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Locally Convergent Fixed-Point Method for Solving Time-Stepping Nonlinear Field Problems

Emad Dlala, Anouar Belahcen, and Antero Arkkio

Laboratory of Electromechanics, Helsinki University of Technology, FIN-02015 TKK, Finland

Because of its stable solution and despite its slow convergence, the fixed-point technique is commonly used for solving hysteretic field problems. In this paper, we propose a new method for accelerating the convergence of the fixed-point technique in solving time-stepping nonlinear field problems. The method ensures locally convergent iteration in an interval that contains the initial value and the fixed-point solution. We provide a thorough discussion and geometric interpretation to clarify and highlight the principle of the method. We also use a finite-element formulation to test the method by computing the magnetic field of an electric machine. Finally, we assess the efficiency and applicability of the method by a comparative investigation. The method proves to be simple and remarkably fast.

Index Terms—Finite elements, fixed point, iteration, magnetic field, nonlinear material, time-stepping.

I. INTRODUCTION

MANY problems in engineering and science require the solution of nonlinear equations. Several examples of such problems are obtained from the field of electrical engineering. Solving nonlinear field problems is a difficult task, especially when ferromagnetic hysteresis is considered. Thus, iterative schemes, which are often computationally costly and problematic, must be used to cope with nonlinearity. The useful approach, which was introduced first by Hantila [1]–[3], solves nonlinear magnetic problems using the fixed-point iteration in the form $\mathbf{x} = \mathbf{G}(\mathbf{x})$. This approach has been particularly instrumental in the case of hysteretic magnetic media. However, even if overrelaxation is applied [3]–[5], the fixed-point iteration can be very slow, especially when complex geometries are involved. Therefore, the Newton–Raphson method has been widely preferred to solve nonlinear nonhysteretic field problems. If convergent, the Newton–Raphson method converges quadratically and thus is markedly fast. Even though the magnetic constitutive relation is strictly of uniform monotone with no inflection points on the magnetization curve, because the Newton–Raphson method utilizes derivatives, the method is still very sensitive and can easily suffer from instability if used for hysteretic media.

Consequently, when dealing with hysteresis, the most prevailing method introduced by far is the iterative technique of contraction mapping based on the Banach fixed-point theorem [6]–[8]. This technique guarantees the existence and uniqueness of fixed points, and provides a constructive method to find those fixed points. A contraction mapping is Lipschitz continuous and has at most one fixed point. The fixed-point (solution) can be found in some interval $\mathbf{V} = [\mathbf{a}, \mathbf{b}]$ if there exists a positive number (called a contraction factor) $0 \leq \alpha < 1$ such that

$$\|\mathbf{G}(\mathbf{a}) - \mathbf{G}(\mathbf{b})\| \leq \alpha \|\mathbf{a} - \mathbf{b}\|. \quad (1)$$

This mapping is said to be inexpensive, and the concept is used to make the iterated function $\mathbf{x}_{k+1} = \mathbf{G}(\mathbf{x}_k)$ converge to a unique fixed point starting from any point $\mathbf{x}_0 \in \mathbf{V}$. The smaller

the contraction factor α , the faster the method converges to the fixed point.

In this paper, we present a new method of how to compute the function $\mathbf{G}(\mathbf{x})$ for time-stepping nonlinear electromagnetic problems. The method secures a locally convergent iteration and enforces a small contraction factor so that the iterative procedure converges fast with speed comparable to the Newton–Raphson method. The method is applied to the computation of electromagnetic fields in an electric machine using finite-element methods where verification by measurement is performed.

II. THE METHOD

The laws of electromagnetic field problems are governed by Maxwell’s equations. In the scope of nonlinearity, one only needs to write the following:

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3)$$

where

\mathbf{H} magnetic field strength (A/M);

\mathbf{J} electric current density (A/m²);

\mathbf{B} magnetic flux density (T).

The nonlinear relationship between \mathbf{H} and \mathbf{B} in an isotropic material is given as $\mathbf{H} = \mathbf{B}/\mu = \nu\mathbf{B}$, where μ and ν are, respectively, the magnetic permeability and the magnetic reluctivity of the material in the computed region. One can also write the latter relation as

$$\mathbf{H} = \mathbf{F}(\mathbf{B}) \quad (4)$$

where \mathbf{F} may be a hysteretic or single-valued function and should be Lipschitz continuous.

Hantila [1] exploited the Banach fixed-point theorem to formulate the problem in the following manner:

$$\mathbf{H} = \nu_{\text{FP}}\mathbf{B} + \mathbf{M}(\mathbf{B}) \quad (5)$$

where \mathbf{M} is a magnetization-like quantity function of \mathbf{B} . The fixed-point coefficient ν_{FP} is a reluctivity-like quantity, which must be constant during iteration and should be properly chosen to ensure contraction.

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Hantila writes the formulation of (5) in an identical form, $\mathbf{B} = \mu_{\text{FP}}\mathbf{H} + \mathbf{I}$, with regard to the magnetic polarization \mathbf{I} and then the formulation is called the polarization method [3]. The formulation of (5) is also called the B -correction scheme when relation (4) is used, and the H -correction scheme when the relation $\mathbf{B} = \mathbf{R}(H)$ is used. In this work, we focus on the B -correction scheme but a similar treatment can be immediately applied to the H -correction scheme.

For the purpose of illustration, we simplify the problem by writing (2) in its integral form, focusing only on the 1-D case. Suppose the magnetic field H in a core of mean length l is produced by a coil of N_c turns carrying total current $N_c i$, and is given by

$$H = \frac{N_c i}{l}. \quad (6)$$

By substituting (5) in (6), one obtains

$$B = \frac{N_c i / l - M(B)}{\nu_{\text{FP}}} \triangleq G(B). \quad (7)$$

Equation (7) is the Banach fixed-point formula to be solved iteratively. One starts by initializing B (or M) to some value, say B_0 , and then carry on the iteration $B_{k+1} = G(B_k)$ using (4), (5), and (7) until a unique fixed point is reached in which $|B_{k+1} - B_k| < \epsilon$ and ϵ is an imposed tolerance.

The iterative technique of (7) can be also written in the following form:

$$\begin{aligned} B_{k+1} = G(B_k) &= \frac{N_c i / l - (F(B_k) - \nu_{\text{FP}} B_k)}{\nu_{\text{FP}}} \\ &= B_k + \frac{N_c i / l - F(B_k)}{\nu_{\text{FP}}} \end{aligned} \quad (8)$$

which is interpreted as: At each iteration step, B_k is added to the term $N_c i / l - F(B_k)$, which acts as the error.

Here, one assumes that the condition of (1) is automatically satisfied and thus the iteration is converging, but this is not as straightforward. Hantila [1]–[3] made analytical proofs about the choice of the coefficient ν_{FP} so that the function $G(B)$ is contractive. He suggested the use of the following optimal coefficient:

$$\nu_{\text{FP}} = \frac{\nu_{\text{dmin}} + \nu_{\text{dmax}}}{2} \quad (9)$$

where ν_{dmin} and ν_{dmax} are the minimum and maximum slopes (differential reluctivities) of the curve $F(B)$. This method will be referred to as the *global*-coefficient method since it uses the same (global) coefficient in the whole B – H curve and the iteration converges for any starting value B_0 .

On the other hand, there is always some freedom for the choice of ν_{FP} provided that the condition of (1) is satisfied. The convergence of the fixed-point iteration depends only on satisfying this condition and not on the particular technique used to calculate ν_{FP} whereas the speed of convergence does.

The fastest convergence occurs when the contraction factor is close to zero ($\alpha = 0$). This convergence is *quadratic* and dramatically fast. To achieve it, the derivative of the function $G'(B)$ with respect to B in (7) is assigned to zero

$$\frac{dG}{dB} = 1 - \frac{1}{\nu_{\text{FP}}} \frac{dH}{dB} = 0 \quad (10)$$

and this gives the most optimal fixed-point coefficient

$$\nu_{\text{FP}} = \frac{dH}{dB}. \quad (11)$$

The same result is obtained when the derivative of the function $M'(B)$ with respect to B in (5) is assigned to zero.

In time-stepping analysis, since the initial value B_0 for time step n is known from the solution of the *previous* time step and should be sufficiently close to the fixed point, a neighborhood interval U that contains B_0 and fulfills (1) is found by rewriting (11) as

$$\nu_{\text{FP}}^{(n)} = C \frac{dH^{(n-1)}}{dB} = C \nu_{\text{d}}^{(n-1)} \text{ for } C > 1 \quad (12)$$

where $\nu_{\text{FP}}^{(n)}$ and $\nu_{\text{d}}^{(n-1)}$ refer to the fixed-point coefficient and the differential reluctivity in the current and previous time steps, respectively. It follows from (12) that in the neighborhood U , a fixed point exists and the contraction factor α is sufficiently small to guarantee fast convergence. C is a *convergence* factor which must be conveniently chosen to ensure fast convergence such that the function $G(B)$ is strictly contractive in the interval U . The convergence factor C is necessary and must be greater than one because the derivative of the current time step has been approximated from the previous time step.

Method (12) computes a local coefficient ν_{FP} at each time step and produces a locally convergent iteration. Thus, the method is referred to as the *local*-coefficient method.

It should be noted that the local-coefficient method (12) makes use of the information about the differential reluctivities at each “time step.” This is similar to the Newton–Raphson method that updates ν_{d} at each “iteration step.” However, the Newton–Raphson method is principally based on calculating the derivative while the fixed-point iteration is not. Therefore, the local-coefficient method is not sensitive to nondifferentiability, even in case of hysteresis.

III. GEOMETRIC INTERPRETATION AND DISCUSSION

In this section, the local-coefficient method along with the global-coefficient method is geometrically studied. The B – H curve is described by the function $F(B)$ using consistently the same single-valued data of a soft magnetic material (Fig. 1). The only parameter that is possible to modify without changing the essence of the problem is the coefficient ν_{FP} (local-coefficient method.) In Section III-B, the role of the convergence factor C and how to find its optimal value will be clarified and discussed in detail.

For the sake of clarity, the source term $i(t)$ in (8) is expressed sinusoidally by discrete-time analysis as

$$i(n) = \hat{i} \cos(2\pi f n t_s) \quad (13)$$

where \hat{i} is the amplitude of the current, f is the frequency, n represents the time index (time step) integer sequence 0, 1, 2, 3, etc., and t_s is some constant time period. The time index n defines which point on the B – H curve is being used for the fixed-point iteration, and thus, different iterative procedures will be needed at each time step n .

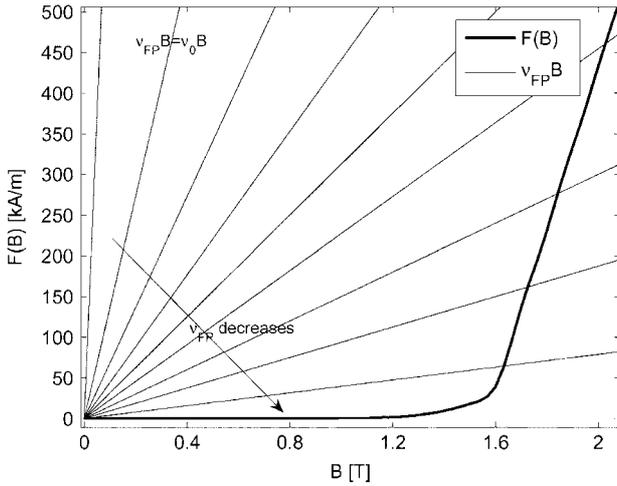


Fig. 1. Impact of the slopes ν_d on the function $M(B)$.

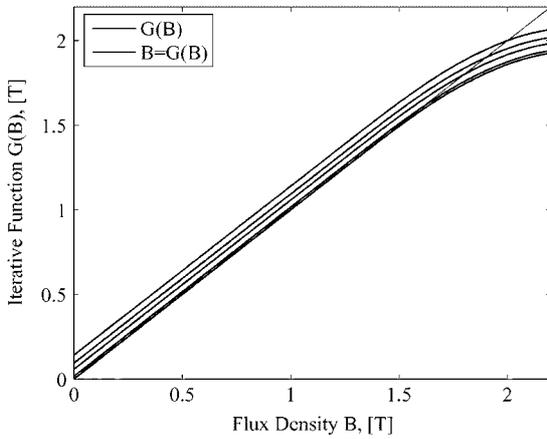


Fig. 2. Similar iterative functions $G(B)$ created at different time steps by the global-coefficient method.

Equation (7) suggests that the shape of the function $G(B)$ follows exactly the shape of $M(B)$. Thus, (5), particularly the coefficient ν_{FP} , plays a vital role in the convergence of (7). On the other hand, the function $M(B)$ is computed through the relation $F(B)$ which is a given and cannot, in principle, be modified. It should be noted that the global-coefficient method restricts the shape of the functions $M(B)$ (and $G(B)$) to be the same for any time step n . Thus, because the coefficient ν_{FP} is kept the same (global) for any n , only a shift to the function $G(B)$ occurs, which is caused by changing $i(n)$ in (7). This property of the global-coefficient method is clearly shown in Fig. 2. Below saturation, the iterative functions $G(B)$ created at different time steps are almost parallel to the line $B = G(B)$, meaning that the slopes (the contraction factors α) of $G(B)$ are very close to one. This latter observation renders the convergence very slow especially in the unsaturated region, and this is the main reason that the fixed-point technique has been widely known to be inefficient for solving nonlinear field problems.

In general, the idea behind the fast convergence of the local-coefficient method and the slow convergence of the global-coefficient method is briefly explained in Fig. 3. When

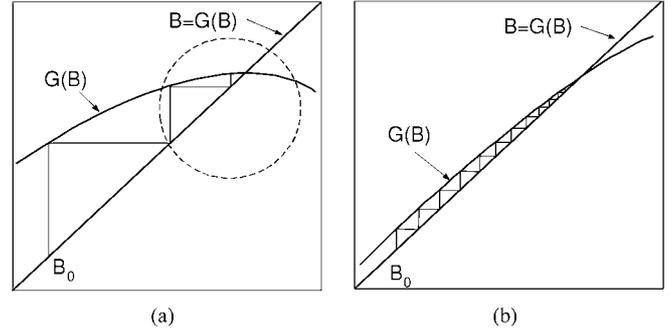


Fig. 3. Comparison of the speed of the fixed-point iteration between the two methods. (a) The local-coefficient method. (b) The global-coefficient method.

the function $G(B)$ is computed from the local-coefficient method (12), the slope of $G(B)$ is close to zero and hence the iteration is fast [Fig. 3(a)]. However, the function $G(B)$ computed from the global-coefficient method (9) in Fig. 3(b) has a slope close to one at the solution interval and hence the solution takes many iterations to converge for the same initial value B_0 used in the case of Fig. 3(a). It is evident that the local-coefficient method creates smaller derivatives of the function $G(B)$ near the fixed-point solution as it is illustrated in Fig. 3(a) by the dotted circle. This feature allows significantly fast convergence.

A. Evaluation of Contractive Functions

Three different cases are here studied in which the instantaneous values of the flux density were deliberately located at three selected points, 2.0, 1.5, and 0.5 T on the B - H curve. The coefficient ν_{FP} is computed from the previous time step by the local-coefficient method (12). The convergence factor C is assigned to some constant value, 2, to keep the demonstration easy to comprehend. The studied cases will demonstrate the usefulness of the local-coefficient method as well as help us understand the proposed techniques. Moreover, a comparison between the local-coefficient method and the global-coefficient method is made in each example to evaluate the methods.

Case 1 [$B(n) = 2.0$ T]: The chosen time step n in this case corresponds to a point lying in the saturation region (Fig. 4). In this region, the local-coefficient method (12) creates an iterative function $G(B)$ like the global-coefficient method (9) with only a minor difference. This was expected because (9) and (12) give relatively close results at saturation. The slopes of the function $G(B)$ for both methods are small, approximately the same, and allow rather fast convergence.

Case 2 [$B(n) = 1.5$ T]: The chosen time step n in this case corresponds to a point lying in the knee region (Fig. 5). In this region, the local-coefficient method (12) creates an iterative function $G(B)$ which is rather different from that created by the global-coefficient method (9). The difference in the speed of the convergence between the two methods is significant in this region. The local-coefficient method shows its effectiveness by making the slope of $G(B)$ small enough, enforcing fast convergence.

Case 3 [$B(n) = 0.5$ T]: The chosen time step n in this case corresponds to a point lying in the unsaturated region (Fig. 6).

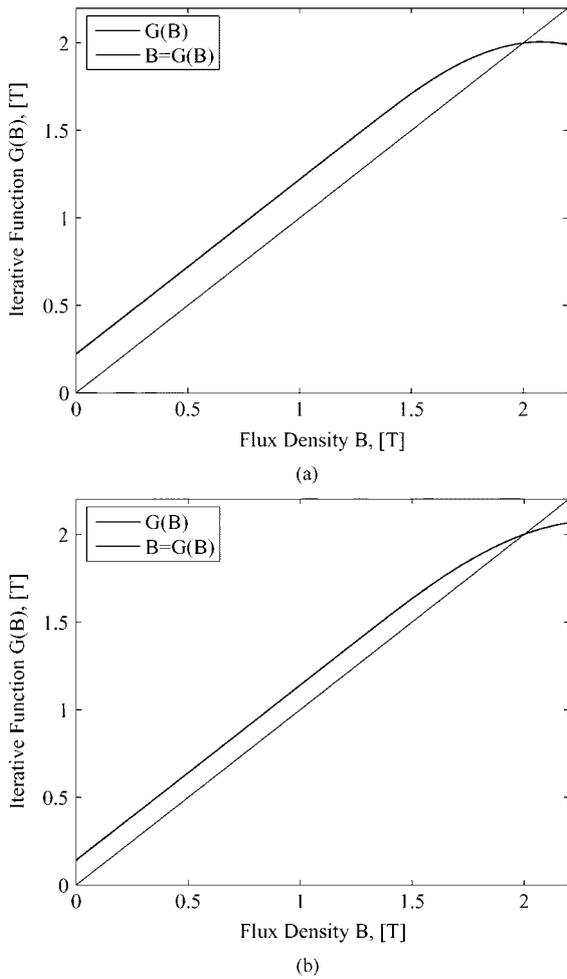


Fig. 4. Comparison of the iterative functions $G(B)$ created at 2.0 T by (a) the local-coefficient method and (b) the global-coefficient method.

In this region, the local-coefficient method (12) creates an iterative function $G(B)$ which is completely different from that created by the global-coefficient method (9). The difference in the speed of the convergence between the two methods is pronounced in this region. The slow convergence of the global-coefficient method is severe as it is shown clearly in Fig. 6(b) by the indistinguishable lines, $G(B)$ and $B = G(B)$. On the contrary, the local-coefficient method shows its absolute effectiveness for bringing the slope of $G(B)$ close to zero, allowing markedly fast convergence [Fig. 6(a)].

There is another issue that has to be dealt with when using the fixed-point technique in solving field problems with hysteretic media. Because the principle of contractive mapping requires the function $G(B)$ to be Lipschitz continuous, the hysteresis model must allow continuity at the reversal points. For example, the data of the discrete Preisach model (the Everett table) [8], [9] need to be extrapolated in order to ensure continuity of the reversal curves.

B. Role of the Convergence Factor C

The convergence factor C and its role in ensuring fast convergence must be given special attention. A scenario in which using the convergence factor C is a necessity is shown in Fig. 7. In the

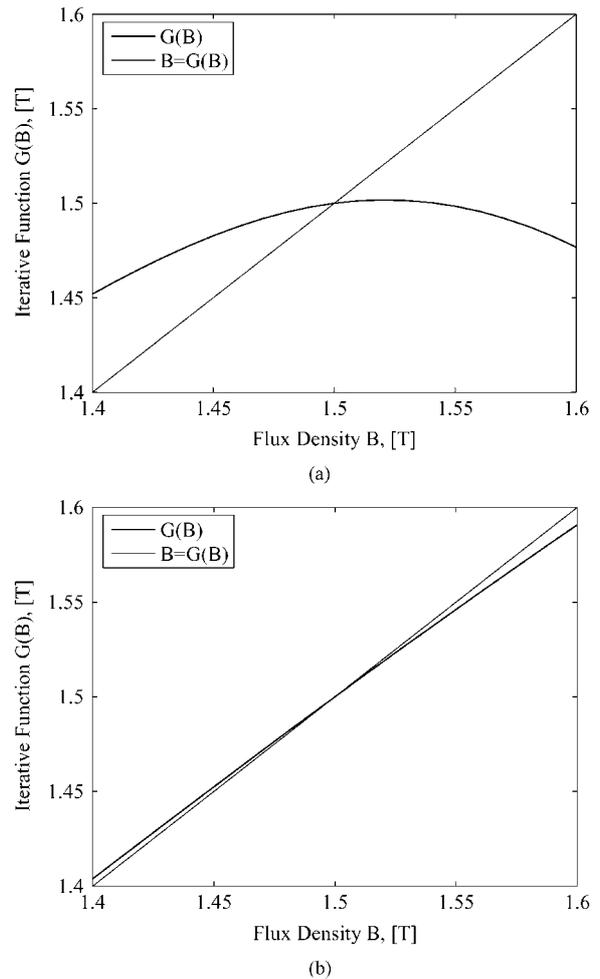


Fig. 5. Comparison of the iterative functions $G(B)$ created at 1.5 T by (a) the local-coefficient method and (b) the global-coefficient method.

neighborhood of the fixed-point solution, the function $G(B)$ is very steep ($\alpha > 1$) and may not be contractive. Thus, the convergence factor C guarantees that the whole neighborhood U is locally contractive.

The convergence factor C is dependent on the time step size and on the temporal behavior of the system involved. Thus, it is not straightforward to find its optimal value, which ensures the fastest convergence. However, a simple technique, adopted in this work, using linear search was found to be effective. In this technique, for any problem, one may start by giving C some value slightly greater than one and then test the convergence for a complete simulation. If converged, the value of C is fixed; if not, C is increased by a small value δ until the appropriate value of C is obtained. For simple problems, it is generally found that C would take values in the range of $1 < C < 2$. On the other hand, in more complicated situations, such as in electrical machines transients, the factor C is usually greater than 2. In these types of problems, a time-stepping scheme such as Crank–Nicholson’s is often used. The initial values that result from the time-stepping scheme from the previous time step cannot always be close to the fixed-point solution due to the system dynamics. Therefore, the interval U is enlarged by increasing C in order to account for these effects.

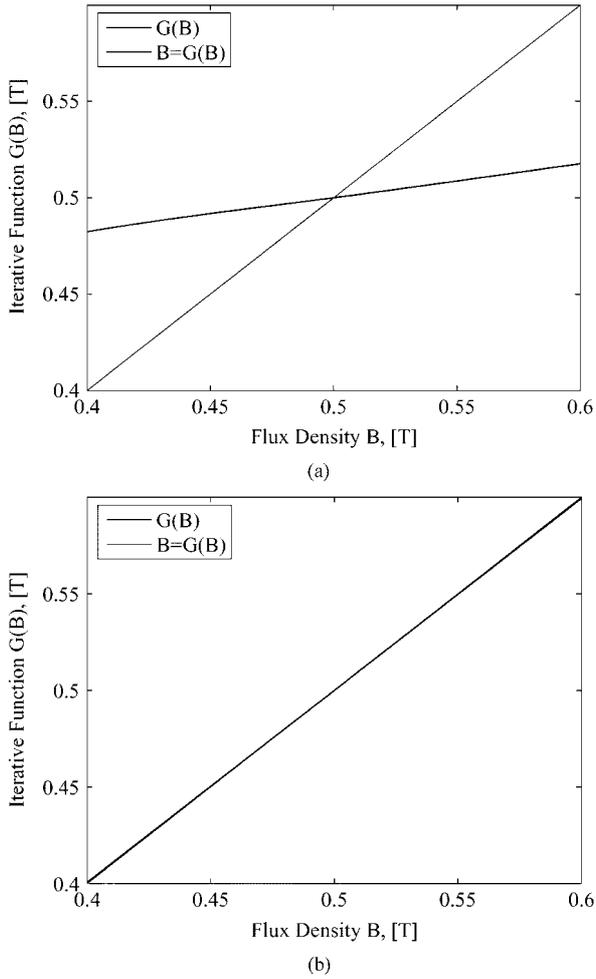


Fig. 6. Comparison of the iterative functions $G(B)$ created at 0.5 T by (a) the local-coefficient method and (b) the global-coefficient method.

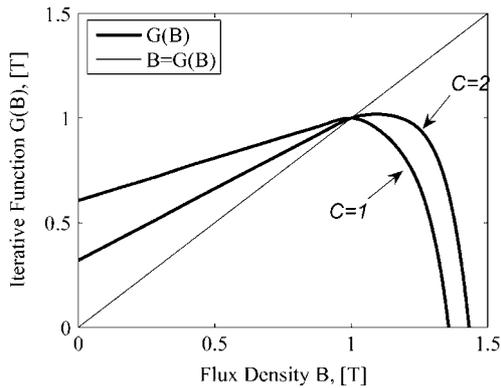


Fig. 7. Role of the factor C in the convergence of the fixed-point iteration.

IV. TIME-STEPPING FINITE-ELEMENT ANALYSIS

Significant improvements are expected to be made on the speed of the fixed-point iteration as a result from using the local-coefficient method, especially when finite-element methods are involved. Instead of keeping a global coefficient ν_{FP} fixed in the entire mesh, a local coefficient ν_{FP} is adaptively computed in each element or more specifically in each integration point in an element.

We briefly describe a quasi-static magnetic system (electric machine) in a 2-D time-stepping finite-element procedure. We only focus on the formulation of the field equations by the fixed-point method while the circuit equations representation will be omitted. A complete description of the methods used in the finite-element solution of the magnetic field with circuit equations is available in [10] and [11].

By using (2) and (5) with the magnetic vector potential \mathbf{A} as the unknown, and considering the eddy currents generated in a material with conductivity σ , one can obtain

$$\nabla \times \nu_{FP}(\nabla \times \mathbf{A}) + \sigma \frac{\partial \mathbf{A}}{\partial t} = \mathbf{J} - \nabla \times \mathbf{M}. \quad (14)$$

If a 2-D approach is performed, then one can write $A = \sum_{i=1}^{N_{tot}} N_i a_i$ in which A is the z -component of \mathbf{A} , N_{tot} is the total number of nodes in the mesh, and a_i is the vector potential in node i whose shape function is N_i . After making some mathematical manipulations, applying Galerkin weighted-residual method over the entire solution region Ω (cross section of the machine), and respecting the boundary conditions, the following system of differential equations of N_f (free nodes) unknowns resulted from (14):

$$\sum_{i=1}^{N_{tot}} S_{ij} a_i + \sum_{i=1}^{N_{tot}} T_{ij} \dot{a}_i = \sum_{i=1}^{N_p} D_{ij} i_i - P_j \quad (15)$$

since

$$S_{ij} = \int_{\Omega} \nu_{FP} \nabla N_i \cdot \nabla N_j d\Omega, \quad T_{ij} = \int_{\Omega} \sigma N_i N_j d\Omega,$$

$$D_{ij} = \int_{\Omega} \beta_i N_j d\Omega, \quad P_j = \int_{\Omega} (\nabla \times \mathbf{M})_z N_j d\Omega$$

where $j = 1, \dots, N_f$, matrix $\mathbf{D} = \{D_{ij}\}$ is associated with the source, β_i describes the orientation of the coil sides in the phase i , and N_p is the number of phases.

In comparison with the Newton–Raphson formulation [10], (14) has a remarkable advantage. The standard Newton–Raphson method requires that the global matrix $(\mathbf{S} + \mathbf{T})$ be updated at each “iteration step.” In contrast, when the global-coefficient method is applied, all matrices in (15) remain unchanged and they need to be factored once and for all except the column \mathbf{P} , which is updated at each iteration step k . However, this is not true when applying the global-coefficient method to a rotating machine in which the mesh in the air gap is modified at each “time step” and hence the global matrix needs to be updated anyway to allow for the rotation.

When using the local-coefficient method, all matrices in (15) remain unchanged during “iteration” (at each time step) except the column \mathbf{P} , which is updated at each iteration step k .

Thus, in case of applying the global-coefficient method or the local-coefficient method in rotating machines, one acquires a system of equations that remains unchanged during iteration and only differs in its right-hand side. In this case, for each time step n , one only needs to factor the global matrix once, and use the factors repeatedly in the iterative calculation of the vector \mathbf{a} .

TABLE I
RESULTS OF THE COMPUTATION TIME

Test results	Element order	LCM	GCM	NRM
Iterations per time-step	1 st	15	64	5
	2 nd	16	67	6
CPU-time per time-step (sec)	1 st	0.104	0.296	0.089
	2 nd	0.449	1.750	0.440
Total CPU-time (sec)	1 st	1872.3	5342.4	1603.8
	2 nd	8095.5	31502.9	7925.0

LCM, GCM, and NRM stand for the local-coefficient method, the global-coefficient method, and the Newton-Raphson method, respectively.

The latter advantage remains active even after the system of differential equations (15) is solved using the Crank–Nicholson time-stepping scheme.

$$\begin{aligned} [\mathbf{S}^n + \frac{2}{\Delta t} \mathbf{T} \quad \mathbf{D}] \begin{bmatrix} \mathbf{a}^n \\ \mathbf{i}^n \end{bmatrix} &= [\mathbf{P}^n + \mathbf{P}^{n-1}] \\ &+ [-\mathbf{S}^{n-1} + \frac{2}{\Delta t} \mathbf{T} \quad \mathbf{D}] \begin{bmatrix} \mathbf{a}^{n-1} \\ \mathbf{i}^{n-1} \end{bmatrix}. \end{aligned} \quad (16)$$

The column \mathbf{P} nonlinearly depends on \mathbf{B} and hence on \mathbf{A} , or simply on the nodal values of the vector potential \mathbf{a} . Therefore, the system of (16) must be solved iteratively. In the first time step ($n = 0, t = 0$), initial values for $k = 0$ are assumed to the vector \mathbf{P}^0 by putting $\mathbf{M}_0^0 = 0$ in each integration point. Then for $n > 1$, one should put $\mathbf{M}_0^n = \mathbf{M}^{n-1}$. The stopping criterion $\|\mathbf{a}_{k+1} - \mathbf{a}_k\|_1 < \epsilon$ is adopted.

In 2-D problems, because there are two partial derivatives for x and y directions, the optimal value of the local-coefficient ν_{FP} is found as

$$\nu_{FP}^{(n)} = C \frac{\frac{\partial H_x^{(n-1)}}{\partial B_x} + \frac{\partial H_y^{(n-1)}}{\partial B_y}}{2}, \quad C > 1. \quad (17)$$

The analysis and derivation of this equation are analogous to the 1-D case. However, we are currently conducting a research on the general case in which the 2-D and 3-D problems are studied and analyzed in detail [12].

V. NUMERICAL RESULTS

In this section, the numerical procedures introduced in Sections II and IV are implemented for the computation of magnetic fields in a two-pole 30 kW induction motor. In effect, more comprehensive modeling of the machine than that shown in Section IV was performed, including the incorporation of the circuit equations in the rotor and stator, equations of the motion, and other phenomena (see [10]).

The local-coefficient method and the global-coefficient method are examined in the numerical procedures. In addition, for comparison purposes, the Newton–Raphson method [10] is applied with the same-stopping criterion ($\epsilon = 1 \times 10^{-5}$). The same Fortran codes are used in the finite-element analysis for both the fixed-point methods and the Newton–Raphson method, differing only in the formulation that each method requires. Furthermore, all simulations were performed in the same 3.2 GHz computer.

The starting of the motor was the case chosen to test the codes. The motor was supplied from a sinusoidal source and the number of time steps used per period was 500 in all simulations. For each method, a number of 2204 first-order and second-order

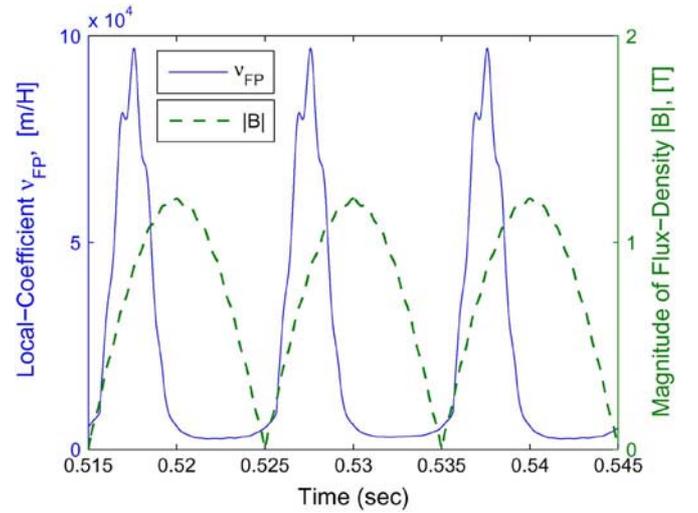


Fig. 8. The development of the local-coefficient ν_{FP} and the magnitude of the flux density $|\mathbf{B}|$ in the steady-state at a point in the stator yoke.

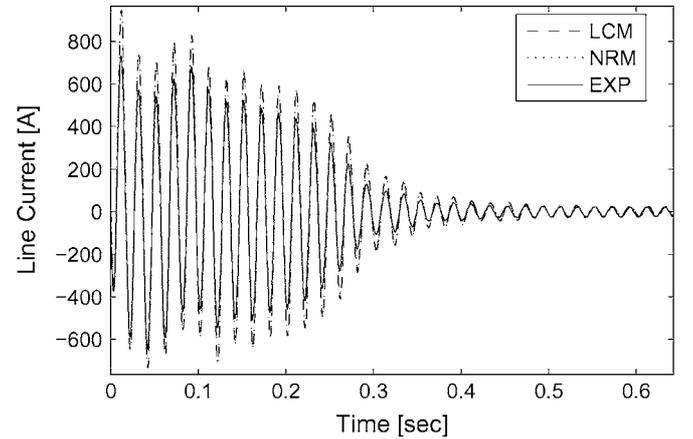


Fig. 9. Waveforms of the motor line current computed by using the local-coefficient method (LCM) and the Newton–Raphson method (NRM) compared with experimental data (EXP). LCM and NRM waveforms are indistinguishable since they are almost identical.

elements were used in which 1135 and 4473 nodes were generated, respectively. The nonlinear function $\mathbf{F}(\mathbf{B})$ was assumed to be nonhysteretic single-valued. The convergence factor C was optimally found to be 7 allowing fast convergence.

The rotor slot opening of the motor is closed, which makes the flux density rise to values higher than 2.5 T; thus, a more difficult nonlinear problem is faced. Table I shows comparative results obtained by implementing the methods for first-order (1st) and second-order (2nd) element simulations. The average number of iterations per time step, the average CPU-time spent on a time step, and the total CPU-time spent on the entire simulation are tabulated. Clearly, the local-coefficient method and the Newton–Raphson method were converging with comparable speed. The local-coefficient method was superior particularly when second-order (2nd) elements were used; the superiority is ascribed to the following advantage: having to factor the global matrix only once in each time step. The global-coefficient method was evidently slow. Fig. 8 depicts the steady-state development of the coefficient ν_{FP} at some point in the stator yoke. The flux density in the same point is superposed

in the same figure. The figure indicates the effectiveness of the local-coefficient method in adapting the fixed-point coefficient \mathcal{U}_{FP} according to the changes of the flux density.

The behavior of one of the motor line currents computed by using the local-coefficient method and the Newton–Raphson method is demonstrated in Fig. 9. The waveforms resulting from both methods are quite identical and relatively in agreement with the measurement.

VI. CONCLUSION

The local-coefficient method proposed in this work showed excellent improvements in speeding up the convergence of the fixed-point technique used for solving time-stepping nonlinear field problems.

The idea of the local-coefficient method is based on calculating the derivative of the B – H curve with respect to B near the fixed-point solution. The local-coefficient method seeks a solution only in a local contractive interval at which the contraction factor is smaller than at all other feasible intervals in its vicinity. The method begins by computing the local-coefficient from the previous time step and generates a sequence of iterations until the solution is reached.

In this work, the local-coefficient method of the fixed-point iteration has been applied to solve the nonlinear magnetic problem in finite-element analysis using single-valued magnetic data. It has been shown that the local-coefficient method is a promising technique that has the potential to compete with the Newton–Raphson method for solving nonlinear electromagnetic problems. Further study should be conducted in the future to test the method within the media of hysteresis.

Using finite-element analysis, the global-coefficient method may have an advantage over the local-coefficient method. The global matrix remains untouched and needs to be factored once in the whole simulation when the global-coefficient method is applied. On the other hand, when the local-coefficient method is applied, the global matrix remains unchanged during iteration and needs to be factored at each time step. Nonetheless, in rotating machines, the two methods require the global matrix to be updated at each time step because of the rotor motion.

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Manuscript received February 20, 2007; revised July 20, 2007. Corresponding author: E. Dlala (e-mail: emad.dlala@tkk.fi).

Emad Dlala was born in Libya in 1976. He received the B.Sc. degree in electrical power engineering from Seventh of April University, Sabrata, Libya, in 1999 and the M.Sc. degree (with distinction) in electromechanical energy conversion from Helsinki University of Technology, Finland, in 2005. He is currently pursuing the Ph.D. degree at the Laboratory of Electromechanics, Helsinki University of Technology. His doctoral study deals with dynamic hysteresis modeling of ferromagnetic steel sheets in electric machines.

Before he joined his M.Sc. study, he had been at an industrial program during 2000–2002 where he was first trained in Malta and then worked in Germany for Fritz Werner Industrie-Ausrustungen GmbH. His research interests include numerical analysis of electromagnetic field problems, and measurement and modeling of magnetic material modeling.

Anouar Belahcen was born in Essaouira, Morocco, in 1963. He received the B.Sc. degree in physics (Licence es-science) from the University Sidi Mohamed Ben Abdellah, Fes, Morocco, in 1988 and the M.Sc. (Tech.), LisTech, and Doctorate degrees from Helsinki University of Technology, Finland, in 1998, 2000, and 2004 respectively.

From 1996 to 1998, he was a Research Assistant at the Laboratory of Electromechanics, Helsinki University of Technology. From 1998 to 2004, he was Research Scientist and since 2004 he has been working as a Senior Researcher at the same Laboratory. His research interests deal with the numerical modeling of electrical machines, especially material modeling and magnetic forces.

Antero Arkkio was born in Vehkalahti, Finland, in 1955. He received the M.Sc. (Tech.) and D.Sc. (Tech.) degrees from Helsinki University of Technology, Finland, in 1980 and 1988, respectively.

He has been Professor of electrical engineering (Electromechanics) at Helsinki University of Technology (HUT) since 2001. Before his appointment as Professor, he was a Senior Research Scientist and Laboratory Manager at HUT. He has worked with various research projects dealing with modeling, design, and measurement of electrical machines.