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Hysteresis Modeling Based on Symmetric Minor Loops

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This paper presents a method for modeling symmetric minor loops in hysteretic magnetic media. The method employs symmetric minor loops combined with certain segments of the initial magnetization curve to identify the classical Preisach model. The proposed method is simple and effective. It exploits the symmetric positions of the reversal points lying on the initial magnetization curve and, therefore, guarantees monotonic symmetric change of the Preisach model input used in the calculation of the Everett function. A further advantage of the method is that it permits the use of the numerical Preisach model and omits the need of higher-order reversal curves. The method is applied to the prediction of symmetric minor loops of semi-hard magnetic material by using the classical scalar Preisach model. The simulation and measurement results show the high accuracy of the method and validate the proposed approach.

Index Terms—Classical Preisach model, magnetic hysteresis, semi-hard magnetic material, symmetric minor loops.

I. INTRODUCTION

DESPITE the numerous studies concerning hysteresis modeling, it is difficult to find a model that would accurately and generally describe this phenomenon in different magnetic materials. Among all the present models, the Preisach model is the most common choice in different applications of hysteresis modeling involving magnetism. Because of its capability of providing relatively accurate predictions, in addition to its generality and robustness, the Preisach model is a prevailing hysteresis model in the area of magnetic field analysis. Several procedures have evolved in order to identify a Preisach distribution function. Some apply the solution of sets of equations and some utilize the experimental data of certain curves such as first or higher-order reversal curves. The significant problems of identification and implementation using experimental data have been addressed by Mayergoyz [1].

From several Preisach-type models, the classical Preisach model (CPM) in its scalar form has prompted many researchers to improve its accuracy and applicability. It is often expressed by the following formula:

$$y(t) = \Gamma[u](t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) [\gamma_{\alpha\beta} u](t) d\alpha d\beta. \quad (1)$$

where $y(t)$ corresponds to the output of the Preisach model. $[\gamma_{\alpha\beta} u](t)$ is an operator sometimes referred to as an elementary Preisach hysteron since it is a basic block from which the Preisach operator $\Gamma[u]$ will be constructed. Here, α and β correspond to increasing and decreasing values of the input $u(t)$. $\mu(\alpha, \beta)$ is a distribution function (also called the density function or the Preisach measure).

This paper introduces a new method to identify the CPM defined by (1). The method is intended to model minor hysteresis loops made by continuously changing the field in a cyclic manner around the *origin*, and hence these minor loops will be

symmetric. The accurate modeling of these loops in applications of *nonrotating* hysteresis, such as hysteresis loss calculations in transformers and inductors, is important and should be carefully handled. In this paper, since the field is nonrotating, only the scalar hysteresis model is presented. To account for *rotating* fields, such as the ones revolving in electric rotating machines, *nonsymmetric* behavior is usually encountered and the extension of the scalar model to a vector model is necessary. The vector model described in [2] consists of angularly distributed scalar models and its identification is based on the scalar model. The investigation of the vector model is out of the scope of this work and left for future pursuit.

The accurate modeling of nonrotating hysteresis taking nonsymmetric behavior into account is not the main objective of this work; however, we will examine our method to model nonsymmetric curves and see how it is prone to these curves. Such modeling requires sophisticated techniques which may consider the complexity of the magnetization curves caused by changing the field in a cyclic manner around different locations in the B - H plane. Although these techniques, such the ones introduced in [3] and [4], utilize extensive amounts of data, yet, they have not reached a satisfactory level of accuracy to model symmetric and nonsymmetric minor loops. Nevertheless, they may still be regarded as approximate ones.

This work was initiated and motivated by the need to model hysteresis in Fe-Cr semi-hard magnetic material (SHMM). This type of material resembles the hard magnetic material in the way that it exhibits a relatively wide hysteresis loop and its magnetization changes sharply and abruptly around the coercive field. It is noted that for such materials, the modeling accuracy of symmetric minor loops is prone to the position of the reversal points. This has brought up the idea of utilizing symmetric minor loops in order to obtain monotonic symmetric positioning of the reversal points lying on the initial magnetization curve. It will be shown that the proposed model is simple and able to predict cyclic symmetric minor loops accurately; furthermore, it has the advantage of skipping the complexity associated with the substantial enlargement of the input-output data in case of using nonlinear models.

II. DEVELOPMENT OF THE MODEL IDENTIFICATION

The proposed method takes advantage of employing symmetric minor loops in the identification of the CPM rather than employing first-order reversal curves commonly used in the Preisach identification.

The identification method [5] of the CPM based on the first-order reversal curves has received a wide success in modeling ferromagnetic hysteresis, especially for soft magnetic materials. It is founded on the assumption that there is a distinct analogy among the magnetization curves situated in the different regions of the B - H plane. The modeling problem has further advanced to utilize higher order reversal curves for the identification process of both the CPM and the nonlinear Preisach model (the generalized model) [3]. These curves represent the dependence of the magnetization on the magnetic field obtained for several reversals of the magnetic field. The generalization of the CPM to the nonlinear model is achieved by assuming that the distribution function μ is dependent on the current value of the input $u(t)$. The comparison made in [6] showed that the results obtained by using the nonlinear model give more accurate predictions than the CPM when the reversal values of the magnetic fields are smaller than the coercive fields; however, when the field is larger than the coercive fields, the accuracy of the two models is not satisfactory.

According to their definition, the first-order reversal curves are obtained by starting at a saturation point (positive or negative) on the major loop and then reversing the field at a certain point (H_r, B_r) (see Fig. 1). In this type of material (SHMM), difficulties associated with measuring the first-order reversal curves because of the abrupt change of hysteresis curves near the coercive field were encountered. Consequently, measuring higher order reversal curves for SHMM would be a laborious task and, therefore, an alternative method of identification should be considered.

In the context of this work, we assume that only the major loop and the symmetric minor loops are available. Although we have managed—after a lot of effort—to measure the first-order reversal curves for the SHMM sample, we keep our assumption to use only the major loop and the symmetric minor loops in the identification process. That is because—for hard and semi-hard magnetic materials in particular—successful measuring of first-order reversal curves by a conventional experimental setup cannot be guaranteed. On the other hand, our experience with SHMM showed relatively easy measurements of the major and the symmetric minor loops. However, for the sake of comparison, we will utilize the measured first-order curves (Fig. 1) in the implementation of the CPM.

In this type of material (SHMM), the likelihood of having sufficiently accurate modeling by using the CPM identified from the first-order curves would be obviously low due to the restricted range of the reversal points $H_{r_0}, H_{r_1}, \dots, H_{r_n}$ inside the major loop (Fig. 1) which govern the calculations of the *Everett function* $E(\alpha, \beta)$ according to the change of the parameters α and β in the Preisach plane. In other words, the first-order reversal points cannot pass into the major loop where the minor loops are generated (this is a direct result of the sharp change of the magnetization around the coercive field), and therefore only

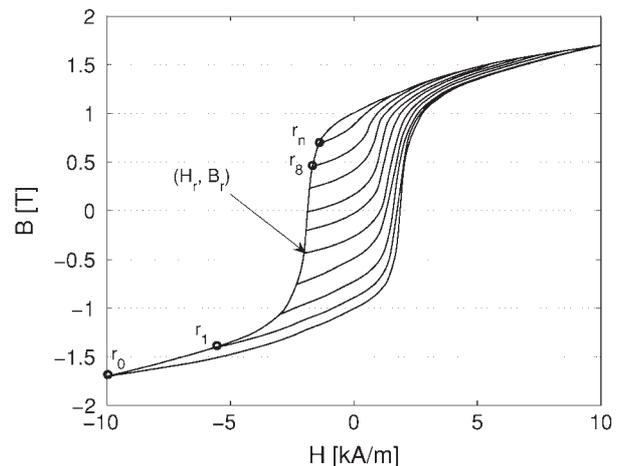


Fig. 1. First-order reversal curves measured on a sample of Fe-Cr semi-hard magnetic material.

limited area inside the major loop could be improved by the use of first-order reversal curves; and for the reversal points which could expand inside the major loop the congruency problem would then become a major drawback.

Therefore, the development of a technique through which an action of ferromagnetic material (its magnetization M or flux density B) can be accurately predicted for any symmetric changes in the external magnetic field H is needed. The objective of this study is the establishment of a hysteresis model with accurate symmetric minor loops. The utility of using these loops in the identification problem emerges as a plausible advantage to enhance their modeling accuracy.

III. THE IDENTIFICATION METHOD

A combination of a family of measured symmetric minor loops (only lower or upper parts) with certain segments of the initial magnetization curve will be realized. This combination of curves will shape an analogous form of the first-order reversal curves but the field reversal is made along the initial magnetization curve (an anhysteretic curve) and for this reason these curves will be called the *anhysteretic reversal curves*. In principle, the Preisach model purely identified from the symmetric minor loops can satisfactorily predict symmetric hysteretic behavior; however, in order to enhance nonsymmetric modeling, a greater amount of data collected from the anhysteretic reversal curves will lead to more accurate hysteresis modeling.

A. Definition of the Anhysteretic Reversal Curves

The anhysteretic reversal curves can be either measured or constructed from the symmetric minor loops and the initial magnetization curve. The definition of the symmetric minor loops must satisfy Madelung's rule which states the following.

"In Fig. 2, if some point m_1 of the initial magnetization curve becomes a reversal point, then the reversal curve that starts from point m_1 reaches point m_2 , which is symmetric with respect to the point m_1 about the origin o ." This qualitative rule with other useful regularities were reported in the early twentieth century known as Madelung's rules and also restated in [7]. So the definition can be also interpreted in a slightly different way as was

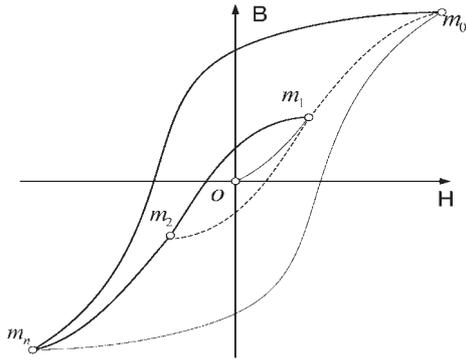


Fig. 2. Madelung's rule of regularity and its meaning to the anhysteretic reversal curve.

reported by Bertotti in [8]: "If one applies a cyclic field around the origin of variable amplitude to the demagnetized state, one obtains the set of symmetric minor loops. The line connecting the loop tips is known as the initial magnetization curve." The latter observation will reduce a lot of effort in the identification method, i.e., the initial magnetization curve can be determined by interpolating in between the tips of the symmetric minor loops and the major loop.

The observation how to create the initial magnetization curve is important and useful, first, for locating the *anhysteretic reversal points*, which represent the starting points of the anhysteretic reversal curves, and second for composing the anhysteretic reversal curves. It is noticed that the anhysteretic reversal points are lying on the initial magnetization curve and the extension curves needed for the symmetric minor loops to reach the point m_n are identical segments of the initial magnetization curve.

The measurement of an anhysteretic reversal curve may be achieved by starting the field from the origin and then reversing it at some point, say m_1 , and continuing until reaching the saturation point m_n (see Fig. 2). With the availability only of the symmetric minor loops, an anhysteretic reversal curve is obtained by adding a portion of the initial magnetization curve to the upper (or lower) part of a symmetric minor loop in order to reach the point m_n . That is, one half of the symmetric minor loop, say m_1 - m_2 , will be added to the segment m_2 - m_n , which is copied from the initial magnetization curve. Noticing that the segment m_2 - m_n by itself represents another anhysteretic reversal curve will be used in the identification process likewise.

B. Identification of the Classical Preisach Model

The Preisach distribution function or the Everett surface can be easily determined from the anhysteretic reversal curves (ARCs). The procedure is similar to that when using first-order curves and can be briefly described as follows. Suppose that a cyclic change of the magnetic field forms a symmetric minor loop as illustrated in Fig. 3(a). y_{α_1} represents the output corresponding to α_1 . The input is now decreased monotonically to a value β_1 and the corresponding output is described as $y_{\alpha_1\beta_1}$. The corresponding α - β diagram (the Preisach plane) is shown

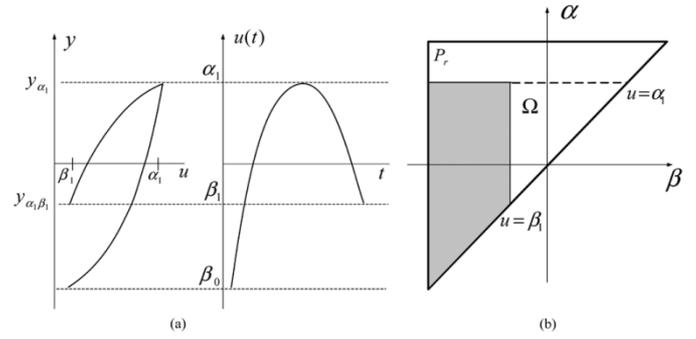


Fig. 3. Representation of a cyclic symmetric loop in the Preisach plane.

in Fig. 3(b). To derive the distribution function in terms of the ARCs, we introduce a function $E(\alpha_1, \beta_1)$

$$E(\alpha_1, \beta_1) = \frac{1}{2}(y_{\alpha_1} - y_{\alpha_1\beta_1}) \quad (2)$$

which represents the change in magnetization as the magnetic field changes from α_1 to β_1 . Equation (2) can be also represented as

$$E(\alpha_1, \beta_1) = \iint_{\Omega(\alpha_1, \beta_1)} \mu(\alpha, \beta) d\alpha d\beta. \quad (3)$$

In case we would like to find the distribution function $\mu(\alpha_1, \beta_1)$, we can take the double derivative with respect to α and β on both sides of (3)

$$\begin{aligned} \mu(\alpha_1, \beta_1) &= \frac{\partial^2}{\partial \alpha_1 \partial \beta_1} E(\alpha_1, \beta_1) \\ &= -\frac{1}{2} \frac{\partial^2 y_{\alpha_1\beta_1}}{\partial \alpha_1 \partial \beta_1}. \end{aligned}$$

If $y_{\alpha\beta}$ could be identified for all points in the Preisach plane P_r , it is clear on physical grounds that the surface $E(\alpha, \beta)$ formed of all points $y_{\alpha\beta}$ should be smooth. This surface could then be differentiated to obtain the distribution function

$$\mu(\alpha, \beta) = \frac{\partial^2 E(\alpha, \beta)}{\partial \alpha \partial \beta}. \quad (4)$$

However, in order to avoid the double numerical differentiation of $E(\alpha, \beta)$ to obtain $\mu(\alpha, \beta)$, the function $E(\alpha, \beta)$ itself is used to obtain the expression for the output $y(t)$, rather than (4). This helps to avoid amplifying errors in the experimental data and simplifies the numerical implementation of the Preisach model.

C. Numerical Preisach Model

The implementation of the Preisach model (1) will be made numerically following the procedure described in [9, pp. 45–58]. The output of the numerical Preisach model, which corresponds to the flux density B , is expressed by

$$y(t) = 2 \sum_{k=1}^N (E(\alpha_k, \beta_{k-1}) - E(\alpha_k, \beta_k)) - E(\alpha_0, \beta_0) \quad (5)$$

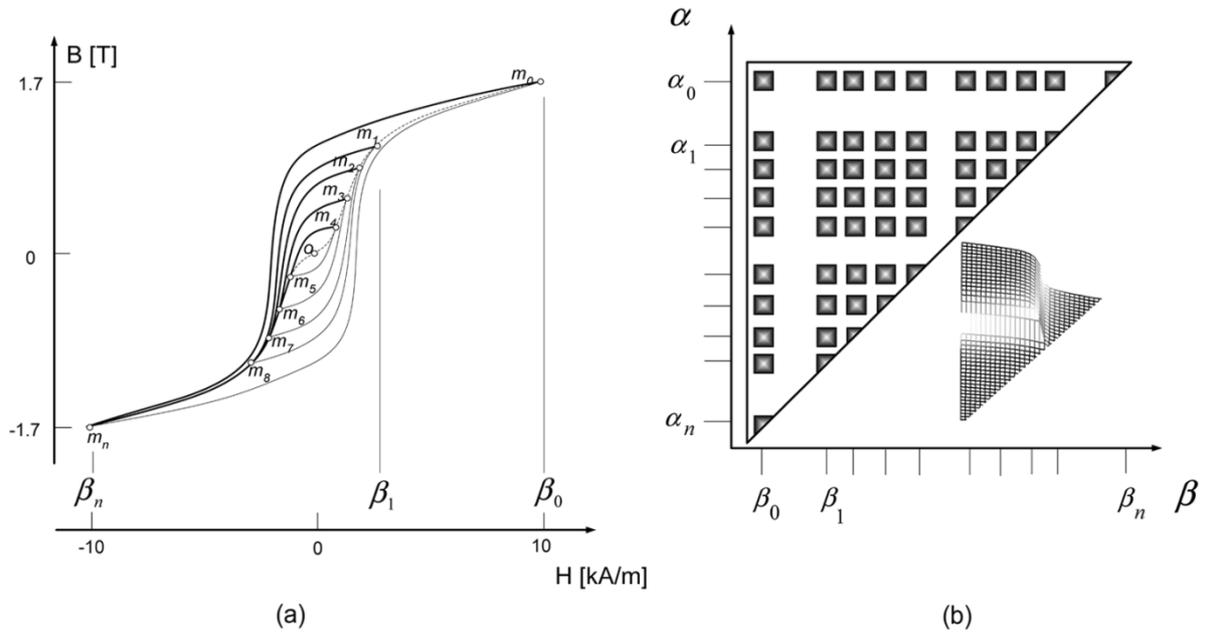


Fig. 4. Representation of the constructed anhysteretic reversal curves in the discrete Preisach plane.

which corresponds to the same formula given in [2], (7) and originally derived in [5]. The function $E(\alpha_k, \beta_k)$ is computed from the Everett table (surface); α_k and β_k represent the sequence of local extrema and they are decreasing and increasing sequences of α and β coordinates of interface vertices, respectively; N is the number of horizontal links made in the Preisach plane.

The input range $u(t)$, which corresponds to the magnetic field intensity H , is discretized into $(n+1)$ subranges of order pairs $\{u_i\}_{i=0, \dots, n}$ resulting in $(1/2)(n+1)(n+2)$ ARC data points in the Preisach plane for all pairs (u_i, u_j) with $j \leq i$ (see Fig. 4). A smooth approximation surface $\tilde{E}(\alpha, \beta)$ is then fit to these data points, and this surface can be differentiated to obtain an approximate distribution surface $\tilde{\mu}$.

The preprocessing of the Preisach model is to work out a surface (or table) from the anhysteretic reversal curves in order to compute the function $E(\alpha, \beta)$ during the process of modeling by using (5). Fig. 4(a) shows the construction of the anhysteretic reversal curves from the measured symmetric minor loops of SHMM. Equation (2) was used to determine the data points from the anhysteretic reversal curves (the thick curves). Each ARC would correspond to a specific reversal point and therefore a specific number of the parameters α and β in the Preisach plane. The discrete Preisach plane for $n=9$ containing 10 subdivisions and 55 data points generated to make the Everett surface is shown in Fig. 4(b). These points represent the weights of the corresponding ARCs $m_0-m_n, m_1-m_n, \dots, m_8-m_n$ initiated from the anhysteretic reversal points m_0, m_1, \dots, m_8 ; respectively, and terminated at the point m_n .

D. Remarks on the Method

The model is required to predict the symmetric branching which occurs inside the major loop on the basis of the information provided by the anhysteretic reversal curves. Regardless of the number of the reversal curves used, it is noted that the data

offered by the anhysteretic reversal curves are less in amount than that offered by the first-order reversal curves. However, it is natural from the type of the information gathered to expect that the model based on the anhysteretic reversal curves will predict symmetric minor loops more accurately.

The modeling of the symmetric branching does not require one to start from the anhysteretic state. This is similar to the case when using first-order reversal curves. There is no restriction to start from the ascending (or descending) branch of the major loop where the first-order reversal points are located. The function $E(\alpha_0, \beta_0)$ is precalculated in (5), which corresponds to the Everett weight of the demagnetized state, and so we can choose to start from any point we wish (see Fig. 8). The method does not either enforce restrictions on the type of branching whether it is symmetric or nonsymmetric. Nevertheless, it is clear that the modeling of the symmetric branching will be more accurate due to the symmetric shape of the Everett surface calculated from the anhysteretic reversal curves.

IV. RESULTS AND DISCUSSION

In the preceding section, the CPM was identified by using the method of the anhysteretic reversal curves. To judge the accuracy of the method, we provided a set of measured symmetric minor loops not used in the identification process and compared them to their model prediction. Additionally, for comparison, the identification method based on the first-order reversal curves was implemented in the CPM likewise. It is not accidental that in the two cases in Figs. 5 and 6 the method of the anhysteretic reversal curves showed more accurate results than that of the first-order reversal curves (FODs). This is related to the fact that using first-order reversal curves leads to accumulate most of the reversal points around the coercive field and with significant differences in the magnetic flux densities. This results in large errors when calculating the Everett function $E(\alpha, \beta)$ in the discrete Preisach plane as the weights of the precalculated points

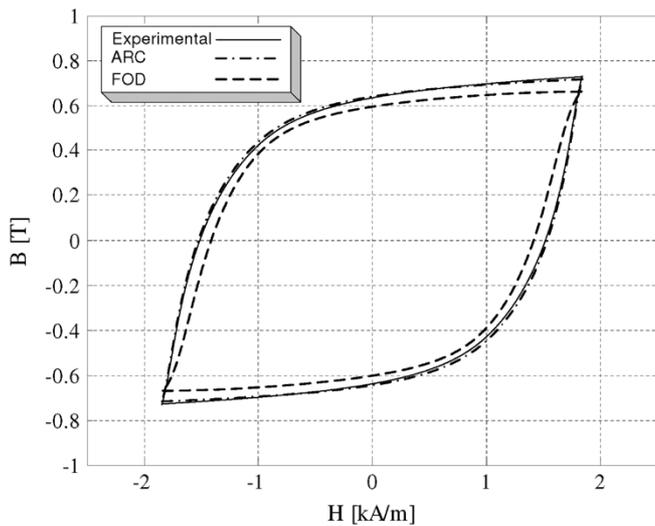


Fig. 5. Symmetric minor loop with $H = -1800$ A/m predicted by the methods based on, the anhysteretic reversal curves (ARC), and the first-order reversal curves (FOD), compared with the experimental results.

of the Everett surface jump sharply with inaccurate values. On the other hand, using the anhysteretic reversal curves makes the distribution of the reversal points more uniform and symmetric (there is monotonic matching between the position of the reversal points and their Everett function) and therefore consideration of the sharp change of the reversal points is asserted. To verify that the latter significantly improves the modeling accuracy of the symmetric minor loops, we have chosen two cases where the symmetric minor loops were situated in two different positions inside the major loop. First, the starting field value of the loop was very close to the coercive field ($H \simeq H_c = -1800$ A/m); and second, the starting field value of the other loop was $H = -1500$ A/m. It is evident that when the starting field value was $H = -1800$ A/m (see Fig. 5) both methods using the FODs and ARCs gave similar good results compared with the experimental ones; however, better modeling still obviously observed in the ARC method. In the second case, the difference becomes distinct that the ARC prediction was of greater accuracy than that of the FOD. This has occurred because at this value ($H = -1500$ A/m) the corresponding reversal point in the FOD would be, according to Fig. 1, interpolated between the values $H_{r8} = -1480$ A/m and $H_{r9} = -1510$ A/m, which would have in effect a smaller value of the function E than what it should be. Conversely in the ARC, the value $H = -1500$ A/m using the symmetry property $E(\alpha, \beta) = E(-\alpha, -\beta)$ would correspond, according to Fig. 4, to the reversal point interpolated between the values $H_{m2} = 1490$ A/m and $H_{m3} = 1530$ A/m, which would approximately contain the required accurate value of the function E .

In Fig. 7, a measurement of symmetric minor loops caused by applying a cyclic external field was carried out to verify the robustness of the method. The results in Fig. 7 show that the presented method was adequate for predicting symmetric minor loops in terms of both modeling quality and rigidity.

The ability of the model to predict nonsymmetric loops was also examined as indicated in Fig. 8. The field was changed in a cyclic manner in the upper half of the B - H plane and

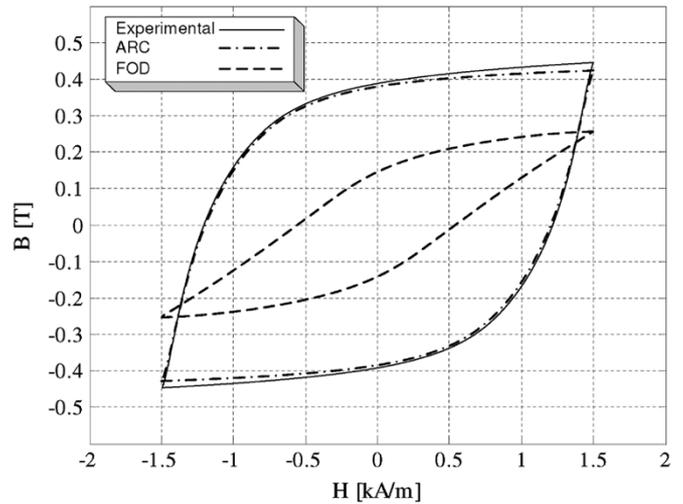


Fig. 6. Symmetric minor loop with $H = -1500$ A/m predicted by the methods based on, the anhysteretic reversal curves (ARC), and the first-order reversal curves (FOD), compared with the experimental results.

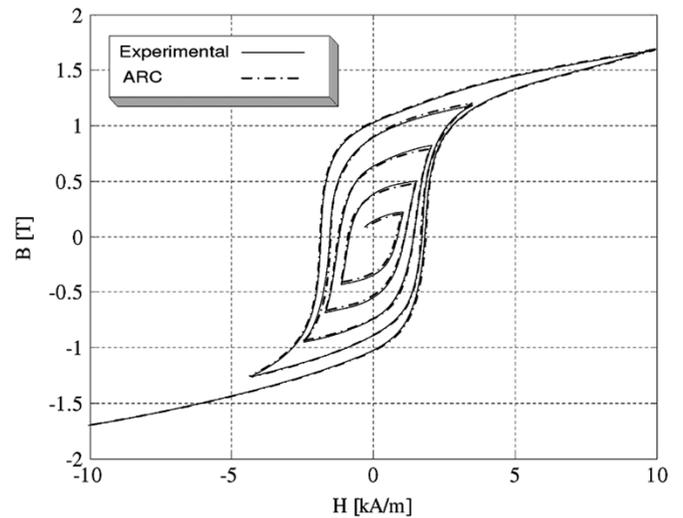


Fig. 7. Symmetric minor loops predicted by the method of the anhysteretic reversal curves (ARC) and compared with experimental data.

not around the origin. It is clear that the predictions of the model based on the ARCs were not as accurate as in the case of symmetric loops. This occurred because the data gathered from the ARCs were more usable and suitable to predict symmetric minor loops rather than nonsymmetric loops. In case we were concerned about modeling accurate nonsymmetric and symmetric minor loops, the use of the nonlinear model [3] identified by the method of the anhysteretic reversal curves with more higher order reversal curves would be adequate. This will be investigated in future work. Fig. 8 also shows that one can start modeling from a hysteretic state. Thus, the model does not impose limitations from where to start modeling whether it is the anhysteretic state or not.

V. CONCLUSION

This work dealt with a new method to identify the classical scalar Preisach model. The method is specially made for pre-

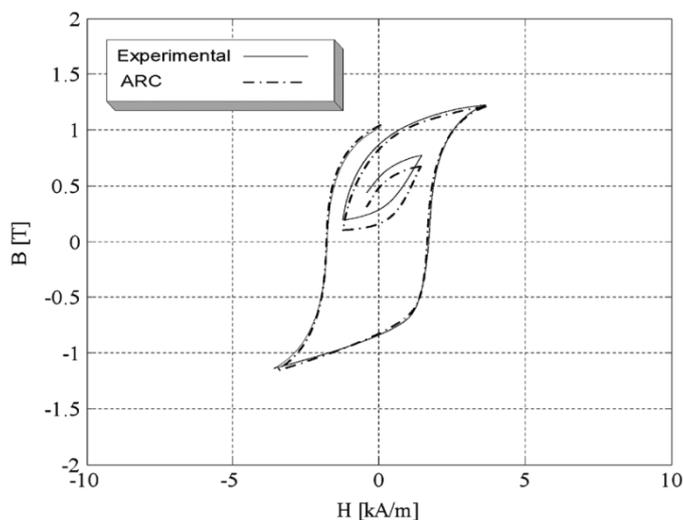


Fig. 8. Nonsymmetric minor loops predicted by the method of the anhyseretic reversal curves (ARC) and compared with experimental data.

dicting symmetric minor loops in magnetic hysteretic media. The choice of replacing the first-order reversal curves by the anhyseretic reversal curves to model symmetric loops was of practical use to fill the gap of the limited reversal points in the major loop, thereby improving the type of coverage provided by each anhyseretic reversal curve.

The accuracy of the presented method was verified by comparing the results with experimental data as well as with the method of the first-order reversal curves. A remarkably good

agreement was observed between the simulated and experimental symmetric curves. The combination of the symmetric minor loops and the initial magnetization curve results in a robust, relatively simple and computationally fast model that accurately predicts symmetric hysteresis behavior in semi-hard magnetic material.

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