

# Conditions for waveguide decoupling in square-lattice photonic crystals

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We study coupling and decoupling of parallel waveguides in two-dimensional square-lattice photonic crystals. We show that the waveguide coupling is prohibited at some wavelengths when there is an odd number of rows between the waveguides. In contrast, decoupling does not take place when there is an even number of rows between the waveguides. Decoupling can be used to avoid cross talk between adjacent waveguides. © 2004 American Institute of Physics. [DOI: 10.1063/1.1794906]

## I. INTRODUCTION

Two-dimensional photonic crystals are promising candidates for implementing integrated optical components.<sup>1</sup> Optical waveguiding in two-dimensional photonic crystals is achieved by introducing line defects in the structure that is otherwise periodic in two dimensions.<sup>2</sup> Two parallel waveguides can be used as a directional waveguide coupler.<sup>3-7</sup> On the other hand, it might be desirable to decouple the two waveguides to minimize cross talk between them, for instance, when envisioning closely packed photonic wires in integrated optical circuits.<sup>8</sup>

We study the coupling between two parallel waveguides in a square-lattice photonic crystal and find that the decoupling of the waveguides depends on the number of rods between the waveguides. If there is an odd number of rods between the waveguides, they are decoupled at a defined wavelength. In case of an even number of rods, the waveguides are coupled at all wavelengths. Previous studies such as Ref. 5 have considered an even number of rods and therefore not demonstrated the decoupling behavior.

## II. GEOMETRY

The studied geometry is a two-dimensional photonic crystal of cylindrical dielectric rods in a square lattice in air. The dielectric constant of the rods is taken to be  $\epsilon=8.9\epsilon_0$ , their radius  $r=0.2a$ , and the lattice constant  $a=512$  nm. The photonic crystal has a large TE (electric field aligned with the cylinders) band gap around  $\omega=0.8\pi c/a$ , where  $a$  is the lattice constant of the crystal. With  $a=512$  nm, this gap is in the wavelength range from 1100 nm to 1600 nm. Two parallel waveguides are formed in the structure by removing two parallel rows of rods. The number of rods between the two waveguides is varied. In Fig. 1(a) and 1(c) we show two examples of the geometries, i.e., one and two rows of rods between the waveguides. We have considered one to seven rows of rods between the waveguides.

The guided eigenmodes supported by this geometry have two possible parities with respect to the symmetry axis between the waveguides. These modes can be classified by the parity of the  $z$  component of the electric field  $E_z$ . Following directly from Maxwell's equations, the parity of  $H_x$  is always opposite to the parity of  $E_z$ , and the parity of  $H_y$  is always the same as that of  $E_z$ . According to the parity of  $E_z$ , the two eigenmodes can be labeled "even" and "odd." Here  $z$  is the

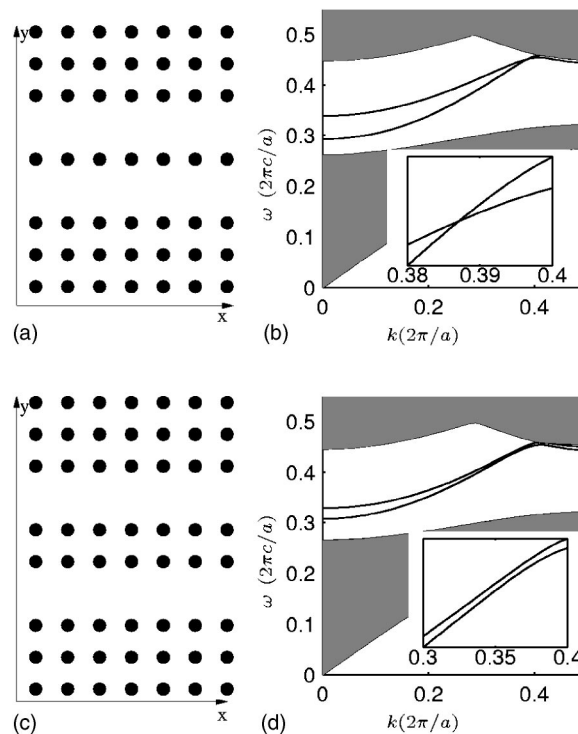


FIG. 1. Geometries and band structures for one [(a) and (b)] and two [(c) and (d)] rows of dielectric rods between two parallel waveguides. The  $z$  direction points out of the plane. Enlarged parts of the band structures are shown in the insets of (b) and (d) to illustrate that the bands cross in (a) but do not cross in (b).

direction of the cylinders (out of plane),  $x$  is along the waveguides, and  $y$  is orthogonal to the cylinders and the waveguides.

### III. COUPLING OF PARALLEL WAVEGUIDES

When the system only supports two guided modes, any signal  $\Psi(x, y, t)$  with a definite frequency  $\omega$  propagating in the system can be written as a superposition of these two eigenmodes

$$\Psi(x, y, t) = \psi_E(x, y, t) \exp(ik_E x) + \psi_O(x, y, t) \exp(ik_O x). \quad (1)$$

Here,  $\psi_E$  and  $\psi_O$  stand for even and odd eigenmodes and  $k_E$  and  $k_O$  for the corresponding values of  $k$ . The spatial dependence of  $\psi_E$  and  $\psi_O$  is lattice periodic. This kind of a superposition gives rise to beating between the eigenmodes. The plane wave terms in Eq. (1) are in the same phase when  $x=0$  and in the opposite phase when  $x=\pi/|k_O - k_E|$ . The beating wavelength is therefore

$$\kappa = \frac{2\pi}{|k_O - k_E|}. \quad (2)$$

When the eigenmodes are in the same phase, their superposition has most of its energy in one of the waveguides and when in opposite phase, in the other. The propagating signal oscillates between the two waveguides with the characteristic wavelength  $\kappa$  given above. This is the mechanism applied, e.g., in waveguide couplers. Note that coupling can be realized also by defects between the waveguides<sup>9</sup> or coupling can be between a waveguide and a defect.<sup>10</sup>

The beating wavelength  $\kappa$  becomes infinite when  $k_E = k_O$ . This means that there is no energy transfer between the waveguides, i.e., the waveguides are decoupled. For the  $k$  values to be identical, the bands of the even and odd eigenmodes have to cross. If they avoid crossing,  $\kappa$  is always finite and the two waveguides cannot be decoupled.

### IV. CONDITIONS FOR DECOUPLING

We have calculated the band structures of two parallel waveguides in a square-lattice photonic crystal with the MIT Photonic Bands<sup>11</sup> program. The band structures for the case of one and two rods between the waveguides are shown in Fig. 1(b) and 1(d), respectively. It can be seen that the bands for the even and odd eigenmodes cross in Fig. 1(b), but do not cross in Fig. 1(d). We calculated the band structures for one to seven rows between the waveguides and found that, for geometries with an odd number rods between the waveguides, the bands for odd and even eigenmodes cross, whereas they never cross when there is an even number of rods between the waveguides.

We calculated the coupling wavelengths  $\kappa$  [Eq. (2)] from the band structures (Fig. 1) and also with the finite-difference time-domain (FDTD) method.<sup>12</sup> Results from both methods are shown in Fig. 2 for the same geometries as considered in Fig. 1. There is a singularity, corresponding to decoupling, in case there is an odd number of rows between the waveguides [see Fig. 2(a)]. In Fig. 2(b) the value of  $\kappa$  is always finite. We found such behavior for one to seven rows of rods between the waveguides. We performed the same calculations with

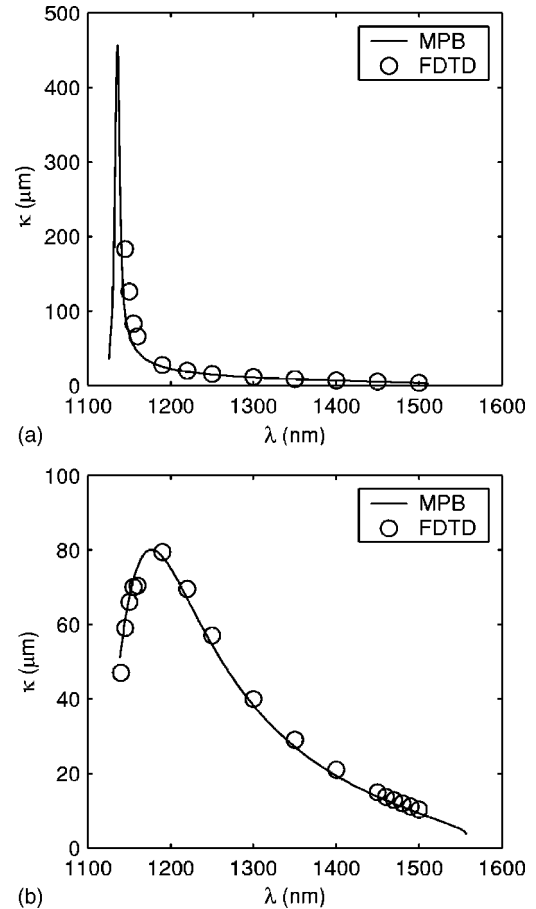


FIG. 2. The coupling distance  $\kappa$  as a function of the wavelength of the light propagating in the waveguides, for the geometries with one (a) and two (b) rows of rods between the waveguides. The solid curve is calculated using the MIT Photonic Bands program and the circles are calculated by FDTD.

FDTD for one to four rows between the waveguides. The calculations using the band structures and FDTD simulations are in excellent agreement.<sup>13</sup>

In order to explain the strong effect of the geometry one has to consider the field distributions of the eigenmodes. The  $E_z$  and  $H_y$  components of the odd eigenmode have a node in the symmetry plane of the structure. The parity of the  $H_x$  component is the opposite of the parities of the  $E_z$  and  $H_y$  components. When there is an odd number of rods between the waveguides, the nodes of the  $E_z$  and  $H_y$  components of the odd eigenmode are in the center of a dielectric rod. For the even eigenmode, the  $E_z$  and  $H_y$  components are nonzero at the symmetry plane. It is known that the more the fields are inside the material of high dielectric constant, the smaller the energy. Thus at small values of the wave vector, the energy of the even eigenmode is smaller than that of the odd eigenmode. The bands cross at some value of the wave vector. This is because the relative power of the  $H_y$  component compared to the power of the  $H_x$  component increases with increasing values of the wave vector. Then  $H_y$  starts to determine the effective parity of the mode. Thus at large values of the wave vector the effective parities of the eigenmodes change and thus the bands cross. When there is an even number of rods between the waveguides, the node of the odd eigenmode is in air and the effective parity does not have

such an effect on the energies of the eigenmodes. In this case the odd eigenmode has a lower energy at all values of the wave vector. This explanation corresponds to the behavior of the eigenmodes in the particular geometry considered in this paper.

## V. CONCLUSIONS

In general, our findings demonstrate that symmetry properties of a photonic crystal waveguide pair, especially parity effects, can be used to design the waveguide properties, for instance, to produce complete decoupling. In this sense photonic crystal waveguides possess an additional degree of freedom compared to traditional dielectric waveguides.

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<sup>13</sup>Our calculations also confirm that an approximative formula for the coupling wavelength  $\kappa$  given by Kuchinsky *et al.* (Ref. 8) works extremely well for our geometries.