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On Crosstalk and Noise in an Optical Amplifier With Gain Clamping by Vertical Laser Field

Jani Oksanen and Jukka Tulkki

***Abstract*—We have calculated the transient behavior and noise figure of a semiconductor optical amplifier (SOA) with the gain clamped by a vertical cavity laser (VCL). The characteristic behavior of the more conventional gain-clamped SOAs and SOAs with no gain-clamping is also studied and compared with the vertically gain-clamped amplifier. The calculations are based on a numerical stochastic rate equation model including several forward- and backward-propagating channels that are coupled to the vertical laser field through the active medium. The noise model takes into account the input noise, randomly amplified spontaneous emission, and random gain. Numerical simulations have been carried out to study the relaxation oscillations, crosstalk, and noise in a system with a strong input signal switched ON and OFF while observing the output signals, VCL photon density, and carrier density. Results show that the VCL field captures most of the disturbances, in agreement with available experimental data.**

***Index Terms*—Amplifier, crosstalk, gain clamping, linear optical amplifier (LOA), random gain, shot noise.**

I. Introduction

GAIN-CLAMPED semiconductor optical amplifiers (GCSOA) offer many advantages compared with the conventional semiconductor optical amplifiers (SOAs). Their response is more linear and they suffer less from crosstalk-related effects. Conventionally the gain clamping has been achieved by fabricating two distributed Bragg reflector (DBR) mirrors on both ends of the amplifier so that the amplifier forms a cavity for the frequencies selected by the DBRs [1], [2]. However, the fabrication of DBRs makes the amplifier structure more complex and also the amplifier response less ideal. The advantage both the SOA and GCSOA have over the fiber optical amplifiers is their integrability, which in case of the SOA comes with the cost of low performance.

A different approach to gain clamping was introduced recently by Genoa Corporation [3]. In their amplifier structure (called linear optical amplifier, LOA) the laser field is perpendicular, rather than parallel, to the signal and vertical in the actual amplifier structure (see Fig. 1). The physics of the LOA differs from that of the parallel approach by a distinctive monotonously decreasing position dependence of the vertical laser field and a position-independent inversion in steady state.

This paper presents a position-dependent stochastic rate equation model allowing for the analysis of the propagation of several signal and noise channels in forward and backward directions in the amplifier. The vertical stabilizing laser field and the carrier density in the system are taken into account with the standard rate equations that are modified to include the effect of the propagating signals. The model is also applicable to conventional GCSOA and SOA when proper boundary conditions are used and the vertical field is removed. The model is used to evaluate the transient behavior of the carrier density, photon densities, and total gain of the three types of optical amplifiers. The dependence of gain fluctuations on the input power, on gain saturation, and on injection current as well as the noise figure of the three types of amplifiers are also studied.

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The authors are with the Laboratory of Computational Engineering, Helsinki University of Technology, Helsinki FIN-02015 HUT, Finland.

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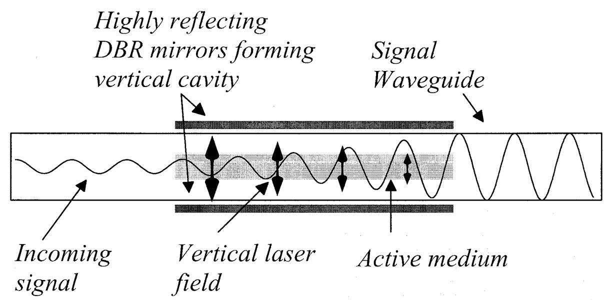

Fig. 1. Schematic representation of the LOA including a waveguide and a vertical microcavity with highly reflecting DBRs.

II. LOA Model

The gain clamping by vertical laser can be realized by combining a vertical microcavity laser (VCL) with a built-in waveguide in a direction perpendicular to the cavity (Fig. 1). The stochastic rate equation model describing the behavior of the carrier density $n(x,t)$ and the photon densities of the vertical laser field $L(x,t)$, the signal $S_i^\rho(x,t)$, and the noise $N_i^\rho(x,t)$ can be written in the form

$$\frac{\partial n}{\partial t} = \frac{I}{qhwl} - vG_L L - v\sum_{i,\rho}\{G_i\,(S_i^\rho + N_i^\rho)\}_r - \frac{n}{\tau} \tag{1}$$

$$\frac{\partial L}{\partial t} = v(G_L - \alpha_L)L + \eta_L \frac{n}{\tau_r} \tag{2}$$

$$\frac{\partial S_i^\rho}{\partial t} = -\rho v \frac{\partial S_i^\rho}{\partial x} + vG_i S_i^\rho \tag{3}$$

$$\frac{\partial N_i^\rho}{\partial t} = -\rho v \frac{\partial N_i^\rho}{\partial x} + v\{G_i\,(N_i^\rho + S_i^\rho)\}_r + \left\{\eta_i \frac{n}{\tau_r}\right\}_r - vG_i S_i^\rho, \tag{4}$$

where curly brackets with subindex r denote that the effect of the enclosed entity is considered to be random when the noise

TABLE I
RATE EQUATION PARAMETERS. THE CARRIER DENSITIES ARE AVERAGED OVER THE WAVEGUIDE CROSS SECTION

Symbol	Value	Explanation
I	0.14 A	Effective injection current
h	0.3 μm	Waveguide height
w	3 μm	Waveguide width
l	1 mm	Amplifier length
v	$3.0^{-1}c$	Speed of light
τ	0.5 ns	Carrier lifetime
τ_r	1 ns	Spont. em. carrier lifetime
ρ	$\{-1, 1\}$	Propagation direction ($\pm x$)
	VCL	
$G_L(n)$	Eq. (5)	Modal gain
G_L^{max}	1×10^5 m^{-1}	Max. modal gain
n_L^{nom}	2×10^{23} m^{-3}	Transparency density
η_L	10^{-4}	Spont. em. coupling
α_L	3×10^4 m^{-1}	Cavity losses
	Signal channel i	
$G_i(n)$	Eq. (5)	Modal gain
G_i^{max}	10000 m^{-1}	Max. modal gain
n_i^{nom}	2.13×10^{22} m^{-3}	Transparency density
η_i	10^{-5}	Spont. em. coupling
α_s	5 000 m^{-1}	Waveguide loss

is included in the calculations. (For a list of physical quantities appearing in (1)–(4), see Table I.) Equation (1) states that the carrier density increases by injection and reduces by the stimulated emission to the vertical laser field and signal (and noise) and the combined effect of spontaneous emission and nonradiative recombination. The vertical laser mode is described by the standard rate equation model (2), where the photon density changes due to the effects of the gain and the losses of the cavity (single-mode operation assumed). In addition, the coupling of the spontaneous emission to the vertical mode is accounted for.

In (1)–(4) the signal traveling in direction ρ (see Table I) is divided in different channels, denoted by the subindex i. These channels are further separated into the deterministic signal channel S_i^ρ and quantized noise channel N_i^ρ, with the total photon density of the mode being $S_i^\rho + N_i^\rho$. The equation for signal (3) includes the derivative along the direction of propagation responsible for displacing the signal along the amplifier and the term describing the average amplification. All random effects of the signal are included in the corresponding noise channel (4). The noise propagates with the signal, is amplified by the (random) gain $\{G_i\}_r$, and is generated by the shot noise of the input signal and the spontaneous emission coupled to the signal channel. The noise channel thus accounts for the additional photon shot noise generated by the random amplification of the signal.

The values given to the amplifier parameters correspond qualitatively to the amplifying materials described in [3] and [4]. The values are collected in Table I.

The approximate dependence of the average gain on the carrier density is expressed with the help of maximum gain $G_i^{\max}$, the transparency carrier density $n_i^{\rm nom}$, and the waveguide loss α_s as

$$G_i(n) = G_i^{\max}\frac{n - n_i^{\rm nom}}{n + n_i^{\rm nom}} - \alpha_s. \tag{5}$$

The treatment of the noise is based on the particle picture of photons: at a given time there can only be an integer number of photons in any given volume of the amplifier. In this picture, the spontaneous and stimulated emission are random processes that can create photons only in integer numbers. For sufficiently small distances Δx (or times $\Delta x/v$), the probability of emitting more than one photon (per incoming photon in case of stimulated emission) becomes negligible. This is the basis for forming the Kolmogorov equation for the probability density for general birth–death–immigration (BDI) processes for analytic analysis of the amplifiers [5]–[8]. Unfortunately these analytic solutions cannot be used directly, because (1)–(4) cannot be solved analytically. However, the problem can be solved numerically if the probabilities for generating one spontaneously emitted photon per unit distance and one stimulated photon per distance per incoming photon are known.

Next, we generate the random variables needed in the equations. The possible values of the term responsible for spontaneous emission in the discretized version of (4) along the signal path $\partial x/\partial t = \pm v$ can be written in the form

$$\begin{aligned}\frac{\Delta N_i^\rho}{\Delta x}_{|sp} &= \left\{\eta_i \frac{n}{v\tau_r}\right\}_r \\ &= \begin{cases} \frac{1}{V\Delta x}, & \text{if } r < \eta_i \frac{nV}{v\tau_r}\Delta x \\ 0, & \text{otherwise}\end{cases}\end{aligned} \tag{6}$$

where $r \in [0, 1[$ is a uniformly distributed random variable and $\eta_i nV\Delta x/v\tau_r$ is the probability that one photon is emitted along a path of length Δx, provided Δx is small enough. V is the volume of the piece of the amplifier where the photon generation is studied.

For stimulated emission, there are three possibilities for an incoming photon: absorption, stimulated emission, or, most probably, no change of the photon state. The associated values of $\{G_i(N_i^\rho + S_i^\rho)\}_r$ are

$$\begin{aligned}\frac{\Delta N_i^\rho}{\Delta x}_{|\rm st} &= \{G_i(N_i^\rho + S_i^\rho)\}_r \\ &= \begin{cases} \frac{-N_p}{V\Delta x}, & \text{if } r < (\alpha + \alpha_s)\Delta x \\ \frac{N_p}{V\Delta x}, & \text{if } r \ge 1 - \gamma\Delta x \\ 0, & \text{otherwise}\end{cases}\end{aligned} \tag{7}$$

This applies to a single incoming photon (ie $N_p = 1$). In case of multiple input photons, the output generated by each photon is calculated separately, as the stimulated emission processes are assumed independent and the probabilities remain the same. The parameters α and γ in (7) are the absorption and emission coefficients for the signal mode. The values of the absorption and emission coefficients α and γ are obtained from

$$\gamma = G_{\max} a \left(\frac{\frac{G'}{G_{\max}} + 1}{1 + a}\right)^2 \tag{8}$$

$$\alpha = \gamma - G' \tag{9}$$

where $G' = G_i + \alpha_s$ and a is the mass ratio of an electron and a hole (we approximate $a = 1$). Equations (8) and (9) arise directly from the simple quantum mechanical approach, taking into account the effect of filled/unfilled electron and hole states. Using this approach, the modal gain (excluding the scattering loss) can be written as $G' = \gamma - \alpha = G_{\max}[f_c(1-f_v) - f_v(1-f_c)]$, where f_c is the probability of an occupied conduction band state and $1 - f_v$ is the probability of an occupied hole state of appropriate energy. Identifying the first term with γ and the second with α and setting $1-f_v = af_c$ gives the above relations for the emission and absorption coefficients.

For a large number of input photons N_p we have used the Gaussian distribution to approximate the output. If the variance of the output photon number was larger than 15, the Gaussian approximation was used. This condition typically corresponded to $N_p \approx 200$.

Finally, we recapitulate the characteristic figures of the noise properties of an amplifier [9]. The signal-to-noise ratio (SNR) of an optical signal is defined using the mean photon number $\langle n_s \rangle$ and its variance during a time interval T_s as

$$\mathrm{SNR} = \frac{\langle n_s \rangle^2}{\mathrm{Var}(n_s)}. \tag{10}$$

For a coherent signal, the variance is equal to the mean photon number, and the SNR becomes

$$\mathrm{SNR}_{\mathrm{coh}} = \langle n_s \rangle = \frac{P_{\mathrm{signal}} T_s}{\hbar\omega}. \tag{11}$$

Note that the SNR depends on the integration time (bandwidth) of the signal.

The amount of noise the amplifier adds to the signal is described by the noise figure (NF) of the amplifier. It is defined by

$$\mathrm{NF} = \frac{\mathrm{SNR}_{\mathrm{input}}}{\mathrm{SNR}_{\mathrm{output}}}. \tag{12}$$

Since the expected number of photons at output can be expressed using the expected total gain G_{tot} of the amplifier as $\langle n_s \rangle_{out} = G_{tot} \langle n_s \rangle_{in}$, the NF can be written in an alternative form

$$\mathrm{NF} = \frac{\mathrm{Var}(n_s)_{\mathrm{out}}}{G_{\mathrm{tot}}^2 \mathrm{Var}(n_s)_{\mathrm{in}}}. \tag{13}$$

The NF of an optical amplifier has a theoretically predicted lower bound given by $NF_{lb} = 2 - 1/G_{\mathrm{tot}} \approx 2$ [6]. This lower bound is found when spontaneous emission is not taken into account and the inversion is complete (no absorption).

III. Results

The most interesting phenomenon in the operation of the LOA is the coupling between light and matter, resulting in the relaxation oscillation patterns in the amplifier carrier density, the VCL laser field and the propagating signal. These operational characteristics were studied by constructing a system with two forward-propagating signal channels. The probe signal is fairly weak while the pump signal is stronger. This corresponds to a WDM system with one channel (the probe) interacting with the other channels of the system. This approach allows also comparison with the experimental results published by Genoa [3].

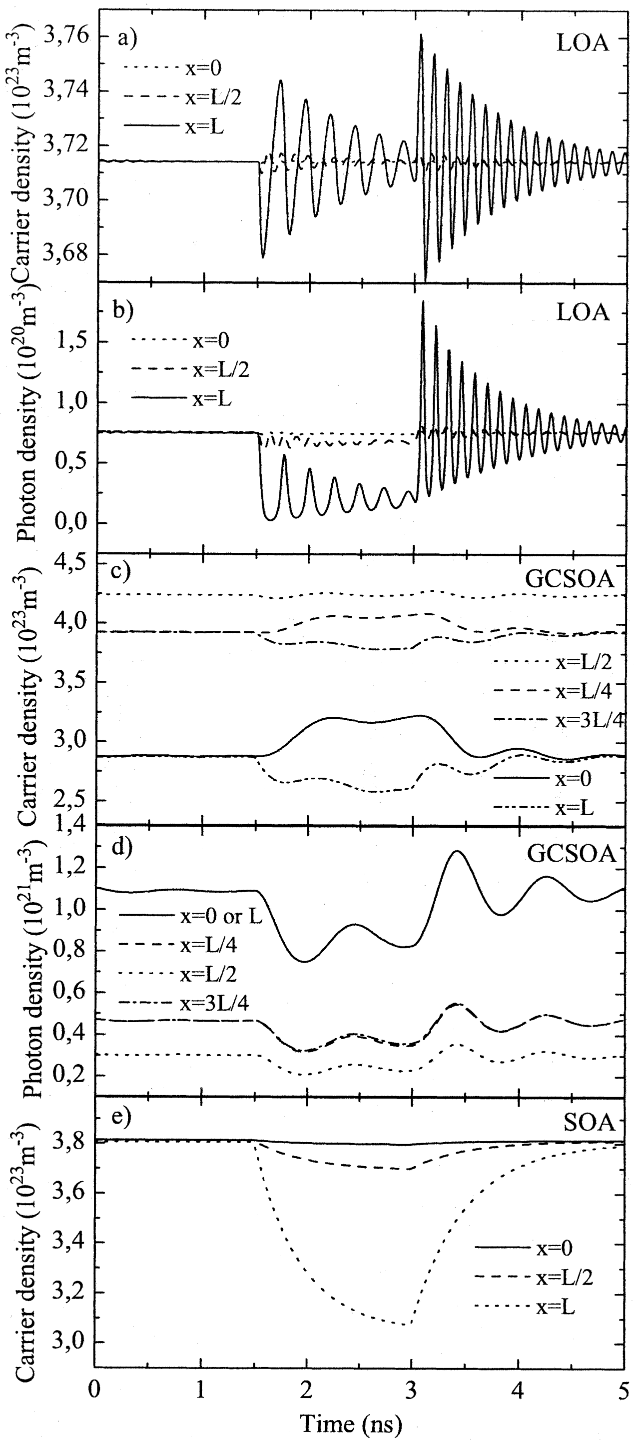

Fig. 2. Carrier density and photon density of the stabilizing laser field for the LOA and the GCSOA and the carrier density for the SOA as a function of time for a 0.124-mW pump signal switched ON at 1.5 ns and OFF at 3 ns. The densities are plotted at different positions along the amplifier. (a) and (b) show the densities for the LOA, (c) and (d) shows the densitites for the GCSOA, and (e) shows the densities for the SOA.

Similar constructions were also made for a conventional GCSOA and an SOA. The GCSOA was configured to have a reflectance of 0.02 at the ends of the waveguide for one channel that formed the gain clamping laser field (the mirrors

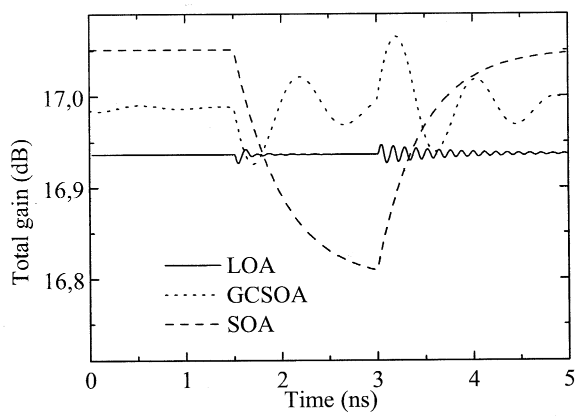

Fig. 3. Total gain of the amplifiers as a function of time for a 0.124-mW pump signal switched ON at 1.5 ns and OFF at 3 ns. The effect of the pump signal is seen to be the smallest for the LOA, where the perpendicular laser field is able to capture the disturbances more effectively than the parallel field of the GCSOA.

are assumed to be 100% transparent for the signal channels). For the SOA, the effective injection current was dropped to 0.11 A to obtain a gain close to 17 dB. In both these cases, the vertical laser field was turned off.

The transient behavior was studied by switching the pump signal (of power 0.124 mW) ON at $t = 1.5$ ns and OFF at $t = 3$ ns. The probe signal had a continuous input power of 1.25 μW. The effects of the power switching on the photon density and carrier density of the amplifiers are shown in Fig. 2 for the LOA and the reference structures. Fig. 2(a) shows the relaxation oscillations in the carrier density of the LOA at the input, in the middle of the amplifier and at the output. Fig. 2(b) shows the corresponding VCL photon densities. In Fig. 2(c) and (d), the carrier density and the lasing mode photon density of the reference GCSOA are plotted at five locations along the GCSOA. Fig. 2(e) shows the corresponding carrier densities in the SOA.

The total gains for these amplifiers are plotted in Fig. 3. The LOA experiences the lowest fluctuations in the gain because of the effective capture of the perturbances by the vertical laser field. The total gain experienced by the probe signal stays constant within 0.02 dB when the pump signal is switched ON or OFF. Defining the crosstalk as the ratio of the maximum deviation from the mean probe power to the mean probe power [3] results in crosstalk of −26 dB. The crosstalk can also be defined as the ratio of the deviation of the probe from its mean to the pump power. Using this definition gives a crosstalk of −46 dB.

The maximum and minimum values of the total gain of the LOA after the rising (time period from 1.5 to 3.0 ns) and falling (time period from 3.0 to 5.0 ns) edges of the pump signal are shown in Figs. 4 and 5 as functions of the pump input power and the maximum modal gain, respectively (the total gain is constant). The gain variations in Fig. 4 are linear for small pump powers, but for larger powers the variation is affected by the frequency of the relaxation oscillations (the photon density is averaged over a period of 0.01 ns), extinction of the vertical laser mode, and the resulting turn on jitter of the vertical mode. Fig. 4 shows also for reference how large a portion of the amplifier is saturated with a steady state input power corresponding to the pump power. From Fig. 5 we simply notice that the stronger the gain saturation is, the smaller the gain deviations become. This

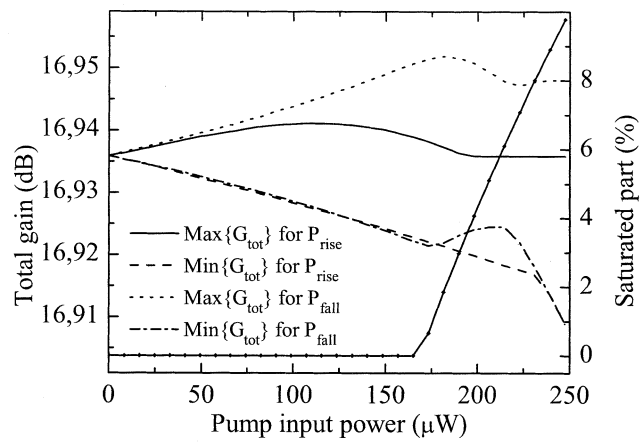

Fig. 4. Effect of the pump input power on the total gain: the maximum and minimum values of the total gain after the rising (time period from 1.5 to 3.0 ns) and falling (time period from 3.0 to 5.0 ns) edge of the pump as a function of the pump input power. The axis on the right shows approximately how large a portion of the amplifier would have the VCL mode turned OFF in case of a steady pump signal.

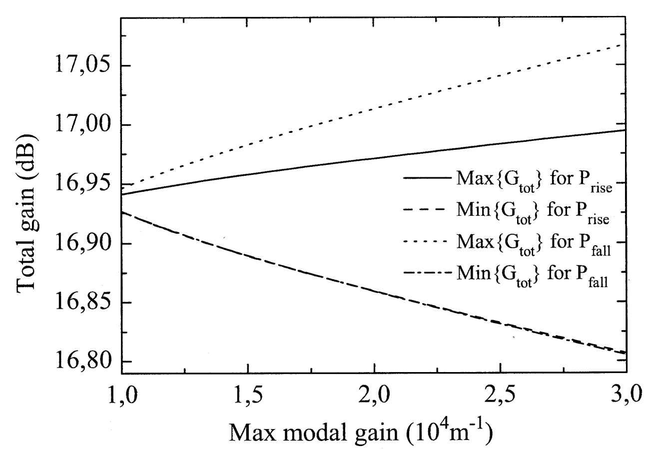

Fig. 5. Effect of the gain saturation on the total gain: the maximum and minimum values of the total gain after the rising (time period from 1.5 to 3.0 ns) and falling (time period from 3.0 to 5.0 ns) edge of the pump as a function of the maximum modal gain.

is very natural, because then the effect the relaxation oscillations have on the gain is small.

Fig. 6 shows yet another picture of the maximum and minimum values of the total gain as a function of the injection current. The gain fluctuations become gradually smaller as the laser field along the amplifier gets strong enough not to be overcome by the signal. When the laser field is strong enough and does not get saturated anywhere, the fluctuations become quite independent of the injection current.

The results shown in Figs. 2–6 were free of noise. The noise of the amplifier consists of the three sources: the amplification of the input noise, random gain, and amplified spontaneous emission (ASE). The mean ASE noise power per channel is found to be of the order of 6 μW. We have calculated the noise figures for coherent input signals as a function of input power. The results are plotted in Fig. 7.

The noise figures do not show pronounced differences for the different types of amplifiers. However, there are a few general

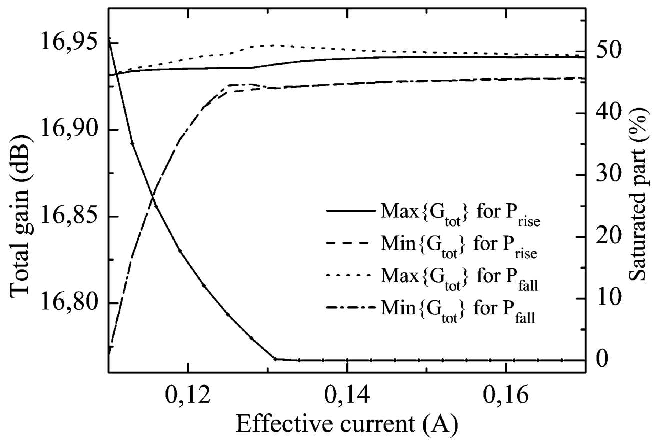

Fig. 6. Effect of the injection current on the total gain: the maximum and minimum values of the total gain after the rising (time period from 1.5 to 3.0 ns) and falling (time period from 3.0 to 5.0 ns) edge of the pump as a function of the injection current. The axis on the right shows approximately how large a portion of the amplifier would have the VCL mode turned OFF in case of a steady state pump signal.

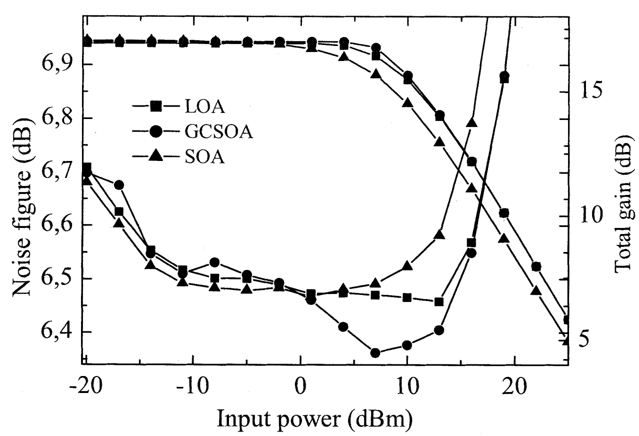

Fig. 7. NF (left axis) and gain saturation (right axis) of the amplifiers as a function of input power. The noise figures of these amplifier configurations are qualitatively very similar, except for the small dip in the GCSOA case and the early rise of the SOA NF due to the smaller injection current.

conclusions that can be drawn on the basis of Figs. 2 and 7. For small signal levels, the GCSOA has the largest NF. This is due to the longitudinal gain profile of the GCSOA, which has a local gain minimum at the amplifier input. For larger input signal levels, the situation is reversed, and the NF of the GCSOA becomes the smallest of the three amplifiers as the gain coefficient at the input becomes larger (see Fig. 2 for a qualitative picture). The performance of the LOA, on the other hand, stays steady throughout its unsaturated operation range, with the higher NF at the lower input levels caused by the proportionally larger effect of the spontaneous emission fluctuations. Finally, the NF of the SOA is slightly smaller than that of the LOA, because the inversion of the SOA is always strongest at the input. For strong saturation, the NF of all the amplifiers starts to rise rapidly. This regime starts earlier for the SOA than for the two other amplifiers because of the lower input current.

Even though the differences in the noise figures shown in Fig. 7 are not very pronounced in our amplifier configurations, there are fundamental reasons for them. Of special interest is the fact that the NF of the LOA is fundamentally lower (for reasonable input signal levels) than the NF of the GCSOA because of the position dependence of the GCSOA carrier density.

The theoretical steady-state estimates we calculated using the analytic photon point process solutions of [6] and the amplifier parameters defined earlier (giving an NF of 6.5 dB for the nonsaturated LOA when spontaneous emission is neglected) agree well with the values of our analysis.

IV. Conclusion

The results obtained here show that a stochastic rate equation model is able to account for the most salient physical aspects of the amplifier operation. The results agree with the experimental data published in [3] and the analytic BDI processes of [6], even though some of the parameters are based on qualitative description of the reference structures. The results suggest that the noise figure of the LOA is marginally larger than the NF of conventional SOA and for small input signal powers slightly smaller than the NF of the traditional DBR based gain-clamped amplifiers [1], even when the effect of the DBR mirrors on the signal is not accounted for.

The crosstalk of the amplifier depends substantially on the operating point and becomes small as the differential gain for the channel frequencies gets small (then the influence of the relaxation oscillations on the amplifier gain becomes small as well). Hence, it is advantageous to operate the amplifier close to gain saturation at the signal frequencies. According to the theory of BDI processes, the minimum excess noise due to random gain is also achieved with complete inversion. For spontaneous emission, the noise minimum occurs naturally when no electron–hole pairs are present.

The LOA model studied here is different from the conventional traveling wave amplifier models because of the inclusion of the equations of the vertical laser field and because it allows the use of the instantaneous random values of the photon number.

The basic structure of the LOA has already been applied to wavelength conversion with good results [10]. In addition, a number of applications, such as switches making use of the vertical laser field, may be possible to realize with the structure, such as switches making use of the vertical laser field. We believe that the stochastic rate equation model discussed in this work is useful in exploring potential applications of similar semiconductor structures.

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Jani Oksanen, photograph and biography not available at the time of publication.

Jukka Tulkki, photograph and biography not available at the time of publication.