An Analytical Model of Metamaterials Based on Loaded Wire Dipoles

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***Abstract*—In this paper, we present a model of straight wire dipole scatterers loaded by arbitrary lumped loads in the dipole center. These particles can be used as inclusions of custom-design composite materials or electrically controllable metamaterials. The model, together with the Clausius–Mossotti formula and an analytical model for reflection and transmission in dipole arrays, allows to find loads required to realize the desired electromagnetic properties of the composite and estimate its realizability with the use of various bulk loads.**

***Index Terms*—Artificial materials, composite sheet, effective permittivity, wire dipole.**

I. Introduction

USUAL artificial dielectrics use simple-shape inclusions like metal spheres or needles. The possibilities for the material design are clearly limited by the electromagnetic properties of the inclusions. If one wants to have more freedom in the design and control of the inclusion properties, one of the natural choices is to use small (in wavelength) wire dipole antennas loaded by bulk impedance circuits. Indeed, a small piece of wire scatters as an electric dipole, imitating a molecule in a usual dielectric, and by appropriately choosing the load, its polarizability as a function of the field frequency can be tuned to provide the required design parameters of the metamaterial. We call these artificial media *metamaterials* because the loads can contain electronic circuits that can be electrically or optically controllable. In contrast to composite materials like artificial dielectrics, the effective material properties cannot be derived from only inclusion material, concentration, and shape.

Probably for the first time artificial media formed by loaded wire dipoles attracted much interest with respect to the studies of chiral media for microwave applications (see, for example, a review in [1]). Conducting particles of the helical shape were used to design chiral microwave composites. The canonical helix [2] is a combination of a short dipole antenna and a small loop antenna connected so that one of the antennas is the load for the other. Analytical models for the polarizabilities of these loaded antennas were developed in [3]. That and similar models were used to study possible realizations of more complex bianisotropic [4], nonlinear [5], and other complicated metamaterials [6], [7]. Another application for arrays of loaded dipoles is in the control and electrical tuning of frequency selective surfaces [8]–[10]. The loads, usually capacitors or inductors, have been used to shift the half-wavelength resonant frequency of wire dipoles [9]. With the use of negative-resistance circuits, wave amplification can be achieved [10]. Numerical methods like the periodical method of moments have been used to analyze such loaded arrays [10].

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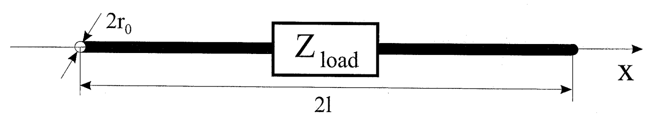

Fig. 1. Geometry of a loaded conducting wire.

This paper is devoted to the problem of metamaterial synthesis, with the goal to find the load impedances needed to realize the desired effective electromagnetic properties of composites from small loaded wire dipoles. Obviously, to attack such problems an analytical model of the composite response is needed. The analytical model of loaded wire dipoles of [3] has been successfully used for several applications, however, its accuracy is limited due to the assumption of triangular current distribution along the antenna. Although this is a very good approximation for a short transmitting dipole antenna, in case of an arbitrary load this assumption can be invalid. In this paper, we derive a more general and accurate formula for the inclusion polarizability. Furthermore, with the help of the Clausius–Mossotti formula, we solve for the load impedance required to realize any given effective polarizability of the artificial material.

II. Polarization of Loaded Wire Dipoles

Let us consider a thin metal wire with the total length $2l$, wire radius $r_0 \ll l$, that is loaded by a bulk impedance $Z_{\rm load}$ in its center (see Fig. 1). We are mostly interested in the analytical modeling of the polarization of the wire at low frequencies when the particle is much smaller than the wavelength ($2l \ll \lambda$) by an external electromagnetic field $E_{\rm inc}$ directed along the wire axis. In particular, we need to know the electric dipole moment induced in this "artificial molecule" by the external field. Although in many applications the inclusions are small compared to the wavelength, we will first develop a model without this restriction and then consider the simplified formulas for small particles.

A. Particle Polarizability

As is well-known from the antenna theory (e.g., [11]), in the transmitting regime the current distribution along a thin wire excited by a point voltage source at its center is approximately sinusoidal

$$I_{\rm trans}(x) = I_{\rm trans}(0) f_t(x), \qquad f_t(x) = \frac{\sin k(l-|x|)}{\sin kl} \quad (1)$$

and in the receiving mode, for a short-circuited antenna

$$I_{\rm rec}(x) = I_{\rm rec}(0) f_r(x), \qquad f_r(x) = \frac{\cos kx - \cos kl}{1-\cos kl}. \quad (2)$$

Here, $I_{\rm trans}(0)$ and $I_{\rm rec}(0)$ are the current amplitudes at the antenna center that depend on the source power, and k is the wavenumber in the surrounding medium. The distribution of current along a wire antenna illuminated by an electromagnetic wave and loaded by an arbitrary load is a linear combination of these two functions. In the low-frequency limit the current distributions are modeled by simpler functions

$$f_t(x) = 1 - \frac{|x|}{l}, \qquad f_r(x) = 1 - \frac{x^2}{l^2} \quad (3)$$

as can be seen from the Taylor expansion of (1) and (2).

For a receiving wire antenna loaded by $Z_{\rm load}$ in its center we can find the current in the antenna center as

$$I(0) = \frac{1}{Z_{\rm load}+Z_{\rm inp}}\,\mathcal{E} = \frac{1}{Z_{\rm load}+Z_{\rm inp}}\int_{-l}^{l} E_{\rm inc}(x) f_t(x)\,dx. \quad (4)$$

Here, $Z_{\rm inp}$ is the input impedance of the wire antenna, and $\mathcal{E}$ is the induced electromotive force. We stress that the electromotive force is determined by the distribution of the current on the antenna *in the transmitting mode*, the distribution function $f_t(x)$. If the external electric field is uniform along the wire, the integral expressing the electromotive force in (4) reads

$$\mathcal{E} = E_{\rm inc}\frac{2\tan(\frac{kl}{2})}{k} \approx E_{\rm inc} l \quad (5)$$

where the approximation is valid for $|k|l \ll 1$.

Let us rewrite (4) as

$$I(0) = \frac{\mathcal{E}}{Z_{\rm inp}} - \frac{\mathcal{E}}{(Z_{\rm load}+Z_{\rm inp})}\frac{Z_{\rm load}}{Z_{\rm inp}} = \frac{\mathcal{E}}{Z_{\rm inp}} - \frac{Z_{\rm load} I(0)}{Z_{\rm inp}}. \quad (6)$$

In this form, the total current is presented as a superposition of two contributions: the first term is the current induced in the short-circuited antenna by the incident field [formula (4) with $Z_{\rm load}=0$], and the second term is the current generated by an equivalent voltage source $V_0 = -Z_{\rm load} I(0)$ at the position of the load. The first part of the current is induced by the incident field directly, and the second part is "scattered" by the antenna load.

Now we can apply (1)–(2) and find the total current distribution along the loaded wire

$$I(x) = \frac{\mathcal{E}}{Z_{\rm inp}} f_r(x) - \frac{\mathcal{E}}{(Z_{\rm load}+Z_{\rm inp})}\frac{Z_{\rm load}}{Z_{\rm inp}} f_t(x) \quad (7)$$

or, in terms of the current in the load $I(0)$

$$I(x) = I(0)\frac{Z_{\rm inp}+Z_{\rm load}}{Z_{\rm inp}}\left[f_r(x) - \frac{Z_{\rm load}}{Z_{\rm inp}+Z_{\rm load}} f_t(x)\right]. \quad (8)$$

Consideration of two limiting cases is instructive. If $Z_{\rm load}\to 0$, we get

$$I(x) = I(0) f_r(x) \quad (9)$$

as should be in the short-circuit case. If $Z_{\rm load}\to\infty$, we have

$$I(x) = \frac{\mathcal{E}}{Z_{\rm inp}}\left[f_r(x) - f_t(x)\right]. \quad (10)$$

Note that in this case $I(0)=0$. For short wires in this case along the two arms there is a quadratic distribution with the quarter amplitude at the two separate parts of the antenna. The charge is distributed according to the linear law, and the distribution is the same on the two arms. The current distributions are illustrated by Fig. 2 for some various load impedances.

The knowledge of the current distribution makes it possible to calculate the induced electric dipole moment p:

$$p = \int_{-l}^{l} q(x) x\,dx = -\frac{1}{j\omega}\int_{-l}^{l}\frac{dI(x)}{dx} x\,dx = \frac{1}{j\omega}\int_{-l}^{l} I(x)\,dx \quad (11)$$

where $q(x)$ is the charge distribution. In this paper, we restrict the analysis to the dipole moment of the particle. Higher-order moments can be easily calculated by appropriate integrations, since the charge distribution is known. Let us only note that the first nonzero higher-order moment is of the third order (because the charge distribution is antisymmetric), and all magnetic moments are zeros. After a simple integration we find the polarizability α defined as $\alpha = p/E_{\rm inc}$

$$\alpha = \left[\frac{\frac{\sin(kl)}{k} - l\cos kl}{1-\cos kl} - \frac{1-\cos kl}{k\sin kl}\,\frac{Z_{\rm load}}{Z_{\rm inp}+Z_{\rm load}}\right]\frac{4\tan(\frac{kl}{2})}{j\omega k Z_{\rm inp}}. \quad (12)$$

Here we have assumed that $E_{\rm inc}$ is uniform over the wire volume and used formula (5). In the quasi-static regime the formula for the polarizability (12) simplifies to

$$\alpha = \left[\frac{4}{3} - \frac{Z_{\rm load}}{Z_{\rm inp}+Z_{\rm load}}\right]\frac{l^2}{j\omega Z_{\rm inp}} = \left(\frac{4Z_{\rm inp}+Z_{\rm load}}{Z_{\rm inp}+Z_{\rm load}}\right)\frac{l^2}{3j\omega Z_{\rm inp}}. \quad (13)$$

For $Z_{\rm load} = -Z_{\rm inp}$, we have a resonance where the polarizability becomes infinitely high, as expected. Close to this resonance, the current distribution along short wires is nearly triangular, since the second term in (7) dominates. This is the reason why a simple model of [3] that assumes the triangular current distribution gives very good estimations near the particle resonance. That simple model can be also useful, since the resonant region is the most interesting for applications: resonant particles can be strongly excited. We can also observe that for $Z_{\rm load} = -4Z_{\rm inp}$ the polarizability becomes zero (this takes place due to a specific current distribution, such that the dipole moments induced at the two arms of the wire cancel out, see Fig. 3). One can conclude that it is possible to effectively tune the polarizability of a wire by varying its load.

The range of realizable properties of small scatterers is naturally restricted by the limitations in the design of electrically small loads with required input impedances. Realization of *passive* loads with, for example, large values of inductance can be impractical, but at least with the help of active electronic circuits this restriction can be significantly relaxed.

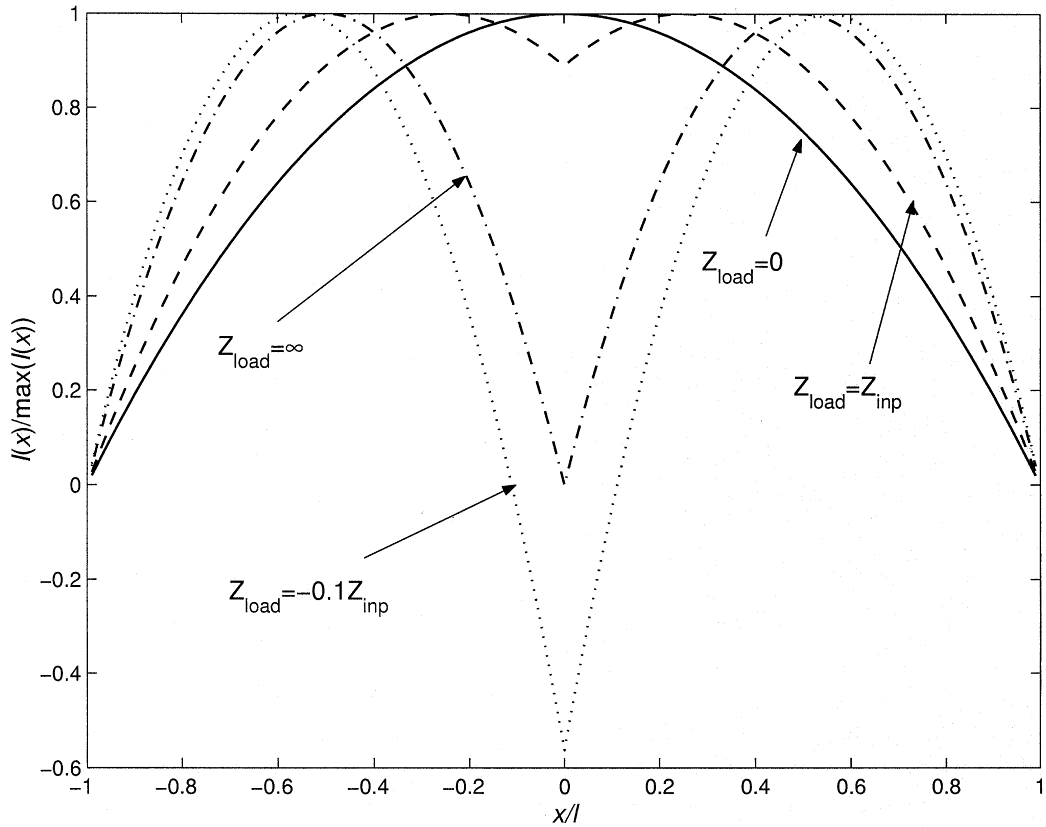

Fig. 2. Distribution of current along a short wire scatterer for different load impedances $Z_{\rm load}$.

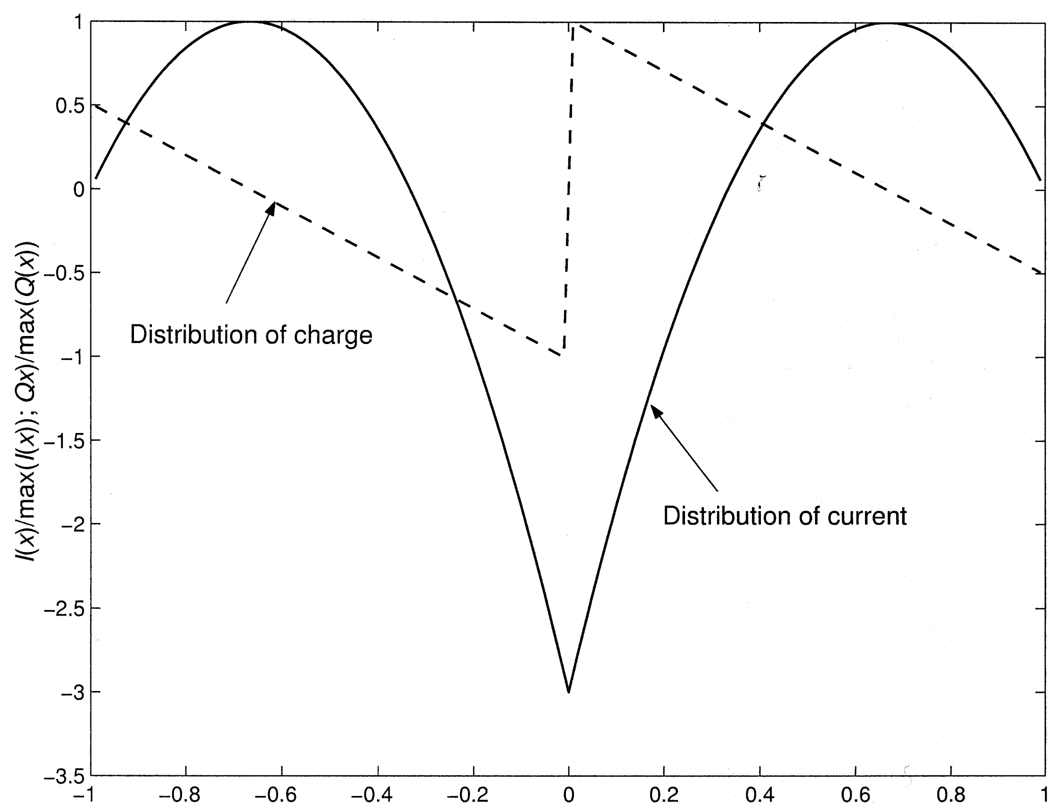

Fig. 3. Distributions of charge and current for wire scatterers loaded by $Z_{\rm load} = -4Z_{\rm inp}$. In this case, the total induced dipole moment of the particle is zero.

B. Dissipation in Loaded Wire Dipoles

Power dissipation in the bulk load is of course taken into account by the above formulas for the polarizability, simply the load impedance is a complex number if there is some dissipation in the load. To account for losses in the wires of the dipoles, however, more analysis is needed, mainly because the current distribution along an antenna with nonideally conducting wires is different from the ideal case.

In fact, the result for the current distribution along any loaded wire receiving antenna expressed by (7) or (8) holds for lossy wire antennas if one finds $Z_{\rm inp}^{\rm lossy}$, $f_r^{\rm lossy}(x)$, and $f_t^{\rm lossy}(x)$ with the wire resistance taken into account. We start the derivation from the input impedance of a wire antenna with lossy wires. Although this is a classical problem, the derivation is given here, since the same steps will be used in the following calculation of the particle polarizability.

If losses are small, there is no dramatic change in the current distributions. A lossy transmitting antenna may be represented in this case as an ideal antenna excited by two sources: a usual point source and a distributed source corresponding to the additional voltage drop caused by the wire resistance. Hence, the total EMF exciting the ideal antenna is

$$\mathcal{E}^{\rm tot} = \mathcal{E} - \int R_w I_{\rm trans}^{\rm lossy}(x) f_t(x)\, dx \tag{14}$$

where R_w is the wire resistance per unit length, $I_{\rm trans}^{\rm lossy}(x) = I(0) f_t^{\rm lossy}(x)$ is the current distribution on the lossy antenna. From the other hand, $\mathcal{E}^{\rm tot} = I(0) Z_{\rm inp}$ and $\mathcal{E} = I(0) Z_{\rm inp}^{\rm lossy}$, therefore, we get

$$Z_{\rm inp}^{\rm lossy} = Z_{\rm inp} + \int R_w f_t^{\rm lossy}(x) f_t(x)\, dx \approx Z_{\rm inp} + \int R_w f_t^2(x)\, dx \tag{15}$$

which is the first-order correction to the antenna impedance. For short dipoles the last integral is

$$\int_{-l}^{l} R_w f_t^2(x)\, dx = \frac{2l}{3} R_w \tag{16}$$

if the wire losses are uniform along the dipole.

First-order corrections to the current distributions may be found by the same approach if the following two additional antenna excitation problems are solved: excitation by a distributed EMF proportional to $f_t(x)$, and excitation by a source proportional to $f_r(x)$. Assuming that the normalized current distributions $\delta_t(x)$, $\delta_r(x)$ are known for these two cases, one writes

$$I(0) f_t^{\rm lossy}(x) \approx \frac{\mathcal{E}}{Z_{\rm inp}} f_t(x) - \frac{I(0)\delta_t(x)}{Z_{\rm inp}} \int R_w f_t^2(x)\, dx \tag{17}$$

and

$$f_t^{\rm lossy}(x) \approx \frac{Z_{\rm inp}^{\rm lossy}}{Z_{\rm inp}} f_t(x) - \frac{Z_{\rm inp}^{\rm lossy} - Z_{\rm inp}}{Z_{\rm inp}} \delta_t(x). \tag{18}$$

Analogously, for the receiving regime the total EMF applied to the ideal antenna is

$$\mathcal{E}^{\rm tot} = \mathcal{E}^{\rm inc} - \int R_w I_{\rm rec}^{\rm lossy}(x) f_t(x)\, dx \tag{19}$$

and

$$I(0) f_r^{\rm lossy}(x) \approx \frac{\mathcal{E}^{\rm inc}}{Z_{\rm inp}} f_r(x) - \frac{I(0)\delta_r(x)}{Z_{\rm inp}} \int R_w f_r(x) f_t(x)\, dx. \tag{20}$$

Hence,

$$f_r^{\rm lossy}(x) \approx \frac{Z_{\rm inp} + Z_R}{Z_{\rm inp}} f_r(x) - \frac{Z_R}{Z_{\rm inp}} \delta_r(x) \tag{21}$$

where

$$Z_R = \int R_w f_r(x) f_t(x)\, dx. \tag{22}$$

For short dipole antennas

$$Z_R = \frac{5l}{6} R_w. \tag{23}$$

Finally, to find the corrected current distribution the obtained $Z_{\rm inp}^{\rm lossy}$, $f_r^{\rm lossy}(x)$ and $f_t^{\rm lossy}(x)$ should be substituted into (8).

For short dipole antennas the functions $\delta_t(x)$ and $\delta_r(x)$ can be approximately calculated:

$$\delta_t(x) = \left(1 - \frac{x^2}{l^2}\right)\left(1 - \frac{x^2}{2l(l+|x|)}\right) \tag{24}$$

$$\delta_r(x) = \left(1 - \frac{x^2}{l^2}\right)\left(1 - \frac{x^2}{5l^2}\right). \tag{25}$$

The calculation is done by the induced EMF approach, assuming triangular distribution of the current induced by a point EMF inserted at an arbitrary position of the dipole wire. Finally, the polarizability can be determined after the appropriate integration of the current distribution. After some algebra, the result read

$$\alpha^{\rm lossy} = \frac{l^2}{j\omega Z_{\rm inp}^{\rm lossy}} \left\{ \frac{4}{3} - \frac{Z_{\rm load}}{Z_{\rm inp}^{\rm lossy} + Z_{\rm load}} + \frac{R_w l}{6 Z_{\rm inp}} \left[\frac{Z_{\rm load}}{Z_{\rm inp}^{\rm lossy} + Z_{\rm load}} + \frac{4}{15} \right] \right\}. \tag{26}$$

Note that the additional term proportional to the wire resistance per unit length R_w becomes large near the particle resonance. However, the main part of the polarizability is also large near this frequency or frequencies, and the ratio between them is still proportional to the wire resistance $R_w l$.

III. Metamaterials Based on Artificial Wire Dipoles With Loads

Suppose that a material with a certain effective permittivity is required for a specific application. Let us try to design a metamaterial with this property using random or regular arrays of small loaded dipole antennas.

A. Volume Composite

In general, three arrays of antennas are needed, which should be arranged in space so that the required response is provided for electric fields along all directions (Fig. 4).

To model metamaterials built of small loaded antennas we will use the Maxwell Garnett approximation, assuming that the particle size is small compared to the wavelength, so that the inclusions can be modeled by electric or magnetic dipoles. The other assumption is that the distances between inclusions are

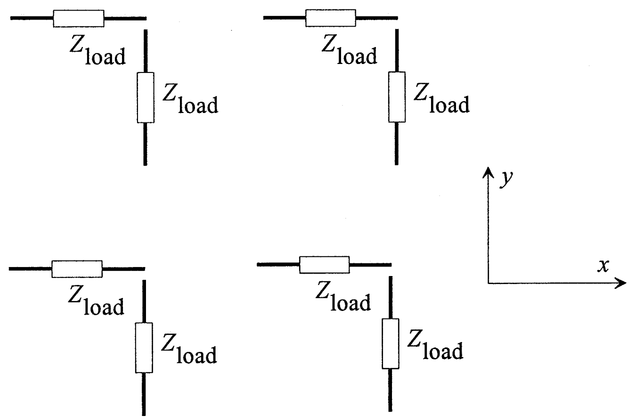

Fig. 4. A spatial arrangement of small loaded wire dipoles. Spatial distribution can be regular or random. Particles in the x-y plane are shown. Electric dipoles along x and y provide the required electric properties for electric fields in this plane. In general, there is also an array of electric dipoles along z, that is not seen on the picture.

large compared to the particle size. Under these circumstances, the well-known Clausius–Mossotti formula

$$\frac{\epsilon_{\rm eff}-\epsilon_0}{\epsilon_{\rm eff}+2\epsilon_0}=\frac{n\alpha}{3\epsilon_0} \quad (27)$$

can be used to connect the effective permittivity $\epsilon_{\rm eff}$ of a composite with the polarizability of the individual inclusion α and the inclusion concentration (the number of inclusions in the unit volume) n. (ϵ_0 is the matrix permittivity). Note that this approximation can be also used in the regime where the particles are resonant and their polarizabilities are rather large in the absolute value. This follows from the analysis presented in [12] and has been numerically verified for example in [13] and [14].

To determine what inclusion loads $Z_{\rm loads}$ are needed to provide the desired response of the composite, we substitute the loaded wire dipole polarizability (13) in (27) and solve for the load impedance. The result reads

$$Z_{\rm load}=\frac{(\epsilon_{\rm eff}-\epsilon_0)\frac{3\epsilon_0}{\epsilon_{\rm eff}+2\epsilon_0}Z_{\rm inp}-\frac{4nl^2}{3j\omega}}{\frac{nl^2}{3j\omega Z_{\rm inp}}-(\epsilon_{\rm eff}-\epsilon_0)\frac{3\epsilon_0}{\epsilon_{\rm eff}+2\epsilon_0}}. \quad (28)$$

Let us compare the values of the two terms in the denominator of (28). The second term, $(\epsilon_{\rm eff}-\epsilon_0)(3\epsilon_0/(\epsilon_{\rm eff}+2\epsilon_0))$, equals $n\alpha$, see (27). The first term, $nl^2/3j\omega Z_{\rm inp}$, also contains most of the terms in the expression for $n\alpha$ (13), only the resonant term $(4Z_{\rm inp}+Z_{\rm load})/(Z_{\rm inp}+Z_{\rm load})$ is missing. This means that in the resonant regime when this term is large in the absolute value, the second term in the denominator of (28) dominates over the first one, and the formula can be simplified as

$$Z_{\rm load}=-Z_{\rm inp}+\frac{4nl^2(\epsilon_{\rm eff}+2\epsilon_0)}{9j\omega\epsilon_0(\epsilon_{\rm eff}-\epsilon_0)}. \quad (29)$$

One can view this impedance as that of a series connection of two loads. The impedance of the first one is the negative of the input impedance of the antenna $Z_{\rm inp}$. The second one depends on the required effective permittivity. So, we can say that the frequency dispersion due to the shape and size of the metal wire is first compensated, and then the desired behavior is created by an additional load impedance.

The input impedance of a short wire dipole is capacitive, so we can substitute $Z_{\rm inp}=1/(j\omega C_{\rm wire})$ in (28) and write[1]

$$Z_{\rm load}=\frac{1}{j\omega}\frac{(\epsilon_{\rm eff}-\epsilon_0)\frac{3\epsilon_0}{(\epsilon_{\rm eff}+2\epsilon_0)C_{\rm wire}}-\frac{4}{3}nl^2}{\frac{nl^2C_{\rm wire}}{3}-(\epsilon_{\rm eff}-\epsilon_0)\frac{3\epsilon_0}{\epsilon_{\rm eff}+2\epsilon_0}}. \quad (30)$$

At this point we can observe that for the design of "dispersion-free" lossless metamaterials (with a frequency independent and real effective permittivity) the load circuit should be a *capacitor*. The value of this required capacitance is obviously

$$C_{\rm load}=\frac{\frac{nl^2C_{\rm wire}}{3}-(\epsilon_{\rm eff}-\epsilon_0)\frac{3\epsilon_0}{\epsilon_{\rm eff}+2\epsilon_0}}{(\epsilon_{\rm eff}-\epsilon_0)\frac{3\epsilon_0}{(\epsilon_{\rm eff}+2\epsilon_0)C_{\rm wire}}-\frac{4}{3}nl^2}. \quad (31)$$

Clearly, this value can be both positive and *negative*, depending on the required $\epsilon_{\rm eff}$. The last case can be realized only using active circuits. This is in harmony with the fact that dispersion-free materials are not physically realizable using only passive elements.

In particular applications, the required materials are often frequency dispersive, and some specific frequency dispersion is desired. A typical example is the design of radar absorbing materials. We invite the reader to substitute the desired frequency behavior of the effective permittivity into (28) to see what kind of loads would realize that material as an array of loaded wire dipoles.

B. Composite Sheets

Next, let us consider a single periodical array of small loaded dipole particles. Our goal here is to determine the loads required to realize desired reflection and transmission coefficients. Assuming the array period is smaller than about half wavelength, we can make use of an approximate analytical expressions for the reflection and transmission coefficients established in [15]. That model is based on an approximate dynamic theory for the field interaction in infinite arrays of dipoles and results in an equivalent sheet condition for simulation of arrays. The reflection coefficient for electric fields of normally incident plane waves on an array with square cells (array period a) can be written as

$$R_E=-\frac{1}{1+\frac{2Z_g}{\eta}} \quad (32)$$

where the grid impedance Z_g normalized to $\eta/2=0.5\sqrt{\mu_0/\epsilon_0}$ reads

$$\frac{2Z_g}{\eta}=\frac{2a^2}{\eta}\left[\frac{3Z_{\rm inp}}{l^2}\frac{(Z_{\rm inp}+Z_{\rm load})}{(4Z_{\rm inp}+Z_{\rm load})}-\frac{\eta k^2}{6\pi}\right]+\frac{j}{2}\left(\frac{\cos kR_0}{kR_0}-\sin kR_0\right). \quad (33)$$

Here, parameter $R_0=a/1.438$, and the polarizability of the loaded wire dipole has been substituted from (13). For lossless scatterers, this simplifies to [15]

$$\frac{2Z_g}{\eta}=j\left[\frac{6a^2}{\eta l^2}{\rm Im}\left\{Z_{\rm inp}\frac{Z_{\rm inp}+Z_{\rm load}}{4Z_{\rm inp}+Z_{\rm load}}\right\}+\frac{1}{2}\left(\frac{\cos kR_0}{kR_0}-\sin kR_0\right)\right]. \quad (34)$$

[1]This result is applicable for regular lattices where the radiation resistance should be excluded from the input impedance.

The transmission coefficient is

$$T_E = 1 + R_E = \frac{\frac{2Z_g}{\eta}}{1+\frac{2Z_g}{\eta}} \tag{35}$$

which follows from the fact that in our case there is no magnetic current in the sheet, thus, the tangential electric field is continuous.

Now we can easily solve for the required load impedance in terms of the reflection coefficient. From (32) and (33) we find

$$Z_{\rm load} = \frac{\frac{6a^2}{\eta l^2}Z_{\rm inp} - 4Q}{Q - \frac{6a^2}{\eta l^2}Z_{\rm inp}} Z_{\rm inp} \tag{36}$$

where

$$Q = -\left(1+\frac{1}{R_E}\right) + \frac{(ka)^2}{3\pi} - \frac{j}{2}\left(\frac{\cos kR_0}{kR_0} - \sin kR_0\right). \tag{37}$$

The same formula actually determines the required load if the transmission coefficient is given, because $T_E = 1 + R_E$.

IV. Conclusion

This paper gives a simple analytical model for polarizabilities of straight loaded wire dipoles. The present analysis extends the previous result of [3] to arbitrary (not necessarily resonant) particles with the help of a more accurate model of the current distribution function. In addition, in the new model there is no restriction on the wire length. An analytical formula for the imaginary part of the polarizability that is responsible for power absorption in nonideally conducting wire is also given. The particle model allows us to derive a design formula that gives the required load impedance for realizing of a material with the desired dispersion law in a definite frequency band. If one does not impose strict restrictions on the load impedance, allowing the load circuit to be active or controllable by external fields, including nonelectromagnetic, it appears that in principle arbitrary effective permittivity dispersion can be realized this way (again, in a limited frequency band). Although the system always remains causal, the limitations like the Kramers-Kronig relation or the Foster theorem (for lossless particles) can be overcome in this design. This is possible because in the derivation of the Kramers-Kronig relation besides causality it is also required that the system be linear and that the memory effects decay in time. As a simple example, a power generating medium can be realized by loading our inclusions with active loads, which clearly violates the Kramers-Kronig relation.

However, actual realizations can be difficult, if at all possible. Even for a quite simple requirement of the effective permittivity that does not depend on the frequency at all, we have seen that the required load can be a *negative* capacitance. This load is of course realizable by impedance inverter circuits (the particles become active in the sense that there are power sources to feed the inverter circuits), but the actual realization can be difficult, mainly due to potential instabilities of the impedance inverters when connected to the wire dipoles. We can finally conclude that the present theory provides a general approach to the design of a very general class of metamaterials, and much work is still needed on the road to realize such active and controllable media with desired properties. We hope that the results of this paper clearly demonstrate the power of the metamaterial concept that allows great flexibility in the design of artificial materials and thin sheets.

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