Auto-controlled algorithm for slotted ALOHA

J.H. Sarker and S.J. Halme

Abstract: A multiple power level transmission system increases the maximum throughput by selecting the power levels randomly, showing the same behaviour as the standard slotted ALOHA, where the throughput decreases abruptly at higher traffic load conditions. An auto-controlled slowly decreasing throughput algorithm for slotted ALOHA is developed. Herein, a mobile terminal can transmit any packet at a higher probability at the lower power levels that increases the probability of only one packet at the highest power levels and thereby augmenting the capture probability. If the traffic load is high, the probability of interfering packets at the same slot increases. Consequently, the transmission probability at the lower power levels increases and the probability of exactly one packet at the highest power levels remains unaltered. As a result, the system is automatically controlled, since it does not need any extra information about the network. Therefore, it is very attractive from an implementation point of view for the emerging mobile networking environment. Extensive comparisons show that the proposed auto-controlled algorithm demands remarkable throughput performance compared with the traditional random power level selection algorithm particularly with a very high retransmission probability.

1 Introduction

Slotted ALOHA is a widely used random access protocol, used both independently and as part of different multiple access protocols due to its simplicity and low delay (under light load) for bursty traffic. It is well known that the slotted ALOHA channel throughput increases initially with increasing aggregate traffic generation rate. The throughput reaches its maximum value at a certain point of the aggregate traffic generation rate and it starts decreasing with a further increase in the aggregate traffic generation rate. Excessive collisions occur if the offered traffic load crosses the capacity or the maximum throughput of the channel. Repeated collisions waste the bandwidth of the channel. Reducing the retransmission probability to randomise the retransmission attempt over a long time interval is the general solution to this problem.

Consider a system having M active terminals. Assume that n out of M terminals have a packet for retransmission. Therefore, $M-n$ stations transmit originating packets. Then aggregate traffic is

$$G = (M - n)p_n + np_r \tag{1}$$

where p_n and p_r represent originating and retransmission packet generation probabilities respectively. In equilibrium, the incoming traffic is equal to the outgoing traffic and hence $S = (M-n)p_n$. Using this value in (1), we obtain

$$G = S + np_r \tag{2}$$

Throughput of slotted ALOHA, by the help of (2) provides

$$S = (S + np_r)\exp(S + np_r) \tag{3}$$

IEE Proceedings online no. 20030237

doi:10.1049/ip-com:20030237

Paper first received 8th May and in revised form 29th August 2002

J. H. Sarker was with Helsinki University of Technology and is now with the Information Technology Department, Espoo-Vantaa Institute of Technology, Vanha maantie 6, FIN-02600 Espoo, Finland

S. J. Halme is with the Communications Laboratory, Helsinki University of Technology, P.O. Box 3000, FIN-02015 HUT, Finland

Fig. 1 depicts the numerical results of throughput–stability characteristics for different values of retransmission probabilities. A slotted ALOHA channel is stable if the load line intersects (nontangentially) the throughput curve exclusively at one point [1]. Clearly, a stable operation can be obtained by decreasing the retransmission probability.

Different kinds of retransmission probabilities, especially the familiar and widely accepted exponential back-off retransmission probability, for different values of p_r are analysed in [2]. The selection of a constant value for this exponential retransmission probability p_r is difficult, particularly at a dynamic load condition [3]. Besides, the original version of slotted ALOHA with an infinite number of users having a constant value of p_r is inherently unstable [4]. Therefore, a dynamic selection of the exponential retransmission probability p_r is the solution to this problem for pure slotted ALOHA with an infinite number of users [5]. Several algorithms are proposed for the stabilised slotted ALOHA system with a dynamic selection of (exponential retransmission probability) p_r [6–7]. The selection of the dynamic retransmission probability always deserves some feedback information that makes the implementation difficult. To solve this problem, randomised slotted ALOHA (RSA) is introduced [8], where the system is always stable even with the long-range inaccurate estimated number of ready terminals (number of originating mode plus number of retransmission mode terminals). A modern radio receiver can capture a packet even when more than one packet is transmitted in the same time slot and thus the channel throughput is increased, which is not considered in RSA. The multiple power level transmission reception system is one type of capture system.

In multiple power level transmission systems, a mobile terminal can transmit packet at any of the power levels from 1 to N as depicted in Fig. 2. The possibility of transmitting packets at multiple power levels and receiving the packet with the highest power level was first introduced and analysed by Metzner [9]. Shacham [10] devised the details of throughput and delay performance analysis. In this approach, the higher classes have the advantage over

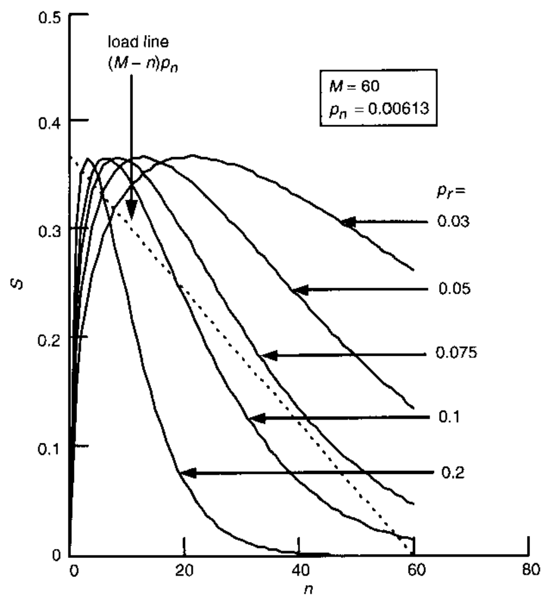

Fig. 1 *Stable and unstable operations*

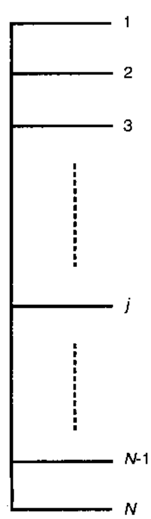

Fig. 2 *Multiple power levels*

lower classes, which is unfair. To make the system fair and efficient, random power levels selection with capture was developed [11].

If users transmit packets randomly without the knowledge of other users, the probability of transmitting more than one packet at the highest power level increases and thus the throughput of the system decreases, rendering the system unstable [12]. The main concern of ALOHA based networks is stability, especially at higher traffic load conditions. One solution to this problem is the introduction of power-level division multiple access (PDMA) [12], where a sequentially assigned power level-based multiple access scheme is used. In this scheme, every terminal is attributed a slot and a power level in every frame. The allocated power level should be one level higher than the previous power level assigned in the previous frame.

In the original version of PDMA, one terminal can capture another terminal to a greater extent in a given cycle (block), which is not fair. A fair PDMA scheme is introduced in [13] using the permutation method to allocate the power levels and time slots to every mobile terminal so that each terminal can capture another terminal equally. Note that one cycle (block) consists of a number of frames and power levels. From the stability point of view this scheme is attractive, but each mobile has to know which power level is assigned to it and in which slot. Therefore, from the implementation point of view, PDMA is not so attractive.

A generalised multicopy ALOHA scheme is also introduced to expedite the throughput at higher traffic load conditions [14]. Excellent work regarding the power level selection probabilities that result in the maximum throughput is analysed in [15]. Unfortunately, this scheme needs to know the number of mobile terminals working in the system. Besides that, the system is unstable at very high traffic load if the number of power levels is small.

The main intention of the aforementioned stability improvement schemes [12–15] is to transmit only one packet to the highest power level especially when the traffic load is high. We contrive a very simple solution to this problem involving packet transmission to pre-selected power levels with a higher probability at lower power levels. The proposed scheme does not reach the maximum throughput, but rather maintains a throughput close to the maximum and can increase that level up to a possible higher level, without any extra feedback information, making the system automatically controlled, and thus easy to implement.

2 Auto-controlled algorithm

Let us concentrate on the general jth power level (Fig. 2), $j=1, 2, .., N$, where N is the lowest power level and 1 is the highest power level. In multiple power level transmission systems, a user can transmit the packet at any power level with a certain probability. Regarding the general jth power level, a packet can be captured by the radio receiver successfully if and only if all interfering packets in the same slot are at lower power levels (i.e. from $(j+1)$ to N power levels), and exactly one packet is transmitted at the jth power level. Assuming that the jth power level is sufficiently high so that if there is a packet at the $(j+1)$th power level, a receiver can decode the packet at the jth power level successfully.

Let P_x [Success$|j$] be the probability of transmitting a packet to the jth power level received successfully, where X ($X=$ RS, ACLEq1 or ACLNoEq1) defines two kinds of transmission probabilities. RS stands for random selection, ACLEq1 means, auto-controlled (AC) with $L=1$ and ACLNoEq1 means auto-controlled with $L\neq 1$. The description of RS and AC are provided later. The average number of packets transmitted at all N power levels is G packets per time slot. Consequently, P_x[Success$|j$] can be defined as

$$P_x[\text{Success}|j] = \sum_{m=0}^{\infty} P[Overlap/m] \times P_x[\text{Success/all } m \text{ packets have lower power than } j] \quad (4)$$

For any kind of transmission

$$P[Overlap/m] = \exp(-G)\frac{G^m}{m!} \quad (5)$$

and

$$P_x[\text{Success/all } m \text{ packets have lower power than } j] = \left(\sum_{k=j+1}^{N} p_{X,k}\right)^m \quad (6)$$

Therefore, we can write

$$P_x[\text{Success}|j] = \exp(-G)\sum_{m=0}^{\infty}\frac{G^m}{m!}\left(\sum_{k=j+1}^{N} P_{X,k}\right)^m$$
$$= \exp(-G)\exp\left\{G\sum_{k=j+1}^{N} P_{X,k}\right\}$$
$$= \exp(-G)\exp\left\{G\left(1-\sum_{k=1}^{j} P_{X,k}\right)\right\}$$
$$= \exp\left(-G\sum_{k=1}^{j} P_{X,k}\right) \quad (7)$$

The probability of success of a packet transmitted at all N power levels can be defined from total probability theory as

$$P_X[\text{Success}] = \sum_{i=1}^{N} P_X[\text{Success}|j] * P[\text{Probability of transmitting a packet to the } j\text{th power level}]$$
$$= \sum_{j=1}^{N} P_X[\text{Success}|j] * P_{X,j}$$
$$= \sum_{j=1}^{N} P_{X,j}\exp\left\{-G\sum_{k=1}^{j} P_{X,k}\right\} \quad (8)$$

Finally, the general throughput with the multiple power level approach is

$$Sx = GP_X[\text{Success}]$$
$$= G\sum_{j=1}^{N} P_{X,j}\exp\left\{-G\sum_{k=1}^{j} P_{X,k}\right\} \quad (9)$$

Two different kinds of transmission possibilities are considered next: traditional random selection (RS); and auto-controlled (AC).

2.1 Random selection (RS)

In the case of random power level selection, each power level is selected by a random choice. Thus, the probabilities for transmitting at different power levels are equal and given by

$$P_{RS,i} = \frac{1}{N} \quad (10)$$

This type of transmission probability is well known and defined in [11, 12].

According to the AC algorithm (will be defined next), the packet is transmitted at lower power levels with a higher probability and at higher power levels with a lower probability. The aforementioned random selection (RS) does not fall in that category. Since, the RS power level does not belong to the AC algorithm then we compare the proposed AC algorithm with this traditional RS.

2.2 Why exponential?

It is well known that the probability of success decreases exponentially with increasing traffic load especially at higher traffic load conditions. As the number of terminals in retransmission mode increases exponentially, the probability of success reaches the minimum value zero and the system falls into deadlock. A stable system can be achieved by decreasing the retransmission probability, but the corresponding packet delay may be too large to be acceptable. The main purpose of the proposed AC algorithm is to keep the packet success probability high at high traffic load conditions without decreasing a high retransmission probability (or more precisely with a very high retransmission probability). The total number of active terminals is equal to the number of originating mode terminals plus retransmission mode terminals. For a given number of active terminals, if the number of retransmission mode terminals increases, the number of terminals in the originating mode decreases. If packet transmission at lower power levels increases exponentially with increasing traffic load, the probability of transmitting exactly one packet at the highest power levels remains the same and the channel is able to keep the success probability at an acceptable level. Therefore, we need the packet transmission probability at different power levels that should also be exponentially distributed. Results presented in [16] suggest that an exponential type of transmission policy is able to control the channel automatically. An exponential type probability distribution is considered here by considering the area of an annular region. Note that the proposed probability is not optimised, only a proposal for the AC algorithm and other similar probability distributions can solve the stability problem.

2.3 Auto-controlled (AC) approach

Consider the N annular system as shown in Fig. 3. Any mobile terminal selects the first power level with a probability proportional to the fractional area of the innermost annular region. The radius of the innermost circle is rL. In the same way, any mobile terminal transmits a packet at the second power level with the fractional probability of the second annular region. The width of the second annular is rL^2. In general, the packet transmission at the jth power level is the fractional probability of the jth inner annular region. Referring to Fig. 3, the fractional area of the jth annular region is

$$P_{AC,j} = \frac{\int_{\sum_{k=1}^{j-1} L^k}^{r\sum_{k=1}^{j} L^k} 2\pi x dx}{\pi\left(r\sum_{k=1}^{N} L^k\right)^2} = \frac{\left(\sum_{k=1}^{j} L^k\right)^2 - \left(\sum_{k=1}^{j-1} L^k\right)^2}{\left(\sum_{k=1}^{N} L^k\right)^2}$$
$$= \frac{(L^2-1)L^{2(j-1)} - 2(L-1)L^{j-1}}{(L^N-1)^2} \quad L \neq 1 \quad (11)$$

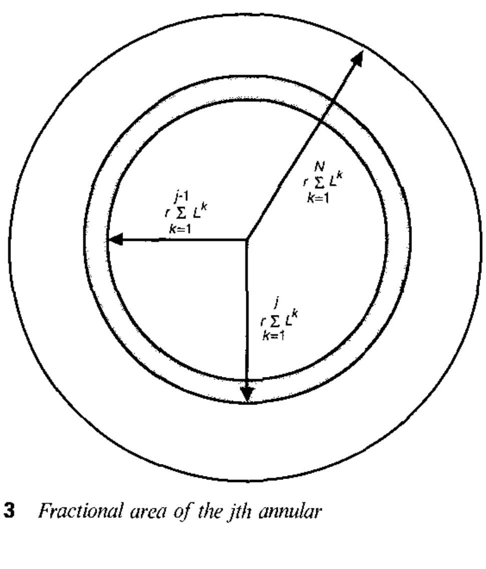

Fig. 3 *Fractional area of the jth annular*

If the widths of all the annular regions are equal, i.e. $L = 1$, (11) can be solved, either by setting $L = 1-\varepsilon$ and letting $\varepsilon \to 0$ or by using L'Hôpital's rule. Combining (10) and (11) along with the prescribed value of L, the packet distribution probability at the jth power level reads

$$P_{X,j} = \begin{cases} \frac{1}{N} & \text{for } X = \text{RS} \\ \frac{2j-1}{N^2} & \text{for } X = \text{ACLEq1} \\ \frac{(L^2-1)L^{2(j-1)} - 2(L-1)L^{j-1}}{(L^N-1)^2} & \text{for } X = \text{ACLNoEq1} \end{cases} \quad (12)$$

Therefore, for each scheme we have

$$\sum_{k=1}^{j} P_{X,k} = \begin{cases} \frac{j}{N} & \text{for } X = \text{RS} \\ \frac{j^2}{N^2} & \text{for } X = \text{ACLEq1} \\ \frac{(L^j-1)^2}{(L^N-1)^2} & \text{for } X = \text{ACLNoEq1} \end{cases} \quad (13)$$

and finally, the throughput for each scheme can be obtained combining (7)–(13)

$$S_x = \begin{cases} \frac{G}{N}\exp\left(-G\frac{1}{N}\right) + \ldots + \frac{G}{N}\exp\left(-G\frac{j}{N}\right) + \ldots + \frac{G}{N}\exp(-G) \\ \text{for } X = \text{RS} \\ G\frac{1}{N^2}\exp\left(-G\frac{1}{N^2}\right) + \ldots + G\frac{2j-1}{N^2}\exp\left(-G\frac{j^2}{N^2}\right) + \ldots \\ +G\frac{2N-1}{N^2}\exp(-G) \\ \text{for } X = \text{ACLEq1} \\ G\frac{(L-1)^2}{(L^N-1)^2}\exp\left\{-G\frac{(L-1)^2}{(L^N-1)^2}\right\} + \ldots \\ +G\frac{(L^2-1)L^{2(j-1)} - 2(L-1)L^{j-1}}{(L^N-1)^2}\exp\left\{-G\frac{(L^j-1)^2}{(L^N-1)^2}\right\} + \ldots \\ +G\frac{(L^2-1)L^{2(N-1)} - 2(L-1)L^{N-1}}{(L^N-1)^2}\exp(-G) \\ \text{for } X = \text{ACLNoEq1} \end{cases} \quad (14)$$

Each approach contains N terms and shows standard slotted ALOHA throughput if $N = 1$. The first part of (14) shows the traditional random power level selection and traffic arrival at any of N power levels and is equal to G/N. Consider the last term of all types of algorithms: the success probability is equal to $\exp(-G)$. At higher traffic load conditions, the probability of success is almost equal to zero and the contribution to the overall throughput is negligible (Fig. 4). Regarding the initial terms of the RS and AC algorithms: the probability of success with the AC algorithm is high especially at high traffic load conditions in comparison with the traditional RS power level. Fig. 4 shows the results for probability of success as well as the throughput contribution of each component with three multiple power levels at a very high traffic load $G = 15$. This is evidence that the AC algorithm achieves a higher throughput by using the multiple power levels more technically than the traditional RS algorithm.

The throughput comparison of the traditional RS and proposed AC algorithms is shown in Fig. 5 which clarifies that the AC algorithm can provide a higher throughput especially at higher traffic conditions, thus increasing the stability.

3 Stability with a very high retransmission probability p_r

Stability is the main concern in this paper, and a slotted ALOHA channel is stable if its load line intersects (nontangentially) the equilibrium contour in exactly one place [1]. The maximum new packet transmission probability can be obtained from the load line and is $p_{n,max}$ ($= S_{max}/M$). The traditional RS multiple power level transmission system shows the same behaviour as standard slotted ALOHA, where the throughput decreases abruptly. Stable operation can be obtained by decreasing retransmission probability as discussed in Section 1. This avoids the instability of the channel, ensuring satisfactory throughput at the expense of increased delay. Another problem is that the new packets experience less delay than the packets that have already suffered collisions. The proposed AC algorithm shows stable operation with a very high retransmission probability (Fig. 6). The value of L has to be increased with increasing retransmission probability to keep the channel stable (Fig. 6*b*). In this case, the throughput curve intersects the load line at a point where the number of retransmission mode terminals is higher. Thus the system is stable with a high retransmission probability, but at the expense of a higher average number of retransmission mode terminals.

4 Conclusions

An auto-controlled slotted ALOHA algorithm in which the throughput decreases slowly is proposed. Since the mobile terminals transmit packets without the knowledge of the current loading condition of the network, the

N=3, G=15								
	RS		AC					
			L=1		$L \neq 1$			
i	$\exp\left(-G\frac{j}{N}\right)$	$\frac{G}{N}\exp\left(-G\frac{j}{N}\right)$	$\exp\left(-G\frac{j^2}{N^2}\right)$	$G\frac{2j-1}{N^2}\exp\left(-G\frac{j^2}{N^2}\right)$	$\exp\left\{-G\frac{(L^j-1)^2}{(L^N-1)^2}\right\}$		$G\frac{(L^2-1)L^{2(j-1)}-2(L-1)L^{j-1}}{(L^N-1)^2}\exp\left\{-G\frac{(L^j-1)^2}{(L^N-1)^2}\right\}$	
					L=1.1	L=1.3	L=1.1	L=1.3
1	6.74×10^{-3}	0.034	0.189	0.315	0.254	0.39	0.348	0.367
2	4.54×10^{-5}	2.27×10^{-4}	1.27×10^{-3}	6.36×10^{-3}	2.39×10^{-3}	6.85×10^{-3}	0.011	0.028
3	3.06×10^{-7}	1.53×10^{-6}	3.06×10^{-7}	2.55×10^{-6}	3.06×10^{-7}	3.06×10^{-7}	2.74×10^{-6}	3.06×10^{-6}
S_X		0.034		0.321			0.359	0.395

Fig. 4 *Probability of success and throughput comparisons of traditional RS and proposed AC algorithm*

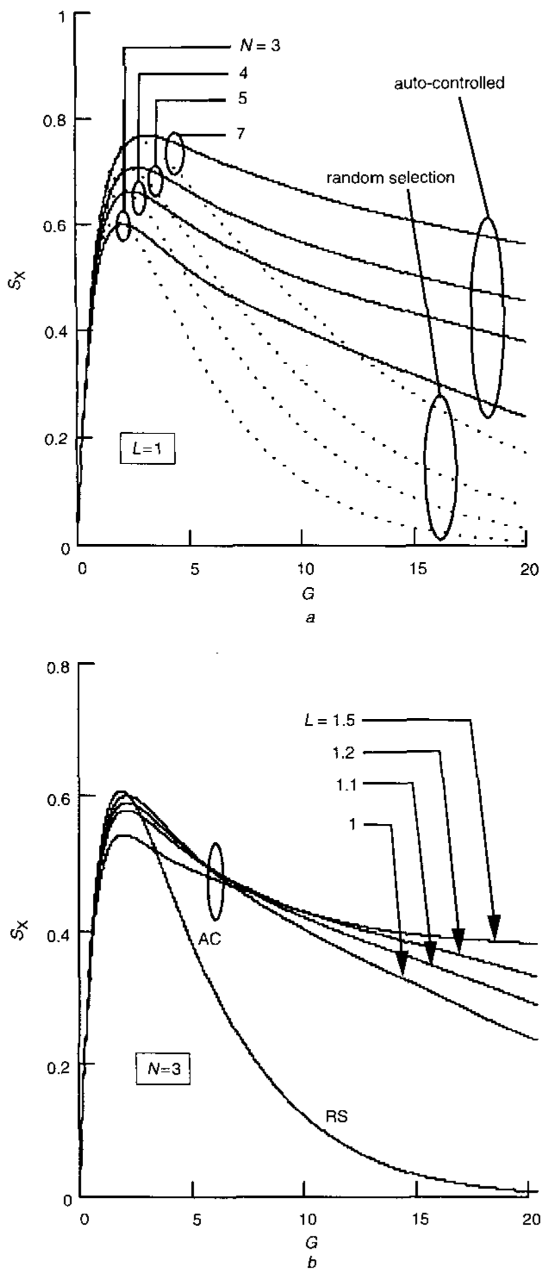

Fig. 5 *Throughput comparison of RS and AC*
a $L=1$; $N=3, 4, 5, 7$
b $N=3$; $L=1, 1.1, 1.2, 1.5$

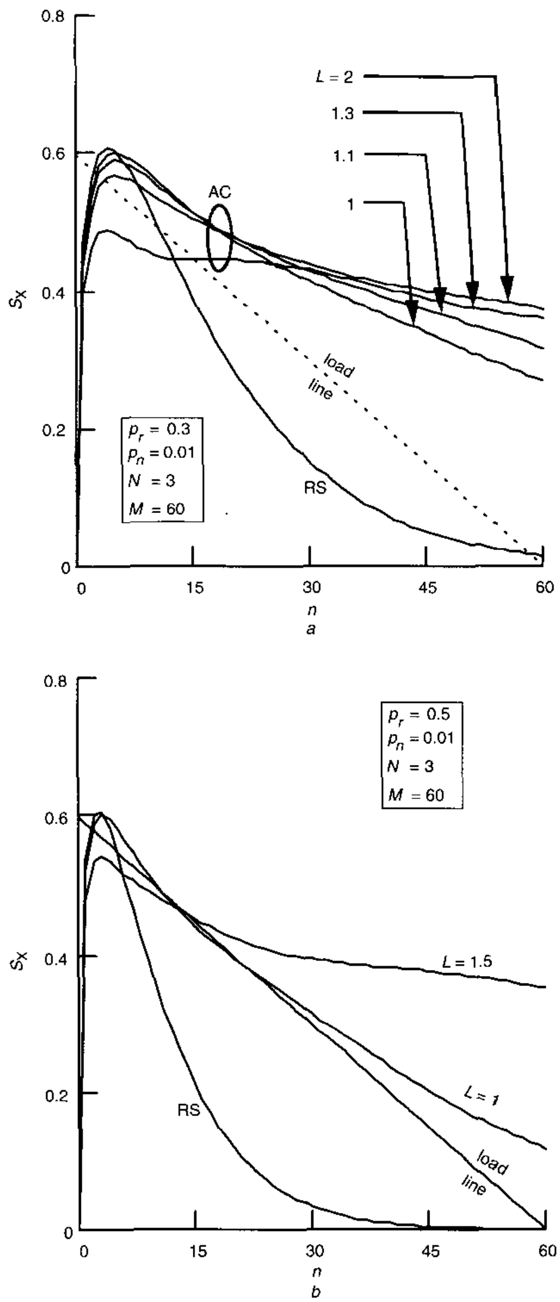

Fig. 6 *Stability*
a Consideration with RS and AC
b Obtained with higher value of *L*

implementation is definitely simpler. For a given number of power levels, the probability of more than one packet at the highest power levels increases if the traffic load is very high. If the mobile terminals transmit packets at higher power levels with lower probabilities, the probability of only one packet at the highest power levels increases. Consequently, enhancement of the packet success probability in each slot is achieved, keeping the system stable at a higher traffic loading condition. The rest of the packets fall automatically to lower power levels due to the higher selection probabilities of those power levels.

An auto-controlled packet transmission algorithm can keep the channel stable with a very high retransmission probability, thus decreasing the average packet delay.

5 References

1 KLEINROCK, L., and LAM, S.S.: 'Packet switching in a multiaccess broadcast channel: Performance evolution', *IEEE Trans. Commun.*, 1975, **23**, (4), pp. 410–423
2 RAYCHAUDHURI, D., and JOSEPH, K.: 'Performance evaluation of slotted ALOHA with generalized retransmission backoff', *IEEE Trans. Commun.*, 1990, **38**, (1), pp. 117–122
3 TASAKA, S.: 'Performance analysis of multiple access protocols' (MIT Press, Cambridge, 1986)
4 KLEINROCK, L.: 'Queueing Systems Vol. 2: Computer Applications' (Wiley, New York, 1976)
5 LAM, S.S., and KLEINROCK, L.: 'Packet switching in multiaccess broadcast channel: Dynamic control procedure', *IEEE Trans. Commun.*, 1975, pp. 891–904
6 JEONG, D.G., and JEON, W.S.: 'Performance of an exponential backoff scheme for slotted-ALOHA protocol in local wireless environment', *IEEE Trans. Veh. Technol.*, 1995, pp. 470–479

7 ALCOBER, J., and CASARES, V.: 'Throughput and delay analysis of the ideal window stabilized ALOHA', *Eur. Trans. Telecommun.*, 1999, **10**, (5), pp. 505–511
8 BING, B.: 'Stabilization of the randomized slotted ALOHA protocol without the use of channel feedback information', *IEEE Commun. Lett.*, 2000, **4**, (8), pp. 249–251
9 METZNER, J.J.: 'On improving utilization in ALOHA networks', *IEEE Trans. Commun.*, 1976, **24**, (4), pp. 447–448
10 SHACHAM, N.: 'Throughput-delay performance of packet switching multiple-access channel with power capture', *Perform. Eval.*, 1984, **4**, pp. 153–170
11 LEE, C.C.: 'Random signal levels for channel access in packet broadcast networks', *IEEE J. Sel. Areas Commun.*, July 1987, pp. 1026–1034
12 SHIMAMOTO, S., ONOZATO, Y., and TESHIGAWARA, S.: 'Performance evaluation of power level division multiple access (PDMA) scheme'. Proceedings of IEEE ICC'92, pp. 1333–1337
13 LEUNG, Y.-W.: 'Fair power level division multiple access', *IEEE Commun. Lett.*, 1997, **1**, (3), pp. 61–63
14 LEUNG, Y.-W.: 'Generalised multicopy ALOHA', *Electron. Lett.*, 1995, **31**, (2), pp. 82–83
15 LAMAIRE, R.O., KRISHNA, A., and ZORZI, M.: 'On the randomization of transmitter power levels to increase throughput in multiple access radio systems', *Wirel. Netw.*, 1998, **4**, pp. 263–277
16 SARKER, J.H., and HALME, S.J.: 'A stability improved algorithm for multiple power levels packet transmission system in cellular environment'. Proceedings of Int. Conf. Telecomm. ICT'98, Greece, 1998, pp. 198–202