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Why is Shadow Fading Lognormal?

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Abstract—Received signal power in mobile wireless communications is often modelled as a product of three factors: distance-dependent average path loss, lognormal variation in the local mean power level (shadow fading), and small-scale fading due to movement in the order of a wavelength. Of these three factors, the least well understood is the shadow fading, the lognormality of which is usually explained as a result of multiplication of large number of random attenuating factors in the radio channel. This model is not credible from physical propagation point of view, and is also partly contradicted by measurements. In this paper, we propose an *additive* cluster-based model for shadow fading. We assume that the received signal is a superposition from several scattering clusters. It is shown that if the distribution of the local mean cluster powers satisfy certain mild statistical conditions, the shadow fading of the received signal will be approximately lognormal. We also present measurement results that support the theory.

I. Introduction

It is common practice in radio communications that amplitude variation of a received radio signal is modelled as a product of path loss, shadow fading and small-scale fading [1]. Several models exist for path loss, including the variants of Okumura-Hata and Walfisch-Ikegami formulas. Likewise, numerous statistical models have been proposed for small-scale fading, most notably the Rayleigh, Rice and Nagakami- m probability laws. Shadow fading has been empirically observed to obey approximately lognormal distribution in a wide variety of propagation environments, yet few theories for the lognormality have been presented. This is somewhat startling since, from mobile communications system point of view, shadow fading plays a key role in interference modelling as well as the analysis of handover algorithms, slow power control, macrodiversity and network planning tools and simulation.

The conventional textbook explanation for the lognormality of shadow fading is the *multiplicative* model which assumes that there are several multiplicative random factors attenuating the received signal, and the logarithm of their product approaches the Gaussian distribution for a sufficiently large number of such factors. This hypothesis has some shortcomings, however. First of all, it has been shown in [2] that convergence of the sum-of-logs to Gaussianity requires a large number of

factors¹, which is difficult to justify from physical propagation point of view. Second, it was remarked in [3] that, should the multiplicative shadow fading model be correct, the path loss itself should increase exponentially with distance. This disagrees with empirical evidence, which suggests a $\sim d^m$ path loss law with distance d . The decay exponent m is typically found in the range 3...5, depending on the environment. Third, we point out that, under the multiplicative model, there should exist systematic experimental evidence of an increase in the shadow fading standard deviation with distance, since the number of random multiplicative attenuations is expected to increase, on average, as the distance between receiver and the transmitter increases. However, most measurement studies indicate that the standard deviation is approximately distance independent. Clearly, the classical ‘textbook theory’ for shadow fading does not fully agree with measurements. For this reason, other distributions have also been proposed to model shadow fading, including the gamma distribution [4]. Without a physical basis, however, it is difficult to justify using one arbitrary probability model over another. Therefore, it is important to investigate the underlying physical process in order to find theoretically justified probability models.

Some studies to explain the lognormality of shadow fading have appeared in literature. In [5] it was shown that in urban environment lognormality may be caused by randomness in building heights alone. Somewhat contradictively, [6] reports that not only random rooftop heights are sufficient to produce lognormal local mean, but also randomized building construction and effect of foliage would be required. The results in [5], [6] focus on the path loss model dependent shadow fading, to be discussed in Section II. Therefore, they cannot explain the lognormality of shadow fading observed in continuous-time signal power measurement. A groundbreaking hypothesis was presented in [3], [7], where it was argued that time-variant multiple scattering in the propagation channel can cause the signal level variations usually perceived as lognormal. Interestingly, measurement-based evidence to support the theory in forest and urban propagation scenarios was also presented. In [8], it was shown that in a line-of-sight microcell few dominant reflections lead to slow variations in the signal power that can be modelled using Nagakami- m or lognormal distribution.

¹For Rayleigh distributed factors more than 30 terms were required using the Kolmogorov-Smirnov test with 5% significance level. Similar conclusions were drawn for other distributions.

This interesting paper is, to our knowledge, the only one that proposes an additive model as a basis for shadow fading. Unfortunately, the results therein were not generalized to other environments, nor were any analytical justifications given. This is exactly the novelty and main contribution of the present paper. We present an *additive cluster-based model* for shadow fading of a narrowband radio signal, and provide a simple analytical justification for its lognormality.

II. Path loss independent and dependent shadow fading

In literature two types of shadow fading are discussed, usually interchangeably. We, however, make distinction between path loss dependent and path loss independent shadow fading. *Path loss dependent* shadow fading is typically a result of regression analysis on a signal level measurement represented on a distance-path loss scatter plot. In other words, a path loss law, typically of form $A + m \log(d)$, is fitted to the measurement, and the residual error of the model fit is called shadow fading. This type of shadow fading depends on the form of the path loss law and the fitting method, since, for a given measurement record, a different path loss model would surely result in a different shadow fading (i.e., residual error) distribution. This appears to lead to the uncomfortable conclusion that in order to analyze the shadowing phenomenon we first need to fix the path loss model.

By *path loss independent* shadow fading we mean the variation in the local mean signal power in a continuous-time signal power measurement. The local area signal power is usually estimated with some type of sliding window processing, e.g. using a sliding median. Our focus is on this type of shadow fading, since it is independent of the path loss model, and has also been noticed to follow approximately lognormal distribution.

III. An explanation of lognormality of shadow fading based on an additive cluster-based signal model

We assume that the received signal can be described as a superposition of reflections or diffractions from N clusters², where N is assumed to be constant. This assumption is valid if the area under examination is not too large. The appearance of multipath components in clusters is supported by numerous measurements, e.g. [9]–[13]. In Fig. 1 we show received signals from two clusters extracted from a channel measurement to be discussed in more detail in Section IV. It can be seen that the cluster signals exhibit fading in small and large distance scales. It is therefore conceivable that not only the composite signal – i.e., complex sum of signals from all clusters – but also individual clusters show large-scale power trends, which could be called shadow fading. The observed fading characteristics of a cluster will naturally depend on the spatial-temporal resolution of the measurement equipment and processing methods used.

The narrow-band channel impulse response sampled at a point x can be written as

²Since clusters are typically identified visually, a cluster is usually subjectively defined as a set of multipath components with ‘similar’ delays and arrival angles.

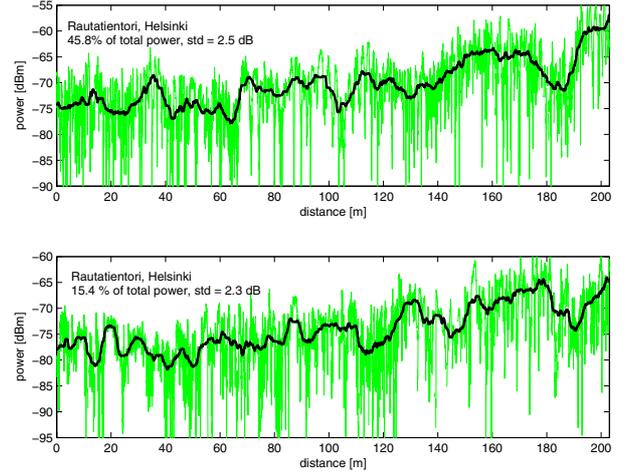


Fig. 1. Signals from two clusters extracted from an outdoor urban macrocell measurement. In addition to small-scale fading, individual clusters exhibit large-scale fading.

$$h(x) = \frac{1}{\sqrt{\mu_N}} \sum_{n=1}^N w_n(x) g_n(x). \quad (1)$$

The w_n 's denote real-valued, positive weight factors signifying the amplitude of the n th cluster. The g_n 's denote independent, identically distributed circularly symmetric zero-mean unit-variance complex Gaussian random variables, which characterize the small-scale varying component of the received signal³. Without loss of generality, we assume the usual power normalization $E[|h(x)|^2] = 1$, or equivalently, $E[\sum_{n=1}^N w_n^2] = \mu_N$. The model (1) forms also basis for a number of geometric stochastic channel models, in which each cluster has its own random angular, delay, and amplitude characteristics [10], [14], [15].

The cluster amplitudes, $w_n(x)$, are assumed to change slowly with respect to the Gaussian processes, $g_n(x)$, as x changes. This is a reasonable assumption as the change in cluster powers can be thought to be caused by slow changes in the cluster scattering cross-section seen by the receiver. The variation of $g_n(x)$, on the other hand, is due to summation of multiple phases due to receiver movement, and takes place in about a wavelength's distance scale. Based on this observation we can further assume that each $w_n(x) \approx w_n$ within a distance scale of several wavelengths (called ‘local area’, see Fig. 2). Since the processes $g_n(x)$'s are assumed independent, we can write

$$h(x) \sim \sqrt{\frac{1}{\mu_N} \sum_{n=1}^N w_n^2} g(x), \quad (2)$$

where $g(x)$ is a zero-mean complex Gaussian random variable with unit variance, and ‘ \sim ’ denotes equivalence in distribution. In other words, the probability distributions of the right-hand sides of (1) and (2) are equivalent within a local area. The

³The assumption of complex Gaussianity is not crucial, but it simplifies the presentation of the main idea. It is possible to derive similar results for a more general sum-of-sinusoids model.

major advantage of (2) over (1) is its multiplicative form, which allows interpreting the statistics of the shadow fading random variable $m = \mu_N^{-1} \sum_{n=1}^N w_n^2$ in terms of the statistics of the cluster powers.

We remark that the right-hand side of (2) is a spherically invariant random process [16]. Such a process has been also proposed for modelling of radio channels [17], [18], although no physical interpretation was given.

The assumption $w_n(x) \approx w_n$ in (2) can be justified if the correlation distance of the shadow fading process is much larger than that of the small-scale fading signal. Typical values for the spatial correlation distances have been measured to be about five meters in urban microcells and about 50-500 meters for urban and suburban macrocells. As the small-scale fading takes place in a distance scale of about one wavelength, the approximation (2) is hence usually justified.

The local mean power, from dB scale data, within a local area is

$$\begin{aligned} E[20 \lg(|h(x)|)] &= E[10 \lg(m) + 20 \lg(|g(x)|)] \quad (3) \\ &= 10 \lg \left(\underbrace{\mu_N^{-1} \sum_{n=1}^N w_n^2}_{m_{dB}} \right) - 2.51 \end{aligned}$$

It can be seen that the second term of (3) is a log-Rayleigh random variable, whose expected value is -2.51 dB [19, App. 2]. The first term in (3) is local mean power, which is, by definition, constant within a local area. We are interested in the distribution of $m_{dB} = 10 \lg(m)$ as the receiver moves within the extended local area (Fig. 2), where path loss and N are assumed constant. Consequently, we neglect the mean of the log-Rayleigh variable. Such constant mean value is indistinguishable from path loss in a practical shadow fading measurement, where only the shape and standard deviation of the shadow fading distribution are of interest.

We now show that, under certain assumptions on the statistics of the cluster powers $\{w_n^2\}_{n=1}^N$, m_{dB} can be approximated with a Gaussian distribution. The following assumptions are made:

- A1: (Central limit theorem) We assume that the distribution of $m = \mu_N^{-1} \sum_{n=1}^N w_n^2$ converges to the Gaussian distribution as $N \rightarrow \infty$.
- A2: $E[m] = 1$ for all N .
- A3: $\text{var}[m] \rightarrow 0$ as $N \rightarrow \infty$, i.e., m has asymptotically vanishing variance.

From the following theorem it follows that m_{dB} has approximately Gaussian distribution for large enough N .

Theorem 1: Consider a continuously differentiable function $y = g(x)$. Assume that the above assumptions hold for a random variable m . Then the random variable $y = g(m)$ has asymptotically Gaussian distribution with mean $g(1)$ and variance $\text{var}[m][g'(1)]^2$.

Proof: For a mathematical proof, see e.g. [20, Prop. 6.4.1]. The simple intuition is that when the variance of m is small, its pdf is concentrated in the neighborhood of $E[m]$, where $\ln(x)$ can be approximated by its tangent. Since a linear transformation of a Gaussian random variable results in another Gaussian random variable, the theorem follows. ■

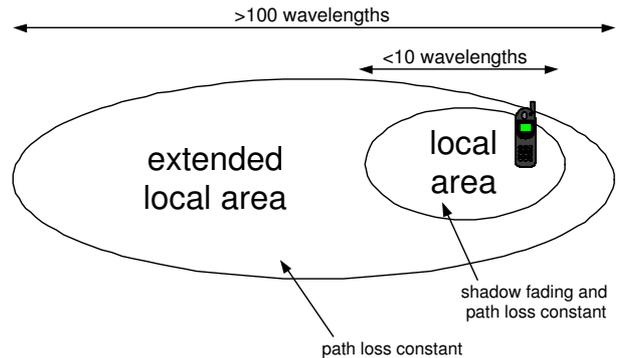


Fig. 2. Local area is defined as a region where path loss and shadow fading are constant. Extended local area, which consists of several overlapping local areas, is a region where only path loss is constant. The number of clusters, N , is also assumed constant within the extended local area. The dimensions of the areas are indicative values based on measurements in an urban macro cell environment.

Theorem 1 says that m_{dB} can be approximated with a Gaussian distribution with zero-mean and variance $100(\ln 10)^{-2} \text{var}[m]$. In practice, measuring cluster power distributions is very difficult. The significance of the result lies in the fact that no matter what distributions the cluster powers may have, the shadow fading of the resulting sum signal (1) will have approximately lognormal distribution. The explanation of lognormality of shadow fading is, however, based on the additive signal model (1), which is more plausible from physical radio propagation viewpoint than the classical multiplicative shadow fading model.

Remarks: The most important assumption in Theorem 1 is the central limit theorem (A1), which, however, holds under very mild conditions. For example, the cluster powers $\{w_n^2\}_{n=1}^N$ need not be statistically independent random variables as there exist central limit theorems for dependent random variables [21].

IV. Numerical examples

IV-A. Simulation examples

In Fig. 3 we show an example of four different cluster power distributions. We assume $N = 4$ and iid pdfs for $\{w_n^2\}$. Histograms and best-fit Gaussian pdfs are shown for chi-squared, Weibull, gamma, and lognormally distributed cluster powers. It can be seen that, with the possible exception of gamma pdf, the resulting shadow fading distributions resemble the Gaussian pdf. Of course, increasing N would make the Gaussian approximation better. The sum of lognormal cluster powers gives a particularly good fit, since a sum of lognormal variates can be well approximated as lognormal [22]. We stress that our point is *not* to argue that real-world cluster powers follow any one of the pdfs used here; our purpose is merely to illustrate how, by virtue of Theorem 1, the apparent lognormality of shadow fading can result from almost *any* cluster power distribution. The selected pdfs serve only as illustrative examples.

IV-B. Measurement example

Measurement results on the cluster shadow fading distributions appear not to be available in literature, probably due to many

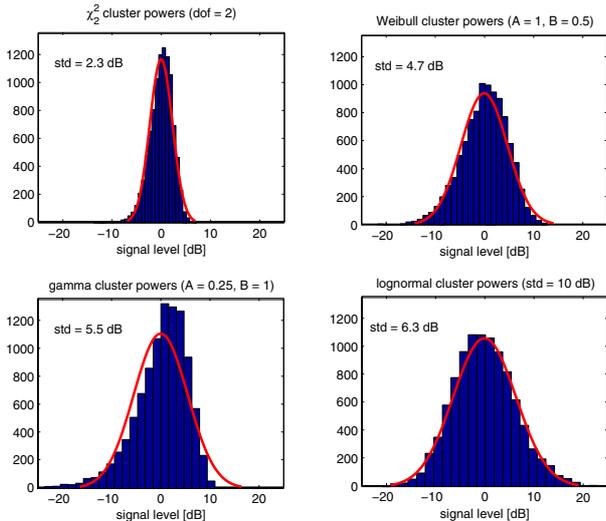


Fig. 3. Examples of shadow fading distributions resulting from different cluster shadow fading distributions. In all cases the number of clusters is four ($N = 4$), and the cluster powers $\{w_n^2\}_{n=1}^4$ have iid pdfs. The best-fit Gaussian pdf is also shown for comparison. The parameters A and B with gamma and Weibull pdfs correspond to the notation of MATLAB's corresponding random number generators (`weibrnd` and `gamrnd`).

practical difficulties in both measurements and data processing (e.g. cluster extraction and tracking from measurement data over long measurement distances). In what follows, we present novel measurement results on cluster shadow fading distributions based on an earlier study in [12], [23]. The measurement was conducted in downtown Helsinki on 2.1 GHz frequency at 120 MHz bandwidth. The fixed single-antenna transmitter was located on a roof at about 25 meter height. The receiver was moving at 1.5 meter height in non-line-of-sight about 200 meter distance over a square. The distance between transmitter and receiver was 300 meters and stayed approximately constant over the measurement route. The receiver antenna array consisted of 32 dual-polarized patch antenna elements located on a spherical surface. The measurement environment has been described in [23] and the measurement system and data postprocessing method are detailed in [24]. A total of 10 clusters were identified from measurement data by visual inspection [23].

The local area power of each cluster was estimated using sliding median with window width of 40 wavelengths (200 time samples, or 6 meters). With most clusters, a slight residual power trend remained in the local mean signal, see Fig. 1 for an example. The trend was removed using a piecewise linear fit with a breakpoint distance of 500 wavelengths.

Fig. 4 shows the histograms of zero-meaned local area powers of the cluster. Three strongest clusters carry more than 75% of the total power. The empirical cluster shadow fading distributions are unimodal with standard deviations ranging from 1.6 to 4.3 dB's. A lognormal model for cluster shadow fading appears to be a reasonable first choice, although longer measurement runs should be evaluated for more reliable conclusions.

To test the ideas presented in Section III, we conducted the following experiment. We added the shadow fading signals of

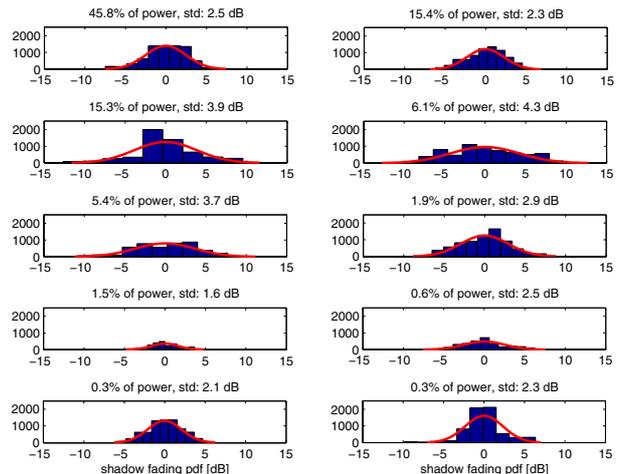


Fig. 4. Measured shadow fading histograms of the ten extracted clusters. The best-fit Gaussian pdf is also shown for comparison.

the strongest and the second strongest cluster. The histogram of the shadow fading power was then plotted. The procedure was repeated by cumulatively summing powers of all clusters so that the weakest cluster was summed last. The results for each step are shown in Fig. 5. We observe that as the number of clusters increases the shadow fading distribution of the composite signal converges to a unimodal distribution resembling a Gaussian pdf. The standard deviation also decreases as the number of clusters summed increases, as is to be expected from theory. The shadow fading distribution of the original signal is shown for comparison in lower subplot of Fig. 6. In the upper subplot of the same figure we show the reconstructed signal obtained by summing the shadow fading processes of the ten clusters. There is clear similarity with shadow fading process of the original signal, although small differences can also be seen. The differences are explained by the limitations of the measurement system and data processing procedures. First, about 7.4% of the power of the original signal could not be clustered. The residual power, i.e., the error signal in Fig. 6, is attributed to short-time sporadic reflections and diffuse scattering. Second, due to limited dynamic range of the measurement system, weaker clusters often fade below the noise floor, in some cases reappearing at a later time instant. Therefore, the cluster power signals have many discontinuities, which complicates sliding window processing. Considering the many challenges in data processing – probably the main reason why studies such as this have not been published before – the results from measurement analysis are quite illuminating. In particular, Fig. 6 confirms that the shadow fading processes of individual clusters are the reason for the shadow fading of the composite signal.

V. Conclusion

We have presented a simple additive cluster-based model to explain the experimentally observed lognormality of shadow fading. It was shown that under very mild conditions (essentially central limit theorem) on the individual cluster shadow fading statistics, the shadow fading of the composite signal will have approximately lognormal distribution. This explanation

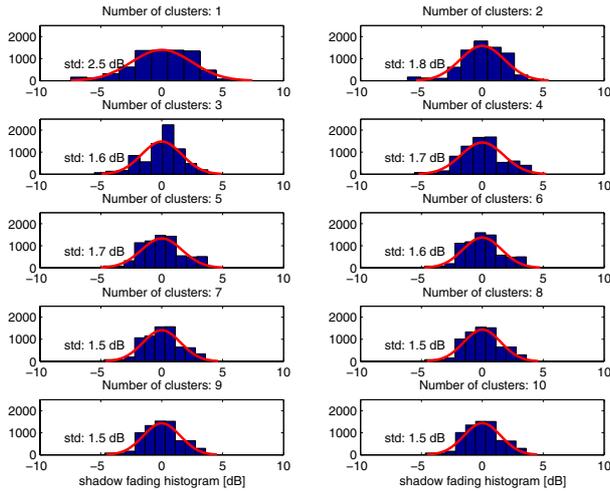


Fig. 5. Shadow fading histograms resulting from summing cluster signals one at the time. The best-fit Gaussian pdf is also shown for comparison.

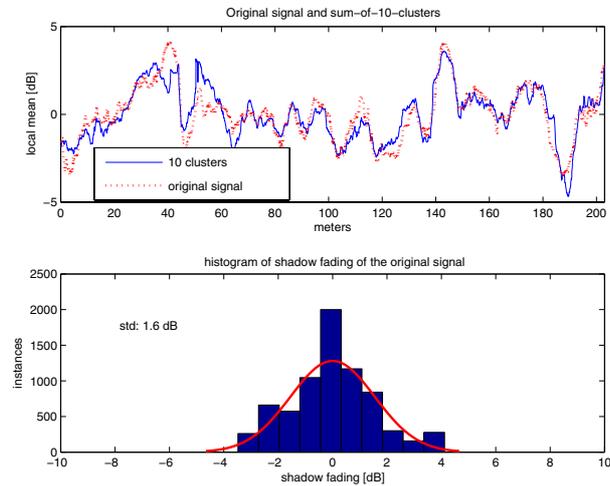


Fig. 6. Shadow fading process of the original measured signal, and signal reconstructed by adding the ten clusters.

is more plausible than the traditional multiplicative model entrenched in the radio propagation literature. Preliminary measurement results on cluster shadow fading support our theoretical findings, although, in order to make more general conclusions, longer measurement routes should be analyzed.

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