

## **Paper VI**

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# Risk Premium and Robustness in Design Optimization of Simplified TMP Plant

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## Abstract

This paper illustrates issues related to optimal design under uncertainty in a simplified TMP (thermomechanical pulp) plant design case. Uncertainty in the case study is due to four dynamic scenarios of the paper machine pulp demand serviced by the designed TMP plant. Both a risk premium approach and a multi-objective optimization technique were employed. In the latter the worst-case scenario (representing the highest cost) was taken as the robustness measure of the design, and the design parameters were determined as a trade-off between the optimum of the mean cost model (i.e. the stochastic model) and of the worst-case scenario. The TMP model is a general example of an industrial case having parallel on/off production units and time-variant productions costs. Therefore, the design case could also be interesting for other fields of chemical industry than paper manufacturing, and the optimization procedures can be applied for risk premium and robustness studies in general dynamic optimization cases.

## 1. Introduction

In papermaking, TMP (thermomechanical pulp) plant has to satisfy the pulp demand of the paper machine. Design optimization of the simplified TMP plant includes the number of refiners ( $N_{\text{Ref}}$ ) and the storage tank volume ( $V_{\text{tank}}$ ) as design parameters. The optimization is genuinely a dynamic problem having paper machine demand and production costs, and thus — when optimally operated — also the number of active refiners varying in time. In the TMP plant design, the optimum of the total costs is found via a subtask of minimizing the capital costs and the production costs in operations and scheduling optimization.

The TMP design optimization is a MINLP (mixed-integer non-linear programming) problem since it has both a discrete,  $N_{\text{Ref}}$ , and a continuous,  $V_{\text{tank}}$ , design parameter. The operational optimization subproblem has integer decision variables (number of active refiners in time) affecting the continuous state of intermediate tank volume through process dynamics. The tank volume is constrained to stay between a minimum and a maximum volume.

In the operational optimization, the task is to schedule startups and shutdowns of refiners in order to minimize the production cost when the demand of the paper machine and the price of the electricity are known over a given time horizon.

## 2. Optimization procedure

### 2.1 Operations and scheduling optimization

In general, the operations optimization task is to find suitable set point trajectories for the controllers. As the controllers are omitted from our simplified TMP system model, no setpoint optimization is included in the study. However, the refiner scheduling optimization can also be considered as operations optimization with refiner activity set point trajectory as binary valued (on/off) function of time.

In this case, the operations optimization over a time horizon of some one hundred decision time intervals took approximately one minute by using a low-end PC and Matlab environment and the simulated annealing algorithm (Otter and van Ginneken, 1987).

### 2.2 Design optimization

The MINLP problem in the TMP case is simple in that the NPV (net present value) per capital employed can be determined by first treating both the design parameters ( $N_{Ref}$  and  $V_{tank}$ ) as discrete ones and then interpolating a continuous cost function  $Cost = f(V_{tank})$  for the optimal number of refiners. Consequently, no advanced MINLP solvers are needed.

### 2.3 Objective function

With a given scenario of the paper machine TMP demand, the production schedule can be optimized and with a given probability distribution of all scenarios ( $p^S$ ), the operational costs as a function of  $n(t)$  and  $V(t)$  can be calculated. By adding the capital costs, the optimal values for the decision-making amongst the studied design alternatives ( $N_{Ref}$ ,  $V_{tank}$ ) are obtained.

DESIGN LEVEL:

$$\min_{N_{Ref}, V_{tank}} \left\{ \sum_1^S p^S g\{n^{(o)}(t; N_{Ref}, V_{max})\} + C_{capital}(N_{Ref}, V_{tank}) \right\} \quad (1)$$

subject to

OPERATIONS LEVEL:

$$n^{(o)}(t; N_{Ref}, V_{max}) = \arg \min_{n(t)} g\{n(t)\} \quad (2)$$

$$g\{n(t)\} = \sum_{t=1}^{100} h_i n(t) + h_{up} n_{up} + h_{down} n_{down} \quad (3)$$

$$\text{s.t. } \frac{dV}{dt} = n(t) f_{Ref} - f_{PM}^S \quad (4)$$

$$0 < V(t) < V_{max} \quad (5)$$

$$n(t) \in [0, 1, \dots, N_{Ref}] \quad (6)$$

where  $f_{Ref}$  is production capacity of one refiner,  $f_{PM}^S$  is paper machine demand and  $h_i$  refers the daytime and night time electricity costs per refiner at each time interval.

The capital cost can be written as

$$C_{capital} = C_{Ref} + C_{tank} = N_{Ref} C_{Ref,i} + C_0 (V_{tank} / V_0)^\alpha \quad (7)$$

where  $C_{Ref}$  is the capital cost of a refiner, which, in real cases, is a function of the capacity (in MW) of the refiner. In the tank cost, the exponent  $\alpha$  is usually from about 0.6 to 0.7 (Biegler and Grossmann, 1999) and  $C_0$  and  $V_0$  are the base capacity and base cost, respectively.

## 2.4 Case calculations

In the over-all design optimization, four different demand scenarios of the paper machine were considered, Figure 1. All scenarios are expected to be equally likely, i.e.  $p^S = 0.25$ .

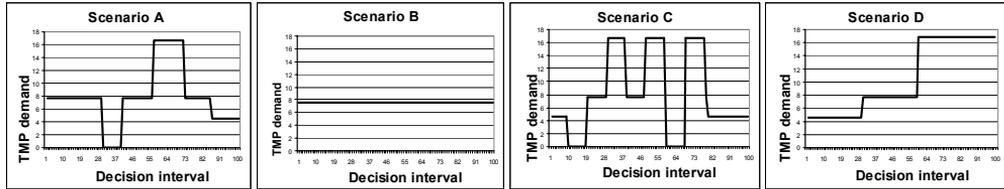


Figure 1. Demand scenarios of the paper machine in the simplified TMP case calculations.

Parameter values for the TMP model to be optimized are shown in Table 1. The time horizon is divided into  $I=100$  decision intervals, where  $\Delta t=30$  min, corresponding to a total of approximately two-day time period for the case calculations.

Table 1. Parameter values for the simplified TMP model.

Model Parameter	Value
TMP demand	Units per decision interval ( $\Delta t = 30$ min)
- average A-C / D	7.6 / 10.4
- min – max	0 – 16.5
Number of refiners	$N_{Ref} = 3, 4, 5, 6$
Refiner production	3.6
Tank volume	Units
- maximum, $V_{max}$	$V_{max} = 20, 30, 50, 70, 100, 200, 400$
- minimum, $V_{min}$	5% $V_{max}$
- initial volume, $V_0(I=0)$	15% $V_{max}$
- end volume, $V_{end}(I=100)$	25% $V_{max}$ - 35% $V_{max}$
Electricity costs	Units per decision interval
- night time	3
- daytime	5
- up/down costs	3

The over all feasible region covered in the optimization was obtained by combining the feasible regions of all scenarios resulting the following:

$$N = 3 \rightarrow V > 200; N = 4 \rightarrow V > 100; N = 5 \rightarrow V > 30 \text{ and } N = 6 \rightarrow V > 30.$$

The non-feasible regions were due to the fact that paper machine demand could not be satisfied. These values were considered to have infinite large cost values and thus omitted from the optimization.

In the design optimization, the interest on capital costs was neglected, and thus the capital cost was simply the annual depreciation. The number of the depreciation years,  $m$ , for the refiners and the storage tank was studied in the range of  $m = [1, 2, 4, 10, 20]$ . The capital cost due to refiners,  $N_{Ref} = [3, 4, 5, 6]$ , is calculated as:

$$C_{Ref} = \frac{1}{m} N_{Ref} C_{Ref,i} \quad (8)$$

where  $m$  is the number of years for depreciation, and  $C_{Ref,i} = 200$  is the cost of one refiner (in the units relative to the two-day time period of electricity costs). Similarly, the capital cost due to the intermediate tank,  $V_{tank} = 20 - 400$ , is calculated as:

$$C_{tank} = \frac{1}{m} b V_{tank}^{0.7} \quad (9)$$

where  $m$  is the number of years for depreciation and  $b = C_0/V_0^{0.7} = 10$  is the relative unit cost of the tank.

### 3. Optimal design and the effect of a risk premium

In the design with a risk premium, the expected value based on the probabilities of all scenarios was calculated for each discrete design parameters,  $N_{Ref}$  and  $V_{tank}$ . The risk premium was defined proportional to the standard deviation of the operational costs under the four equally likely scenarios, with a proportionality factor  $a = 0 \dots 3$ . The objective function to be minimized is expressed as

$$C_{tot} = \sum_1^S p^S g(n(t)) + C_{capital} + a\sigma^S. \quad (10)$$

Figure 2 shows that with these parameters the risk premium affects the design only at intermediate number of depreciation years. It was obvious that the number of the depreciation years (affecting the importance of capital costs *versus* operational costs) had a strong effect on the optimal design alternative. At 10 years depreciation time, the risk premium factor had an influence on the optimal design increasing both the number of refiners,  $N_{Ref}$ , and the continuous design parameter,  $V_{tank}$ .

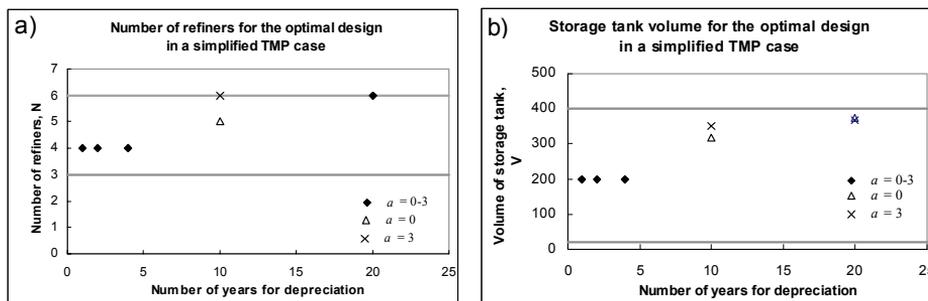


Figure 2. Risk premium weighing effect ( $a = 0 - 3$ ) on the optimal design in the simplified TMP case with different number of years for depreciation: (a) number of refiners and (b) storage tank volume.

The optimum of the design parameters was found in the studied range of  $N_{Ref}$  and  $V_{tank}$  for the depreciation times 1 – 10 years, where the capital costs dominates the total costs.

As for the 20 years of depreciation time, the optimum number of refiners is observed to reach the maximum ( $N_{\text{Ref}} = 6$ ) indicating that the parameter range should probably be larger than studied. It was therefore calculated that the minimum costs in operational optimization (based on maximum production rate and storage tank volume at low energy price period) are to be found amongst the number of refiners up to  $N_{\text{Ref}} = 13$  and  $V_{\text{tank}} = 550$ . However, the studied range ( $N_{\text{Ref}} = 3 - 6$ ,  $V_{\text{tank}} = 20 - 400$ ) was adequate to find the optimal design parameters when the depreciation time of the relative capital costs is restricted to maximum of 10 years.

## 4. Robust Design Optimization

The robust optimization study was based on the worst-case scenario analysis (Suh and Lee, 2001). The best robust solution was chosen amongst the Pareto optimal design alternatives.

### 4.1 Multiobjective optimization

In the MOO (multiobjective optimization) method expected cost and robustness measure are simultaneously optimised. The robust model is based on the stochastic model (Eq. 10, with  $a = 0$ ) having an additional objective of controlling the variability of performances of individual scenarios. The worst-case scenario is taken as the objective variable for the robustness measure and a decision-making procedure is applied to choose the best robust design alternative for the case study.

### 4.2 Decision-making

The best robust solution for the decision-making (with the studied model parameter range and the depreciation years  $m = 10$ ) was found by using an  $L^2$ -metric method, where the robust model parameters are obtained nearest to the ideal point, Figure 3.

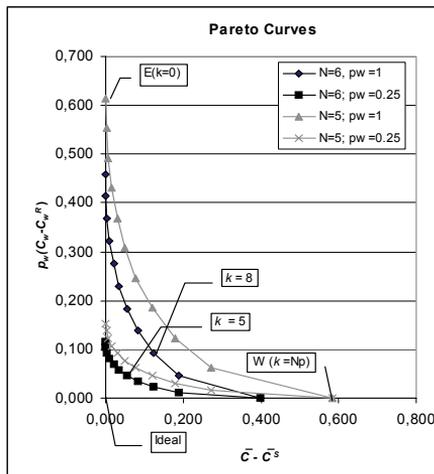


Figure 3. Robust optimization of the simplified TMP case with four different demand scenarios of the paper machine. The best robust solutions based on the worst-case analyses are found with  $N = 6$  and with the indexes  $k = 5$  and  $k = 8$  for the different scaling factors (probability of the worst-case scenario)  $p_w = 0.25$  or  $p_w = 1$  (no scaling), respectively. The optimum solution with  $p_w = 0.25$  corresponds the storage tank volume of  $V_{\text{tank}} = 350$ .

The scaling factor,  $p_w$ , is a function of a probability of the worst-case scenario, i.e.  $p_w = 0.25$ . The result was also calculated without any scaling ( $p_w = 1$ ). The selected robust optimal solution is closer to the stochastic model solution  $E(k=0)$  for smaller  $p_w$  and closer to the worst case analysis solution  $W(k=N_p)$  for larger  $p_w$ .

## 5. Results and Discussion

For the simplified TMP design case, stochastic model gives  $N = 5$  refiners and  $V_{\text{tank}} = 320$  for the optimal design. Both the risk premium study and the robust optimization study prefer  $N = 6$  refiners. In both studies, the optimum storage tank volume is  $V_{\text{tank}} = 350$ . Design optimization with a stochastic model does not take into account any uncertainty in design.

The TMP design alternatives are based on the over all feasibility region of all scenarios. However, if one of the scenarios is strongly restricting the feasibility region and, in addition, is quite infrequent, the scenario can be omitted from the optimization. This might cause a situation where the TMP line is temporarily unable to produce pulp for the paper mill. Then the question to be answered is, what will be an additional cost of such a scenario, and in more general, how the extra cost should be handled in the design optimization.

## 6. Conclusions

In the paper, the optimization with a risk premium, stochastic optimization and robust optimization were compared. The TMP plant design, even though extremely simplified, had all the characteristics of mixed-integer and dynamical design problems. Therefore, the study offers a suitable application for comparison of the design principles when uncertainty is considered in decision making of optimal design parameters.

## 7. References

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