. J. Park, and K. Bian, "PSUN: An OFDM scheme for coexistence with pulsed radar," in *2015 International Conference on Computing, Networking and Communications (ICNC)*, February 2015, pp. 984–988.

[105] S. S. Raymond, A. Abubakari, and H. S. Jo, "Coexistence of Power-Controlled Cellular Networks With Rotating Radar," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 10, pp. 2605–2616, October 2016.

[106] B. Li and A. Petropulu, "MIMO radar and communication spectrum sharing with clutter mitigation," in *2016 IEEE Radar Conference (RadarConf)*, May 2016, pp. 1–6.

[107] L. Zheng, M. Lops, X. Wang, and E. Grossi, "Joint Design of Overlaid Communication Systems and Pulsed Radars," *IEEE Transactions on Signal Processing*, vol. 66, no. 1, pp. 139–154, January 2018.

[108] R. A. Romero and K. D. Shepherd, "Friendly Spectrally Shaped Radar Waveform With Legacy Communication Systems for Shared Access and Spectrum Management," *IEEE Access*, vol. 3, pp. 1541–1554, August 2015.

[109] A. Leshem and A. Nehorai, "Information Theoretic Radar Waveform Design for Multiple Targets," in *2006 40th Annual Conference on Information Sciences and Systems*, March 2006, pp. 1408–1412.

[110] Y. Yang and R. Blum, "MIMO radar waveform design based on mutual information and minimum mean-square error estimation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 1, pp. 330–343, January 2007.

[111] R. A. Romero, J. Bae, and N. A. Goodman, "Theory and Application of SNR and Mutual Information Matched Illumination Waveforms," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 2, pp. 912–927, April 2011.

[112] D. Guo, S. Shamai, and S. Verdú, *The Interplay Between Information and Estimation Measures*, ser. Foundations and trends in signal processing.

[113] J. Tang, N. Li, Y. Wu, and Y. Peng, "On Detection Performance of MIMO Radar: A Relative Entropy-Based Study," *IEEE Signal Processing Letters*, vol. 16, no. 3, pp. 184–187, March 2009.

[114] S. Kay, "Waveform design for multistatic radar detection," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 45, no. 3, pp. 1153–1166, July 2009.

[115] E. Grossi and M. Lops, "Space-Time Code Design for MIMO Detection Based on Kullback-Leibler Divergence," *IEEE Transactions on Information Theory*, vol. 58, no. 6, pp. 3989–4004, June 2012.

[116] M. M. Naghsh, M. Modarres-Hashemi, S. ShahbazPanahi, M. Soltanalian, and P. Stoica, "Unified Optimization Framework for Multi-Static Radar Code Design Using Information-Theoretic Criteria," *IEEE Transactions on Signal Processing*, vol. 61, no. 21, pp. 5401–5416, November 2013.

[117] Y. Li, H. Li, Y. Sun, and N. Zheng, "Waveform design based on J-divergence for MIMO radar detection," in *2017 IEEE 2nd Advanced Information Technology, Electronic and Automation Control Conference (IAEAC)*, March 2017, pp. 971–975.

[118] M. M. Naghsh, M. Modarres-Hashemi, A. Sheikhi, M. Soltanalian, and P. Stoica, "Unimodular code design for MIMO radar using Bhattacharyya distance," in *2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2014, pp. 5282–5286.

[119] T. Kailath, "The Divergence and Bhattacharyya Distance Measures in Signal Selection," *IEEE Transactions on Communication Technology*, vol. 15, no. 1, pp. 52–60, February 1967.

[120] E. Laz, "Optimal cost allocation in centralized and decentralized detection systems using Bhattacharyya distance," in *2017 IEEE Radar Conference (RadarConf)*, May 2017, pp. 1170–1173.

[121] A. D. Maio, M. Lops, and L. Venturino, "Diversity-Integration Tradeoffs in MIMO Detection," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 5051–5061, October 2008.

[122] A. D. Maio and M. Lops, "Design Principles of MIMO Radar Detectors," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 3, pp. 886–898, July 2007.

[123] A. Khawar, A. Abdelhadi, and C. Clancy, "Target Detection Performance of Spectrum Sharing MIMO Radars," *IEEE Sensors Journal*, vol. 15, no. 9, pp. 4928–4940, September 2015.

[124] Y. Cui, V. Koivunen, and X. Jing, "Interference alignment based precoder-decoder design for radar-communication co-existence," in *2017 51st Asilomar Conference on Signals, Systems, and Computers*, October 2017, pp. 1290–1295.

[125] ——, "IA Based Spectrum Sharing for MIMO radar and communication systems," in *2018 IEEE 19th Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2018.

[126] D. W. Bliss, "Cooperative radar and communications signaling: The estimation and information theory odd couple," in *2014 IEEE Radar Conference*, May 2014, pp. 0050–0055.

[127] F. Zanier and M. Luise, "Fundamental issues in time-delay estimation of multi-carrier signals with applications to next-generation GNSS," in *2008 10th International Workshop on Signal Processing for Space Communications*, October 2008, pp. 1–8.

[128] F. Zanier, G. Bacci, and M. Luise, "Criteria to Improve Time-Delay Estimation of Spread Spectrum Signals in Satellite Positioning," *IEEE Journal of Selected Topics in Signal Processing*, vol. 3, no. 5, pp. 748–763, October 2009.

[129] A. R. Chiriyath, B. Paul, G. M. Jacyna, and D. W. Bliss, "Inner Bounds on Performance of Radar and Communications Co-Existence," *IEEE Transactions on Signal Processing*, vol. 64, no. 2, pp. 464–474, January 2016.

[130] P. Comon and C. Jutten, *Handbook of Blind Source Separation: Independent component analysis and applications*. Academic Press, 2010.

[131] W. Wiesbeck, L. Sit, M. Younis, T. Rommel, G. Krieger, and A. Moreira, "Radar 2020: The future of radar systems," in *2015 IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, July 2015, pp. 188–191.

[132] K. V. Mishra, A. Zhitnikov, and Y. C. Eldar, "Spectrum Sharing Solution for Automotive Radar," in *2017 IEEE 85th Vehicular Technology Conference (VTC Spring)*, June 2017, pp. 1–5.

[133] A. Droitcour, V. Lubecke, J. Lin, and O. Boric-Lubecke, "A microwave radio for Doppler radar sensing of vital signs," in *IEEE MTT-S International Microwave Symposium Digest*, vol. 1, May 2001, pp. 175–178.

[134] C. Li, J. Cummings, J. Lam, E. Graves, and W. Wu, "Radar remote monitoring of vital signs," *IEEE Microwave Magazine*, vol. 10, no. 1, pp. 47–56, February 2009.

[135] C. Li, V. M. Lubecke, O. Boric-Lubecke, and J. Lin, "A Review on Recent Advances in Doppler Radar Sensors for Noncontact Healthcare Monitoring," *IEEE Transactions on Microwave Theory and Techniques*, vol. 61, no. 5, pp. 2046–2060, May 2013.

[136] I. Arai, "Survivor search radar system for persons trapped under earthquake rubble," in *APMC 2001. 2001 Asia-Pacific Microwave Conference*, vol. 2, December 2001, pp. 663–668.

[137] G. Grazzini, M. Pieraccini, F. Parrini, A. Spinetti, G. Macaluso, D. Dei, and C. Atzeni, "An ultra-wideband high-dynamic range GPR for detecting buried people after collapse of buildings," in *Proceedings of the XIII International Conference on Ground Penetrating Radar*, June 2010, pp. 1–6.

The ever-increasing demand for radio spectrum requires future wireless systems to share this limited available physical resource. Extensive adaptability of both systems and a new design paradigm are expected. Coexistence among radar and communications systems is proposed in this thesis as a solution to the problem of congested spectrum use. The coexistence methods proposed in this thesis are based on waveform optimization and adaptation and are applicable also to other systems sharing radio spectrum. The need for future radar systems to be fully adaptive and cognitive and take advantage of all available degrees of freedom for optimizing their performance is met by multicarrier waveforms. A generalized multicarrier radar model able to describe most commonly used radar waveforms as well as novel ones in an easy and customizable way is proposed. Using such model novel multicarrier waveforms with improved radar performance are proposed. It is also shown that the radar system can improve its performance by cooperating with the coexisting communications systems.

BUSINESS +
ECONOMY

ART +
DESIGN +
ARCHITECTURE

SCIENCE +
TECHNOLOGY

CROSSOVER

DOCTORAL
DISSERTATIONS