GENERALIZATION OF CRITICAL TRANSMISSION RANGE FOR CONNECTIVITY TO WIRELESS MULTIHOP NETWORK MODELS INCLUDING INTERFERENCE

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ABSTRACT

The connectivity of wireless multihop networks has mostly been studied neglecting interference, assuming that all network nodes have a common transmission range within which they can form direct links. Under this assumption, the critical transmission range for connectivity of a given network equals the greatest edge length in the Euclidean minimum spanning tree of the network nodes. While the impact of interferences on the percolation phenomena of infinite networks has been studied [1], little is known about the graph connectivity of finite networks under more realistic network models. In this paper, we generalize the critical range for connectivity to two network models presented earlier which are both based on a minimum required signal-to-noise-and-interference ratio for successful communication. As these models have more than one free parameters, the critical range generalizes into a boundary in the space of these parameters; we show how to determine this boundary for a given network. The connectivity boundary implies tradeoffs between different parameters dictating network performance. Our results allow studying the connectivity of interferences-limited networks by simulation and give insight on the sensitivity of connectivity to different network parameters.

KEY WORDS

ad hoc networks, physical model, connectivity, network performance

1 Introduction

The connectivity of wireless multihop networks has been a topic of extensive research for which graph theory provides a natural analytical framework. Applying graph theory to these networks requires defining when a single link is connected. This boils down to issues on the physical layer: the quality of reception of a radio transmission depends on the signal-to-noise-and-interference ratio (SINR) at the receiver. From an information-theoretic point of view, any positive SINR enables successful communication; only the achievable rate of communication depends on the SINR. From this viewpoint, any link and hence any given network can always be said to be connected.

To say that some link is not connected therefore requires us to set a minimum value for the SINR corresponding to a required minimum rate of communication. Such a minimum value may also be well motivated by technical issues, such as the existence of proper coding schemes within a given communication framework. This kind of condition for successful communication was e.g. used under the name Physical Model in studying the capacity of wireless networks in [2].

Under a simple network model that neglects all interferences and instead of the SINR only considers the SNR, the ratio of the received signal power to that of a constant-level ambient noise, such a minimum value for the SNR can easily be translated into a transmission range, i.e., the maximum distance from a transmitter that, given the transmission power, allows a reception exceeding the minimum SNR. This simple range-based *Boolean model* has been the most popular one in the research literature on connectivity. In particular, under the common assumption that all network nodes can achieve the same transmission range under the Boolean model, it was pointed out in [3] that for a given network, the *critical range* for connectivity, i.e. the required minimum value for this transmission range to make all node pairs connected in a multihop fashion, equals the greatest edge length in the Euclidean minimum spanning tree for the nodes.

In this paper, we study the graph connectivity of wireless multihop network models that also take interferences into account. More precisely, we generalize the notion of the critical range for connectivity to two network models, presented in [1] and [4], that are both based on the Physical Model mentioned above and differ from each other essentially by the assumed medium access method employed. Like relied on by the definition of the critical range under the Boolean model, we also assume that all network nodes employ some common constant transmission power. As these models have more than one free parameters, the critical range for connectivity generalizes into a boundary in the space of those parameters; we present algorithms that, for a network with given node locations, determine this boundary. In the context of both models, we see that the requirement of a connected network imposes a boundary condition resulting in tradeoffs between different parameters dictating network performance.

Thus far, in the context of network models that account for interference, little has been done to address connectivity as defined in graph theory, namely, the requirement that all node pairs be connected through the network. The aim in the paper [1] presenting the first model of interest to us was to study the percolation properties, i.e. the existence of infinite connected components in infinite networks, of that model. As for the analysis of the second model in [4], the authors defined connectivity as a dynamic property allowing the successful transfer of packets between all node pairs at some positive average rate. This definition can be regarded as coincident with the omnipresent information-theoretic connectivity referred to in the beginning of this section. Both of these studies differ from the present paper in that they focused on infinite random networks – which rules out studying graph connectivity based on one common link quality constraint – whereas we only consider networks with a finite number of nodes.

This paper is comprised of the following parts. In the next section we introduce the two network models that we will study. The generalization of the critical range for the first model motivated by CDMA is presented in Section 3. The connectivity of the second model assuming a slotted-Aloha medium access scheme is examined in Section 4. We conclude in Section 5.

2 Network models

2.1 A CDMA network

We will first consider the conceptually simpler one of the two network models which, as shown in the next section, allows easy and unambiguous determination of the connectivity boundary. This model was introduced and named the Signal To Interference Ratio Graph (STIRG) in [1]. According to this model, node j located at point x_j in the plane can successfully receive the signal transmitted by node i at x_i with power P_i if and only if the SINR at the reception exceeds some threshold $T > 0$:

$$\frac{P_i L(x_i - x_j)}{N_0 + \gamma \sum_{k \neq i,j} P_k L(x_k - x_j)} \geq T. \qquad (1)$$

Here, N_0 is the power of the background noise on the frequency channel utilized by the network and $L(x)$ is the attenuation function in the wireless medium. The factor $0 \leq \gamma \leq 1$ weighting the interference power sum is motivated by the partial orthogonality of CDMA codes and can be interpreted as the inverse of the processing gain of the system. As in [1], we neglect unidirectional links because of their low utility and therefore define the connectivity of a link as in an undirected graph where there is an edge between node i and j if and only if (1) also applies with i and j interchanged.

As in [1], we also make the assumption that every node transmits constantly at some common power: $P_i \equiv P$, which can be considered a rather unrealistic assumption. Further, although not at all necessary, we will restrict ourselves to the commonly used power-law attenuation function

$$L(x_i - x_j) = l(||x_i - x_j||) = (C||x_i - x_j||)^{-\alpha}, \qquad (2)$$

to allow easy scaling of any configuration of network nodes to arbitrary physical node densities. Here, $C > 0$ sets the scale and $\alpha > 2$.

2.2 A slotted-Aloha network

Next, we will examine the more realistic model which, as discussed in [4], also has notable practical appeal in terms of implementation. The medium access scheme used under this model is slotted Aloha: time is slotted, and each node is allowed to transmit in any slot with a fixed medium access probability p. Keeping to the assumption of the power-law attenuation function (2) as in [4], as well as one common transmission power P, we suppose that node j at x_j is able to receive node i's transmission successfully in any time slot – producing what is chosen as the unit throughput from i to j over this time slot – if

$$\frac{P(C||x_i - x_j||)^{-\alpha}}{N_0 + \sum_{k \neq i,j} e_k P(C||x_k - x_j||)^{-\alpha}} \geq T, \qquad (3)$$

where e_k is the indicator variable of the event that node k is allowed to transmit in that time slot. The individual permissions to transmit are independent among both nodes and time slots, so that the variables $\{e_k\}$ in every time slot are independent Bernoulli-distributed random variables with parameter p.

Section 4 will show that this model has more dimensions of free parameters than the previous one when defining graph connectivity, but it turns out that the connectivity requirement will constitute a boundary condition for a tradeoff between the delay and throughput of the network links and allows for an optimization between the two as desired.

3 The connectivity boundary in the CDMA network model

We will now derive the connectivity boundary in the space of free parameters for a network with given node locations x_k and some fixed attenuation exponent α, under the first network model. Applying the power-law attenuation function (2), we may write (1) in the form

$$\frac{||x_i - x_j||^{-\alpha}}{C^\alpha \frac{T}{P/N_0} + T\gamma \sum_{k \neq i,j} ||x_k - x_j||^{-\alpha}} \geq 1, \qquad (4)$$

where we are now able to recognize the two free parameters $C^\alpha \frac{T}{P/N_0} \stackrel{def}{=} A$ and $T\gamma \stackrel{def}{=} B$. The above is then equivalent to

$$A \le -B \sum_{k \ne i,j} ||x_k - x_j||^{-\alpha} + ||x_i - x_j||^{-\alpha}. \quad (5)$$

Hence, in a given network, the condition for node j successfully receiving node i's transmission is satisfied on and below the descending line in the B/A-plane with slope $-\sum_{k \ne i,j} ||x_k - x_j||^{-\alpha}$ and intercept $||x_i - x_j||^{-\alpha}$, both of which we may calculate. Since our attenuation function is reciprocal, i.e. $L(x_i - x_j) = L(x_j - x_i)$, which can also be said to hold in reality on a fixed frequency, the resulting condition for an edge existing between nodes i and j in the undirected STIRG is determined by this common intercept and the steeper one of the two slopes calculated in both directions (i.e., the receiver under more noise).

The domain in the B/A-plane in which the given network is connected then lies below a connected curve consisting of segments of descending lines, representing the constraints of different links that are critical for network connectivity at each point; this curve is the connectivity boundary. It can be found as follows. First calculate the slope and intercept of the line for each individual link. Start at $\gamma = 0$ implying $B = 0$. This makes the model coincide with the Boolean model, meaning that that the critical link is found as the longest edge in the Euclidean minimum spanning tree of the nodes. The descending line corresponding to this link determines the boundary of the connectivity region in the plane as long as no line of the other links is crossed. When this happens, we may determine the new critical link using the following simple rule:

- When a line is crossed from above, i.e. the connectivity region of another link is entered when tracing the current line, the corresponding appearing link is the critical link from this point onwards if it connects the two network partitions separated by the current critical link. Otherwise, the critical link does not change.

- When a line is crossed from below, i.e. the connectivity region of another link is left when tracing the current line, the corresponding disappearing link is the critical link from this point onwards if the remaining links no longer form a single connected graph. Otherwise, the critical link does not change.

As an example, Figure 1 shows the resulting connectivity boundary of the network in a unit square shown in Figure 2. Note how the interference experienced by node 6 at a central location affects the result. Note also that the choice of any physical distance for the side of the square domain can be incorporated into the attenuation scaling factor C. The connectivity boundary defines the constraints for the various parameters that must be satisfied to guarantee a connected network topology.

In this model, we assumed that all other nodes transmit constantly. It would of course be more reasonable to assume that at least half of the nodes are receiving instead of transmitting at any instant. In general, if we assume that on average every kth node is transmitting, we should regard γ as a general interference thinning factor $\gamma = \tilde{\gamma}/k$, where $\tilde{\gamma}$ is the actual code orthogonality factor.

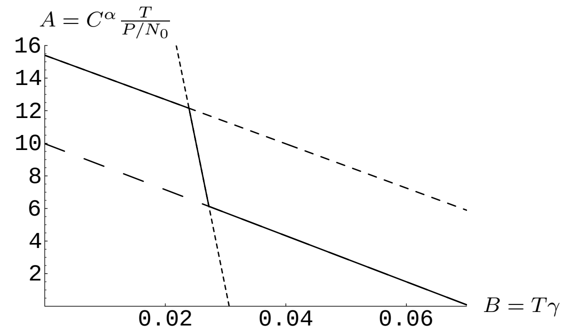


Figure 1. The connectivity domain of the network of Figure 2 when $\alpha = 3$. The solid line indicates the border of the domain. Taking $A = 0$ implies neglecting background noise; the case $\gamma = 0 \Rightarrow B = 0$ implies neglecting interference.

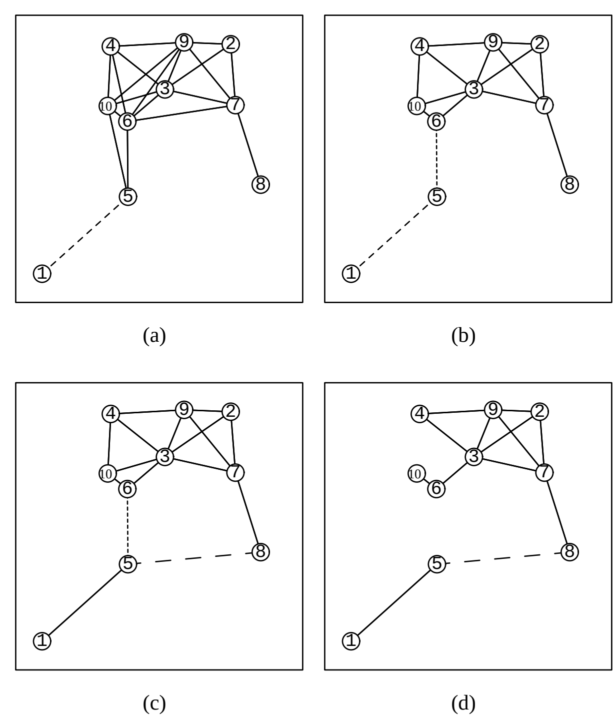


Figure 2. An example network in the unit square. The prevailing topology with the identified critical links have been drawn at each of the vertices of the border in Figure 1. The dashing of the links corresponds to that used in Figures 1 and 3.

4 Connectivity under the slotted-Aloha network model

4.1 The connectivity boundary

We then extend the definition of the connectivity boundary to the second network model. We assume, as in the previous section, that the network node locations are given and the attenuation exponent α is fixed. In addition, we assume that the medium access probability p of this model has also been fixed.

Under this model, the equivalent of (4) is

$$\frac{||x_i - x_j||^{-\alpha}}{C^\alpha \frac{T}{P/N_0} + T \sum_{k \neq i,j} e_k ||x_k - x_j||^{-\alpha}} \geq 1, \qquad (6)$$

where the sum $\sum_{k \neq i,j} e_k ||x_k - x_j||^{-\alpha}$ is a random variable independent in every time slot, having a discrete probability distribution with generally 2^{n-2} distinct possible values, with n denoting the number of all the network nodes. Given the assumed information, we may calculate this distribution.

In addition to the free parameters $A = C^\alpha \frac{T}{P/N_0}$ and T, we then define a third parameter, the *link confidence* q. For a given q, we may calculate the q-quantile of the distribution of the above random sum for each directed link $i \to j$; this quantile gives the level below which the random sum describing the scaled interference remains with confidence (probability) q. The difference from the previous section is that instead of the function $f_{ij}(\gamma) = \gamma \sum_{k \neq i,j} ||x_k - x_j||^{-\alpha}$ in (4), which is linear in its argument $0 \leq \gamma \leq 1$, we now have the nonlinear function $F_{ij}^{-1}(q)$ where $F_{ij}(\cdot)$ is the cumulative distribution function of the random sum $\sum_{k \neq i,j} e_k ||x_k - x_j||^{-\alpha}$. (To be exact, we define the inverse function of this discrete-valued cumulative distribution as $F_{ij}^{-1}(q) = \min\{t : F_{ij}(t) \geq q\}$.) Because of this nonlinearity, the parameter q can no longer be incorporated into the second free parameter with T but has to be treated as a separate, third parameter. Note however that the two functions $f_{ij}(\gamma)$ and $F_{ij}^{-1}(q)$ coincide at argument value 1.

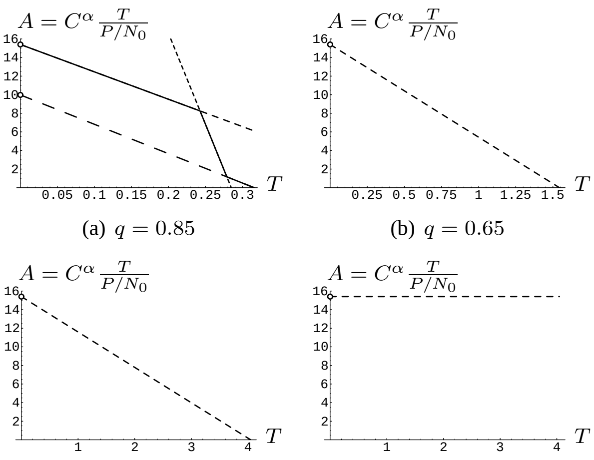
The connectivity boundary is a surface in the space of the three free parameters $A = C^\alpha \frac{T}{P/N_0}$, T, and q, a cross-section of which with fixed q looks similar to Figure 1: the equivalent of (5) is now

$$A \leq -F_{ij}^{-1}(q) \cdot T + ||x_i - x_j||^{-\alpha}, \qquad (7)$$

which, with fixed q, is satisfied on and below the descending line in the T/A-plane with slope $-F_{ij}^{-1}(q)$ and intercept $||x_i - x_j||^{-\alpha}$. As with the previous model, we define the condition for nodes i and j being bidirectionally connected to be determined by the steeper slope, i.e., the greater interference with the given confidence q.

With any fixed q, the cross-section of the connectivity boundary in the T/A-plane is found exactly as in the previous section, by tracing along the critical links. The reason why the longest edge in the Euclidean minimum spanning tree is again the critical link as $T \to 0$ is that in this limit, the above condition (7) for every link is dominated by the intercept $||x_i - x_j||^{-\alpha}$ which is a monotonically decreasing function of the link distance.

To demonstrate, we examine again the example network of Figure 2. We assume that $\alpha = 3$ and take $p = 0.1$, the latter representing a magnitude found suitable for the medium access probability in [4]. Figure 3 shows the connectivity domain in the T/A-plane with q fixed to different values; it is easy to see that $q = 1$ leads to the domain of Figure 1 with T in the place of B. The connectivity sur-



(a) $q = 0.85$ (b) $q = 0.65$

(c) $q = 0.45$ (d) $q = 0.40$

Figure 3. The connectivity domain of the network of Figure 2 when $\alpha = 3$ and $p = 0.1$, with the boundary indicated by a solid line where needed. Note that in this case, there is no interference in an arbitrary time slot with probability $(1 - 0.1)^8 \approx 0.43$, hence the zero-slope with $q = 0.40$.

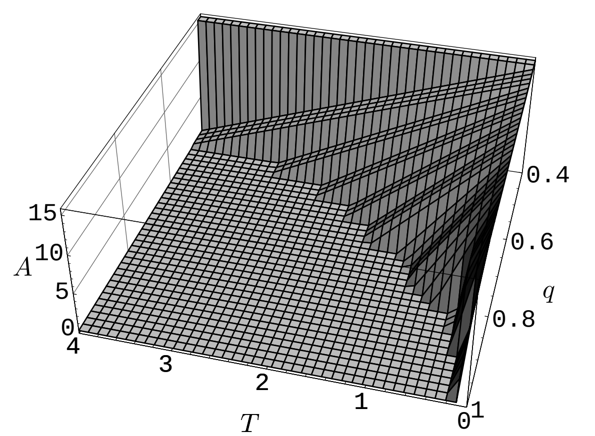


Figure 4. The surface below which the network of Figure 2 is connected when $\alpha = 3$ and $p = 0.1$. The cross-section of the surface is as in Figure 3(d) for all $q \leq (1 - 0.1)^8 \approx 0.43$.

face of the network in the space of all three parameters is depicted in Figure 4.

4.2 Delay-throughput tradeoff

The link confidence q defines when we consider a pair of nodes directly connected: we say that there is a link from any node i to any other node j only if, with given parameters $A = C^\alpha \frac{T}{P/N_0}$ and T, the probability $\tilde{q}$ that (3) holds in a random time slot is at least q. If we assume that the conditions for a completely successful transmission from i to j are that (i): node i is allowed to transmit, (ii): node j does not transmit in the same time slot, and finally that (iii): the states of the remaining nodes are such that (3) holds, then due to the nodes' independent operation the number of time slots needed for one successful transmission from i to j obeys a Geometric distribution with parameter $p(1-p)\tilde{q}$. Furthermore, for the critical link in a network with many nodes it is reasonable to assume that $\tilde{q} \approx q$. Hence, requiring a higher link confidence means requiring a lower maximum average link delay in the network, whereas allowing a lower link confidence means allowing a higher maximum average link delay.

On the other hand, the probability $p(1-p)\tilde{q}$ is also the proportion of time slots with successful transmissions from node i to j over time. The defined unit throughput of each successful transmission in turn depends on the SINR threshold T: if we take as a reference the Shannon capacity of a channel with Gaussian noise and interference and a given SINR, this is proportional to $\log(1 + SINR)$. Then the minimum time-averaged link throughput in the network is proportional to $q \log(1+T)$.

The two parameters q and T that together determine the maximum average link delay and the minimum link throughput are bound together by the connectivity constraint, which dictates that the greatest achievable T depends on the required q: $T_{\max} = T_{\max}(q)$. As an example, assume that we require the link confidence $q = 0.85$ from the network of Figure 2. Then $T_{\max}(q = 0.85)$ is determined by the intersection of the boundary in Figure 3(a) and the straight line $A = \frac{C^\alpha}{P/N_0} T$ rising from the origin with slope $\frac{C^\alpha}{P/N_0}$ (assumed given). Thus, for a given network, the required link confidence q can be increased from zero to some positive value without sacrificing minimum link throughput (with our example network, we know that this value is at least $q = (1-0.1)^8$). The maximum of the minimum link throughput with respect to q marks the beginning of Pareto-optimal combinations of maximum link average delay and minimum link throughput, meaning that neither quantity can be improved without making the other quantity worse: beyond this maximizing value of q, a delay-throughput tradeoff must then be made according to design preferences.

Finally, we note that one might just as well define the greatest achievable q given T, i.e. $q_{\max}(T)$, but whether or not this is defined with given T now depends on $\frac{C^\alpha}{P/N_0}$: for instance, in the case of our example network with $\frac{C^\alpha}{P/N_0} = 8$ and $T = 2$, no value of q can satisfy the connectivity constraint.

4.3 Example simulations

We now demonstrate the application of the above analysis to studying the performance of random slotted-Aloha networks under the requirement of connectivity. We assume that $\alpha = 3$ and $p = 0.1$ as above, as well as a negligible background noise, i.e. $N_0 = 0 \Rightarrow A = 0$, which results in independence of the scaling factor C and transmission power P. Constraining the link delays so that the average number of time slots needed for a successful transmission on any valid link should be no more than 20, we wish to determine a SINR threshold T as high as possible, under the requirement that the valid links in a network with ten nodes distributed independently and uniformly at random in a square domain should form a connected topology with high probability.

Thus, by the properties of the Geometric distribution we have $q \geq 1/(0.1 \cdot 0.9 \cdot 20) \approx 0.56$, with the lowest value maximizing the feasible T in all cases. Figure 5 shows the results of determining $T_{\max}(q = 1/(0.1 \cdot 0.9 \cdot 20))$ from 10000 such random networks: we may conclude, e.g., that setting $T = 1$ results in a roughly 95%-probability of a connected network, whereas $T = 1/2$ gives a very high connectivity probability.

4.4 Efficient approximation for determining the distribution of interference

The computational task of determining the distribution of the interference from a given network of n nodes, for receiver node j given the transmitter node i, entails evaluating all the possible 2^{n-2} values of the sum

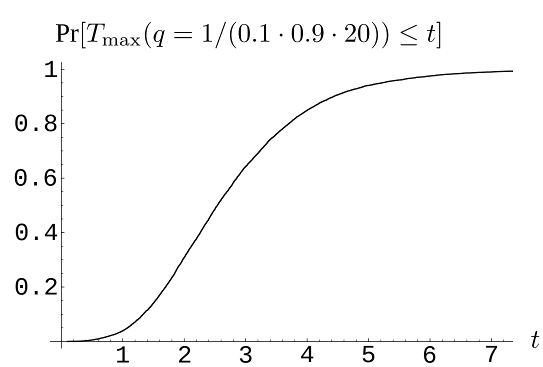


Figure 5. Empirical cumulative distribution function of $T_{\max}(q = 1/(0.1 \cdot 0.9 \cdot 20))$ determined from 10000 random networks with ten nodes distributed independently and uniformly at random in a square-shaped domain, when $\alpha = 3$, $p = 0.1$, and $N_0 = 0$.

$\sum_{k\neq i,j} e_k||x_k - x_j||^{-\alpha}$ and their probabilities. To be able to apply these methods to networks with even a moderate number of nodes, an efficient approximation method is therefore needed. For this purpose, it is useful to note that given the number of active transmitters m among the other $n-2$ nodes in a time slot (which is independent for each time slot and Bin$(n-2,p)$-distributed), the conditional expectation of the interference is

$$\mathrm{E}\left[\sum_{k\neq i,j} e_k||x_k - x_j||^{-\alpha}\middle|\sum_{k\neq i,j} e_k = m\right] = \frac{m}{n-2}\sum_{k\neq i,j}||x_k - x_j||^{-\alpha}.$$

The distribution of this conditional interference can therefore be used to approximate that of the interference itself.

However, this approximation is hindered by dominating terms $||x_k - x_j||^{-\alpha}$ resulting from near-by nodes, since all terms contribute equally to all values of the above conditional expectation. An improved approximation can be achieved by determining the distribution of the interference from such dominating nodes exactly and convoluting this distribution with that of the conditional expected interference from the remaining nodes. Our criterion for such a dominating node $h \neq i,j$ is that $||x_h - x_j||^{-\alpha} \geq f\sum_{k\neq i,j}||x_k - x_j||^{-\alpha}$, for some fixed fraction f.

Figure 6 shows an example of how these approximations relate to the exact distribution.

5 Summary and discussion

We generalized the critical transmission range for connectivity of the commonly used Boolean network model to two models presented earlier that both take interferences into account. The conceptually simpler model is motivated by CDMA and leads to an unambiguous definition of the connectivity boundary in the space of two free parameters. The other model assumes a slotted-Aloha medium access scheme and, due to the varying interference from one time slot to the next, requires defining an additional parameter, the link confidence. With both models, we showed how to determine the connectivity boundary for a given network. The constraints imposed by the requirement of connectivity on the network parameters imply tradeoffs between different performance quantities.

Our results can be used to study by simulation the connectivity of wireless multihop networks under more realistic modelling assumptions than those leading to the Boolean model. The concept of the connectivity boundary also allows analyzing the interdependence of different networks parameters under the requirement of network connectivity.

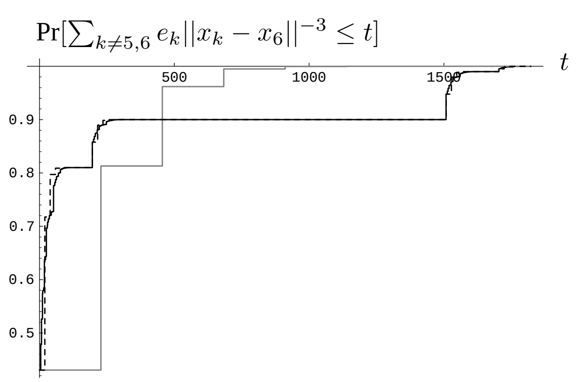


Figure 6. The cumulative distribution of the scaled interference at node 6, excluding node 5 in the network of Figure 2. Solid line: exact distribution, evaluated at $2^{10-2} = 256$ points; gray line: conditional expectation, evaluated at 9 points; dashed line: the hybrid approximation with $f = 20\%$, evaluated at 28 points. The medium access probability has again been taken to be $p = 0.1$.

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