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Magnetodynamic vector hysteresis model of ferromagnetic steel laminations

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Abstract

This article introduces a magnetodynamic vector hysteresis model for predicting alternating and rotational iron losses. The model is based on the well-known Mayergoyz model, whereas the identification of the proposed model requires no further modification of the measured scalar data. The model can be used for a wide variety of frequencies, enhancing its generality and use. A 1D finite-element formulation is performed to investigate the properties of the model and its ability to reproduce the experimental data. The calculated rotational losses and the modeled dynamic loops have been found to be in accordance with what has been observed in measurement.

Keywords: Vector hysteresis; Magnetodynamic modeling; Time-stepping finite-element method; Rotational loss separation

1. Introduction

Since numerical methods became available and easily applied in the computer era, there has been a rapidly growing interest to consider accurate modeling of the core loss phenomena, including hysteresis, eddy current, and excess losses, which all lead to the *magnetodynamic* effects [1–4]. The complex microscopic behavior of these phenomena is hardly understood by the engineering community and usually their macroscopic modeling is of more interest to this community.

The eddy current in the lamination is intrinsically a 3D problem but it can be reduced to 2D by using another 1D model for the lamination depth as was proposed by few researchers [5,6]. In this respect, modeling magnetodynamic effects of ferromagnetic steel sheets has been, and can be, done in various ways. The simplest, perhaps, is to solve the diffusion equation (resulting from Maxwell equations) assuming constant reluctivity along the lamination depth. Such an approach can easily lead to under- or overestimated results [6]. An improved approach would independently model the excess and hysteresis losses, and, then, add them to the eddy-current loss resulting from solving the diffusion equation using single-valued reluctivity. Yet, this approach is far from completion because the core loss phenomena are interdependent and cannot be simply kept apart. The most accurate, impeccable, and systematic way achieved by far solves the diffusion equation numerically applying the hysteretic nonlinearity directly in the lamination [1–4]. However, such a method has been often applied for "alternating" fields, whereas modeling magnetodynamic losses under "rotational" excitations has not yet been commonly investigated [7–9].

Although the "rotational eddy-current" phenomenon has been mentioned quite a few times in literature, it has not nevertheless been extensively studied. Only few researchers, most of whom used analytical methods, have given it considerable importance [10]. It is no surprise that the reason may be attributed to the difficulties surrounding the understanding of the phenomenon or, perhaps, the lack of a general vector model that can rigorously describe the relation between the magnetic field strength $\boldsymbol{H}$ and the magnetic flux density $\boldsymbol{B}$ at varied frequencies. As it is known that even at a given fixed frequency (e.g., rotational static hysteresis), modeling the vector relation between $\boldsymbol{H}$ and $\boldsymbol{B}$ is still considered to be a new subject with many speculations, making the modeling of the rotational eddy-current loss a problem of appreciated difficulties.

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The generalized vector hysteresis model introduced by Mayergoyz [11] for static fields is a prevailing model because it can well represent the ***B*–*H*** behavior. In this article, the authors propose a modification to the generalized vector hysteresis model [11] in order to account for the magnetodynamic rotational quantities. The identification of the proposed model is considered in the article and proven to be simple. A time-stepping 1D finite-element procedure is carried out to solve the magnetodynamic problem under rotational field excitations in which rotational losses are modeled, separated, and analyzed.

2. Magnetodynamic vector hysteresis model

The focus of this article is on developing an isotropic magnetodynamic vector hysteresis model and its integration into 1D finite-element method while the extension of the analysis to 2D is omitted here and remains as a future pursuit. The authors prefer to write the magnetic field quantities of the 1D model in small letters instead of capital letters, which are commonly adopted for 2D problems. The 1D and 2D models are related because the time-varying components of the flux density of the 2D model ($B_x(t)$ and $B_y(t)$) are used to set the boundary conditions of the 1D model [5].

The nonlinear 1D magnetodynamic problem can be formulated using the fixed-point iteration as

$$\nabla \times \nu_{\mathrm{fp}}(\nabla \times \boldsymbol{a}) + \sigma \frac{\partial \boldsymbol{a}}{\partial t} = -\nabla \times \boldsymbol{m}, \tag{1}$$

where σ is the conductivity of the ferromagnetic material and the residual term $\boldsymbol{m}$ is associated with the nonlinearity. Here, $\boldsymbol{a}$ is the magnetic vector potential, which normally has the x and y components perpendicular to the lamination depth z.

If one assumes that the second term in Eq. (1) is negligible ($\sigma\partial(\boldsymbol{a}/\partial t) = 0$), then the isotropic vector hysteresis relation between $\boldsymbol{H}$ and $\boldsymbol{B}$ can be simply characterized by the generalized vector hysteresis model [11] in its inverted version [12] without the need of solving the nonlinear problem (1). Considering N directions along $\boldsymbol{e}_\vartheta$, the output $\boldsymbol{H}$ is directly calculated as

$$\boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{e}_{\vartheta_i} F_{\mathrm{st}}(B_{\vartheta_i}) = \sum_{i=1}^{N} \boldsymbol{e}_{\vartheta_i} H_{\vartheta_i}, \tag{2}$$

where $B_\vartheta = B_x \cos^{1/w}(\vartheta) + B_y \sin^{1/w}(\vartheta)$, and w is a coefficient associated with the rotational loss and can be experimentally identified.

On the other hand, if the "magnetodynamic" vector hysteresis property is taken into account ($\sigma\partial\boldsymbol{a}/\partial t \neq 0$), the *static*, hysteretic function F_{st} must be replaced by a *dynamic*, hysteretic function F_{dy} and Eq. (1) must be solved. However, the dynamic function F_{dy} which involves the solution of Eq. (1) renders the vector model difficult to identify. Therefore, the authors propose to modify Eq. (2) as follows:

$$\boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{e}_{\vartheta_i} \frac{F_{\mathrm{dy}}(b_{\vartheta_i})}{N^\gamma} = \frac{1}{N^\gamma} \sum_{i=1}^{N} \boldsymbol{e}_{\vartheta_i} h_{\vartheta_i}, \tag{3}$$

where γ is a parameter characterizing the magnetodynamic vector hysteresis and can be identified so that the output of the magnetodynamic vector model magnetized along one direction is equal to the output of the magnetodynamic scalar model. The natural simplification in Eq. (3) lies in two aspects. First, the model utilizes the scalar hysteresis data directly without additional modification, which is not the case when using model (2). Second, γ can be identified to be unique for a wide range of frequencies. This means that model (2) is no longer needed even in static field conditions.

The nonlinear problem in the lamination is linearized by defining $m_\vartheta(z,t) = h_\vartheta(z,t) - \nu_{\mathrm{fp}} b_\vartheta(z,t)$, where ν_{fp} is a constant to be chosen appropriately. In the lamination, the dynamic, hysteretic relation is described in the ϑ direction by a viscosity-based model [4,13]:

$$h_\vartheta(z,t) = F_{\mathrm{st}}(b_\vartheta(z,t)) \pm \left| \frac{1}{R} \frac{\mathrm{d}b_\vartheta(z,t)}{\mathrm{d}t} \right|^{1/p}. \tag{4}$$

The first term of Eq. (4) can be, in principle, computed by any static hysteresis model. In this work, the history-dependent model [14], which employs a static family of first-order reversal curves, has been used. The second term of Eq. (4) represents the excess field through the time delay of the magnetic flux behind the field strength. The field strength on the surface $h_\vartheta(z = d/2, t)$ is being used for computing the output of the vector model (3). The dynamic magnetic resistivity R is a material property and the constant p is related to the dependency of the excess loss on the frequency. The signs $\pm$ are switched according to whether the field is increasing or decreasing.

Eq. (1) is discretized using first-order finite-element method and solved by the Crank–Nicholson time-stepping scheme. N system of equations for a_ϑ result and are strongly coupled through the magnetodynamic vector hysteresis model. Since the components a_ϑ are symmetric around the plane $z = 0$, only the segment $[0, d/2]$, instead of $[-d/2, d/2]$, is needed to be discretized in the solution of Eq. (1), saving half of the computation time. The 1D model is subjected to a known magnetic flux per unit length $\boldsymbol{\phi} = \boldsymbol{B}d$. Thus, the magnetic vector potential on the boundary is computed from

$$a_\vartheta\left(z = \frac{d}{2}\right) = \frac{1}{2}\phi_\vartheta = \frac{1}{2} b_\vartheta d = \frac{d}{2}(B_x \cos^{1/w}(\vartheta) + B_y \sin^{1/w}(\vartheta)),$$
$$a_\vartheta(z = 0) = 0. \tag{5}$$

3. Application of the model

In order to identify the magnetodynamic vector model, the characteristics of a soft magnetic steel sheet of thickness, $d = 0.5\,\mathrm{mm}$, conductivity, $\sigma = 2.92 \times 10^6\,\mathrm{S/m}$, and coercivity, $H_c = 57\,\mathrm{A/m}$, have been experimentally obtained. A set of dynamic ***B*–*H*** loops as well as their respective iron losses have been measured in the existence of alternating unidirectional fields for various frequencies at different flux densities. The measurements were made using a digital setup that ensured a sinusoidal flux density waveform.

3.1. Identification of the model

Unlike the identification process of the vector hysteresis model (2), which is difficult and may lead to rather tedious calculations, identifying model (3) is simple and straightforward. In principle, the only parameter to be identified is the magnetodynamic parameter γ. For all cases, the rotational loss coefficient w has been given a constant value, $w = 3$, as it was suggested in Refs. [11,15]. The remaining parameters related to the viscosity-based model (4) have already been dealt with in the past [4,13], and they have been treated similarly here.

Three measured unidirectional dynamic loops were used to identify the model parameters. The parameters are adjusted using a least square algorithm, which best fits the modeled loops to the measured dynamic loops. Since the number of directions N influences the results and the computation time, N has been compromised to ensure relatively fast, accurate modeling. A sinusoidal source was applied on the boundary of the considered geometry in which only the x component was active ($B_y = 0$). The values of the identified parameters have been as follows: $R = 1.0$, $p = 2$, $N = 8$, and $\gamma = 0.78$. In Fig. 1, the modeled and measured dynamic loops are compared. It is clear that the model gives relatively accurate results for low and high frequencies as well as for different inductions.

3.2. Numerical results

The magnetodynamic vector hysteresis model identified above is here applied to predict rotational iron losses at different frequencies. The model is also used to separate the rotational and alternating hysteresis, excess, and classical eddy-current losses according to Refs. [13,16]. Fig. 2 depicts the calculated dynamic loops resulting from imposing rotational elliptic flux (Fig. 2a, $\hat{B}_x = 1.5\,\mathrm{T}$, $\hat{B}_y = 0.9\,\mathrm{T}$). The respective iron losses of this case are indicated in Fig. 3. It is worthwhile to note that the shapes of the trajectories of the calculated dynamic loops have been experimentally evidenced by other researchers [17].

The other situation considered is when the flux is purely rotational (circular, $\hat{B}_x = 1.5\,\mathrm{T}$, $\hat{B}_y = 1.5\,\mathrm{T}$). Fig. 4 shows the trajectories of H_x, H_y, B_x, and B_y, while the corresponding rotational losses of this case are demonstrated in Fig. 5. The rotational losses tend to decrease at higher flux amplitudes B_m (Fig. 5b), agreeing with what has been observed in measurement [17].

The development of the flux waveforms of b_{93} inside the lamination is shown in Fig. 6a. Each waveform in the graph corresponds to the computed flux density in the 1D finite elements, the number of which was 9 (the waveform of the flux on the surface b_s is indicated in the graph.) The averaged value of the flux waveforms is a sinusoid as it has to be. It is clear that at saturation the skin effect becomes negligible. The variation of the flux penetration in the lamination at an instant of time is shown in Fig. 6b.

4. Conclusions

A magnetodynamic vector hysteresis model of ferromagnetic steel laminations has been presented. The model utilizes the well-known Mayergoyz vector model to describe the rotational quantities and applies the concept of the magnetic viscosity to account for the magnetodynamic effects. The simplicity and generality of the model are its main attractions. The model can be used to predict rotational or alternating iron losses for a wide variety of frequencies.

The viscosity-based magnetodynamic model is applied in each direction defined by the vector model. The overall output of the magnetodynamic vector model is the result of the reduced averaged contributions of each direction. The identification process of the model leads to rather simple calculations. The scalar static data of the ferromagnetic material obtained from measurement, the major loop or the first-order reversal curves, are directly employed without modifications. To adjust its parameters, the model requires three dynamic loops measured in unidirectional

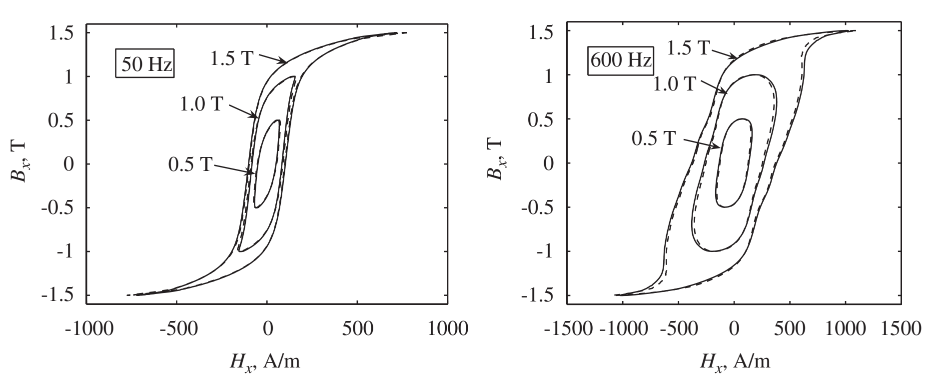

Fig. 1. Measured (solid) and calculated (dashed) unidirectional dynamic loops.

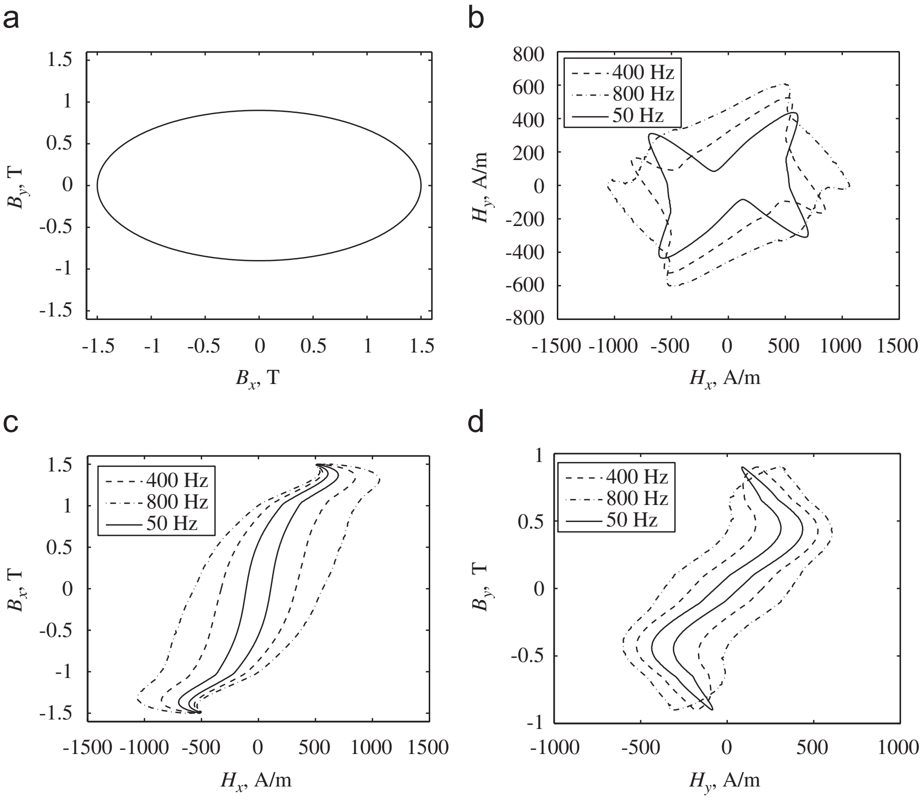

Fig. 2. Dynamic loops calculated under elliptic flux excitation.

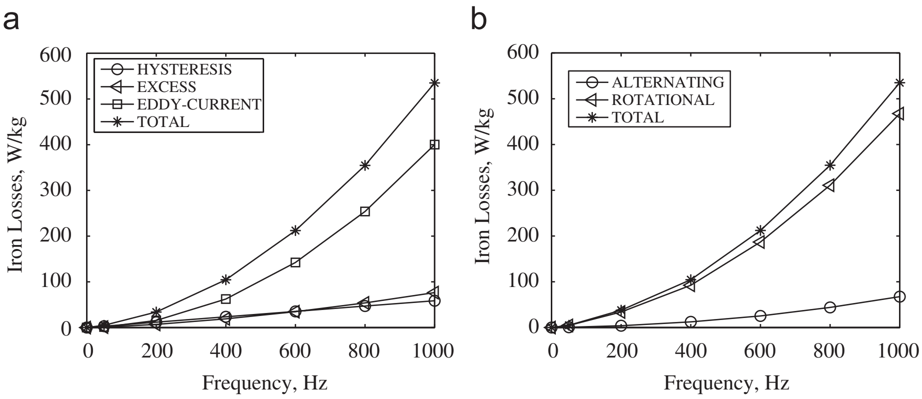

Fig. 3. Separation of the calculated iron losses under elliptic flux excitation shown in Fig. 2a.

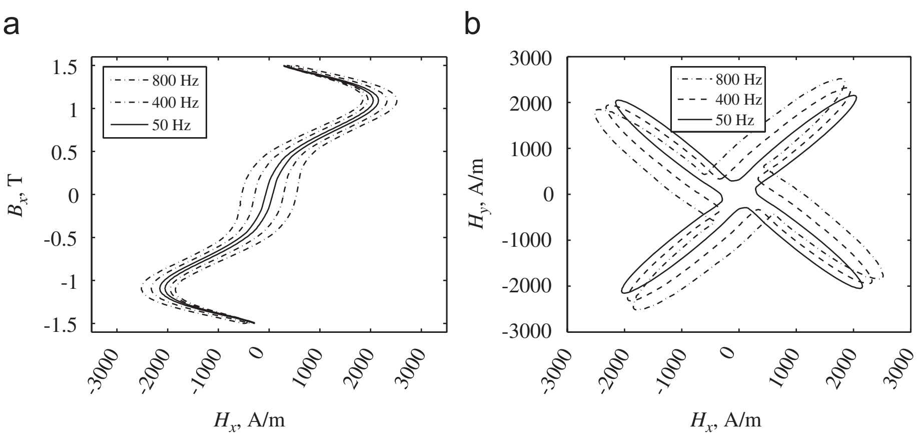

Fig. 4. Dynamic loops calculated under circular flux excitation.

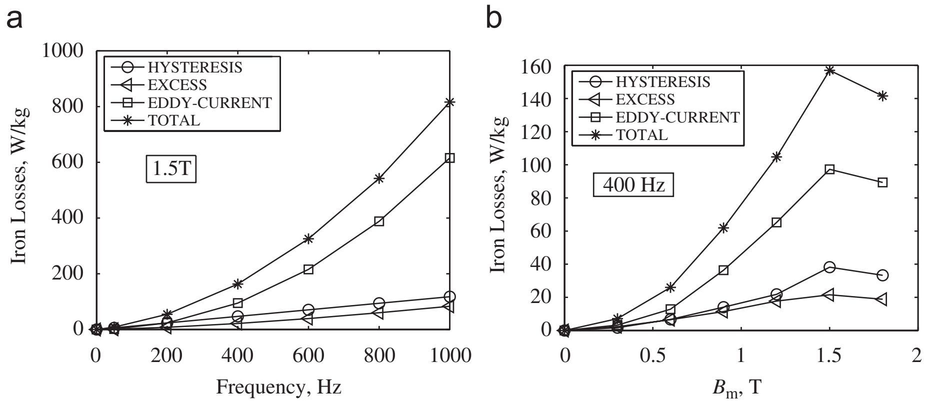

Fig. 5. Separation of the calculated iron losses under circular flux excitation.

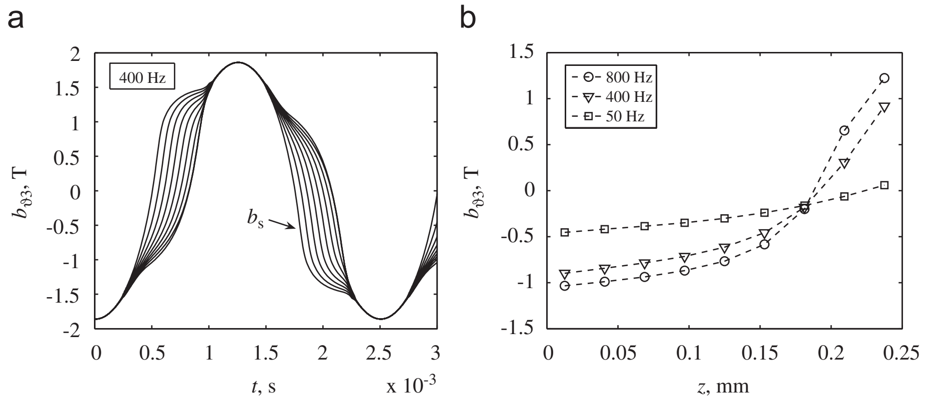

Fig. 6. Flux distribution inside the lamination for ϑ_3.

alternating field. The extension to utilize the rotational losses, obtained from 2D measurements, in tuning the rotational coefficient w is feasible and will be done in a future work.

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References

[1] L.R. Dupre, O. Bottauscio, M. Chiampi, J. Melkebeek, M. Repetto, IEEE Trans. Magn. 35 (1999) 4171.
[2] V. Basso, G. Bertotti, O. Bottauscio, F. Fiorillo, M. Pasquale, M. Chiampi, M. Repetto, J. Appl. Phys. 81 (8) (1997) 5606.
[3] C. Serpico, C. Visone, I.D. Mayergoyz, V. Basso, G. Miano, J. Appl. Phys. 87 (9) (2000) 6923.
[4] S.E. Zirka, Y.I. Moroz, P. Marketos, A.J. Moses, IEEE Trans. Magn. 42 (2006) 2121.
[5] O. Bottauscio, M. Chiampi, D. Chiarabaglio, IEEE Trans. Magn. 36 (2000) 561.
[6] J. Pippuri, A. Arkkio, Combined 2D–1D time-harmonic model for analysis of rotating electrical machines, in: ICEM, Chania, Greece, PTA3-17, September 2006.
[7] O. Bottauscio, M. Chiampi, J. Appl. Phys. 89 (2001) 6728.
[8] S. Spomic, D. Moussaoui, A. Kedous-Lebouc, B. Cornut, J. Magn. Magn. Mater. Appl. Phys. 160 (1996) 147.
[9] L.R. Dupre, F. Fiorillo, C. Appino, A.M. Rietto, J. Melkebeek, J. Appl. Phys. 87 (2000) 6511.
[10] I. Mayergoyz, Nonlinear Diffusion of Electromagnetic Fields, Academic Press, New York, 1998.
[11] I. Mayergoyz, Mathematical Models of Hysteresis and Their Applications, Academic Press, New York, 2003.
[12] E. Dlala, J. Saitz, A. Arkkio, IEEE Trans. Magn. 42 (2006) 1963.
[13] S.E. Zirka, Y.I. Moroz, P. Marketos, A.J. Moses, IEEE Trans. Magn. 41 (3) (2005) 1109.
[14] S.E. Zirka, Y. Moroz, P. Marketos, A.J. Moses, IEEE Trans. Magn. 40 (2004) 390.
[15] T. Matsuo, M. Shimasaki, IEEE Trans. Magn. 43 (4) (2007) 1389.
[16] K. Atallah, D. Howe, IEEE Trans. Magn. 29 (6) (1993) 3547.
[17] G. Bertotti, I. Mayergoyz, The Science of Hysteresis, Academic Press, New York, 2005.