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Load Balancing by Joint Optimization of Routing and Scheduling in Wireless Mesh Networks

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Abstract. In this paper we study load balancing in wireless mesh networks when the MAC layer of the network is modelled by STDMA. We formulate the linear problem for joint optimization of traffic allocation and transmission schedule. Both unconstrained path set, allowing arbitrary routing, and predefined paths are considered. In our numerical examples roughly third of the load of the most congested link can be reduced by load balancing. This reduction in the load decreases the delays of the network as well as increases the reliability of the system if link conditions change suddenly.

1 Introduction

Wireless Mesh Networks (WMNs) are dynamic networks that are constructed from two types of nodes, mesh routers and mesh clients [1]. By supporting multi-hop routing the mesh routers form a mesh backbone which enables cost-effective communication between the clients in the network as well as an access to other networks such as Internet. Originally, multi-hop wireless networks were considered primarily for military use but now there is also a growing interest in commercial exploitation of them.

Familiar from fixed networks, the term *load balancing* refers to optimization of usage of network resources by moving traffic from congested links to less loaded parts of the network based on knowledge of network state. By this approach QoS experienced by the users, such as transmission delay, is improved. Many load balancing algorithms proposed for IP networks, especially along development of new tunnelling technique MPLS (see [2], [3]). However, these algorithms are not directly usable in WMNs, since the interference restricts the simultaneous use of the links. However, by the scheduling the interference can be alleviated, and moreover, the resources of the congested links can be added. Combining scheduling and routing is obviously beneficial in optimization of WMNs.

In this paper we study load balancing problem in wireless mesh networks. In the problem our aim is to find such routing and MAC layer scheduling that the maximum link utilization of the network is minimized. We formulate an LP-problem both for free routing in which data can be split to arbitrarily many routes as well as for constrained routing in which the paths used by data flows are predefined, but the traffic allocation to the paths can be optimized. We compare results of joint optimization of MAC layer and link layer parameters (also known as cross-layer optimization) to equal splitting of traffic onto the shortest paths and to the more conventional scenario where optimization is done separately at two layers. As a result we find that cross-layer approach is required to achieve significant improvements in maximum link utilization.

We assume that the MAC layer of the wireless network is be modelled by Spatial TDMA [4]. In STDMA the transmission resources are divided into time slots and the links that are spatially sufficiently separated, can transmit in the same slot. The set of links transmitting in same slot is called as the transmission mode. In addition, we assume that the interference affects only the set of active links, but the nominal capacity of the links remain fixed.

Optimization of resource usage in wireless mesh networks has gained lot of interest recently. Major part of them focuses on throughput maximization ([5], [6], [7]), where as some papers have studied delay minimization [8] or dimensioning [9]. Our work is most closely related to study of Wu et al. in [6], but has some differences: our view is on load balancing as a routing problem with given demands between source and destination pairs, where as paper [6] concentrates on multicast sessions. In addition, the formulation of the multipath routing problem with a predefined set of routes is missing from the above studies.

The paper is organized as follows: In Section 2 we introduce the wireless network model and, in particular, the interference model that we are assuming. The problem formulations for load balancing with arbitrary routing as well as with the predefined path set are given in Section 3. In section 4 we give numerical examples of load balancing. Finally, section 5 makes a short conclusion of the paper.

2 Modelling the wireless network

Let us consider a wireless mesh network, which consists of N nodes equipped with a single radio. The distance between nodes i and j is denoted by d_{ij}. The communication range of node i is denoted by R_i and the interference range by R'_i. Depending on the interference model, there might exists a link between two nodes i and j in the network. Also the capacity of link l, denoted by c_l, depends on the interference model. We assume that the link exists from node i node j if the receiver of node j is inside the communication range of the transmitter of node i. Let vector $\mathbf{b} = \{b_1, b_2, ..., b_L\}^T$ denote nominal bandwidths of the links without interference and let L denote the number of these links.

Adapting the interference model from [5], the transmission from a transmitter to a receiver is successful if there is not any other node transmitting in the interference range of the receiver simultaneously. That is to say, for successful transmission from node i to j the following constraints should be true:

1. $d_{ij} \leq R_i$.
2. No node k with $d_{kj} \leq R'_k$ is transmitting.

A set of the links that can transmit simultaneously according to the above constraints is called a *transmission mode*. The mode is said to be *maximal*, if we cannot insert any link to the mode without violating the aforementioned rules for successful transmission. Let M denote the number of transmission modes and let $\mathbf{S} \in \mathbb{R}^{L \times M}$ denote the transmission mode matrix, in which element $S_{l,m}$ denotes the capacity of link l in mode m. We assume that the capacities of the active links remain fixed over the transmission modes.

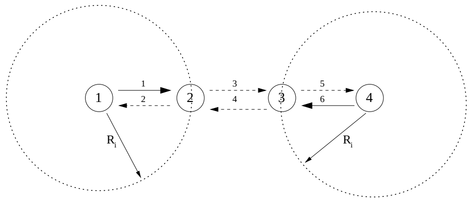

Fig. 1. Transmission mode in a network with 4 nodes and 6 links.

As an example of transmission modes, consider a chain of four nodes with six one-directional links. The distances from each node to the neighboring nodes as well as communication and interference ranges of the nodes are assumed to be 1. For instance, if link 1 is transmitting the only link that can transmit without interference is link 6. The transmission mode of these links is depicted in Figure 1, in which a solid arrow refers to an active link and dashed arrow to non-active link. Altogether, there are four feasible and maximal transmission modes. The corresponding transmission mode matrix is:

$$\mathbf{S} = \mathbf{b}^T \begin{pmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \\ 0\,1\,0\,0 \\ 1\,0\,0\,0 \end{pmatrix} .$$

In general, finding all feasible transmission modes is known to be a hard problem. For example, in a network with 22 links, the feasible transmission modes have to be selected from over 4 million different link combinations. However, the maximal transmission modes can effectively be generated by a simple algorithm which adds non-interfering links to a tree in a depth-first search manner. The proposed algorithm is described in Appendix.

In wireless networks with STDMA type MAC layer, different transmission modes are scheduled in a TDMA manner. The transmission mode matrix can be associated with a schedule, which defines the proportion of time that each mode uses for transmission. Let $\mathbf{t} = \{t_1, t_2, ..., t_M\}^{\mathrm{T}}$ denote the schedule, which element t_m is the transmission time of mode m. The actual capacities of links, denoted by vector $\mathbf{c} = \{c_1, c_2, ..., c_L\}$, can be determined by weighting the capacity of a link in one transmission mode by the length of the mode's time slot in the schedule and taking the sum over all M modes:

$$\mathbf{c} = \mathbf{St}.$$

When scheduling is executed at sufficiently fast time scale, the flows using the links in effect experience a constant link capacity.

3 Load balancing problem

In this section we formulate the load balancing problem in wireless mesh networks with STDMA scheduling. The problem is well-known from the wired networks with fixed capacity (see [2], [3]). In the wireless context the new thing is that, in addition to routing, the resources of the network can be shifted from congested areas to other areas by scheduling. We consider two different formulations of the load balancing problem: In the one formulation we can split traffic to arbitrary many routes, whereas in the second one the path set is predefined.

The network is loaded by traffic flows travelling from origin nodes to destination nodes. Let s_k refer to the origin node and t_k referring to the destination node of OD-pair k. In addition, let K denote the number of OD-pairs and vector $\mathbf{d} = \{d_1, d_2, ..., d_K\}^{\mathrm{T}}$ the mean sending rates of the OD-pairs.

3.1 Paths unconstrained

Let us consider a wireless network described in Section 2. In addition to earlier notation, let $\mathbf{A} \in \mathbb{R}^{L\times N}$ denote the link-node incidence matrix for which $A_{l,n} = -1$ if link l directs to node n, $A_{l,n} = 1$ if link l leaves from node n, and $A_{l,n} = 0$ otherwise; and let $\mathbf{R} \in \mathbb{R}^{K\times N}$ denote the demand matrix for which $R_{k,s_k} = d_k$, $R_{k,t_k} = -d_k$, and $R_{k,n} = 0$ for all other nodes n.

Let $\mathbf{X} \in \mathbb{R}^{K\times L}$ be the traffic allocation matrix, with element $X_{k,l}$ corresponding to the traffic of OD-pair k allocated to link l. The allocation matrix is unknown yet, but will be determined as a solution of the load balancing problem. Given the traffic allocation matrix $\mathbf{X}$, the induced link load vector $\mathbf{y} = \{y_1, y_2, ..., y_L\}^{\mathrm{T}}$ is

$$\mathbf{y} = \mathbf{X}^{\mathrm{T}}\mathbf{e}_K,$$

where vector $\mathbf{e}_K$ is K-dimensional column vector with all elements equalling 1 (same notation is used for other unit vectors also). In general, the mapping between the traffic matrix $\mathbf{X}$ and the link load vector $\mathbf{y}$ is not one-to-one, i.e. while $\mathbf{X}$ determines $\mathbf{y}$ uniquely, the opposite is not true.

Let variable α denote the maximum utilization of the links

$$\alpha = \max_l \frac{y_l}{c_l}.$$

The primal objective in the unconstrained load balancing problem is to minimize the maximum link utilization α by choosing an optimal traffic allocation $\mathbf{X}$ and transmission schedule $\mathbf{t}$. For obtaining linear constraints, we introduce also a new variable $\mathbf{q} = \{q_1, ..., q_M\}$ which elements satisfying $q_m = \alpha t_m$. The linear load balancing problem for unconstrained path set is as follows:

$$\begin{aligned}
&\text{Minimize } \alpha \text{ over } \mathbf{X} \text{ and } \mathbf{q}\\
&\text{subject to the constraints}\\
&\mathbf{X} \geq 0, \mathbf{q} \geq 0,\\
&\alpha = \mathbf{e}_M{}^{\mathrm{T}}\mathbf{q},\\
&\mathbf{X}^{\mathrm{T}}\mathbf{e}_K \leq \mathbf{S}\mathbf{q},\\
&\mathbf{X}\mathbf{A} = \mathbf{R}.
\end{aligned} \tag{1}$$

In the problem above we have five constraints. The first two constraints state that all free variables should be positive. The third one, when divided by α, states that in a schedule the sum of the time proportions should equal to 1, fourth one is the link capacity constraint and the fifth one is the so-called *conservation of flow constraint*, which states that the traffic of each OD-pair incoming to a node has to equal the outgoing traffic from that node.

By problem formulation (1) we can find an unique solution for α, but the load allocation is not necessarily optimal in some sense. For example, if there is a bottleneck in the network, the rest of the network remains unbalanced. Ott et al. present in [3] that the routing $\mathbf{y}$ is *non-dominated* if there does not exist another routing with link-loads $\mathbf{y}'$ with property

$$y_l' \leq y_l \text{ for all } l \text{ and } y_l' < y_l \text{ for at least one } l. \tag{2}$$

For this reason, after finding the minimum of the maximal link utilization α^* by optimizing problem (1), we optimize the traffic allocation and scheduling again in the network, in which the link capacities are reduced by the factor α^*. The optimization problem in the second phase, where our objective is to minimize the overall usage of resources, is as follows:

$$\begin{aligned}
&\text{Minimize } \mathbf{e}_K{}^{\mathrm{T}}\mathbf{X}\mathbf{e}_L \text{ over } \mathbf{X} \text{ and } \mathbf{t}\\
&\text{subject to the constraints}\\
&\mathbf{X} \geq 0, \mathbf{t} \geq 0,\\
&\mathbf{e}_M{}^{\mathrm{T}}\mathbf{t} = 1,\\
&\mathbf{X}^{\mathrm{T}}\mathbf{e}_K \leq \alpha^*\mathbf{S}\mathbf{t},\\
&\mathbf{X}\mathbf{A} = \mathbf{R}.
\end{aligned} \tag{3}$$

As a solution of the second optimization problem we obtain the link loads of each OD-pair as well as the optimal schedule. For each OD-pair k, from the resulting link load vector $\mathbf{X}_k^*$, the used paths and the traffic volumes on the paths can be solved by the following procedure: First we construct a reduced link topology, which includes the links that has traffic in the solution $\mathbf{X}_k^*$, and then calculate all paths between origin node s_k and destination node t_k. As the link load vector $\mathbf{X}_k^*$ is optimal, the paths between the source and destination node are automatically loop-free. Let $\mathbf{Q}_k$ denote the path-link incidence matrix, which element $Q_{k,p,l}$ is 1, if path p uses link l in the new reduced topology and 0 otherwise. Let $\mathbf{X'}_k$ denote the traffic allocation vector to the links in the reduced topology. This new traffic allocation vector can directly be constructed from the original optimal solution $\mathbf{X}_k^*$. Finally, traffic allocation to each path in matrix $\mathbf{Q}_k$, denoted by vector $\mathbf{z}_k$, can be solved from the matrix equation:

$$\mathbf{Q}_k{}^{\mathrm{T}}\mathbf{z}_k = \mathbf{X'}_k. \tag{4}$$

The system described by matrix equation (4) can be over- or underdetermined, because the number of paths often differs from the number of links. However, for us it is sufficient to find one path allocation that induces the link loads of the optimal solution.

3.2 Predefined paths

Now we assume that the paths available for each OD-pair are defined explicitly. This type of formulation is needed in situations where the number of paths is constrained to some value or some paths are known to be preferable in advance.

In the problem each OD-pair k has a predefined set $\mathcal{P}_k$ of the paths available. Let $\mathcal{P} = \cup_{k\in\mathcal{K}}\mathcal{P}_k$ denote the set of all available paths and P the number of all paths. We also introduce a path-link incidence matrix $\mathbf{P} \in \mathbb{R}^{P\times L}$ for all paths, with element $P_{p,l}$ equalling 1 if path p uses link l and 0 otherwise; and OD-pair-path incidence matrix $\mathbf{O} \in \mathbb{R}^{K\times P}$ with element $O_{k,p}$ indicating whether OD-pair k uses path p or not.

As in the previous subsection, let $\mathbf{X} \in \mathbb{R}^{K\times P}$ denote the allocation matrix of traffic of each OD-pair onto paths. To ensure that all traffic of each OD-pair is allocated, the traffic allocation matrix must satisfy:

$$(\mathbf{X} * \mathbf{O})\mathbf{e}_P = \mathbf{d},$$

where operator $*$ means element-wise multiplication of two matrices. Given $\mathbf{X}$, the link load vector $\mathbf{y}$ is

$$\mathbf{y} = \mathbf{e}_K^{\mathrm{T}}\mathbf{X}\mathbf{P}.$$

The objective in the load balancing problem with predefined path set is to minimize the link utilization coefficient α by choosing an optimal traffic allocation $\mathbf{X}^*$ and a scaled transmission schedule $\mathbf{q}^*$. The first phase of the problem, the minimization of the maximum link utilization is formulated as follows:

$$\begin{aligned}
&\text{Minimize } \alpha \text{ over } \mathbf{X} \text{ and } \mathbf{q}\\
&\text{subject to the constraints}\\
&\mathbf{X} \geq 0, \mathbf{q} \geq 0\\
&\alpha = \mathbf{e}_M^{\mathrm{T}}\mathbf{q},\\
&\mathbf{e}_K^{\mathrm{T}}\mathbf{X}\mathbf{P} \leq \mathbf{S}\mathbf{q},\\
&(\mathbf{X} * \mathbf{O})\mathbf{e}_P = \mathbf{d}.
\end{aligned} \tag{5}$$

Having optimal α^* from the first phase, the second phase, minimization of capacity usage, is as follows:

$$\begin{aligned}
&\text{Minimize } \mathbf{e}_K^{\mathrm{T}}\mathbf{X}\mathbf{P}\mathbf{e}_L \text{ over } \mathbf{X} \text{ and } \mathbf{t} \\
&\text{subject to the constraints} \\
&\mathbf{X} \geq 0, \mathbf{t} \geq 0 \\
&\mathbf{e}_M^{\mathrm{T}}\mathbf{t} = 1, \\
&\mathbf{e}_K^{\mathrm{T}}\mathbf{X}\mathbf{P} \leq \alpha^*\mathbf{S}\mathbf{t}, \\
&(\mathbf{X} * \mathbf{O})\mathbf{e}_P = \mathbf{d}.
\end{aligned} \tag{6}$$

The pathwise solution is not necessarily unique if the paths share common links. In addition, note that if path set $\mathcal{P}$ contains all possible paths between the origin and destination nodes, the path-constrained load balancing problem is similar to problems of the previous subsection.

4 Results

In this section we present numerical results for the load balancing problems presented in the previous section. To get an idea about good routing, we study first load balancing in a very small network with 4 nodes. After that more realistic network examples are studied. Since the gains achieved by cross-layer approach in resource allocation depends heavily on interaction between the topology and demands, we present results for both backbone and access type networks as well as for many OD-pair combinations. The feasible transmission modes are calculated beforehand once and for all. The optimal traffic allocation and the schedule can be then solved by an LP-solver with relatively small computational efforts.

Small network example Let us consider a 2x2 grid network with $N = 4$ nodes and $L = 8$ links. The distances between the neighbor nodes as well as transmission ranges are assumed to be 1. The network is loaded by two OD-flows; one from node 1 to node 4 and another one from node 4 to 3. The mean volumes of the flows are 1 unit, whereas the nominal capacities of the links are 3 units. There are 4 transmission modes in the network. The modes are depicted in Figure 2.

We consider three different optimization policies; shortest path routing with MAC layer optimization, separate optimization of two layers, routing and scheduling, and joint optimization of them. Using single shortest paths, traffic of the first OD pair is routed through nodes 1-2-4 and the second through nodes 4-3 (see the left hand side of Figure 3). This routing induces link load vector $\mathbf{y} = \{1, 0, 0, 1, 0, 0, 0, 1\}^T$. To allocate required resources to these two paths, transmission modes 1, 2 and 3 have to be used. It is easy to see that the optimal schedule is $\mathbf{t} = \{1/3, 1/3, 1/3, 0\}^T$ producing actual capacities $\mathbf{c} = \{1, 1, 1, 1, 0, 1, 0, 1\}^T$. The maximum load is then 1. The problem in the traffic allocation produced by

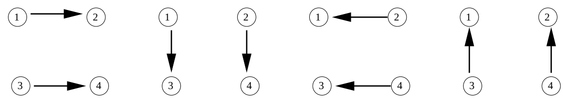

Fig. 2. Transmission modes in a 2x2 grid network.

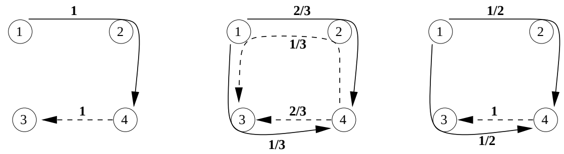

Fig. 3. Traffic allocations in a 2x2 grid network. Left: shortest path routing, middle: optimization at two layers, right: joint optimization.

shortest path routing is that in each schedule only one of two links is in use and thus the overall capacity usage is not optimal.

In the second approach we first balance the load without knowledge of the interference and then optimize the schedule. To obtain a balanced network, we move traffic from primary paths to secondary paths (not used by shortest path routing) until the maximum load does not decrease anymore (see the middle of Figure 3). Resulting traffic allocation is $\mathbf{y} = \{2/3, 2/3, 1/3, 2/3, 0, 1/3, 1/3, 2/3\}^T$. The schedule that minimizes the maximum load is $\mathbf{t} = \{2/7, 2/7, 2/7, 1/7\}^T$ and the corresponding maximum load is $7/9 \approx 0.78$.

Finally, we find that due to interference it is not optimal to spread traffic of OD-pair 2 to route 4-2-1-3 since there are bottlenecks in two different schedules. Instead, in optimal routing and scheduling, traffic of OD pair 1 is routed evenly over paths 1-2-4 and 1-3-4, but OD pair 2 uses only the shortest path 4-3 (see the right hand side of Figure 3). This produces quite uneven load distribution $\mathbf{y} = \{1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 1\}^T$ but using the schedule $\mathbf{t} = \{1/4, 1/4, 1/2, 0\}^T$ load can be balanced. The resulting maximum load is $3/4 = 0.75$.

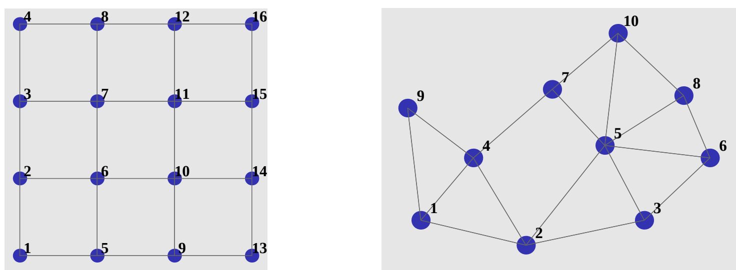

Fig. 4. Network topologies. Left: 4x4 grid topology, right: mesh topology.

4x4 Grid topology Next we study load balancing in more realistic-size networks (see the topologies in Figure 4). In the 4x4 grid topology there are 16 nodes, 48 links and 2934 maximal transmission modes. Generation of these modes by the DFS algorithm presented in Appendix takes 7 minutes (the algorithm implemented by Mathematica).

We study two different traffic scenarios. In the first scenario we consider that all nodes are identical and they form a wireless backbone network. To load the network, we generate random OD pairs, the source and destination nodes of which are randomly selected and the mean volumes are uniformly distributed between 0 and 1. In the second scenario we consider an access network, where all traffic is sent to one common access point located at the edge of the grid. We generate again random OD pairs, but now only source nodes and the mean volumes are random, whereas the destination node is the same for all OD pairs.

First we compare the optimal resource allocation with unconstrained routing to the scenario where traffic is optimally allocated to predefined paths with different lengths. Then the optimal resource allocation is compared to shortest path routing (referred as SP), optimization of the two layers separately (referred as 2-layer) and to so-called Equal Cost Multipath (ECMP) policy, in which traffic is distributed evenly to all shortest paths.

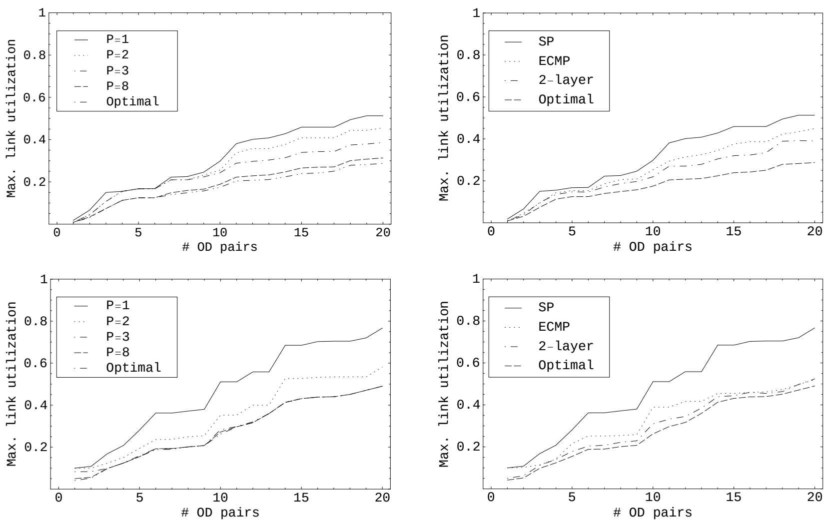

Fig. 5. The maximum link utilization as a function of the number of OD pairs. Top: backbone grid, bottom: access grid. Left: Path set constrained. Right: SP, ECMP, 2-layer optimization and optimum.

On the left hand side of Figure 5 we depict the maximum link utilization as a function of the number of random OD pairs in the backbone (top) and the access grid network (bottom), when the number of predefined paths varies from 1 to 8. Since finding N optimal paths is an integer optimization problem, the paths are just selected randomly from shortest to longest. From the figures we can see that adding one or two alternative paths improves the performance of the system. However, to obtain results very close to the optimum, we need considerably many paths. The optimal routing is compared to the single shortest path routing, ECMP and 2-layer optimization on the right hand side of the Figure 5. The maximal, minimal and mean differences between optimal resource allocation and other policies are also shown in Table 1. We can see that in some cases ECMP policy, for example, provides the same maximum link utilization as the optimal resource allocation, but on average the results of ECMP policy are 28% worse.

In general, as the backbone and access network cases are compared, the link utilizations are higher in the latter case, since there is a bottleneck around the access point. However, as seen from Table 1, by allocating the resources optimally the maximum link utilization can still be decreased 43 % on average as compared to shortest path routing.

Table 1. The difference between the optimal resource allocation and other policies.

	SP	ECMP	2-layer
Backbone grid			
Min	25 %	0 %	14 %
Max	53 %	37 %	28 %
Mean	42 %	28 %	22 %
Access grid			
Min	35 %	5 %	3 %
Max	58 %	58 %	17 %
Mean	43 %	19 %	9 %
Mesh network			
Min	15 %	14 %	17 %
Max	43 %	42 %	42 %
Mean	32 %	32 %	31 %

Mesh topology The last studied network is a mesh network (also used in [9]). In the network, there are two access points (nodes 2 and 6) as well as two relay nodes (nodes 4 and 5). The number of links is 34 and the number transmission modes 168, and the nominal capacities of the links are 100 units. The set of source and destination pairs is fixed and contains 18 OD pairs, but the mean volume of each OD pair is random. In Figure 6, the maximum link utilization is depicted as a function of the mean traffic volume of the OD pairs. The performance of load balancing as a function of the number of predefined paths is close to the grid network. However, as seen from the right side of Figure 6, in the mesh network the joint optimization of routing and scheduling is the only approach that can fully exploit the resources of the network. The mean difference between optimal resource allocation and other methods is approximately 32 % (see the last three rows in Table 1).

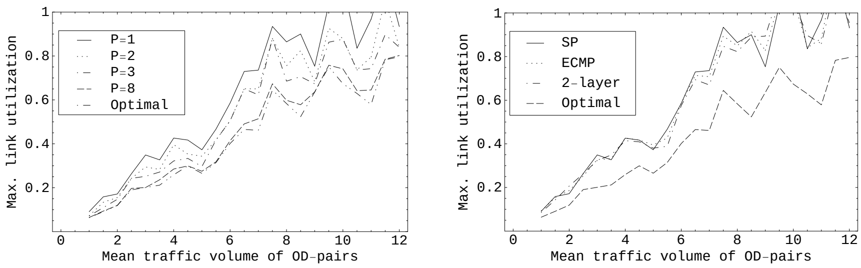

Fig. 6. The maximum link utilization as a function of the mean traffic volume in the mesh network. Left: Path set constrained. Right: SP, ECMP, 2-layer optimization and optimal.

5 Conclusion

In this paper we have studied load balancing in wireless mesh networks by formulating the linear optimization problem for the both unconstrained and predefined path sets and finding the optimal values for traffic allocation as well as the schedule. The main difference of load balancing in WMNs with STDMA as compared to traditional wired networks is that actual link capacities observed by the users are not fixed. In WMNs, in addition to taking advantage of spatial load balancing by moving traffic from loaded links to other routes, congestion can temporally be alleviated by allocating longer time shares for loaded links. Optimization of these two dimensions has to be done jointly to achieve the best results. In our numerical examples, when the joint optimization was other approaches, the average decrease in the maximum load was roughly 30%.

In this paper we considered only a simple boolean interference model. However, the LP formulations for balancing load can be used for other models just by modifying capacity matrix **S**.

Acknowledgements

I like to thank Prof. Jorma Virtamo and Dr. Aleksi Penttinen from TKK for their helpful advices and comments.

Appendix: Depth-first search algorithm

The algorithm searches all maximal transmission modes in a DFS manner starting from link 1 as the root and ending to link L. The modes are returned as a list M, in which element M_m consists of the links in mode m. In addition to earlier notations, let s_l and t_l denote the starting and destination node of link l, respectively.

Algorithm 1 DFS

$M \leftarrow \emptyset$ {set of feasible and maximal transmission modes}
$a \leftarrow \{0\}$ {A candidate for transmission mode}
while $a \neq \{L\}$ **do**
  $ind \leftarrow a(last) + 1$ {index ind set to value of the last element of a plus one}
  $a \leftarrow a \setminus \{a(last)\}$ {the last element of a dropped}
  for $i = ind$ to $i = L$ **do**
    if $(d_{s_l,t_i} > R'_l) \wedge (d_{s_i,t_l} > R'_i),\ \forall l \in a$ **then**
      $a \leftarrow a \cup \{i\}$ {if no interference, link i added to transmission mode a}
    end if
  end for
  if $a \nsubseteq M(j),\ \forall j$ **then**
    $M \leftarrow M \cup \{a\}$ {if not already a subset of some mode, candidate a added to M}
  end if
end while

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