Self-interference Mitigation in Full-duplex Relays: Blind Adaptive Techniques and Optimal Linear Filter Design

Emilio Antonio Rodríguez
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# Abstract

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**Abstract**

Wireless communication systems need to satisfy the increasing demand for high data rates, low latency and ubiquitous connectivity. Besides, efficient resource management and a high spectral efficiency are also paramount. Multi-hop communication via relays is an attractive and cost-effective solution for cooperative communication and coverage extension.

Full-duplex technology, i.e., simultaneous transmission and reception in the same frequency band, offers twice the spectral efficiency than its half-duplex counterpart, where transmission and reception take place in different time slots. Full-duplex technology makes possible for devices to operate directly on the waveform samples without synchronizing to the network, thus allowing to reduce the end-to-end latency and posing itself as an ideal solution for wireless systems. However, the implementation of a full-duplex device has several practical issues. Of utmost importance is the self-interference, which causes the transmitted signal to interfere with the received signal, leading to a severe drop of the signal-to-noise ratio. To fully take advantage of the full-duplex technology, self-interference must be mitigated.

This dissertation addresses the issue of self-interference mitigation for different relay protocols and configurations. The first theme deals with adaptive blind algorithms for self-interference cancellation. Assuming no available training information or synchronization with the network, we propose algorithms that yield residual self-interference below noise level. The second theme considers the optimization of a full-duplex relay link subjected to limited dynamic range, i.e., impairments at transmission and reception sides. Considering available channel state information, we design optimal linear filters at each node under the maximum signal-to-noise ratio and the minimum mean square error criteria.

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full-duplex, relays, wireless communication

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Preface

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Helsinki, November 1, 2017,

Emilio Antonio Rodríguez
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<th>Description</th>
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<tbody>
<tr>
<td>AD</td>
<td>Analog-to-digital</td>
</tr>
<tr>
<td>AF</td>
<td>Amplify-and-forward</td>
</tr>
<tr>
<td>AOA</td>
<td>Angle-of-arrival</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
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<tr>
<td>BS</td>
<td>Base station</td>
</tr>
<tr>
<td>CF</td>
<td>Compress-and-forward</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic prefix</td>
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<tr>
<td>CSI</td>
<td>Channel state information</td>
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<tr>
<td>D</td>
<td>Destination node</td>
</tr>
<tr>
<td>DA</td>
<td>Digital-to-analog</td>
</tr>
<tr>
<td>DF</td>
<td>Decode-and-forward</td>
</tr>
<tr>
<td>DF-SGD</td>
<td>Decode-and-forward stochastic gradient descent</td>
</tr>
<tr>
<td>DF-RLS</td>
<td>Decode-and-forward recursive least squares</td>
</tr>
<tr>
<td>FD</td>
<td>Full-duplex</td>
</tr>
<tr>
<td>FF</td>
<td>Filter-and-forward</td>
</tr>
<tr>
<td>FF-IAS</td>
<td>Filter-and-forward instantaneous autocorrelation shaping</td>
</tr>
<tr>
<td>FF-WIAS</td>
<td>Filter-and-forward weighted instantaneous autocorrelation shaping</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite impulse response</td>
</tr>
<tr>
<td>FT</td>
<td>Fourier transform</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
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<tr>
<td>HD</td>
<td>Half-duplex</td>
</tr>
<tr>
<td>IFT</td>
<td>Inverse Fourier transform</td>
</tr>
<tr>
<td>IIR</td>
<td>Infinite impulse response</td>
</tr>
<tr>
<td>LMS</td>
<td>Least mean squares</td>
</tr>
<tr>
<td>LSQI</td>
<td>Least squares with quadratic inequality constraint</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-input multiple-output</td>
</tr>
<tr>
<td>MISO</td>
<td>Multiple-input single-output</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean square error</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum mean square error</td>
</tr>
<tr>
<td>MVDR</td>
<td>Minimum variance distortionless response</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal frequency division multiplex</td>
</tr>
<tr>
<td>PA</td>
<td>Power amplifier</td>
</tr>
<tr>
<td>R</td>
<td>Relay node</td>
</tr>
<tr>
<td>RR</td>
<td>Relay-to-relay</td>
</tr>
<tr>
<td>RD</td>
<td>Relay-to-destination</td>
</tr>
<tr>
<td>RLS</td>
<td>Recursive least squares</td>
</tr>
<tr>
<td>S</td>
<td>Source node</td>
</tr>
<tr>
<td>SD</td>
<td>Source-to-destination</td>
</tr>
<tr>
<td>SDP</td>
<td>Semidefinite programming</td>
</tr>
<tr>
<td>SFN</td>
<td>Single frequency network</td>
</tr>
<tr>
<td>SI</td>
<td>Self-interference</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-interference-plus-noise ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-input single-output</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>SR</td>
<td>Source-to-relay</td>
</tr>
<tr>
<td>ULA</td>
<td>Uniform linear array</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$0$</td>
<td>All-zero matrix</td>
</tr>
<tr>
<td>$\mathcal{CN}(\mu, \Gamma)$</td>
<td>Circularly-symmetric complex normal distribution with mean $\mu$ and covariance $\Gamma$</td>
</tr>
<tr>
<td>$\text{C}_R[n]$</td>
<td>Impulse response of the cancellation filter at the relay</td>
</tr>
<tr>
<td>$\text{diag}{X}$</td>
<td>Transformation into diagonal matrix</td>
</tr>
<tr>
<td>$\mathbb{E}{\cdot}$</td>
<td>Expectation operation</td>
</tr>
<tr>
<td>$\text{F}_R[n]$</td>
<td>Impulse response of the transmit filter at the relay</td>
</tr>
<tr>
<td>$\text{F}_S[n]$</td>
<td>Impulse response of the transmit filter at the source</td>
</tr>
<tr>
<td>$\text{G}_D[n]$</td>
<td>Impulse response of the receive filter at the destination</td>
</tr>
<tr>
<td>$\text{G}_R[n]$</td>
<td>Impulse response of the receive filter at the relay</td>
</tr>
<tr>
<td>$\text{H}_\text{RD}[n]$</td>
<td>Impulse response of the relay-to-destination channel</td>
</tr>
<tr>
<td>$\text{H}_\text{RR}[n]$</td>
<td>Impulse response of the relay-to-relay (self-interference) channel</td>
</tr>
<tr>
<td>$\text{H}_\text{SD}[n]$</td>
<td>Impulse response of the source-to-destination channel</td>
</tr>
<tr>
<td>$\text{H}_\text{SR}[n]$</td>
<td>Impulse response of the source-to-relay channel</td>
</tr>
<tr>
<td>$I$</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>$L_{\text{CR}}$</td>
<td>Order of the cancellation filter at the relay</td>
</tr>
<tr>
<td>$L_D$</td>
<td>Order of the receive filter at the destination</td>
</tr>
<tr>
<td>$L_{\text{FR}}$</td>
<td>Order of the transmit filter at the relay</td>
</tr>
<tr>
<td>$L_{\text{GR}}$</td>
<td>Order of the receive filter at the relay</td>
</tr>
</tbody>
</table>
List of Symbols

\( L_{RD} \) Order of the relay-to-destination channel
\( L_{RR} \) Order of the relay-to-relay (self-interference) channel
\( L_S \) Order of the transmit filter at the source
\( L_{SD} \) Order of the source-to-destination channel
\( L_{SR} \) Order of the source-to-relay channel
\( m \) Number of independent data streams
\( M_R \) Number of transmit antennas at the relay
\( M_S \) Number of transmit antennas at the source
\( N_D \) Number of receive antennas at the destination
\( N_R \) Number of receive antennas at the relay
\( P_R \) Transmit power of the relay
\( P_S \) Transmit power of the source
\( x[n] \) Generic signal
\( \|x[n]\| \) Euclidean norm
\( \alpha_{RD}^2 \) Estimation noise power of the relay-to-destination channel
\( \alpha_{RR}^2 \) Estimation noise power of the relay-to-relay (self-interference) channel
\( \alpha_{SR}^2 \) Estimation noise power of the source-to-relay channel
\( \delta_R \) Dynamic range parameter at the relay
\( \delta_S \) Dynamic range parameter at the source
\( \epsilon_D \) Dynamic range parameter at the destination
\( \epsilon_R \) Dynamic range parameter at the relay
\( \sigma_D^2 \) Noise power at the destination
\( \sigma_R^2 \) Noise power at the relay
List of Publications

This dissertation consists of an overview and of the following publications which are referred to in the text by their Roman numerals.


V Emilio Antonio-Rodríguez and Roberto López-Valcarce, “ Cancelling Self-interference in Full-duplex Relays without Angle-of-arrival Information”, in IEEE International Conference on Acoustics, Speech


Author’s Contribution

Publication I: “An Adaptive Feedback Canceller for Full-duplex Relays based on Spectrum Shaping”

The author of this dissertation assisted with technical knowledge, obtained the simulation results and assisted during the redaction of the manuscript.

Publication II: “Wideband Full-duplex MIMO Relays with Blind Adaptive Self-interference Cancellation”

The author of this dissertation presented the idea, developed the technical solution, obtained the simulation results, redacted every draft of the manuscript and its final version. The remaining authors provided technical feedback and assisted with the redaction of the manuscript.


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Author’s Contribution


The author of this dissertation presented the idea, developed the technical solution, obtained the simulation results, redacted every draft of the manuscript and its final version. The remaining authors provided technical feedback and assisted with the redaction of the manuscript.

Publication V: “Cancelling Self-interference in Full-duplex Relays without Angle-of-arrival Information”

The author of this dissertation presented the idea, developed the technical solution, obtained the simulation results, redacted every draft of the manuscript and its final version. The remaining authors provided technical feedback and assisted with the redaction of the manuscript.

Publication VI: “Autocorrelation-based Adaptation Rule for Feedback Equalization in Wideband Full-duplex Amplify-and-forward MIMO Relays”

The author of this dissertation presented the idea, developed the technical solution, obtained the simulation results, redacted every draft of the manuscript and its final version. The remaining authors provided technical feedback and assisted with the redaction of the manuscript.


The author of this dissertation presented the idea, developed the technical solution, obtained the simulation results, redacted every draft of the manuscript and its final version. The remaining authors provided technical feedback and assisted with the redaction of the manuscript.
Publication VIII: “SINR Optimization in Wideband Full-duplex MIMO Relays under Limited Dynamic Range”

The author of this dissertation presented the idea, developed the technical solution, obtained the simulation results, redacted every draft of the manuscript and its final version. The remaining authors provided technical feedback and assisted with the redaction of the manuscript.

Publication IX: “Subspace-constrained SINR Optimization in MIMO Full-duplex Relays under Limited Dynamic Range”

The author of this dissertation presented the idea, developed the technical solution, obtained the simulation results, redacted every draft of the manuscript and its final version. The remaining authors provided technical feedback and assisted with the redaction of the manuscript.

Publication X: “Robust Filter Design for Full-duplex Relay Links under Limited Dynamic Range”

The author of this dissertation presented the idea, developed the technical solution, obtained the simulation results, redacted every draft of the manuscript and its final version. The remaining authors provided technical feedback and assisted with the redaction of the manuscript.
1. Introduction

1.1 Motivation and Scope of the Dissertation

The search for larger data rates and a higher spectral efficiency in wireless communications has motivated the development of the full-duplex (FD) protocol, wherein transmission and reception of data take place in the same frequency band. This is in contrast to the half-duplex (HD) protocol, where transmission and reception take place in non-overlapping time slots. Single-frequency networks (SFN) make use of the FD protocol to reduce the required bandwidth.

The FD protocol constitutes an efficient allocation of the system's resources and, in theory, achieves twice the spectral efficiency of the HD protocol. However, an actual implementation of an FD device is faced with several practical issues. One of the major implications of the FD protocol is a new type of distortion, the so-called self-interference (SI) [1–7]. Simultaneous transmission and reception in the same frequency band, together with insufficient isolation between antenna arrays, causes a significant part of the transmitted signal to be fed back into the device, thus interfering with the received signal. The power imbalance between SI and received signal could reach up to 100 dB [8,9], therefore, the success of the FD technology lies in its ability to mitigate the SI.

An additional impairment in any FD device is the dynamic range, i.e., the fidelity of the system between analog radio frequency (RF) and digital baseband domain [10–12]. It includes distortion-inducing operations such as digital-to-analog (DA) and analog-to-digital (AD) conversion, frequency downconversion and upconversion, as well as the power control
in the power amplifier (PA). As a consequence, this type of distortion is a 
function of the received/transmitted signal and, together with the power 
imbalance, it significantly affects the performance of FD devices [10–12]. 
Furthermore, dynamic range makes the design of FD systems difficult 
and involved. The distortion resulting from the limited dynamic range is 
modeled as a set of additional noise sources whose power depends on the 
signal statistics.

The motivation of this dissertation is to study and develop novel tech-
niques that enable an efficient implementation of FD devices. As ex-
plained before, to fully take advantage of the higher spectral efficiency 
of the FD protocol, SI must be mitigated. At the same time, SI mitigation 
should not interfere with the normal operation of the device and, in 
the ideal case, SI mitigation should allow the device to operate as if it 
were free of interference. The existing SI mitigation techniques can be 
classified into two main categories: SI cancellation, where a replica of the 
SI interferes destructively with the actual SI, and SI suppression, where 
some spatial degrees of freedom are designated in order to block the SI 
from looping back into the system. In addition to mitigation, the design 
must consider the limited dynamic range of the system, which introduces 
signal-dependent noise distortion. A design that takes into account the ef-
fects of dynamic range results in a higher performance, as shown in [10]. 
The second part of this dissertation considers a relay link subjected to lim-
ited dynamic range and proposes different design methods that aggregate 
and take into account the aforementioned impairments of an FD device.

1.2 Contributions of the Dissertation

This dissertation presents contributions to the implementation of FD de-
vices, developed in two different lines of research:

- **Blind adaptive SI cancellation algorithms.** Assuming no training 
sequence, we develop blind algorithms for SI cancellation that do 
not modify the normal operation of the FD device and yield residual 
SI below the noise level. The cancellation algorithms are able to 
mitigate the SI and simultaneously equalize the source-to-relay (SR) 
channel.
• **Optimal linear filter design.** Assuming available channel state information (CSI), we develop methods for linear filter design on FD links subjected to limited dynamic range under the minimum mean square error (MMSE) and the signal-to-noise ratio (SNR) criteria. The cases resulting in non-convex optimization problems are solved by using an alternating procedure.

Concretely, Publications I, II, IV-VII make contributions in the field of blind algorithms for SI cancellation. Publication I considers a single-input single-output (SISO) amplify-and-forward (AF) relay and presents a blind algorithm for SI cancellation that adjusts the signal autocorrelation based on spectrum restoration. Publication IV extends the idea to a multiple-input single-output (MISO) AF relay in which the angle-of-arrival (AOA) of the signal of interest is known. Publication V presents an algorithm able to estimate the AOA of the signal of interest and perform SI cancellation without any training information. Publications VI and VII consider a multiple-input multiple-output (MIMO) relay and present the blind cancellation algorithm for the AF, filter-and-forward (FF) and decode-and-forward (DF) cases, respectively. Publication II considers a MIMO relay case, presents and analyzes both power minimization and recursive least squares (RLS) based algorithms for blind SI cancellation in the AF, FF and DF cases.

The contributions in optimal design of linear filters for FD relay links are contained in Publications III and VIII-X. Publication III presents a method for filter design in an AF relay link where a direct path between the source node (S) and the destination node (D) exists, i.e., a source-to-destination (SD) channel exists. The resulting recursive and non-convex problem is solved by using an alternating procedure after introducing reasonable approximations and constraints into the design. Publications VIII and IX consider a DF relay link with limited dynamic range in which transmission and reception filters are designed to maximize the signal-to-interference-plus-noise ratio (SINR) criterion. In Publication X, we consider the case where limited CSI is available. Relay filters are designed as to minimize the mean square error (MSE) for the worst case scenario.
1.3 Structure of the Dissertation

The dissertation is structured into five chapters comprising this introductory part and ten original publications with contributions in adaptive filtering for SI cancellation in relay systems and optimal filter design in relay links under limited dynamic range. Chapter 2 provides an overview of FD relay systems, their different operation modes and characteristics, their transmission protocols and how they affect the design of the SI mitigation techniques. Chapter 3 describes the problem of SI cancellation in FD relays without training information and presents the proposed cancellation algorithms for different relay configurations. The chapter starts by presenting the fundamentals of adaptive filtering and the system model. Next, cancellation algorithms for MIMO DF and AF relays are presented. To conclude, cancellation algorithms for MISO AF relays with and without training information are also introduced. Chapter 3 ends with a summary of the conclusions. In chapter 4, we study the problem of optimal linear filter design for an FD link subjected to limited dynamic range. The chapter starts by elaborating on the particularities of the problem and presenting the system model. Next, we introduce two different methods for SNR optimization in an FD relay. The chapter follows with the optimization of the MSE of an AF FD relay link with direct connection between source and destination, SI and limited dynamic range. We finalize the chapter with a robust method for MSE optimization and a summary of the conclusions. Chapter 5 draws the conclusions and outlines future research lines.
2. Full-duplex Relay Systems

2.1 Introduction

Wireless communication systems face the challenge of meeting the ever-increasing demand for higher data rates and ubiquitous connectivity while maintaining a reasonable power consumption and a high spectral efficiency. In addition, as the number of available services continues to increase, bandwidth remains a scarce resource [9, 13, 14]. This issue has caused a need for more sophisticated transmission techniques and has driven the research community into developing new transmission protocols with a high spectral efficiency, such as the FD protocol, that allows for simultaneous transmission and reception in the same frequency band. The implementation of an FD protocol entails some technical challenges, primarily the problem of SI mitigation, which is the major focus of this dissertation.

Wireless networks must provide with connectivity to a dynamic number of users spreading across a certain area. In order to modify/extend the coverage area while preserving a high network efficiency and avoiding a costly deployment, new and flexible hardware solutions shall be introduced. Relays are devices that solve the problems of coverage area extension and performance improvement without investing on a base station (BS) [14–18]. From a business point-of-view, although a significant number of relays are needed to substitute a single BS, relays can reduce operator costs while maintaining the network requirements for performance and coverage. Specifically, the usage of medium/high power relays results in a cost reduction of 30% when compared to the usage of a BS, as reported
in [19]. For that reason, relays are an attractive and cost-efficient solution for wireless networks. Relays range from repeaters, or signal amplifiers, to full regenerative devices with multilayer functionalities.

This chapter presents an overview of the technology behind FD communication and its application to relays. Next, we detail the different operating modes of a relay and describe the problem of SI, i.e., the distortion due to simultaneous transmission and reception in the same frequency band. Finally, we provide a classification of the different techniques for SI mitigation, as well as a survey of previous literature.

2.2 Relay Devices

The advent of relay communications can be traced back to the first broadcast systems employing repeaters to provide service in the shadow zones of a main transmitter. A repeater simply amplifies and forwards the received signal to the next element of the network. Although providing the system with a redundant signal path, neither the repeater collaborates with the main transmitter, nor the destination exploits the redundant signals from the repeater and the main transmitter. In that sense, repeaters are non-cooperative devices.

The use of relays as proper cooperative devices came after the seminal work in [20] and the development of cellular networks. This fostered the introduction of new devices, so-called relays, that not only extend the coverage area but also increase the throughput of the network. Formally, in the context of wireless communications, a relay is a device of limited processing power designed to extend the coverage area and/or improve the end-to-end performance by collaborating with the BS and the final destination.

As technology brings more computing power into devices, research on relays has considered the use of more complex techniques, such as spatial diversity, spatial multiplexing, orthogonal frequency division multiplexing (OFDM) modulation and multi-user communications. As a consequence of the increasing computing power, the physical layer in a relay is of similar complexity to that in a BS. Significant implementation differences between a relay and a BS lie in the features of upper layers [2, 21–30].
Relays range from repeater-like devices that just amplify and forward the information to error-correcting devices that implement data decoding-regeneration-encoding. A relay is modeled by two characteristics:

- The **relaying mode** specifies the rules for when transmission and reception take place.

- The **relaying protocol** specifies how the information is processed before being forwarded.

Next, we provide an overview of the different relaying modes and relaying protocols considered in the literature, their advantages and disadvantages as well as their associated technical challenges.

### 2.3 Relaying Modes

The strategy by which a relay passes on the information to the next element of the link will impact not only the end-to-end performance but also the technical challenges during design. Relays typically operate under one of the following transmission modes:

- In **Full-Duplex mode**, FD, the information is received and transmitted simultaneously.

- In **Half-Duplex mode**, HD, the information is received and transmitted in different time slots.

An FD mode can be implemented by using orthogonal frequency bands for reception and transmission, in which case twice the bandwidth is required. This results in a spectral efficiency similar to that of an HD mode. In this dissertation, we only consider the so-called **in-band FD mode** [5, 7], where transmission and reception take place in the same frequency band. For convenience, we refer to the in-band FD mode as simply FD mode. Several alternatives have been proposed in the literature, such as a hybrid combination of both modes [31–33] and a virtual FD mode over an HD mode [34–37], though most of the communications systems implement either an FD or an HD mode. An HD mode divides time into non-overlapping time slots for transmission and reception, thus avoiding simultaneous usage of the same physical channel. During the first time
slot, the relay receives data from the source, while during the second time slot, the relay transmits data towards the destination. The consequences of using an HD mode are a reduction in the spectral efficiency, because the data reaches destination in two time slots, and the need for the relay to be synchronized with the elements of the network. An FD mode based on orthogonal frequency bands circumvents the need for time division, but introduces cochannel interference without increasing the spectral efficiency, due to the need for extra bandwidth.

On the other hand, an (in-band) FD mode enables a simultaneous transmission and reception of the information using the same frequency band. Ideally, this results in twice the spectral efficiency of its HD counterpart and the asynchronous operation of the relay [38]. However, due to non-ideal isolation between antennas, an FD mode introduces the problem of SI, wherein the transmitted signal feeds back into the relay and interferes with the received signal.

2.4 Relaying Protocols

As defined in Sec. 2.2, a relay protocol specifies how the information is processed before transmission. According to the complexity of the operations, the different relay protocols can be classified as [9,31,39–43,43–48]:

- In a filter-and-forward protocol, FF, the information undergoes a transformation, typically a linear filter, that shapes the spectrum and spatial distribution before transmission.

- In an amplify-and-forward protocol, AF, the information undergoes a spatial transformation. Since the spectrum is not altered, the AF protocol constitutes a special case of the FF protocol.

- In a decode-and-forward protocol, DF, the information is decoded, regenerated, and re-encoded before transmission.

In the literature, other relay protocols have been proposed, such as compress-and-forward (CF) [20,49] or a hybrid AF/DF [50]. The CF protocol quantizes the received signal and transmits an encoded version of it. The CF protocol is similar to the DF protocol.
Table 2.1. Comparison between relay protocols.

<table>
<thead>
<tr>
<th>Protocol</th>
<th>Complexity</th>
<th>Processing delay</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>Low</td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>AF</td>
<td>Low</td>
<td>Negligible</td>
<td>Low</td>
</tr>
<tr>
<td>DF</td>
<td>High</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

A comparison between the different protocols can be done in terms of the following three parameters: complexity, i.e., the number of operations per sample; performance, i.e., results under some figure of merit, and processing delay, i.e., the time it takes for the information to go from the input to the output of the relay.

In terms of performance, the DF protocol exhibits the best results, while the FF protocol outperforms the AF protocol in most of the cases [41, 51]. Performance is a trade-off between complexity and processing delay. The protocol with the least complexity and processing delay is the AF protocol, followed by the FF protocol. The DF protocol generally shows the longest processing delay due to the decoding-regeneration-encoding operations. The processing delay is important in systems exploiting modulations with a cyclic prefix (CP), because the relative delay between the different propagation paths should be shorter than the CP duration [1]. Table 2.1 summarizes the different characteristics of the relay protocols.

### 2.5 The Problem of Self-interference

Regardless of the relay protocol, the higher spectral efficiency of the FD mode comes at a price: the unavoidable SI arising from simultaneous transmission and reception on the same physical channel [1–7, 52–55].

Several causes contribute to the presence of SI in an FD relay. First, the reduced dimension of the relay limits the separation between the transmit/receive antenna arrays, therefore decreasing the achievable natural isolation between antennas. In a single antenna implementation, circulators leak part of the transmitted signal into reception, thus contributing to the presence of SI. Second, since the relay is designed to extend the coverage area, the signal is heavily amplified. The imbalance between transmit and received power can reach 100 dBs [2, 10, 56, 57]. Both effects...
mentioned above make of SI the largest source of distortion in FD devices by several orders of magnitude [56]. Therefore, to ensure an optimal performance of the relay, SI must be mitigated. The SI mitigation scenario differs from the typical noise/interference reduction scenario in that the distortion is several orders of magnitude larger than the information signal and, as a consequence, conventional noise reduction techniques cannot be applied. On the other hand, the digital baseband transmitted signal can be assumed known, which helps the SI mitigation. As explained in the following chapters, the SI is, under some circumstances, correlated with the information signal, which makes SI mitigation even more difficult to implement and requires of a deep analysis of the problem.

In the next section, we describe the SI mitigation methods found in the literature and we classify them according to their domain of application.

2.6 Mitigation of Self-interference

Any well-designed mitigation method aims to reduce the SI power in the system as much as possible. At the same time, mitigation should cause minimal distortion of the information signal. Several SI mitigation methods have been proposed in the literature, which, based on their domain of application, can be classified as follows:

- **Analog-domain SI mitigation** includes any method preventing the electromagnetic energy to feed back into the relay [56, 58, 59]. The main techniques found in this category are antenna isolation and analog cancellation. The design goal of antenna isolation is to block radiation leakage between antennas. Analog cancellation estimates and subtracts a replica of the strongest path(s) between antennas, circulators and other analog elements.

- **Digital-domain SI mitigation** includes any method operating after digital conversion of the signal [1, 26, 29, 60, 61]. Techniques found in this category are digital cancellation and spatial suppression. Digital cancellation estimates the SI channel and subtracts a replica of the SI. Spatial suppression uses spatial redundancy to transmit the signal in modes of minimal energy of the SI channel [1–3, 8, 26, 38, 62–69].
As mentioned in the previous section, power imbalance between the received signal and SI can reach up to 100 dBs. In other words, a typical system requires of 100 dBs of SI mitigation. A solution consisting of only analog-domain mitigation methods is not enough to meet that requirement, because high mitigation values are difficult to achieve for a bandwidth spanning over more than a few megahertz and for different carrier frequencies [4]. Consequently, a residual SI under noise level can only be obtained by combining analog and digital-domain mitigation methods.

This dissertation focuses on mitigation methods in the digital domain. Chapter 3 studies the problem of blind adaptive digital cancellation while Chapter 4 studies optimal linear filter design combining digital cancellation and spatial suppression.

2.6.1 Mitigation Methods in Time-domain and Frequency-domain

According to the type of processing, mitigation takes place in the time domain or in the frequency domain. While time-domain mitigation processes the signal on a sample-by-sample basis, mitigation in the frequency domain processes one symbol at a time.

Frequency-domain mitigation goes hand in hand with the use of an OFDM modulation, wherein the common assumption is that the relative delay between the different paths of the SI channel is shorter than the CP duration. This eliminates inter-symbol and inter-carrier interference and, therefore, simplifies the equalization. Due to the reduced physical dimensions of the relay, such assumption is reasonable. OFDM relies on the Fourier transform (FT) and inverse Fourier transform (IFT) for its easy implementation.

Mitigation in the frequency domain takes place after FT. As a consequence, the relay demands synchronization with other elements of the network (timing recovery) and any pre-FT operation works under the presence of SI. This has some implications: there is an unavoidable delay of at least one single OFDM symbol, and mitigation has to be independently performed for each subcarrier. The complexity depends on the number of subcarriers and the number of antennas [1]. Mitigation in the time domain, typically in the form of a linear filter, can be placed at any point of the reception chain and introduces no extra processing delay.
In other words, time-domain mitigation can work directly with waveform samples without the need to synchronize with other elements of the network (the detection of an OFDM symbol introduces a processing delay and requires from synchronization with the network). The complexity of this method depends on the number of antennas and the multipath delay [70].

### 2.6.2 Cancellation versus Suppression

As explained above, any SI mitigation method in the digital domain falls into two categories:

- **Digital cancellation** consists in generating a replica of the SI to cause destructive interference at reception, in a similar fashion to cancellation in the analog-domain [38, 62, 67–70].

- **Digital suppression** consists in transmitting the signal into directions of minimal energy of the SI channel. Suppression can only be used in multiantenna devices [1, 26, 28, 58, 60].

Fig 2.1 depicts a relay incorporating digital SI cancellation (top) and digital SI suppression (bottom). The relay has $M_R$ transmit and $N_R$ receive antennas. The block $\mathcal{D}$ denotes the relay protocol and $H_{RR}[n]$ is the SI channel. Filters $C_R[n]$, $F_R[n]$ and $G_R[n]$ are the cancellation, transmit and reception filters, respectively. A cancellation architecture consists of the filter $C_R[n]$. Since the filter is fed by the signal before transmission,
SI cancellation is equivalent to an identification problem whose solution is $C_R[n] = -H_{RR}[n]$. While a cancellation architecture uses destructive interference in time domain to remove the SI, a suppression architecture exploits the available degrees of freedom to minimize the SI, i.e.,

$$\min_{\{G_R[n], F_R[n]\}} \|G_R[n] \ast H_{RR}[n] \ast F_R[n]\|^2$$

(2.1)

which, under some circumstances, results in $G_R[n] \ast H_{RR}[n] \ast F_R[n] = 0$. This requires that $M_R \neq N_R$, and the order of $G_R[n]$ and $F_R[n]$ to be sufficiently large. In contrast to a cancellation architecture, suppression can reduce noise at transmit and reception sides [1].

Both architectures are suitable for time and frequency domain, but a solution to (2.1) might be computationally demanding and, therefore, suppression is normally performed in the frequency domain. The problem transforms into the following matrix decomposition per subcarrier

$$\min_{\{G_R, F_R\}} \|G_R H_{RR} F_R\|^2$$

(2.2)

whose solution is related to the SVD decomposition of $H_{RR}$. The combination of cancellation and suppression into a single architecture yields better mitigation results, as reported in [1]. When the design of the relay protocol and SI mitigation are decoupled, SI mitigation must not interfere with the relay’s normal operation, and, in the ideal case, it will provide the relay with a signal free of SI. In Chapters 3 and 4, we present SI mitigation methods that work independently of the relay protocol and also SI mitigation methods jointly designed with the relay protocol.

### 2.7 Summary

We presented an overview of FD relays, a classification of the different types of relay protocols, and summarized various implementation issues. An FD mode allows for simultaneous transmission and reception in the same physical channel, doubling the spectral efficiency of an HD mode at the expense of a new type of distortion, the SI. We classified the existing methods for SI mitigation according to the implementation domain, i.e., analog and digital methods. The digital-domain methods were further classified into cancellation and suppression methods that can work either in the time domain or the frequency domain.
Full-duplex Relay Systems
3. Adaptive Self-interference Cancellation in Full-duplex Relays

3.1 Introduction

In this chapter, we study the problem of SI mitigation in FD relays. The simultaneous transmission and reception in the same frequency band results in the presence of SI, which is the major source of interference in FD devices. In order to take advantage of the higher spectral efficiency of the FD mode and to ensure a reasonable performance of the device, the SI shall be mitigated as much as possible. The power imbalance between SI and the incoming signal from the source may reach several tens of dBs, thus making mitigation a fundamental component of any FD system [2, 10, 15, 56, 57, 71].

Ideally, if the residual SI after mitigation is below noise level, its effect is considered negligible. Mitigation is intimately related to the estimation of the SI channel, and, in most cases, a first stage of channel estimation is embedded within the mitigation scheme. Such channel estimation differs from a typical case in the sense that the relay always knows the transmitted signal. Besides, the received signal from the source behaves as interference during channel estimation. Both cancellation and suppression methods need an estimate of the SI channel.

The mitigation scheme must satisfy additional design constraints of the FD device. Firstly, it should not affect the normal operation of the relay, i.e., no interruption or deferring of data transmission, which would cause a loss in spectral efficiency. Some mitigation methods buffer data before transmission or introduce additional delay for mitigation purposes [62–64]. Secondly, the use of a training sequence is, in most cases, not pos-
Adaptive Self-interference Cancellation in Full-duplex Relays

sible. Typically, an FF/AF relay works directly with the waveform samples, without network synchronization. This makes the use of training information unfeasible and calls for a non-intrusive solution without information assistance [51, 72–74].

To fulfill these requirements, we propose the use of a blind adaptive cancellation scheme [75–77]. Our algorithms do not introduce any additional delay and are able to work with waveform samples, i.e., with arbitrary spectrum shapes. As shown in the results, our algorithms mitigate the SI down below noise level, making its impact practically negligible.

The chapter starts with an overview on the fundamentals of adaptive filtering and the system model of the relay. Next, we introduce the cancellation algorithm for a DF MIMO relay, which can be posed as the solution of an optimization problem. We present two algorithms based on the least mean squares (LMS) and the RLS criteria. The case of an FF MIMO relay requires a different approach because of the bias problem, which is a consequence of the correlation between SI and data signal. We explain the reasons why an optimization approach does not result in the desired SI cancellation and we introduce the algorithms that yield SI cancellation in an FF relay. Finally, we consider the case of an FF MISO relay and present two cancellation algorithms that combine beamforming and SI mitigation. We propose an algorithm that exploits spatial side information and we extend the idea to an algorithm that is able to blindly estimate the AOA of the signal [67, 78]. The chapter ends with a summary of the results.

3.2 Adaptive Filtering

In this section, we present an overview of the fundamentals of adaptive filters. Typically, an adaptive filter system consists of a linear filter whose coefficients are controlled and updated in a sample-by-sample fashion. The use of adaptive techniques started with the work by Widrow in the 1960s and subsequent seminal research in later decades [79–82]. The coefficients of an adaptive filter are governed by the so-called adaptation rule, that establishes how to compute the new coefficients based on previous iterations and the input signal of the filter. The adaptation rule is de-
derived from an optimization criterion, which typically minimizes the power of the error signal, denoted by $e[n]$. Matrix $G[n]$ denotes the adaptive filter, whose coefficients are designed under one of the following criteria:

- **The least mean squares** (LMS) criterion minimizes the MSE, i.e.,
  $$G_*[n] = \arg \min_{G[n]} \mathbb{E}\{||e[n]||^2\},$$ [83–86]. The adaptation rule implements an instantaneous estimate of the gradient of $\mathbb{E}\{||e[n]||^2\}$.

- **The recursive least squares** (RLS) criterion minimizes the weighted squared error, i.e.,
  $$G_*[n] = \arg \min_{G[n]} \sum_{i=0}^{n} \lambda^{n-i}||e[n]||^2,$$ with $0 < \lambda \leq 1$ denoting the forgetting factor [79–81, 87, 88].

The resulting adaptation rule depends on the architecture of the filter and the chosen criterion. But, in general, both criteria result in an adaptation rule of the following form

$$G[n+1] = G[n] + F(G[n], e[n], e[n-1], \ldots)$$ (3.1)

where $F$ is a general function, typically non-linear and recursive when $G[n]$ is of infinite impulse response (IIR) type. Additional criteria have been proposed, from which we highlight the affine projection [79, 80].

The differences between LMS and RLS algorithms are substantial and well-known. We compare LMS and RLS algorithms according to three characteristics: complexity (number of operations per new sample), convergence speed (number of iterations required to reach the steady state) and performance (residual power of the error signal after convergence). The RLS algorithm has a higher complexity per sample than the LMS algorithm, but achieves better results in convergence speed and performance. Concretely, the convergence speed and residual error power are independent of the spectral characteristics of the input signal, whereas the LMS algorithm exhibits properties that are heavily dependent on the spectral characteristic of the input signal.

Table 3.1 highlights the different properties of the two adaptive criteria considered in this chapter [79, 80]. The reader is referred to [79, 80] for a thorough analysis of adaptive filtering techniques.
Table 3.1. Comparison between adaptive algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity</th>
<th>Convergence speed</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>RLS</td>
<td>High</td>
<td>Fast</td>
<td>High</td>
</tr>
<tr>
<td>LMS</td>
<td>Low</td>
<td>Slow</td>
<td>Low</td>
</tr>
</tbody>
</table>

3.3 System Model

The equivalent baseband system model for this chapter is represented in Figure 3.1, from which the particular cases in following sections arise. The considered FD MIMO relay link consists of a source node (S) equipped with $M_S$ transmit antennas, a destination node (D) equipped with $N_D$ receive antennas, and an FD relay node (R) equipped with $M_R$ transmit and $N_R$ receive antennas. The relay link supports the transmission of $m \geq 1$ data streams. At any point in time, $n$, S transmits $\hat{s}[n] \in \mathbb{C}^{M_S \times 1}$, D receives signal $d[n] \in \mathbb{C}^{N_D \times 1}$, and R receives $r[n] \in \mathbb{C}^{N_R \times 1}$ while simultaneously transmitting $\hat{r}[n] \in \mathbb{C}^{M_R \times 1}$. The received signals at R and D are given by

$$r[n] = H_{SR}[n] \star \hat{s}[n] + H_{RR}[n] \star \hat{r}[n] + n_R[n] \quad (3.2)$$

$$d[n] = H_{SD}[n] \star \hat{s}[n] + H_{RD}[n] \star \hat{r}[n] + n_D[n] \quad (3.3)$$

where $H_{ij}[n] \in \mathbb{C}^{N_j \times M_i}$, $i \in \{S, R\}$ and $j \in \{R, D\}$, is the $L_{ij}$th-order channel impulse response matrix between nodes $i$ and $j$. In (3.2) and (3.3), all channels are causal whereas $H_{RR}[n]$ is strictly causal, i.e., $H_{RR}[0] = 0$. The processing block denoted by $D$ represents the relay protocol, and (optional) filter $F_R[n] \in \mathbb{C}^{M_R \times M_R}$ is the relay precoding filter. The $L_{CR}$th-order cancellation filter is denoted by $C_R[n] \in \mathbb{C}^{N_R \times M_R}$.

In the system model, vectors $n_R[n] \in \mathbb{C}^{N_R \times 1}$ and $n_D[n] \in \mathbb{C}^{N_D \times 1}$ are the thermal noise sources in R and D, respectively. They follow an additive white Gaussian noise (AWGN) model and their distributions are

$$n_R[n] \sim \mathcal{CN}(0, \sigma_R^2 I) \quad (3.4)$$

$$n_D[n] \sim \mathcal{CN}(0, \sigma_D^2 I) \quad (3.5)$$

where $\mathcal{CN}(x, \Gamma)$ denotes a circularly-symmetric complex normal distribution with mean $x$ and covariance $\Gamma$. Constants $\sigma_R^2$ and $\sigma_D^2$ denote the noise power per receive antenna. Finally, we introduce vector $z_R[n] \in \mathbb{C}^{N_R \times 1}$ to
Adaptive Self-interference Cancellation in Full-duplex Relays

represent all noise sources at R. From Fig. 3.1 we see that \( z_R[n] \) is composed of two different noise sources: one originating from the receive side of R, \( n_R[n] \), while the other originates from the transmit side of R, \( v_R[n] \), and couples back into the relay through \( H_{RR}[n] \), i.e.,

\[
z_R[n] = H_{RR}[n] \ast v_R[n] + n_R[n]
\]

(3.6)

where \( v_R[n] \sim \mathcal{CN}(0, \delta^2 I) \in \mathbb{C}^{M_R \times 1} \) represents the relay transmit noise and models impairments in the transmission process \([1, 89–94]\).

3.4 Adaptive Self-interference Cancellation for Decode-and-forward MIMO Relays

As explained in the introduction, our proposed method decouples SI cancellation and relay design. The decoding block \( D \) is assumed to be designed independently from the SI cancellation, following a signal regeneration procedure that includes several operations such as time-frequency transformation, error correction, frame alignment, etc. These operations introduce delay and, as a consequence, it is reasonable to make the following assumption

\[
E \{ p[n] \hat{p}^H[n - k] \} = 0, \text{ for all } k > 0
\]

(3.7)

where \( p[n] \in \mathbb{C}^{N_R \times 1} \) and \( \hat{p}[n] \in \mathbb{C}^{M_R \times 1} \) denote the input and output of the decoding block in Fig. 3.1, or, in other words, past samples of the regenerated signal are uncorrelated with the incoming signal. This assumption makes it possible to use the LMS and RLS criteria for SI mitigation. Both criteria require an error signal for the adaptation. The error signal is the signal at the input of the decoding block, i.e.,

\[
p[n] = H_{SR}[n] \ast \hat{s}[n] + H_{RR}[n] \ast \hat{r}[n] + C_R[n] \ast \hat{p}[n] + n_R[n]
\]

(3.8)

Using (3.7) and omitting \( F_R[n] \) without a loss of generality, the power of the error signal (3.8) is minimized with respect to \( C_R[n] \) when

\[
C_R[n] = -H_{RR}[n]
\]

(3.9)

or, in other words, minimization of (3.8) and SI are equivalent. This is a result of (3.7) and, as shown in next section, it is not true for an FF/AF relay. We propose two methods for SI mitigation based on the LMS and RLS criteria:
Figure 3.1. System model of a relay link incorporating the SI cancellation architecture.
• DF-SGD (Decode-and-forward stochastic gradient descent) is based on the LMS criterion. The adaptation rule is an instantaneous approximation of the solution to the following optimization problem

$$\min_{\mathbf{C}_R[n]} \mathbb{E}\{\|\mathbf{p}[n]\|^2\}$$ (3.10)

• DF-RLS (Decode-and-forward recursive least squares) is based on the RLS criterion. The adaptation rule is derived from the solution to the following optimization problem

$$\min_{\mathbf{C}_R[n]} \sum_{i=0}^{n} \lambda^{n-i}\|\mathbf{p}[n]\|^2$$ (3.11)

Table 3.2 summarizes the adaption rules for the LMS-based DF-SGD algorithm and the RLS-based DF-RLS algorithm. Matrix $\mathbf{C}_R$ collects the coefficients of $\mathbf{C}_R[n]$ into a single matrix and vector $\hat{\mathbf{p}}[n]$ collects $L_{CR}$ past samples of $\hat{\mathbf{p}}[n]$, i.e.,

$$\mathbf{C}_R = [\mathbf{C}_R[1] \ldots \mathbf{C}_R[L_{CR}]]$$

$$\hat{\mathbf{p}}[n] = [\hat{\mathbf{p}}^T[n] \ldots \hat{\mathbf{p}}^T[n - L_{CR} + 1]]^T$$ (3.12) (3.13)

The major difference between algorithms is that in the DF-RLS algorithm the adaptation rate $\mu$ is substituted by matrix $T[n]$, which controls the adaptation in different directions.

As shown in [70], the higher computational complexity of the DF-RLS algorithm results in faster convergence time and less residual SI than the DF-SGD algorithm. Both algorithms yield residual SI below noise level and reach stationary state after a few tens of samples.

### 3.5 Adaptive Self-interference Cancellation for Filter-and-forward MIMO Relays

In this section we discuss the problem of SI cancellation for MIMO FF relays. The FF protocol (typically a linear filter) is a simpler relaying technique than the DF protocol and has the advantage of introducing less processing delay due to the absence of signal regeneration. In addition, it does not require to be synchronized with other elements of the network, and therefore, cancellation may work directly with waveform samples. An oversampling factor will result in a bandlimited arbitrary spectrum.
Table 3.2. Stochastic gradient descent and recursive least squares based algorithms for SI cancellation in DF relays.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The DF-SGD algorithm</strong></td>
<td>$C_R[n+1] = C_R[n] - \mu p[n] \hat{p}^H[n-1]$</td>
</tr>
</tbody>
</table>
| **The DF-RLS algorithm** | $T[0] = \epsilon^{-1}I, \; 0 < \epsilon \ll 1$
|                     | $K[n] = T[n-1] \hat{p}[n-1]$ |
|                     | $T[n] = \frac{1}{\lambda} \left( T[n-1] - \frac{K[n]K^H[n]}{\lambda + K^H[n] \hat{p}[n-1]} \right)$ |
|                     | $C_R[n] = C_R[n-1] - p[n] \hat{p}^H[n-1] T[n]$ |

shape. On top of that, the short processing delay in an FF relay and the presence of SI makes the equivalent source-relay-destination channel of IIR type. In other words, the SI and the incoming signal are correlated. As explained in the previous section, power minimization of the signal before $D$ and SI cancellation are equivalent, hence an adaptive optimization solution is suitable for the SI mitigation. However, in the case of an FF relay and because of the correlation between SI and data signal, it cannot be posed as the solution to an optimization problem. Due to the IIR channel caused by the relay, the solution requires a more careful analysis of the problem.

Throughout this section we assume that $N_R = M_R = m$. The relay protocol is $D = G_R$, where $G_R$ is a matrix of size $m \times m$, and transmit filter $F_R[n] = I_m \delta[n]$. Function $\delta[n]$ is defined as

$$\delta[n] = \begin{cases} 
0, & n = 0 \\
1, & n \neq 0 
\end{cases} \quad (3.14)$$

which is known as the Kronecker delta function in discrete time. We define the SI present in the relay by $i[n] = H_{RR}[n] * F_R[n] * \hat{p}[n]$.

To understand the problem behind SI mitigation in a MIMO FF relay, note that the power at the input of the relay, $P_r$, has the following expression:

$$P_r = \mathbb{E}\{\|i[n]\|^2\} + \mathbb{E}\{\|r[n] + z_R[n]\|^2\} + 2\Re\{\mathbb{E}\{i^H[n](r[n] + z_R[n])\}\} \quad (3.15)$$

where the last term in (3.15) is due to correlation between the signal coming from the source and the SI.
However, the minimum of $P_r$ does not lead to SI mitigation, because of the correlation between SI and the incoming signal. In particular, the following relation holds

$$\min_{C_R[n]} P_r \leq \mathbb{E}\{\|\hat{r}[n] + z_R[n]\|^2\}$$  \hspace{1cm} (3.16)

where $\mathbb{E}\{\|\hat{r}[n] + z_R[n]\|^2\}$ corresponds to the case of perfect cancellation, i.e., $i[n] = 0$. For $L_{CR}$ sufficiently large, the minimum of (3.16) is attained when the impulse response of the relay is a prediction filter of the signal $\hat{r}[n] + z_R[n]$ and, therefore, minimization of $P_r$ with respect to $C_R$ only results in SI cancellation when $\hat{r}[n] + z_R[n]$ is temporally white. Otherwise, such approach results in a biased solution. As detailed in Publication II, the bias problem is the suboptimal solution obtained by minimizing the relay input power when the incoming signal and SI are correlated.

The solution to the bias problem is explained in [38, 70], and consists in modifying the trajectories of the adaptive algorithm to reach a different stationary point [81, 95]. The idea is to restore the spectrum of the received signal to its original shape, or, equivalently, to adjust the autocorrelation of the transmit signal. While matrix $G_R$ adjusts the output power, filter $C_R[n]$ adjusts the autocorrelation coefficients up to delay $L_{CR}$.

The algorithms make use of statistical information about the data signal, which is reasonably assumed to be known at the relay. Let the following matrices be defined as

$$\mathcal{R}(0) = \mathbb{E}\{s[n]s^H[n]\}$$  \hspace{1cm} (3.17)

$$\mathcal{R}(L_{CR}) = \mathbb{E}\{s[n][s^H[n - 1] \ldots s^H[n - L_{CR}]\}$$  \hspace{1cm} (3.18)

Additionally, we define vector $\hat{p}[n] = [\hat{p}^T[n] \ldots \hat{p}^T[n - L_{CR} + 1]]^T$, which collects past samples of the transmitted signal $\hat{p}[n]$, and constants $\mu > 0$ and $\rho > 0$, which are the learning rates. Table 3.3 summarizes the two algorithms for SI cancellation in an FF relay. As explained in Publication II, the filter-and-forward instantaneous autocorrelation shaping (FF-IAS) algorithm and the filter-and-forward weighted instantaneous autocorrelation shaping (FF-WIAS) algorithm are able to mitigate the SI, but due to the spectrum restoration principle, they are also able to equalize the SR channel. Under some circumstances, after the algorithm converges, the impulse response from $s[n]$ to $\hat{p}[n]$ is $\hat{p}[n] = Qs[n]$ for some unitary
Table 3.3. Instantaneous autocorrelation shaping and weighted instantaneous autocorrelation shaping algorithms for SI cancellation in FF relays.

<table>
<thead>
<tr>
<th>The FF-IAS algorithm</th>
<th>The FF-WIAS algorithm</th>
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<tbody>
<tr>
<td>(G_{R}[n+1] = G_{R}[n] + \rho(\mathcal{R}(0) - \hat{p}[n]\hat{p}^H[n]))</td>
<td>(G_{R}[n] = G_{R}[n-1] + \rho(\mathcal{R}(0) - \hat{p}[n]\hat{p}^H[n]))</td>
</tr>
<tr>
<td>(C_{R}[n+1] = C_{R}[n] + \mu(\mathcal{R}(L_{CR}) - \hat{p}[n]\hat{p}^H[n-1]))</td>
<td>(C_{R}[n] = C_{R}[n-1] + (\mathcal{R}(L_{CR}) - \hat{p}[n]\hat{p}^H[n-1])T[n])</td>
</tr>
</tbody>
</table>

matrix \(Q\), i.e., data signal has been restored. This can be understood as a best-effort solution when the relay has no available side information.

To show the performance of the algorithm, we respectively define the signal-to-interference ratio at the relay (SIRR) and the signal-to-noise ratio at the relay (SNRR) as

\[
SIRR = \frac{\mathbb{E}\{|\hat{f}[n]|^2\}}{\mathbb{E}\{|H_{RR}[n] \ast F_{R}[n] \ast \hat{p}[n]|^2\}} \\
SNRR = \frac{\mathbb{E}\{|\hat{f}[n]|^2\}}{\mathbb{E}\{|z_{R}[n]|^2\}}
\]

(3.19)  (3.20)

Additionally, convergence time is defined as the number of iterations needed from initialization to a point where the residual SI is reduced by more than 25 dB.

The simulation results in Publication II show the residual SI and convergence time of algorithms FF-IAS and FF-WIAS. Both algorithms result in a residual SI below noise level, with FF-WIAS outperforming FF-IAS by 3-4 dB. Regarding convergence time, FF-WIAS is up to 8 times faster than FF-IAS, which sums up to a few thousand samples.

Figures 3.2 and 3.3 show the signal-to-interference ratio (SIR) after mitigation as a function of the SNR and the SIR before mitigation, for both FF-IAS and FF-WIAS algorithms. For all the tested cases, the residual SI lies below noise level. As seen in Figs. 3.2 and 3.3, the noise level affects
Figure 3.2. Signal-to-interference ratio in dB after mitigation as a function of the SI.
Algorithm AF-IAS.

the performance of the algorithm. Note that both algorithms yield similar results when the SNR is low, i.e., SNR $\sim 5$ dB.

Figures 3.4 and 3.5 show the convergence time as a function of the SNR and the SIR before mitigation for both FF-IAS and FF-WIAS algorithms. The difference between algorithms is notable and, in contrast to the residual SI, convergence time only depends on the SIR before mitigation.

3.6 Adaptive Self-interference Cancellation for Filter-and-forward MISO Relays

We now consider the case of a relay equipped with several receive antennas that supports the transmission of a single data stream. From the system model in Figure 3.1, $N_R \geq 1$ and $M_R = m = 1$. Direct application of the FF-IAS algorithm is not possible, therefore new algorithms are needed. In Publications IV and V, we consider the problem of an FF MISO relay with several receive antennas in a uniform linear array (ULA) configuration. We concretely consider the case of a MISO AF relay, i.e., $N_R \geq 1$, $M_S = M_R = N_D = m = 1$, as shown in Fig. 3.6 and Fig. 3.7. The
Figure 3.3. Signal-to-interference ratio in dB after mitigation as a function of the SI. Algorithm AF-WIAS.

Figure 3.4. Convergence time in samples as a function of the SI. Algorithm AF-IAS.
Adaptive Self-interference Cancellation in Full-duplex Relays

Figure 3.5. Convergence time in samples as a function of the SI. Algorithm AF-WIAS.

Figure 3.6. System model of a MISO AF relay incorporating the SI cancellation architecture with AOA.

Figure 3.7. System model of a MISO AF relay incorporating the SI cancellation architecture without AOA.
Adaptive Self-interference Cancellation in Full-duplex Relays

**Table 3.4.** Instantaneous autocorrelation shaping algorithm with angle-of-arrival information for SI cancellation in MISO FF relays.

<table>
<thead>
<tr>
<th>The FF-IAS algorithm with AOA information</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ g_{R}[n+1] = v(\theta_0) + P(g_{R}[n] - \mu_r[n]p^*[n]) ]</td>
</tr>
<tr>
<td>[ f_{R}[n+1] = f_{R}[n] + \eta(R_0 -</td>
</tr>
<tr>
<td>[ c^{(k)}<em>{R}[n+1] = c^{(k)}</em>{R}[n] + \rho (R_k - \hat{r}[n]\hat{r}^*[n-k]) ]</td>
</tr>
<tr>
<td>[ k = 1, 2, \ldots, L_{CR} ]</td>
</tr>
<tr>
<td>[ P = I_{N_R} - v(\theta_0)v(\theta_0)^H ]</td>
</tr>
</tbody>
</table>

**Table 3.5.** Instantaneous autocorrelation shaping algorithm without angle-of-arrival information for SI cancellation in MISO FF relays.

<table>
<thead>
<tr>
<th>The FF-IAS algorithm without AOA information</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ g_{R}[n+1] = g_{R}[n] + \eta G_{R}[n](R_0f_{R} - \hat{p}[n]\hat{r}^*[n]) ]</td>
</tr>
<tr>
<td>[ c^{(k)}<em>{R}[n+1] = c^{(k)}</em>{R}[n] + \rho G^{-1}<em>{R}[n] (R_k f</em>{R} - \hat{p}[n]\hat{r}^*[n-k]) ]</td>
</tr>
<tr>
<td>[ k = 1, 2, \ldots, L_{CR} ]</td>
</tr>
<tr>
<td>[ G_{R}[n] = \text{diag}{g_{R}[n]}, f_{R} = 1_{N_R} ]</td>
</tr>
</tbody>
</table>

Channel \( h_{SR}[n] \) is modeled in the frequency domain as

\[ h_{SR}(\omega) = \sum_{i=0}^{N_p} \beta_i v(\theta_i) e^{-j\omega \Delta_i / T_s} \tag{3.21} \]

where \( 1/T_s \) is the sampling rate, \( N_p \) is the number of propagation paths, and \( \beta_i \) and \( \Delta_i \) are the gain and delay of each path, respectively. Each path has an associated phase \( \theta_i \) and the AOA of the signal of interest is denoted by \( \theta_0 \). Vector \( v(\theta) \in \mathbb{C}^{N_R \times 1} \) is the normalized response of the ULA:

\[ v(\theta) = 1/\sqrt{N_R} \begin{bmatrix} 1 \ e^{j\theta} \ldots \ e^{j(N_R-1)\theta} \end{bmatrix}^H \tag{3.22} \]

First, we assume that the AOA of the signal of interest is known and, therefore vector \( v(\theta_0) \in \mathbb{C}^{N_R \times 1} \). To combine the cancellation algorithm with the spatial redundancy of the relay, we proceed by decoupling both parts. A suitable criterion for the adaptation of the beamformer \( g_{R} \) is the minimization of the power of the array output \( p[n] \), subject to a unity gain constraint on the desired direction:

\[ \min_{g_{R}} \mathbb{E}\{ |p[n]|^2 \} \quad \text{subject to} \quad g_{R}^H v(\theta_0) = 1 \tag{3.23} \]
Adaptive Self-interference Cancellation in Full-duplex Relays

The solution of (3.23) is the minimum variance distortionless response (MVDR) beamformer, and $g_R$ is accordingly updated using the adaptive implementation of the MVDR beamformer, which is called Frost’s rule [96]:

$$g_R[n + 1] = v(\theta_0) + P(g_R[n] - \mu r[n])p^*[n] \quad (3.24)$$

where the projection matrix $P \in \mathbb{C}^{N_R \times N_R}$ is $P = I_{N_R} - v(\theta_0)v(\theta_0)^H$ and $\mu > 0$ is the learning rate. The cancellation algorithm consisting of filter $c_R[n]$ and coefficient $f_R$ follows the same restoration principle as in previous sections, and is given by

$$f_R[n + 1] = f_R[n] + \eta (R_0 - |\hat{r}[n]|^2) \quad (3.25)$$

$$c_R^{(k)}[n + 1] = c_R^{(k)}[n] + \rho (R_k - \hat{r}[n]\hat{r}^*[n - k]) \quad (3.26)$$

for all the filter taps $k = 1, \ldots, L_{CR}$. Constants $R_k$ are the autocorrelation coefficients $R_k = \mathbb{E}\{s[n]s^*[n - k]\}$, $\eta > 0$ and $\rho > 0$ are the learning rates. Table (3.4) summarizes the SI cancellation algorithm with AOA information for MISO FF relays.

A major drawback of the algorithm is the requirement for information about the AOA of the signal of interest, $\theta_0$, which can be difficult to estimate in the presence of SI. As a consequence, an algorithm that jointly estimates the AOA of the signal and mitigates the SI is a more practical solution.

In Publication V, we propose an algorithm that does not require AOA information. The architecture, see Fig. 3.7, differs slightly with respect to Fig. 3.6, where the combination of the $N_R$ branches takes place before cancellation, whereas in Fig. 3.7 the combination takes place after cancellation. Vector $f_R \in \mathbb{C}^{N_R \times 1}$ is fixed and equal to the all-one vector, i.e.,

$$f_R^T = \begin{bmatrix} 1 & \ldots & 1 \end{bmatrix}_{N_R \text{ times}} \quad (3.27)$$

Matrix $G_R \in \mathbb{C}^{N_R \times N_R}$ is diagonal and its diagonal components are given by the vector $g_R \in \mathbb{C}^{N_R \times 1}$, i.e., $G_R = \text{diag}\{g_R\}$. The adaptation equations for $g_R$ and $c_R[n]$ are given by

$$g_R[n + 1] = g_R[n] + \eta G_R[n](R_0 f_R - \hat{p}[n]\hat{p}^*[n]) \quad (3.28)$$

$$c_R^{(k)}[n + 1] = c_R^{(k)}[n] + \rho G_R^{-1}[n](R_k f_R - \hat{p}[n]\hat{p}^*[n - k]) \quad (3.29)$$
for $k = 1, \ldots, L_{\text{CR}}$. Constants $R_k$ are the autocorrelation coefficients $R_k = \mathbb{E}\{s[n]s^*[n-k]\}$, $\eta > 0$ and $\rho > 0$ are learning rates. Note that the inversion operation in (3.29) is trivial because $G_R$ is a diagonal matrix. Table 3.5 summarizes the SI cancellation algorithm for MISO FF relays without AOA information. It is possible to show that, when $h_{\text{SR}}[n]$ is of minimum-phase, the algorithm achieves phase alignment of the signals at the $N_R$ branches. This desirable property is obtained without knowledge about the SR channel.

3.7 Summary

In this chapter we have presented the contributions related to SI cancellation for FD relays. We introduced various blind algorithms for FF relays that offer a tradeoff between complexity and convergence speed. SI mitigation techniques that do not need side information enable the relay to work without synchronization with the rest of the network, i.e., the relay can work directly with waveform samples. This feature is specially important in FF relays, wherein the relay processes the signal without any regeneration mechanism, thus maintaining a minimal processing delay.

The algorithms are based on power minimization, FF-IAS, and RLS criteria, FF-WIAS. While the FF-IAS algorithm requires less computation per sample, it exhibits a worse performance and convergence time than the FF-WIAS algorithm, which in turn needs more operations per new sample. Both algorithms use the same autocorrelation coefficients for simultaneous mitigation and equalization.

We introduced two different algorithms for SI cancellation in MISO FF relays. The first proposed algorithm combines a beamforming and a cancellation algorithm to mitigate the SI and equalize the SR channel, but it requires information about the AOA of the signal of interest. The second proposed algorithm is blind and, therefore, it does not need any side information. In addition to mitigation of the SI and equalization of the SR channel, the algorithm is able to achieve phase alignment of the $N_R$ branches, or, in case that the receive array is an ULA, to estimate the AOA of the signal of interest.
4. Optimal Linear Filter Design for Full-duplex Relay Links

4.1 Introduction

In this chapter we study the problem of linear filter design for FD MIMO relay links under limited dynamic range [10–12, 97]. A linear filter solution, though suboptimal in terms of the bit error rate, constitutes a compromise solution between performance and complexity. The linear property of the convolution operation makes it possible to pose the design problem, under the MSE or the SNR criteria, as a convex problem, for which efficient off-the-shelf solvers exist. However, the presence of a concatenation of several filters in a relay link results in a non-convex problem. A common way to solve this kind of problem is by means of an alternating optimization procedure, in which only one variable is optimized at each iteration, while the remaining variables are kept fixed. For an FD MIMO relay link setting, this mechanism makes possible to approximate a non-convex problem into a set of convex subproblems in each variable. The particular solution depends on the initialization point and may differ from the global optimum. The convergence of the method is ensured whenever individual subproblems are convex in each variable [98].

As seen in Chapter 2, FD devices suffer from SI distortion as a consequence of simultaneous transmission and reception in the same physical channel. Due to the power imbalance of the relay between transmit and receive power, SI can be up to 100 dB larger than the incoming signal, representing the major source of distortion in an FD device [1–4, 8, 9]. Physical isolation and analog cancellation are insufficient to mitigate the SI to a degree where the dynamic range of the receiver is not reduced. Several
models for dynamic range have been proposed [10, 99, 100]. Non-linear models offer a more accurate description of the system, but result in a complicated design difficult to integrate within a SI mitigation scheme. A linear model consisting of a statistically independent noise source whose power is a constant fraction of the received power is able to recreate the dynamic range distortion while being a tractable problem, as explained in [10]. Due to imperfections during AD/DA conversion of the signal and distortion in the PA, any impairment of the modulation process and the demodulation process will limit the dynamic range at both transmission and reception sides [10]. In Publications VIII and IX we propose two methods for the maximization of the SINR in a DF FD relay subjected to limited dynamic range. In Publication VIII we approximate the non-convex objective function by decoupling the relay transmit and receive filters and, in Publication IX, we solve the problem by means of an alternating optimization procedure [98, 101]. Additional constraints are imposed to ensure that no trivial solutions are feasible.

In Publication III, we study the problem of joint design for FF FD relay links with direct propagation between source and destination, and a relay-to-destination (RD) channel [42, 51, 72–74, 102, 102–105, 105–107]. Similarly to Publication IX, the non-convex problem is solved by means of an alternating procedure. The IIR channel caused by an AF/FF relay turns any optimal design intractable. We propose a constrained design that assigns some degrees of freedom to ensure a minimal SI distortion at the relay. In a realistic scenario, the available CSI is imperfect and contaminated by noise, because of the finite and noisy observations during estimation. The deviation of the estimated channel from its actual value, the so-called channel uncertainty, may lead to severe degradation of the performance, which, in the particular case of an FD device, may intensify due to the power imbalance between SI and the incoming signal. In order to present a design that accounts for imperfect CSI, a model of the channel uncertainties is needed. Different models for channel uncertainty have been proposed [108–110]. A common model assumes the estimation error power to be bounded by a maximum value, while a design that accounts for channel uncertainties and minimizes their impact is called a robust design. In Publication X, we propose a method for robust worst-case MSE optimization in an FD relay with limited dynamic range.
4.2 System Model

Figure 4.1 depicts the general system model, from which the particular cases of the following sections arise [70, 111–114]. The MIMO FD link consists of a source node, S, equipped with $M_S$ transmit antennas, a destination node, D, equipped with $N_D$ receive antennas, and a relay node, R, equipped with $N_R$ receive antennas and $M_R$ transmit antennas. At any discrete-time instant, denoted by index $n$, S transmits signal $\hat{s}[n] \in \mathbb{C}^{M_S \times 1}$, D receives signal $d[n] \in \mathbb{C}^{N_D \times 1}$, and R receives signal $r[n] \in \mathbb{C}^{N_R \times 1}$ while simultaneously transmitting signal $\hat{r}[n] \in \mathbb{C}^{M_R \times 1}$. The number of independent data streams is $m \geq 1$. The $L_{ij}$th-order channel between node $i$ and node $j$, where $i \in \{S, R\}$ and $j \in \{R, D\}$, is denoted by $H_{ij}[n] \in \mathbb{C}^{N_j \times M_i}$.

The set of filters under design is $\{F_S[n], G_R[n], C_R[n], F_R[n], G_D[n]\}$, see Fig. 4.1. At S, the $L_{S}$th-order filter $F_S[n] \in \mathbb{C}^{M_S \times m}$ precodes data signal $s[n] \in \mathbb{C}^{m \times 1}$

$$\hat{s}[n] = F_S[n] \ast s[n]$$  (4.1)

At R, the $L_{GR}$th-order filter $G_R[n] \in \mathbb{C}^{m \times N_R}$ filters the post-cancellation signal,

$$p[n] = G_R[n] \ast (r[n] + C_R[n] \ast \hat{r}[n])$$  (4.2)

where the $L_{CR}$th-order filter $C_R[n] \in \mathbb{C}^{N_R \times M_R}$ removes the SI. Block $D$ represents the relay protocol, from which we consider the following,

- **Filter-and-forward.** The relay performs a linear transformation of the received signal before transmission. No regeneration scheme is applied. Concretely, $D = I_m$ and the transmit signal is $\hat{r}[n] = F_R[n] \ast G_R[n] \ast r[n]$.

- **Amplify-and-forward.** This is a particular case of the FF protocol in which $L_{GR} = L_{FR} = 0$, so the linear transformation only shapes the spatial distribution of the transmitted signal, i.e., $\hat{r}[n] = F_R G_R r[n]$.

- **Decode-and-forward.** In this case, $D$ represents an arbitrary decoding-encoding function fulfilling the following assumption: the processing delay introduced by $D$ is long enough to decorrelate both input and output signals, i.e., [70]:

$$E\{p[n]p^H[n-j]\} = 0, \text{ for } j > 0$$  (4.3)
Figure 4.1. System model of a relay link with linear filters.

Optimal Linear Filter Design for Full-duplex Relay Links
Note that this assumption (4.3) is reasonable due to the processing delay introduced by time-to-frequency and frequency-to-time conversion (at least one OFDM symbol), data scrambling, frame alignment and additional upper layer operations.

The $L_{FR}$th-order filter $F_R[n] \in \mathbb{C}^{M_R \times m}$ precodes the output of block $D$, $\hat{p}[n] \in \mathbb{C}^{m \times 1}$, before transmission

$$f[n] = F_R[n] \ast \hat{p}[n] \quad (4.4)$$

Finally, at D, the $L_D$th-order filter $G_D[n] \in \mathbb{C}^{m \times N_D}$ transforms the received signal, $d[n] \in \mathbb{C}^{N_D \times 1}$

$$\hat{d}[n] = G_D[n] \ast d[n] \quad (4.5)$$

We denote the maximum transmit power of S and R by $P_S > 0$ and $P_R > 0$, i.e.,

$$E \{ \| F_S[n] \ast s[n] \|^2 \} \leq P_S \quad (4.6)$$

$$E \{ \| F_R[n] \ast p[n] \|^2 \} \leq P_R \quad (4.7)$$

Thermal noise, which captures the distortion due to electronic imperfections, is modeled as an additive distortion that follows a circularly-symmetric complex normal distribution, spectrally white and statistically independent of the received signal. In the system model, vectors $n_R[n] \in \mathbb{C}^{N_R \times 1}$ and $n_D[n] \in \mathbb{C}^{N_D \times 1}$ are the thermal noise sources in R and D, respectively. They follow an AWGN model and their distributions are

$$n_R[n] \sim \mathcal{CN}(0, \sigma_R^2 I) \quad (4.8)$$

$$n_D[n] \sim \mathcal{CN}(0, \sigma_D^2 I) \quad (4.9)$$

where $\mathcal{CN}(x, \Gamma)$ denotes a circularly-symmetric complex normal distribution with mean $x$ and covariance $\Gamma$. Constants $\sigma_R^2$ and $\sigma_D^2$ denote the noise power per receive antenna. Though thermal noise is able to capture a significant part of the distortion in the system, there are other sources of distortion affecting the performance. Dynamic range captures the accuracy of the system during AD and DA conversion, and modulation/demodulation [10]. Such imperfections are modeled as additional noise sources whose power depends on the feeding signal. Vectors $v_S[n] \in \mathbb{C}^{M_S \times 1}$ and $v_R[n] \in \mathbb{C}^{M_R \times 1}$ model the limited dynamic range at the transmit sides of S and R, respectively. They follow a circularly-symmetric complex normal distribution, they are white and statistically independent of
the transmit signals $s[n]$ and $\tilde{r}[n]$,

\[
v_s[n] \sim \mathcal{CN}(0, \delta_S \text{ diag } \{E\{s[n]s^H[n]\}\}) \tag{4.10}
\]

\[
v_r[n] \sim \mathcal{CN}(0, \delta_R \text{ diag } \{E\{\tilde{r}[n]\tilde{r}^H[n]\}\}) \tag{4.11}
\]

with $0 \leq \{\delta_S, \delta_R\} \ll 1 \{10\}$. Operator $\text{diag}(A)$ yields a square matrix with the same diagonal elements as $A$ and zero elsewhere.

Vectors $w_R[n] \in \mathbb{C}^{N_R \times 1}$ and $w_D[n] \in \mathbb{C}^{N_D \times 1}$ model the limited dynamic range at the receive sides of $R$ and $D$, respectively. They follow a circularly-symmetric complex normal distribution, they are white and statistically independent of the signals prior to digital conversion $\tilde{r}[n]$ and $\tilde{d}[n]$,

\[
w_R[n] \sim \mathcal{CN}(0, \epsilon_R \text{ diag } \{E\{\tilde{r}[n]\tilde{r}^H[n]\}\}) \tag{4.12}
\]

\[
w_D[n] \sim \mathcal{CN}(0, \epsilon_D \text{ diag } \{E\{\tilde{d}[n]\tilde{d}^H[n]\}\}) \tag{4.13}
\]

with $0 \leq \{\epsilon_R, \epsilon_D\} \ll 1 \{10\}$. Note that constants $\{\delta_S, \epsilon_R, \delta_R, \epsilon_D\}$ set an upper limit on the SNR at transmission and reception independently of the signal power. The aggregated noise components at $R$ and $D$ are denoted by $z_R[n] \in \mathbb{C}^{N_R \times 1}$ and $z_D[n] \in \mathbb{C}^{N_D \times 1}$ respectively, which are given by

\[
z_R[n] = n_R[n] + w_R[n] + H_{RR}[n] \ast v_R[n] + H_{SR}[n] \ast v_S[n] \tag{4.14}
\]

\[
z_D[n] = n_D[n] + w_D[n] + H_{SD}[n] \ast v_S[n] + H_{RD}[n] \ast v_R[n] \tag{4.15}
\]

Finally, we assume that higher-order noise terms are negligible, i.e., $\delta_i \epsilon_j \approx 0$, $\delta_i \sigma_j^2 \approx 0$, $\epsilon_i \sigma_j^2 \approx 0$, $\delta_i \delta_j \approx 0$ and $\epsilon_i \epsilon_j \approx 0$, or in other words, noise components only depend on their respective information signal.
4.3 Design Criteria

In order to design filters \( \{ F_S[n], G_R[n], C_R[n], F_R[n], G_D[n] \} \), we consider the following formulae as criteria:

- The post-processing signal-to-interference-plus-noise-ratio at R is
  \[
  \text{SINRR}_R = \frac{\mathbb{E}\left\{ \| G_R[n] \ast H_{SR}[n] \ast F_S[n] \ast s[n]\|^2 \right\}}{\mathbb{E}\left\{ \| G_R[n] \ast (z_R[n] + (H_{RR}[n] + C_R[n]) \ast F_R[n] \ast \hat{p}[n])\|^2 \right\}}
  \] (4.16)

- The post-processing mean square error at R is
  \[
  \text{MSE}_R(\tau) = \mathbb{E}\left\{ \| G_R[n] \ast (H_{SR}[n] \ast F_S[n] \ast s[n] + z_R[n]) - s[n - \tau]\|^2 \right\}
  \] (4.17)

- The post-processing mean square error at D is
  \[
  \text{MSE}_D(\tau) = \mathbb{E}\left\{ \| G_D[n] \ast (H_{EQ}[n] \ast F_S[n] \ast s[n] + z_D[n]) - s[n - \tau]\|^2 \right\}
  \] (4.18)

where constant \( \tau \geq 0 \) is a design parameter [115–117], and matrix \( H_{\text{EQ}}[n] \in \mathbb{C}^{N_D \times M_S} \) is the impulse response of the equivalent channel from S to D.

4.4 SINR Optimization in a Decode-and-forward MIMO Relay

The content of this section summarizes the work in Publications VIII and IX, wherein we consider the DF relay depicted in Figure 4.2. The decoding block \( D \) regenerates the signal before retransmission, therefore, any imprecision or error during regeneration will propagate to the destination. In combination with the power imbalance between data signal and SI at the relay, it is reasonable to off-load the optimal design of the RD hop to the destination (filter \( G_D[n] \)), while the resources from the relay will be dedicated to optimize performance in the relay itself. By considering the signal-to-interference-plus-noise ratio after processing, i.e., SINR, filters \( G_R[n], F_R[n] \) and \( C_R[n] \) are designed as a solution to the following optimization problem

\[
\begin{align*}
\max_{\{ G_R[n], F_R[n], C_R[n] \}} & \quad \text{SINR}_R \\
\text{subject to} & \quad \mathbb{E}\{\| \hat{r}[n]\|^2 \} \leq P_R
\end{align*}
\] (4.19)
From inspection of (4.16), the optimal $C_R[n]$ is obtained as $C_R[n] = -H_{RR}[n]$, i.e., the cancellation filter destructively interferes with the SI given by $H_{RR}[n] * F_R[n] * \hat{d}[n]$. As a consequence, $L_{CR} \geq L_{RR}$, i.e., the number of coefficient taps of $C_R[n]$ must be sufficiently large. Note that, due to limitations of the architecture, the cancellation filter can not mitigate the transmit noise component that feeds back into the relay, i.e., $H_{RR}[n] * v_R[n]$.

Problem (4.19) transforms into

$$
\max_{\{G_R[n], F_R[n]\}} \frac{\mathbb{E}\{\|G_R[n] * H_{SR}[n] * F_S[n] * s[n]\|^2\}}{\mathbb{E}\{\|G_R[n] * z_R[n]\|^2\}}
$$

subject to

$$
\mathbb{E}\{\|F[n]\|^2\} \leq P_R
$$

Problem (4.20) presents some technical challenges. First, the trivial solution $F_R[n] = 0$ is optimal. Note that when $F_R[n] = 0$, transmit noise power $\mathbb{E}\{v_R[n]v_R^H[n]\} = 0$ and (4.20) is maximized. This results in disruption of the RD communication and must be avoided. Ideally, $F_R[n]$ is designed to optimize the performance of the RD hop, e.g., to maximize the SINR at destination. But, in general, such approach demands a use of all the degrees of freedom in $F_R[n]$ (unique solution) or makes (4.20) difficult to solve (non-linear manifold of $F_R[n]$). We propose a compromise solution that ensures a predefined and controlled distortion at destination while designating some of the degrees of freedom in $F_R[n]$ to solve (4.20).

In particular, we tackle trivial solutions by imposing linear constraints on $F_R[n]$. In Publication VIII, these constraints are embedded in $F_R[n]$ by fixing the equivalent RD channel to a targeted value, whereas in Publication IX, we combine the use of channel shortening and subspace projection. Additionally, filters $G_R[n]$ and $F_R[n]$ are coupled through (4.16), which makes (4.20) a non-convex problem. In Publication VIII, we decouple filters $F_R[n]$ and $G_R[n]$ by modifying the objective function, whereas in Publication IX we use an alternating maximization procedure described in Algorithm 1. In what follows, we explain the different approaches used in Publications VIII and IX to solve (4.20).

### 4.4.1 Optimal Solution for the Relay Transmit Filter $F_R[n]$

The trivial solution of (4.20), $F_R[n] = 0$, is unacceptable from a design point of view because it causes disruption of the RD link. Therefore, we must modify (4.20) and introduce new constraints on $F_R[n]$ to prevent trivial solutions and ensure that the received signal at destination undergoes
Algorithm 1 Alternating filter design procedure

1: **Initialization point**: $F_R^{(0)}[n]$, $G_R^{(0)}[n]$.
2: **repeat** for each iteration $i = 1, 2, 3, \ldots$
3: Solve (4.20) with respect to $F_R^{(i)}[n]$ for fixed $G_R^{(i-1)}[n]$.
4: Solve (4.20) with respect to $G_R^{(i)}[n]$ for fixed $F_R^{(i)}[n]$.
5: **until** the convergence criterion is met.

If we impose linear constraints, the resulting problem is equivalent to (4.20) with a change of variable. The new variable, denoted by $w$, lies in a subspace of $F_R[n]$ and is related to $F_R[n]$ by an affine transformation. Using the fact that any filtering operation, due to the linear property of the convolution operation, can be expressed as a matrix multiplication, in Publication VIII we impose linear constraints that equalize the RD channel to a predefined impulse channel response $H_{RD}^{EQ}[n]$, i.e.,

$$H_{RD}[n] \ast F_R[n] = H_{RD}^{EQ}[n] \quad (4.21)$$

The set of solutions of (4.21) is, with respect to $F_R[n]$, a linear subspace whose dimension is $m(M_R(L_{FR} + 1) - N_D(L_{FR} + L_{RD} + 1))$, provided that the convolution matrix of $H_{RD}[n]$ is of full-rank and the following condition holds

$$M_R(L_{FR} + 1) > N_D(L_{FR} + L_{RD} + 1) \quad (4.22)$$

or, equivalently, $M_R > N_D$, i.e., the number of transmit antennas at the relay must be greater than the number of receive antennas at the destination, and $L_{FR} > (M_R L_{RD})/(M_R - N_D) - 1$ sufficiently large. The reduction in degrees of freedom of $F_R[n]$ will also reduce the achievable SINR, so the selection of $H_{RD}^{EQ}[n]$ will impact the performance of both hops. In Publication VIII we do not explore different criteria for selecting $H_{RD}^{EQ}[n]$ and we assume it to be predefined beforehand.

Another technical issue with (4.20) is that $F_R[n]$ and $G_R[n]$ are coupled by expression $\mathbb{E}\{\|G_R[n] \ast z_R[n]\|^2\}$. To decouple $F_R[n]$ from $G_R[n]$, we proceed as follows: since $z_R[n]$ is a function of all the incoming signals at the relay and the high gain of the SI channel, $F_R[n]$ is calculated as to
minimize the SI plus noise that feeds back into the relay,

$$\min_{\{F_R[n]\}} \mathbb{E}\{\|H_{RR}[n] \ast (F_R[n] \ast \hat{p}[n] + v_R[n])\|^2\}$$

subject to

$$\mathbb{E}\{\|\hat{r}[n]\|^2\} \leq P_R$$

$$H_{RD}[n] \ast F_R[n] = H_{RD}^{EQ}[n]$$

(4.23)

As explained in Publication VIII, the linear constraints in (4.23) are equivalent to a change of variable which transforms (4.23) into a least squares problem with a quadratic inequality constraint (LSQI) that can be solved semi-analytically [101]. Once $F_R[n]$ is obtained, the next step is to calculate $G_R[n]$, see section 4.4.2. With this method, Algorithm 1 only takes one iteration.

In Publication IX, we propose a different set of constraints consisting in the combination of channel shortening and subspace projection. Note that constraints (4.21) specify the whole impulse response of the combined channel $H_{RD}[n] \ast F_R[n]$. With the use of a multicarrier modulation that employs a CP, only those coefficients exceeding the CP duration need to be equalized or set to zero. Channel shortening was studied in [118–123] as a time-domain equalization method for multicarrier modulations that aimed to mitigate the paths whose delays exceeded the CP duration. Channel shortening reduces the effective order of $H_{RD}[n] \ast F_R[n]$ by setting those filter taps exceeding a certain delay $\tau$ to zero, i.e.,

$$H_{RD}[n] \ast F_R[n] = 0, n = \tau + 1, \ldots, L_{RD} + L_{FR}$$

(4.24)

The set of solutions of (4.24) with respect to $F_R[n]$ forms a linear subspace whose dimension is $m(M_R(L_{FR} + 1) - N_D(L_{FR} + L_{RD} - \tau))$, provided that the reduced convolution matrix of $H_{RD}[n]$ is of full-rank and the following condition holds

$$M_R(L_{FR} + 1) > N_D(L_{FR} + L_{RD} - \tau)$$

(4.25)

Though (4.24) requires less degrees of freedom than (4.21), it does not prevent trivial solutions. In order to overcome that issue, we impose another set of constraints on $F_R[n]$, which are designed to ensure that the received power at destination is, at least, some fraction of the maximal possible delivered power. The received signal power at D depends on two parameters: the transmit power and the direction of transmission. The transmit power can be adjusted to $P_R$ by scaling $F_R[n]$ by some factor.
As explained in Publication IX, by choosing $F_R[n]$ to lie in a subspace generated by the set of eigenvectors associated with the largest eigenvalues of a correlation matrix depending on the RD channel and the data signal, the received signal power at D will be a fraction of the maximum delivered power. The intersection of the two sets of constraints can be expressed as a subspace projection, and the optimal $F_R[n]$ as the eigenvector associated with the minimum eigenvalue of the correlation matrix, see Publication IX for details. Hence, from (4.20), let $F_R(t)$ be a function of the vector $t$, where $t$ is the solution to

$$\min_t \quad t^HQt$$

subject to $\mathbb{E}\{\|\hat{r}[n]\|^2\} = P_R$  \hspace{1cm} (4.26)

### 4.4.2 Optimal Solution for the Relay Receive Filter $G_R[n]$

After we have obtained the optimal $F_R[n]$, we solve (4.20) with respect to $G_R[n]$, i.e.,

$$\max_{\{G_R[n]\}} \quad \text{SINR}_R$$

Problem (4.27) is identified as a generalized eigenvalue problem for which an analytic solution is possible. Note that in Publication VIII, Algorithm 1 takes a single iteration due to both filters being decoupled. In Publication IX, both filters are coupled so Algorithm 1 takes several iterations to converge.

### 4.4.3 Discussion of the Results

Results show that, under similar conditions, the method in Publication IX outperforms the method in Publication VIII by some dBs. This is a consequence of the joint design of both filters and a more efficient allocation of the degrees of freedom of filter $F_R[n]$. Both methods show a similar behaviour that depends on the dynamic range of the relay. We observe two cases: for a large dynamic range, the ratio $N_R/M_R$ is a good estimator of the achievable SNR, whereas for a low dynamic range, $M_R$ is a suitable indicator of the performance.
4.5 MSE Optimization in a Filter-and-forward MIMO Relay Link

This section comprises the work in Publication III, where we propose a method for optimal linear filter design in an FF relay link under the MMSE criterion. As seen in Chapter 3, the design of an FF relay is significantly more involved than its DF counterpart, due to the correlation between received signal and SI. Additionally, as explained in Chapter 2, an FF protocol yields worse performance than a DF protocol. In spite of that, a relay implementing an FF protocol offers significant benefits in terms of latency and complexity, with respect to its DF counterpart. First, since the relay just filters and forwards the received signal, an FF relay may operate directly with waveform samples. This avoids synchronization with the network and its entailing timing recovery, which would introduce latency and depends on the SNR of the signal. Second, since only a time domain approach is feasible when working with waveform samples, the unavoidable delay of at least one OFDM symbol, consequence of time-to-frequency and frequency-to-time transformations, does not apply. As a result, an FF relay working in the time domain with waveform samples is an attractive solution to keep a low latency across the link.

Let us consider the system model of Fig. 4.3, which supports a single data stream. The relay implements an FF protocol, i.e., \( m = 1 \), \( D = I_m \) and \( C_R[n] = 0 \), see Fig. 4.3. Filter \( G_R[n] \in \mathbb{C}^{M_R \times N_R} \) denotes the convolution \( F_R[n] * G_R[n] \) in Fig. 4.1. Therefore,

\[
\hat{r}[n] = G_R[n] * r[n]
\]

Using the \( \text{MSE}_D(\tau) \), we design filters \( \{f_S[n], G_R[n], g_D[n]\} \) as a solution
Algorithm 2 Alternating filter design procedure

1: Initialization point: $f_S^{(0)}[n], G_R^{(0)}[n], g_D^{(0)}[n]$.
2: repeat for each iteration $i = 1, 2, 3, \ldots$
3: Solve (4.30) with respect to $g_D^{(i)}[n]$ for fixed $G_R^{(i-1)}[n]$ and $f_S^{(i-1)}[n]$.
4: Solve (4.30) with respect to $G_R^{(i)}[n]$ for fixed $g_D^{(i)}[n]$ and $f_S^{(i-1)}[n]$.
5: Solve (4.30) with respect to $f_S^{(i)}[n]$ for fixed $g_D^{(i)}[n]$ and $G_R^{(i)}[n]$.
6: until the convergence criterion is met.

to the following problem

$$\min_{\{f_S[n], G_R[n], g_D[n]\}} \text{MSED}(\tau)$$

subject to

$$\mathbb{E}\{\|\hat{s}[n]\|^2\} \leq P_S$$

$$\mathbb{E}\{\|\hat{r}[n]\|^2\} \leq P_R$$

(4.29)

Problem (4.29) presents some technical challenges. Note that since filters $\{f_S[n], G_R[n], g_D[n]\}$ are coupled, (4.29) is clearly a non-convex problem. As explained in previous section, this issue is solved by means of an alternating procedure where one filter is designed at a given time in a sequential fashion. The IIR channel of the relay is difficult to handle computationally because no closed form is available. This turns (4.29) into a recursive problem for which no efficient solvers exist. In order to partially circumvent the IIR nature of the relay, we introduce the following SI-free constraints:

$$\min_{\{f_S[n], G_R[n], g_D[n]\}} \text{MSED}(\tau)$$

subject to

$$\mathbb{E}\{\|\hat{s}[n]\|^2\} \leq P_S$$

$$\mathbb{E}\{\|\hat{r}[n]\|^2\} \leq P_R$$

$$H_{RR}[n] \ast G_R[n] = 0$$

(4.30)

Constraints $H_{RR}[n] \ast G_R[n] = 0$ are equivalent to making the SI negligible, i.e., $H_{RR} \ast \hat{p}[n] \approx 0$ or, in other words, the relay has an finite impulse response (FIR) duration. Nevertheless, it comes at the price of reducing the number of free parameters in $G_R[n]$. With the SI-free constraints and the alternating procedure of Algorithm 2, (4.30) is a convex problem in each variable, for which efficient solvers exist.
4.5.1 Optimal Solution for the Destination Receive Filter $g_D[n]$

This section solves Step 3 of Algorithm 2. When $G_R[n]$ and $f_S[n]$ are fixed, (4.30) transforms into

$$\min_{\{g_D[n]\}} \text{MSE}_D(\tau)$$

(4.31)

Note that, since $g_D[n]$ is at the receive side of D, its optimal expression will correspond to that of the solution to the normal equations of linear regression [124]. Concretely, the optimal expression for $g_D[n]$ consists of the product of the inverse autocorrelation matrix of the input signal and a function of the autocorrelation of the data signal.

4.5.2 Optimal Solution for the Relay Receive Filter $G_R[n]$

The next step of Algorithm 2 is to design $G_R[n]$ as the solution to (4.30) assuming both $F_S[n]$ and $G_D[n]$ fixed, i.e.,

$$G^*_R[n] = \arg \min_{G_R[n]} \mathbb{E}\{|\hat{d}[n] - \hat{s}[n - \tau]|^2\}$$

subject to

$$\mathbb{E}\{|\hat{r}[n]|^2\} \leq P_R$$

$$H_{RR}[n] * G_R[n] = 0$$

(4.32)

Problem (4.32) is not convex with respect to $G_R[n]$, because $z_R[n]$ exhibits a higher-order relation with $G_R[n]$. In order to transform it into a convex problem with respect to $G_R[n]$, we make use of values of $G_R[n]$ obtained from previous iterations. That way, a higher order dependence with $G_R[n]$ is also avoided. The expected power of $z_R[n]$ is therefore approximated as

$$\mathbb{E}\{||z_R[n]||^2\} \approx \mathbb{E}\{||G_R^{(k)}[n] * n_R[n]||^2\} + \mathbb{E}\{||v_R[n]||^2\}$$

$$+ \mathbb{E}\{||G_R^{(k)}[n] * H_{SR}[n] * v_S[n]||^2\}$$

$$+ \mathbb{E}\{||G_R^{(k)}[n] * w_R[n]||^2\}$$

$$+ \mathbb{E}\{||G_R^{(k-1)}[n] * H_{RR}[n] * v_R[n]||^2\}$$

(4.33)

By using the approximation in (4.33), $\mathbb{E}\{||z_R[n]||^2\}$ has a second-order relation with current iteration $G_R^{(k)}[n]$, and (4.32) can be cast as a constrained least squares problem.

The SI suppression constraints in (4.32), $H_{RR}[n] * G_R[n] = 0$, force the possible solutions to lie within a subspace of reduced dimension. For a nontrivial solution to exist, the number of transmit antennas must, in general, exceed the number of receive antennas ($M_R > N_R$), and the order
of the relay filter must satisfy $L_{GR} \geq N_R(L_{RR} - 1)/(M_R - N_R)$. Since convolution constraints can be written as linear constraints, (4.32) is equivalent to an unconstrained problem after a change of variable. Denoting the new variable as $w$, (4.32) is an LSQI of the form [124]:

$$
\begin{align*}
\mathbf{w}^* &= \arg \min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{T} \mathbf{w} + \Re\{\mathbf{w}^H \mathbf{t}\} + t \\
\text{subject to} \quad \mathbf{w}^H \mathbf{Q} \mathbf{w} + \Re\{\mathbf{w}^H \mathbf{q}\} + q &\leq P_R 
\end{align*}
$$

(4.34)

whose solution is obtained by making use of the theory in [124]. Concretely, (4.34) is solved in several steps. First, by means of the generalized singular value decomposition of $\mathbf{T}$ and $\mathbf{Q}$, an equivalent problem is obtained for which a semi-analytical solution is calculated. Second, the Lagrange multiplier is calculated by finding the roots of a secular equation [124].

4.5.3 Optimal Solution for the Source Transmit Filter $f_S[n]$

As a final step and in order to complete an iteration of the alternating algorithm, we must solve (4.30) with respect to $f_S[n]$ for both $G_R[n]$ and $g_D[n]$ fixed, i.e.,

$$
f_S^*[n] = \arg \min_{f_S[n]} \quad \mathbb{E}\{|\hat{d}[n] - \hat{s}[n - \tau]|^2\}
$$

subject to

$$
\mathbb{E}\{\|\hat{s}[n]\|^2\} \leq P_S
$$

(4.35)

Since $f_S[n]$ is located at node S, its optimal expression will be that of a precoding filter tailored to the channel from the source up to the destination. Similarly to previous step, problem (4.35) can be cast as an LSQI problem and solved by using the theory in [124].

4.5.4 Discussion of the Results

From the simulations in Publication III we can extract the following conclusions. As depicted in Fig. 4.4, the presence of the direct path between source and destination, $H_{SD}[n]$, has a strong impact on the final MSE. When the RD channel does not support a high SNR, a direct path between source and destination can improve the MSE by 5 dB. This improvement drops to a single dB when the RD channel does support a high SNR.

Regarding the influence of the SR and RD hops, we obtain the following results: For low SNR$_{SR}$, increasing SNR$_{RD}$ by 10 dB results in an MSE
improvement of 3 dB, whereas, when $\text{SNR}_{\text{SR}}$ is large, the same operation results in an MSE improvement of 5.5 dB. Therefore, a system where the SR hop supports higher SNR values than the RD hop is preferable. Figure 4.5 compares the influence of the limited dynamic range in the SNR at destination $\text{SNR}_{\text{D}}$, for different values of $\text{SNR}_{\text{SR}}$ and $\text{SNR}_{\text{SD}}$. Dashed lines depict the unlimited dynamic range (u.d.r.) cases, $\delta_S = \epsilon_R = \delta_R = \epsilon_D = 0$, whereas solid lines depict the limited dynamic range (l.d.r.) cases, $\delta_S = \delta_R = -30$ dB and $\epsilon_R = \epsilon_D = -20$ dB. The gap between l.d.r and u.d.r. cases is of approximately 1–2 dB, and it increases alongside $\text{SNR}_{\text{SD}}$. Finally, we underline the difference in performance between an AF and an FF relay. For all the tested cases, the FF relay outperforms the AF relay by 2–3 dB. Consequently, FF protocols have clearly the edge in performance over AF protocols.

4.6 Robust MSE Optimization in a Decode-and-forward MIMO Relay

While in previous sections we have considered perfect CSI available at any time, in a practical implementation, only limited or imperfect CSI is available. A finite number of noisy observations during channel estimation results in the presence of estimation noise. The difference between
Figure 4.5. SNRD versus SNRsr for cases of limited dynamic range (l.d.r.) and unlimited dynamic range (u.d.r.).

Figure 4.6. System model of a DF relay link with the SI mitigation architecture.

CSI and the actual channel impulse response may significantly drop the system performance [108, 109]. A design that accounts for channel uncertainty or limited CSI is able to partially mitigate the performance drop. Several models for channel uncertainty have been proposed [108–110].

In Publication X we adopt a model based on a norm-bounded random distribution of the estimation error. Concretely, let $H$ denote the actual physical channel and $\tilde{H}$ its estimated value. Both quantities are related as:

$$\tilde{H} = H + \Delta$$

(4.36)

where $\Delta$, representing the estimation error, is statistically independent of $H$, has a random distribution and is norm-bounded by $||\Delta||^2 \leq \alpha^2$, with $\alpha^2$ known. A robust design takes into account the uncertainty model to optimize the worst-case MSE, i.e., to minimize the MSE for the value of
Algorithm 3 Alternating filter design procedure

1: **Initialization point**: \( F_S^{(0)}, G_R^{(0)}, F_R^{(0)} \)
2: **repeat** for each iteration \( i = 1, 2, 3, \ldots \)
3: Solve (4.37) w.r.t \( F_R^{(i)} \) for fixed \( F_S^{(i-1)} \) and \( G_R^{(i-1)} \).
4: Solve (4.37) w.r.t \( G_R^{(i)} \) for fixed \( F_S^{(i-1)} \) and \( F_R^{(i)} \).
5: Solve (4.37) w.r.t \( F_S^{(i-1)} \) for fixed \( G_R^{(i)} \) and \( F_R^{(i)} \).
6: **until** the convergence criterion is met.

\( \Delta \) that yields the worst performance [108–110].

Figure 4.6 depicts the system model in the frequency domain of the relay link incorporating filters \( F_S, G_R, C_R \) and \( F_R \). The robust design of these filters is given as the solution to the following optimization problem

\[
\begin{align*}
\min_{\{F_S, G_R, C_R, F_R\}} & \quad \max_{\{\Delta_{SR}, \Delta_{RR}\}} \text{MSE}_R \\
\text{subject to} & \quad \mathbb{E}\{\|\hat{r}[n]\|^2\} \leq P_R \\
& \quad \mathbb{E}\{\|\hat{s}[n]\|^2\} \leq P_S \\
& \quad \|\Delta_{RR}\|^2 \leq \alpha_{RR}^2 \\
& \quad \|\Delta_{SR}\|^2 \leq \alpha_{SR}^2 
\end{align*}
\]  

(4.37)

where \( \Delta_{SR} \) and \( \Delta_{RR} \) denote the channel uncertainties of the SR and SI channels, respectively. Constants \( \alpha_{SR}^2 \) and \( \alpha_{RR}^2 \) represent the uncertainty levels of the SR and SI channels, respectively. The optimal cancellation filter is \( C_R[n] = -H_{RR}[n] \) and the resulting non-convex problem (4.37) is solved by means of the alternating procedure in Algorithm 3. Each sub-problem in Algorithm 3 can be cast as a semidefinite programming (SDP) problem, whose solution is obtained by means of a convex optimization solver. Filter \( F_R \) is designed as the solution to the following SDP:

\[
\begin{align*}
\min_{\{F_R\}} & \quad \max_{\{\Delta_{RR}\}} \text{MSE}_R \\
\text{s.t.} & \quad \mathbb{E}\{\|\hat{r}[n]\|^2\} \leq P_R \\
& \quad \|\Delta_{RR}\|^2 \leq \alpha_{RR}^2 \\
& \quad H_{RD}F_R = H_{RD}^E 
\end{align*}
\]  

(4.38)

where linear constraints \( H_{RD}F_R = H_{RD}^E \) avoid trivial solutions of \( F_R \) by equalizing the equivalent RD channel to \( H_{RD}^E \). Similarly, filter \( G_R \) is de-
signed as the solution to the following SDP:

\[
\begin{align*}
\min_{\{G_R\}} & \quad \max_{\{\Delta_{SR}, \Delta_{RR}\}} \text{MSE}_R \\
\text{s.t.} & \quad \|\Delta_{RR}\|^2 \leq \alpha_{RR}^2 \\
& \quad \|\Delta_{SR}\|^2 \leq \alpha_{SR}^2
\end{align*}
\] (4.39)

To showcase the performance gain of a robust method over a non-robust method, we consider a relay link supporting \(m = 2\) data streams, where S has \(M_S = 4\) transmit antennas, R has \(N_R = 4\) receive antennas and \(M_R = 6\) transmit antennas, and D has \(N_D = 2\) receive antennas. Transmit powers are set to \(P_R = 5\) dB and \(P_S = 5\) dB, respectively. The target channel \(H_{EQ}^{RD} = \sqrt{\frac{1}{N_{D}M}}.\) Thermal noise powers are \(\sigma_{R}^2 = \sigma_{D}^2 = -20\) dB. Limited dynamic range values are \(\epsilon_R \in [-30, -10]\) dB, \(\epsilon_D = -30\) dB, \(\delta_S = \delta_R = -25\) dB. Channel uncertainty is bounded by \(\alpha_{SR}^2 = \alpha_{RR}^2 \in \{-10, -20, -30\}\) dB and \(\alpha_{RD}^2 = -20\) dB. Figure 4.7 shows the worst-case MSE as a function of the limited dynamic range at R and different values of the SR and
SI channel uncertainty. The performance difference between the robust method and the non-robust method depends on the channel uncertainty, e.g., when $\sigma_{\chi}^2 = -10$ dB, $\chi \in \{SR, RR\}$, the robust method outperforms the non-robust method by 6–7 dB, whereas the gap is roughly 1–2 dB when $\sigma_{\chi}^2 = -30$ dB, $\chi \in \{SR, RR\}$. As the dynamic range decreases ($\epsilon_R \approx -10$ dB), the difference between both methods diminishes and approaches zero, regardless of the uncertainty level.

Figure 4.8 shows the worst-case MSE as a function of the source transmit power $P_S$ for various channel uncertainty levels $\sigma_{SR}$ and $\sigma_{RR}$. Figure 4.8 shows the worst-case MSE as a function of the source transmit power $P_S$ and different values of the SR and SI channel uncertainty. In this case, $\epsilon_R = -30$ dB, while the remaining parameters have the same value as in previous simulation. The gap between robust and non-robust method varies with $P_S$, being larger for increasing values of $P_S$, $\sigma_{SR}^2$ and $\sigma_{RR}^2$, e.g., when $\sigma_{SR}^2 = \sigma_{RR}^2 = -10$ dB and $P_S = 10$ dB, the difference between both methods is about 7.5 dB. The performance gap decreases for smaller values of $P_S$. Note the saturation effect on the performance for larger $P_S$, due to transmit noise being proportional to the signal power.
4.7 Summary

In this chapter we studied the problem of optimal linear filter design for an FD relay link. In Publications VIII and IX we presented an iterative algorithm to design transmit and receive filters in a DF relay. The design criterion is to maximize the SINR before the decoding block.

To avoid the trivial solutions of the transmit filter, we introduce linear constraints that equalize the RD channel to a targeted impulse response. In Publication IX, constraints consist of the combination of channel shortening, in which the effective order of the channel is reduced, and projection onto a subspace that ensures a significant fraction of the maximum delivered power at destination. In Publication III, we presented a filter design method, under the MMSE criterion, for an FF relay link with limited dynamic range and direct link between source and destination. The original recursive non-convex problem is solved by means of an alternating procedure and the imposition of SI-free constraints at the relay.

Finally, in Publication X, we considered a DF relay operating in the frequency domain. The system model accounts for channel uncertainties and limited dynamic range. The design criterion is to minimize the worst-case MSE, problem that is solved by using an alternating optimization procedure in which every individual filter can be designed as the solution to an SDP.
5. Conclusions

In this dissertation we presented several methods for SI cancellation and optimal linear filter design in FD relay links. The promise of twice the spectral efficiency of an FD device comes at a price: the unavoidable SI. The SI differs from a conventional interference source in that the level of interference is significantly higher than the received signal and, in the particular case of FF/AF relays, the received signal and the SI are correlated. Another performance-limiting factor is the dynamic range, which introduces a new type of noise distortion. In contrast to the conventional thermal noise, dynamic range is modeled as a noise source that depends on the feeding signal, i.e., the noise power is proportional to the received/transmitted signal power.

In Chapter 3, we studied the problem of SI cancellation in FD MIMO relays. While the problem of SI cancellation in a DF relay can be posed as the solution to an optimization problem, a similar approach results in a biased solution when the relay implements an AF/FF protocol. We overcome this issue by introducing the $R$ coefficients into the adaptation rule. These coefficients use information about the autocorrelation of the transmitted signal to modify the trajectories of the algorithm and eliminate the bias problem, while providing the algorithm with equalization capabilities. We proposed two different algorithms. One based on the LMS criterion, the FF-IAS algorithm, and the other based on the RLS criterion, the FF-WIAS algorithm. They exhibit different complexity, convergence time and performance properties. While the FF-WIAS algorithm implies a higher complexity per sample, it yields better performance and faster convergence time than the FF-IAS algorithm. In contrast to other methods, our algorithms do not introduce additional or artificial delay to
decorrelate both signals, nor they interrupt data transmission. Delay is a critical parameter in a network. In particular, the relative delay between different paths and concretely, the direct path between source and destination and the one through the relay, should be shorter than the CP duration to avoid any interference after FT in the receiver.

In the case of FD MISO relays, we designed two algorithms. First, in order to exploit the potential of multiple receive antennas, we propose an algorithm that uses the AOA of the desired signal as side information. In a practical case, it is difficult to obtain the AOA of the desired signal under the presence of the SI. To overcome the shortcoming of the algorithm, we proposed another algorithm able to estimate the AOA without the use of any spatial information. The results obtained from simulations show a residual SI below noise level, which validates the use of blind adaptive algorithms in the time domain as an attractive option for SI mitigation in FD devices.

In Chapter 4, we studied the problem of optimal filter designs for an FD MIMO relay link subjected to limited dynamic range. In addition to thermal noise, imperfect AD/DA conversion and other RF operations introduce distortion in the form of additional noise sources, which propagate throughout the link into D. Concretely, we presented a method for SINR maximization at the relay input. The architecture includes a cancellation filter as well as transmit and receive filters. To avoid excessive distortion in the transmit side, we imposed linear constraints on the RD hop. These constraints, though suboptimal, conform a linear subspace and, therefore, ensure the convexity of the optimization problem. We proposed two ways to solve the optimization problem, decoupling both filters by approximating the cost function, and using an alternating optimization procedure in which, at each iteration, a single filter is designed while the others remain fixed. We also considered the optimization of the end-to-end performance of a relay link under the MMSE criterion. In a similar way to the aforementioned cases, the solution is obtained by solving a set of LSQI problems. We studied the combined impact of the limited dynamic range at transmission and reception sides of the nodes and the presence of a direct path between S and D. Results show that a direct SD path significantly contributes to the end-to-end performance when the SNR of the SR and RD paths is low. Due to noise propagation, an SR link supporting higher
SNR than the RD link yields better results than the opposite case.

To conclude the chapter, we considered a robust filter design under the MMSE criterion. By using an alternating optimization procedure, the problem of designing a robust filter that takes into account channel uncertainties can be posed as the solution to a set of SDP problems.

From the work presented in this dissertation, we extract the following conclusions regarding the problem of SI mitigation in an FD device. First, in order to achieve a 100 dB mitigation goal, the use of both analog and digital SI mitigation is mandatory. Though analog mitigation is able to reduce SI by several dozens of dBs, it falls short of goal, especially for wideband signals. Nonetheless, digital-domain mitigation presents some issues when applied to different relay protocols. As we have seen in Chapter 3 and in contrast to a DF protocol, when the relay implements an AF/FF protocol, its design needs to cope with the IIR nature of the relay. In conclusion, the AF/FF protocol demands a more complicated SI mitigation than the DF protocol. In Chapter 4, we also studied the problem of limited dynamic range and constrained design. Limited dynamic range modifies filter design by adding additional noise sources and, if not considered in the system model, a penalty in performance of several dBs. Constrained design plays a significant role in the achievable SI mitigation and in the performance of the relay. Concretely, different sets of constraints lead to a wide range of results, see Publication IX, so they should be carefully chosen during design.

When we consider imperfect CSI and limited dynamic range in the relay link, the performance of conventional techniques can significantly degrade. For an FD device, the aggregated effect of dynamic range and channel uncertainty may worsen due to the SI and the transmit noise feeding back into the device. To mitigate this effect we use robust techniques, such as in Publication X, which optimize the MSE, or some other criteria, for the worst case scenario. The performance gap between robust and non-robust techniques is typically of several dBs and is a function of the dynamic range and transmit power of the nodes.

Finally, we identify some open problems and future research lines. As described in Publication II, the proposed algorithms are designed for a spatial multiplexing case. A diversity case has not been considered during the development of the algorithm. Extending the scope of application
of the algorithms to a diversity case is an open problem that may result in improved performance. In Chapter 4, we explored different criteria for SI mitigation and, in particular, different constraints on the transmit filter of the relay. These constraints lead to different performance values, but an open problem is to find the set of optimal constraints under certain criteria. The development of a hybrid FD/HD protocol or a partial FD protocol, i.e., FD operating within a limited time interval or a delimited region of the spectrum, and the implementation of SI mitigation techniques to these protocols, constitute an exciting future research line with several applications.
References


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