

Multi-period steam turbine network optimisation. Part II: Development of a multi-period MINLP model of a utility system

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Abstract

Efficient utility systems are important both due to the impact utility systems have on the economic efficiency of the process industry, as well as the negative environmental effects of energy conversion. In this paper a methodology is used to build a multi-period MINLP model for steam turbine network of a utility system. The methodology consists of several steps, including the building of a simulation model of the system, the development of regression model(s) of the system based on simulation and D-optimal design, and finally the development of a MINLP model.

The results show that the methodology is capable of analysing a relatively complex steam turbine network in a utility system. The systematic methodology can help to make the MINLP model compact and transparent, and thus avoid the major challenges that are connected with solving large (nonconvex) MINLP models. However, the MINLP models developed using the methodology should be regarded as ‘ad hoc’ models, and have certain limitations as the models are not so flexible compared to models where all the units in the utility system are modelled in detail.

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1. Introduction

The development of a simulation based regression model using an evolutionary algorithm for finding D-optimal designs has been presented in Part I. Based on the design generated by the evolutionary algorithm, the simulation is used to generate data for the regression model. The use of regression models makes it possible to build optimisation models of utility systems that are compact and transparent. In this paper the methodology for the regression model is used to build a multi-period MINLP model for the steam turbine network of a utility system. At the same time the complete methodology is demonstrated by applying it to a hypothetical case. The case is an industrial site with four different processes that require steam at different pressure levels.

The analysis process can roughly be divided into the following steps:

- (1) Building a simulation model of the system.
- (2) Deciding the form of the regression model(s).
 - Choose the dependent and independent variables.
 - Choose the mathematical form of the regression model(s) (e.g. linear, polynomial or logarithmic). The form of the regression model is needed to select the optimal experimental design.
 - Choose the number of observations (i.e. simulation runs).
- (3) Building and running the evolutionary algorithm.
- (4) Running the simulation model after the D-optimal design found by the evolutionary algorithm.
- (5) Analyse the regression model.
- (6) Build and solve the MINLP model together with integer cuts to find the optimal solutions.

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Nomenclature

a	first-order coefficients	p	pressure (bar), number of variables in a model
b	second-order coefficients	R^2	the square of the Pearson correlation
BFW	boiler feedwater	W	power (MW)
HP	high pressure	y	binary variable
HRSG	heat recovery steam generator		
LP	low pressure		
m	mass flow (kg/s)	<i>Super- and subscripts</i>	
MINLP	mixed integer nonlinear programming	0	design state
MP	medium pressure	α	state before turbine stage
		ω	state after turbine stage

2. Development of the MINLP model

The purpose of the multi-period MINLP model is to analyse how changes to processes will affect the utility system. The importance of an efficient utility system is increasing, both because of the rising energy prices and due to the desire to reduce the emissions of carbon dioxide and other greenhouse gases. An efficient utility system has both the potential to improve the economic efficiency of the plant, as well as to reduce the negative environmental impact of the plant. This means that when changes to a system are considered, the effects these changes will have on the utility system are important to the decision makers.

The plant being analysed in this work is hypothetical, but it is based on existing industrial processes. The plant with the utility system consists of four chemical processes, labelled A, B, C and D. It has been decided that the processes will be modified to accommodate changes in the raw material flow. The changes will involve modifications to the process equipment and possibly new investments. There are several different options to make these changes. The options chosen will have different effects on the steam demand, and thus the electricity generated in the utility system. The steam demand varies during the year, which complicates the analysis. The different monthly steam demands of the process options are presented in Fig. 1.

The utility system at the plant consists of a gas turbine, a HRSG and two turbines (one turbine with reheat and one without). At the design state the utility system is generating 10.0 kg/s of 15 bar steam, 24.0 kg/s of 6 bar steam, 30.0 kg/s of 2 bar steam. An overview of the utility system model is shown in Fig. 2.

The first turbine is regulated by a regulation stage, while the second turbine is regulated by a regulation valve. Both turbines are back-pressure turbines, while the second turbine have also a condensing stage. Both turbines have extractions of 15, 6 and 2 bar steam.

The objective of the study is to find how to maximise the generation of electricity, while simultaneously choosing the optimal combination(s) of the process options. To solve this problem a MINLP model is developed according to the steps presented in Section 1. The development of

the MINLP model is explained in the following three sections.

2.1. Simulation model

The simulation model of the utility system is built using Prosim from Endat Oy [1]. Prosim is a steady state power plant simulation software package, where a simulation model is comprised by modules that are connected by streams. The full Prosim model of the utility system in this work consists of 72 modules that are connected with 113 streams. The turbines with the extractions are built using series of turbine modules, where each turbine module corresponds to a turbine stage. The first turbine stage of the first turbine is a regulation stage. The number of regulation valves in the regulation stage is 4. The expansion curves for the turbines at the design point are shown in Fig. 3.

The simulation model will be used to generate the data needed for the regression model.

2.2. Regression model and the evolutionary algorithm

The first task when developing the regression model, is to choose the dependent and independent variables. The natural choices of the dependent variable and the independent variables are in this case respectively the generated electricity from the turbines, W , and the mass flows through the turbines, m . The generated electricity will be used in the objective function of the MINLP model, and the mass flows will connect the steam demand to the generation of electricity. Due to the nonlinear behaviour of the turbines, it is not likely that a linear model will be sufficient to model the system (see for instance the paper by Tveit [2]). In general for MINLP models, convex models are preferable to nonconvex [3]. The quadratic form shown in Eq. (1) is chosen in this case:

$$W = a_0 + \sum_{j \in \text{TurbineFlow}} (a_j m_j + b_j m_j^2) \quad (1)$$

where the set TurbineFlow is the set of all the mass flows and extractions, i.e. the independent variables. The quadratic form is chosen since it has the nice property that it

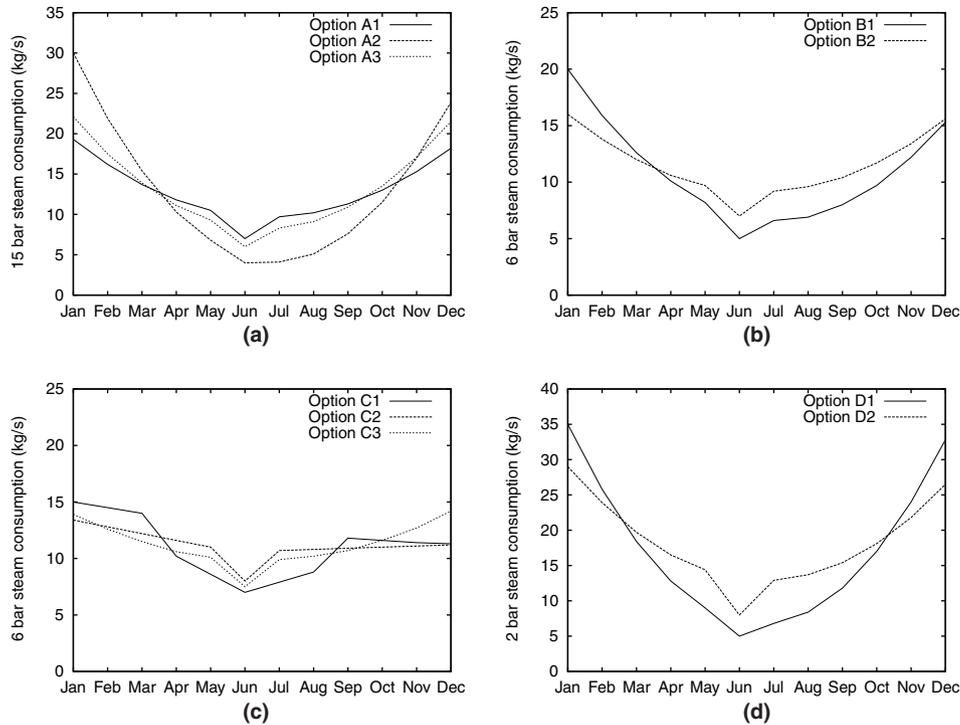


Fig. 1. Steam demand (kg/s) for the different process option for the different processes. (a) Process A: steam demand for options A1, A2 and A3. (b) Process B: steam demand for options B1 and B2. (c) Process C: steam demand for options C1, C2 and C3. (d) Process D: steam demand for options D1 and D2.

is convex if all the second-order coefficients, b_j , are positive. If this is not the case, it is still easy to convexify the function by simple transformations.

Following the steps presented in Section 1, the next step is to choose the number of observations. One way to choose the number of observations, is to find an acceptable upper limit of the time it takes to complete all the simulation runs. In this case an upper limit of 15 h is chosen, resulting in about 75 simulation runs. The reason for the relatively long simulation time is that even if one simulation run per se takes only about 20–30 s, it is very time consuming to set up the simulation model according to the D-optimal design. This is due to the time it takes to get the correct values for the dependent variables. For simulation software with an integrated NLP solver, the time needed to get values of the dependent variables right, could be considerably reduced.

When the form of the regression model and the number of observations are chosen, the next step is to build the evolutionary algorithm in order to find the D-optimal design for the simulation runs. The evolutionary algorithm is the same as the one described in Part I, but it has been expanded to accommodate one more turbine and the additional mass flows and stages. The algorithm is implemented and solved in Matlab [4], with the final simulation run design found after 300 generations with 15 individuals in each generation. After the simulation runs the coefficients of the regression model are fitted to the data using the least absolute deviation criterion. The resulting regression model

is a relatively good fit, with a maximum error of 6.1%, an average error of 0.7%, a standard deviation of 0.01 and the square of the Pearson correlation, R^2 , is 0.99.

2.3. MINLP model

The final step will be to build and solve the multi-period MINLP model. The MINLP model consists of the objective function to be maximised (electricity generation) and the constraints, which are basically mass balances and the constraints for the pressures. The complete 12 period MINLP model consists of 987 single equations and 875 variables, of which 10 are binary. Of the 987 equations, 492 are nonlinear. The objective function is shown in Eq. (2):

$$\text{objective} = \sum_{i \in \text{Periods}} W_i \quad (2)$$

where the set Periods is the set of all the periods and W_i is the regression model shown in Eq. (3):

$$W_i = a_0 + \sum_{j \in \text{TurbineFlow}} (a_j m_{j,i} + b_j m_{j,i}^2) \quad \forall i \in \text{Periods} \quad (3)$$

where $m_{j,i}$ is the massflow of stream j at period i , and the set TurbineFlow is the set of all streams connected to the turbines.

There are different ways of modelling the disjunctions due to the different process options, e.g. the *big-M* or *convex hull* formulations (see for instance the work by Vecchi-

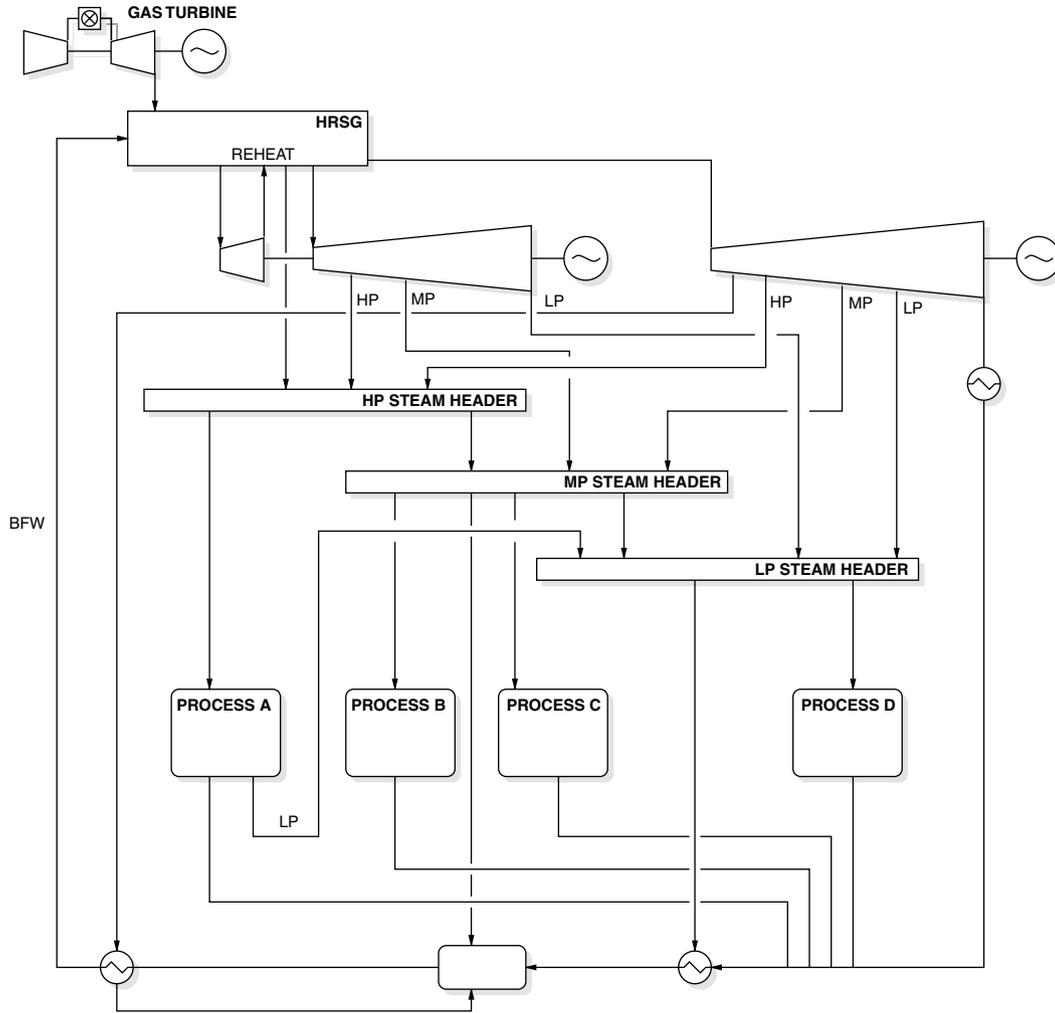


Fig. 2. Overview of the model of the utility system.

etti et al. [5]). The formulation chosen in this work is the big-M formulation shown in the linear equations (4)–(9):

$$m_{(p,i)} - d_{(o,i)} \leq M \cdot (1 - y_o) + s_{(o,i)}^+ \quad \forall (p,o) \in \text{ProOptions and } \forall i \in \text{Periods} \quad (4)$$

$$m_{(p,i)} - d_{(o,i)} \geq M \cdot (y_o - 1) - s_{(o,i)}^- \quad \forall (p,o) \in \text{ProOptions and } \forall i \in \text{Periods} \quad (5)$$

$$s_{(o,i)}^- \leq M \cdot (1 - y_o) \quad \forall (o) \in \text{Options and } \forall i \in \text{Periods} \quad (6)$$

$$s_{(o,i)}^+ \leq M \cdot (1 - y_o) \quad \forall (o) \in \text{Options and } \forall i \in \text{Periods} \quad (7)$$

$$\sum_{(p,o) \in \text{ProOptions}} y_o = 1 \quad \forall p \in \text{Process} \quad (8)$$

$$m_{(p,i)}, s_{(o,i)}^+, s_{(o,i)}^- \geq 0 \text{ and } y_o \in \{0, 1\} \quad (9)$$

where the real variables $s_{(o,i)}^+$ and $s_{(o,i)}^-$ are slack variables, $d_{(o,i)}$ is the steam demand for option o at period i and y_o is a binary variable modelling the choice of the process op-

tions. If $y_o = 1$ the process option is selected. The set Process is the set of the processes A, B, C and D. If the set Options is the set of all the process options, then the set ProOptions is a subset of Process \times Options, and contains only the *feasible* pairs of the processes and the options. The mass flow through the turbines is bounded by the pressure levels. The pressure is calculated using Eqs. (10) and (11):

$$p_{\alpha,s,i} = \sqrt{\left(\frac{m_{i,s}}{m_{0,s}}\right)^2 \cdot (p_{\alpha 0,s}^2 - p_{\omega 0,s}^2) + p_{\omega,s,i}^2} \quad \forall s \in \text{Stages and } \forall i \in \text{Periods} \quad (10)$$

where the set Stages is the set of all the turbine stages, the subscripts α and ω refer to the turbine inlet and outlet stage respectively and the subscript 0 refers to the state at the design point. Eq. (10) is nonlinear and convex with respect to m and p_ω . This is a nice feature, since, as mentioned earlier, nonlinear convex functions are preferable to nonconvex function. The proof of the convexity of Eq. (10) is given in Appendix A.

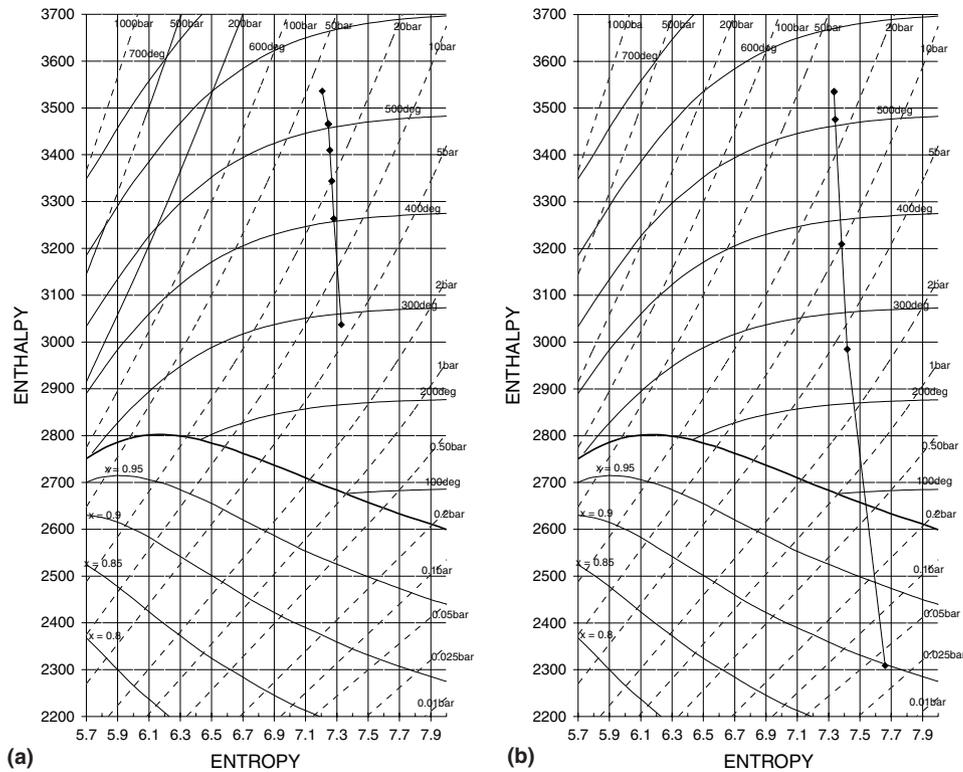


Fig. 3. Expansion curves for the steam turbines at the design point. (a) Expansion curve for Turbine 1 and (b) expansion curve for Turbine 2.

Eq. (11) simply states that the outlet pressure of a stage is equal to the inlet pressure of the next stage:

$$p_{x,(s-1),i} - p_{x,s,i} = 0 \quad \forall i \in \text{Periods and} \\ \forall \{s | s \in \text{Stages and } s \text{ is not the first stage in a turbine}\} \quad (11)$$

In order to find the different optimal solutions, the MINLP model is solved in series with *integer cuts* added to the model for each new solution. The integer cuts exclude the previous solutions from the feasible region. The integer cut for the k th solution is shown in Eq. (12):

$$\sum_{o \in B^k} y_o \leq |B^k| - 1 \quad (12)$$

where the set B^k is the set of all $y_o \in \text{Options}$ that are equal to 1 at the k th solution, and $|B^k|$ is the cardinality of the set B^k . Since all the processes have to have *one and only one* option selected, the cardinality of the set B^k is fixed. Because of this, it is not necessary to include the binary variables that are equal to 0 in the integer cuts.

It should be noted that compared to MINLP models where all units are modelled in detail, the MINLP models developed using the methodology presented in this paper are probably not equally flexible. This means that, if for instance a new objective is found to be important during the optimisation process, most probably a new MINLP model must be constructed to accommodate a new objective. For models where all the units are modelled in detail, this could often be achieved by simply changing the

objective function. Examples of MINLP models where the units are modelled in detail can be found in the work by Bruno et al. [6] and Manninen and Zhu [7]. The trade-off between the complexity and the flexibility of the model is discussed in earlier work by Tveit [8].

The MINLP model is now complete and ready to be solved. The model is implemented in the general algebraic modeling system (GAMS) by GAMS Development Corporation [9]. The MINLP solver used is *SBB* [10]. *SBB* is chosen as the solver, since it is a branch and bound algorithm that works well for MINLP models that are fairly nonlinear and with few integer variables. In this case less than 1.5% of the variables are integer and about 56% of the equations are nonlinear. The NLP solver used together with *SBB* is *CONOPT* [11]. Both *SBB* and *CONOPT* are made by *ARKI Consulting and Development AS*. The solution time for one solution is approximately 50 s on a 700 MHz Pentium-III with 256 MB of RAM.

3. Results and discussion

The optimal solution of the MINLP model found by *SBB* and *CONOPT* is 673 MW and the optimal process options with respect to the electricity generation are A3, B2, C2 and D2. The second best solution is 621 MW, which is achieved with the process options A3, B2, C3 and D2. The average system prices at the Nordic power market exchange Nord Pool in 2004 and 2003 were approximately 28.90 €/MW h and 36.36 €/MW h respectively [12].

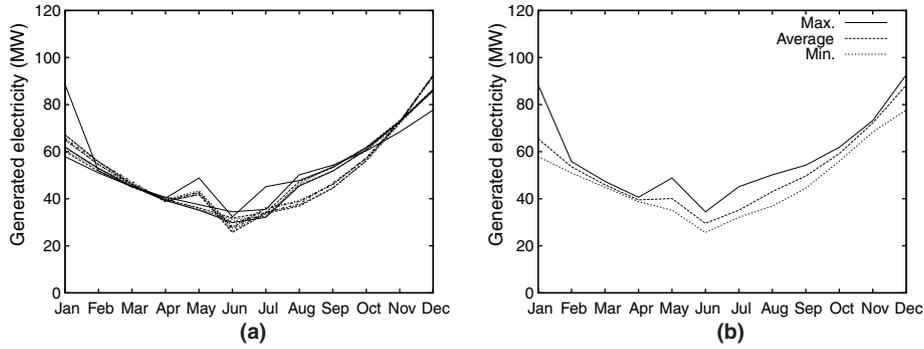


Fig. 4. Optimal solutions for each period. (a) The different optimal solutions (derived using integer cuts) and (b) the maximum, average and minimum value of all the optimal solutions for each period.

Assuming that the plant operates for 8000 h/a, the difference between the best and second best solutions would have accounted to about 1.0 M€ /a in 2004 and 1.3 M€ /a in 2003. Fig. 4 shows the different optimal solutions derived by adding the integer cuts described in Section 2.3 to the MINLP model and solving the model in series.

Fig. 5 shows how the objective function is decreasing with the addition of the integer cuts.

It can be seen that the difference of the electricity generated in the different optimal solutions is significant. It is clear that it will affect the outcome of any economical feasibility analysis of different investment options if these results are not taken into account.

The most time consuming part of the analysis is the development of the simulation model. In many cases a suitable simulation model of the system already exists, which will sufficiently speed up the process. As mentioned in Section 2.2, using simulation software with integrated NLP solver can reduce the time it takes to get values of the dependent variables right. This will either help to speed up the development process, or increase the number of

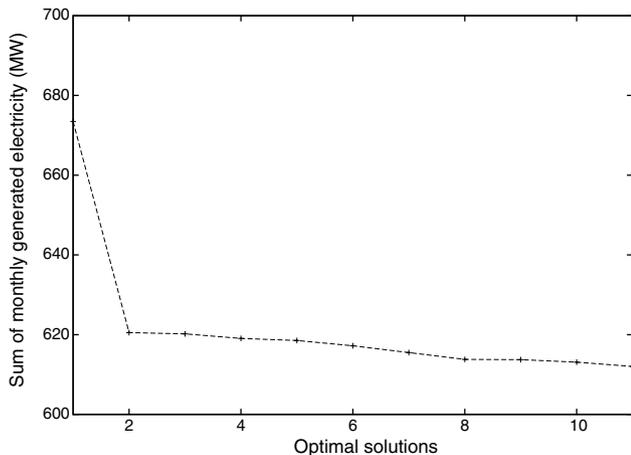


Fig. 5. The decrease in the optimal value of the objective functions as the previous optimal solution is excluded from the feasible region using integer cuts.

observations and potentially improve the quality of the regression model(s).

4. Conclusion

As the importance of an efficient utility system increases, it becomes similarly important to be able to find the optimal operation of the utility system. A systematic methodology to develop MINLP models of utility systems (with focus on steam turbine networks) that are compact and transparent has been proposed. The methodology consists of several steps, including the building of a simulation model of the system, the development of regression model(s) of the system based on simulation and D-optimal design, and finally the development of a MINLP model.

The results reported in this paper have shown that the methodology is capable of analysing a relatively complex steam turbine network of a utility system. The MINLP models developed using the methodology should be regarded as ‘ad hoc’ models, and have certain limitations as the models are not so flexible compared to models where all the units in the utility system is modelled in detail. The systematic methodology can help to make the MINLP model compact and transparent, and thus avoid the major challenges that are connected with solving large (nonconvex) MINLP models.

Appendix A. Proof of convexity for Eq. (10)

With respect to $(m_{(i,s)}, p_{(\omega,s,i)})$, Eq. (10) has the form:

$$f(x, y) = \sqrt{a \cdot x^2 + y^2} \tag{A.1}$$

where x and y corresponds to $m_{(i,s)}$ and $p_{(\omega,s,i)}$ respectively and a is a constant and corresponds to $\frac{1}{m_0^2} \cdot (p_{\omega 0}^2 - p_{\omega 0}^2)$. The eigenvalues of the Hessian matrix of $f(x, y)$ are given in Eq. (A.2):

$$\left[0, \frac{a\sqrt{ax^2 + y^2}(x^2 + y^2)}{a^2x^4 + 2ax^2y^2 + y^4} \right] \tag{A.2}$$

Since both the eigenvalues are positive or equal to zero for all $a \geq 0$, $f(x, y)$ is positive semidefinite and thus convex for all $a \geq 0$. Since $\frac{1}{m_0} \cdot (p_{\omega 0}^2 - p_{\omega 0}^2) \geq 0$, Eq. (10) is convex.

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