Simultaneous estimation of optical anisotropy and absorption in medical optical tomography

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Abstract— **In this paper we propose a possible model for anisotropic light propagation, and present simultaneous reconstruction of anisotropy parameters and optical absorption for optical tomography. The anisotropic model is introduced into the Radiative Transfer Equation (RTE). The most commonly used approximation to the RTE in optical tomography is the Diffusion Equation (DE). In anisotropic case, the diffusion coefficient in the DE assumes a tensor form. We present the diffusion tensor as an eigenvalue decomposition corresponding to the directions and the strength parameters of anisotropy. The numerical approximation is done in two dimensions using the finite element (FE) method. We then consider the inverse problem of reconstructing the optical absorption when the directions of anisotropy are assumed to be known, but the strength may vary. For this estimation to be successful, the strength parameters are reconstructed simultaneously with the absorption. We present numerical examples of cases in which the location of anisotropy is truly known, and in which there is an error in the assumed location.**

I. Introduction

OPTICAL tomography is a relatively new, non-invasive modality for medical applications such as functional imaging of the brain or breast cancer detection. Techniques using near-infrared light have some important advantages over existing modalities. Near-infrared light is non-ionizing and thus harmless to the patient, enabling long term monitoring. Also, the instrumentation can be made relatively light and inexpensive.

Several human tissues, such as the white matter of the brain, muscles, or skin [1], have optical properties that depend not only on location but also on direction. It is known, e.g., that the fibres in the white matter of the brain have direction dependent properties for the diffusion of water [2]. Since the imaging of the human brain is one of the main application targets of optical tomography, it is important to understand how anisotropies effect the image reconstruction. In this paper, we present one approach to model the anisotropic effects of light propagation and some examples of simultaneous reconstruction of optical absorption and anisotropy model parameters.

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II. Light propagation model

Light propagation in the presence of spatially varying absorbing coefficient μ_a and scattering coefficient μ_s is generally described (ignoring polarization and coherence effects) by the Radiative Transfer Equation (RTE) [3]. For a source modulated with angular frequency ω, this is written

$$(\hat{\mathbf{s}}\cdot\nabla+\mu_a+\mu_s-i\omega/c)\,\phi(\mathbf{r},\hat{\mathbf{s}};\omega) = \mu_s\int_{S^2}\Theta(\hat{\mathbf{s}},\hat{\mathbf{s}}')\phi(\mathbf{r},\hat{\mathbf{s}}';\omega)\mathrm{d}^2\hat{\mathbf{s}}'+q(\mathbf{r},\hat{\mathbf{s}};\omega). \quad (1)$$

Here $\phi(\mathbf{r},\hat{\mathbf{s}};\omega)$ is the radiance at position $\mathbf{r}$ with direction of propagation $\hat{\mathbf{s}}$, c is the speed of light, q denotes the source term and the *phase function* $\Theta(\hat{\mathbf{s}},\hat{\mathbf{s}}')$ represents the probability density function for scattering $\hat{\mathbf{s}}'\to\hat{\mathbf{s}}$.

When scattering dominates absorption a common approximation with high fidelity to the physical situation is the *Diffusion Approximation* (DA), which arises from assuming firstly that the spherical harmonic expansion of ϕ can be reasonably approximated by only first order terms, and secondly that the phase function depends only on $(\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}')$ i.e. the angular separation between incoming and scattered radiation, not the absolute direction. Then the DA is written

$$-\nabla\cdot\kappa\nabla\Phi+(\mu_a-i\omega/c)\,\Phi=Q, \quad (2)$$

where $\kappa=\frac{1}{3}\left(\mu_a+(1-g)\mu_s\right))^{-1}$ is the scalar diffusion coefficient, in which g is the mean of the cosine of the scattering angle, and Q presents an isotropic source term.

As mentioned, in many situations it is likely that the scattering phase function has an absolute dependency on direction. In this paper we assume a simple phase function of the form

$$\Theta(\hat{\mathbf{s}},\hat{\mathbf{s}}')=\frac{1}{4\pi}\left(1+3\hat{\mathbf{s}}^{\mathrm{T}}\mathsf{B}\,\hat{\mathbf{s}}'\right), \quad (3)$$

where B is a symmetric positive definite tensor. Inserting this phase function into the RTE, expanding in spherical harmonics and taking terms only up to first order leads to the tensor diffusion equation

$$-\nabla\cdot\mathsf{K}\nabla\Phi+(\mu_a-i\omega/c)\,\Phi=Q, \quad (4)$$

where

$$\mathsf{K} = \frac{1}{3}\left((\mu_\mathrm{a}+\mu_\mathrm{s})\mathsf{I} - \mu_\mathrm{s}\mathsf{B}\right)^{-1}. \tag{5}$$

Note (4) reduces to (2) if $\mathsf{B} = g\mathsf{I}$ which implies $\Theta(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = \frac{1}{4\pi}(1 + g\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}')$.

In this work, we use the so called collimated source approximation [4], where the light source is modeled with a collimated pencil beam perpendicular to the surface. In this approximation, we write the source term as a point source below the surface: $Q = Q_0\delta(\mathbf{r}_s)$, where $\mathbf{r}_s$ is the source position. For the boundary condition, we assume that the inward directed flux in each point on the boundary is zero. Within the diffusion approximation, this assumption leads to a so called Robin boundary condition

$$\Phi + 2\hat{n}\cdot\mathsf{K}\nabla\Phi = 0, \tag{6}$$

where $\hat{n}$ is the outward unit normal vector to the surface. The boundary data consists of measured outward flux Γ_out at points x_m, the optode locations, on the boundary $\partial\Omega$. Using the boundary condition (6), the outward flux within the diffusion approximation is simply $\Gamma_\mathrm{out}(x_m) = \frac{1}{2}\Phi(x_m)$.

III. Numerical approximation

The numerical approximation in this work is based on the finite element computations conducted in a two-dimensional space. For implementing the finite element method, we write the variational formulation of the diffusion equation (4). By multiplying the equation (4) by a test function ψ and integrating by parts over Ω, we arrive at the variational equation

$$\int_\Omega \nabla\psi\cdot\mathsf{K}\nabla\Phi dx + \int_\Omega(\mu_\mathrm{a} - i\omega/c)\psi\Phi dx + \int_{\partial\Omega}\frac{1}{2}\psi\Phi dS = Q_0\psi(\mathbf{r}_s), \tag{7}$$

where the Robin boundary condition (6) and the collimated source condition have been taken into account.

In the finite element approximation, the domain Ω is divided into finite elements and the solution is approximated by nodal-based basis functions,

$$\Phi(x) \approx \sum_{j=1}^{N_\mathrm{n}} \alpha_j\psi_j(\mathbf{r}). \tag{8}$$

where N_n is the number of the nodes in the finite element mesh. By choosing the test function ψ in (7) to be one of the basis functions, we arrive at the matrix equation $A\alpha = \beta$, where A is the $N_\mathrm{n}\times N_\mathrm{n}$ symmetric matrix with entries

$$A_{j,\ell} = \int_\Omega \nabla\psi_j\cdot\mathsf{K}\nabla\psi_\ell dx + \int_\Omega(\mu_\mathrm{a} - i\omega/c)\psi_j\psi_\ell dx + \int_{\partial\Omega}\frac{1}{2}\psi_j\psi_\ell dS, \tag{9}$$

and β is a N_n–vector $\beta_j = Q_0\psi_j(\mathbf{r}_s)$.

In two dimensions, we write the eigenvalue decomposition for the anisotropy matrix $\mathsf{B} = \mathsf{B}(\mathbf{r}) \in \Re^{2\times 2}$ as

$$\mathsf{B}(\mathbf{r}) = U(\mathbf{r})\mathrm{diag}(b_1(\mathbf{r}), b_2(\mathbf{r}))U(\mathbf{r})^\mathrm{T}, \tag{10}$$

where $U(\mathbf{r}) \in \Re^{2\times 2}$ is an orthogonal matrix and we assume the eigenvalues $b_j(\mathbf{r})$ are positive. This decomposition leads to a corresponding decomposition of the diffusion matrix K,

$$\mathsf{K}(\mathbf{r}) = U(\mathbf{r})\mathrm{diag}(\lambda_1(\mathbf{r}), \lambda_2(\mathbf{r}))U(\mathbf{r})^\mathrm{T}, \tag{11}$$

where

$$\lambda_j(\mathbf{r}) = \frac{1}{2(\mu_\mathrm{a}(\mathbf{r}) + (1-b_j(\mathbf{r}))\mu_\mathrm{s}(\mathbf{r}))}, \quad j = 1,2 \tag{12}$$

(see figure 1).

Consider the system matrix A defined in (9). By writing

$$U(\mathbf{r})^\mathrm{T}\nabla\psi_j(\mathbf{r}) = \xi_j(\mathbf{r})\hat{e}_1 + \eta_j(\mathbf{r})\hat{e}_2, \tag{13}$$

where the vectors $\hat{e}_j$ are the Cartesian basis vectors, and making the assumption that the strength of the anisotropy is constant, i.e., $\lambda_j(\mathbf{r}) = \lambda_j = \text{constant}$, $j = 1,2$, we get

$$\int_\Omega \nabla\psi_j\cdot\mathsf{K}\nabla\psi_k dx = \lambda_1\int_\Omega \xi_j\xi_k dx + \lambda_2\int_\Omega \eta_j\eta_k dx. \tag{14}$$

Note that the assumption of constant strength does not mean that the diffusion coefficient is constant, since the principal directions coded in the matrix $U(\mathbf{r})$ may vary. In this connection, we make one further simplifying approximation: We assume that the anisotropy factors λ_j, $j = 1,2$, are independent of the absorption coefficient. This assumption, although not correct, can be justified when the material is scattering dominated, i.e., $\mu_\mathrm{a} \ll \mu_\mathrm{s}$, so in formula (12) the absorption coefficient can be ignored.

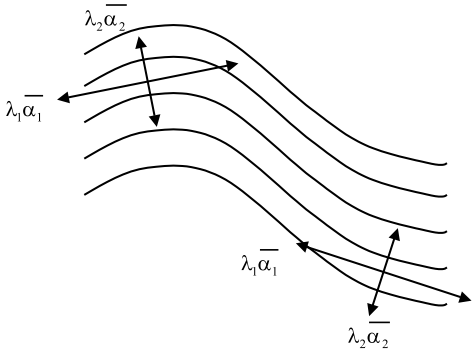

Fig. 1. A schematic illustration of the representation of anisotropy using eigenvalue decomposition. The direction of anisotropic structures is presented by two orthogonal directions confined in the matrix $U(x) = [\vec{\alpha}_1(x), \vec{\alpha}_2(x)]$ and the strength by the corresponding eigenvalues $\lambda_1(x), \lambda_2(x)$.

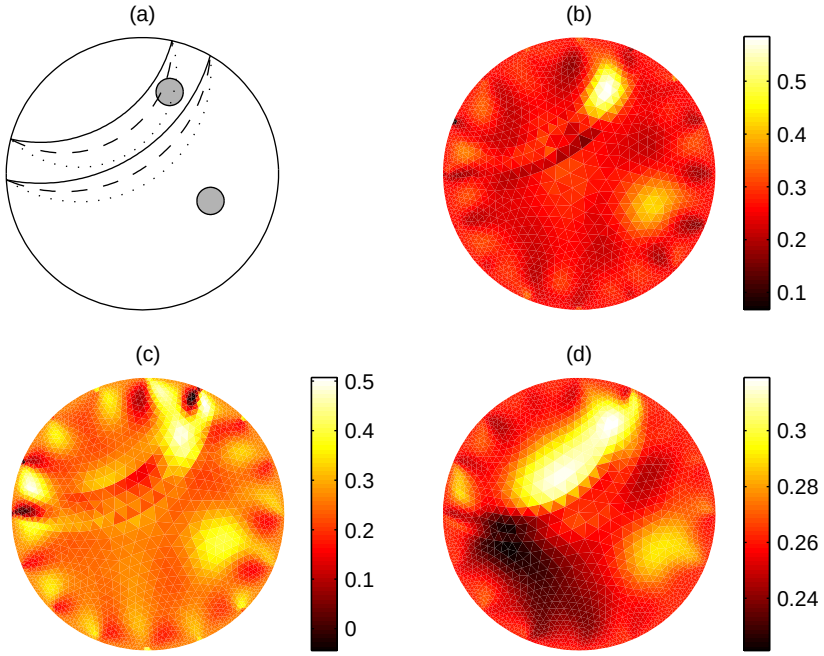

Fig. 2. Reconstruction of the absorption coefficient. (a) Geometry used in the reconstruction is a sphere of a radius of 2 cm with an anisotropic band and two spherical perturbations in the absorption coefficient. In the band (Ω_1) the direction of anisotropy is parallel with the band. Outside the band (Ω_2) material is isotropic. The solid lines denote the true location of the band, and the dashed and dotted lines two false locations of the band. In Ω_1 (λ_1, λ_2)=(0.0250 cm, 0.0083 cm) and in Ω_2 κ=0.0192 cm. In the background μ_a=0.25 cm^{-1} and in the spherical perturbations μ_a=1 cm^{-1}. (b) Reconstruction of μ_a using a model with the true location for the anisotropic band. (c) Reconstruction using the false position denoted by the dashed lines. (d) Reconstruction using the false position denoted by the dotted lines.

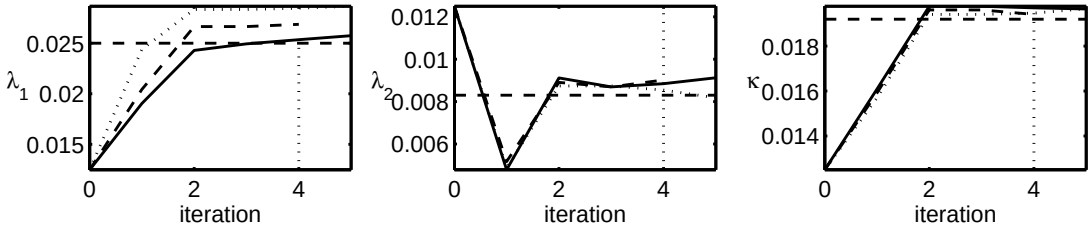

Fig. 3. Values of the parameters $(\lambda_1, \lambda_2, \kappa)$ during iteration. The horizontal dashed lines denote the real value of the parameter. Solid line is for the reconstruction in figure 2 (b), dashed line for figure 2 (c) and dotted line for figure 2 (d). The dotted vertical line denotes the time of the change to the local μ_a basis.

IV. Inverse problem

Next let us consider the following inverse problem of optical tomography: We assume that the principal directions of the anisotropy confined in matrix $U(\mathbf{r})$ are known, but the strength (λ_1, λ_2) of anisotropy is poorly known. The goal is to estimate the absorption coefficient $\mu_a = \mu_a(\mathbf{r})$ based on optical boundary measurements.

The justification for this choice is that it may be possible to make some inference of the principal directions of structural anisotropies in human tissue, e.g., based on anatomical information or other imaging modalities such as functional MRI (Diffusion Tensor Imaging, see [2]). Of course, this type of information is never accurate, and also, correlation between optical and diffusion anisotropy has not been experimentally verified. However, they both depend on structural anisotropy of the matter, and Monte Carlo studies indicate that structural anisotropy gives rise to optical anisotropy [1].

For the estimation of the absorption coefficient to be successful, the incomplete knowledge of the anisotropy need to be taken into concern. Here, the estimation of the constant strength (λ_1, λ_2) of anisotropy in the anisotropic region Ω_1, and the scalar diffusion coefficient κ in the isotropic region Ω_2, is included into the solution. Another possibility would be to use a prior distribution for the uncertain anisotropy parameter values by applying statistical inversion methods [5].

The observation model in this study is

$$y = G(\mu_a, \lambda_1, \lambda_2, \kappa) + n, \qquad (15)$$

where y is vector consisting of boundary measurements, which in this case are the logarithm of the

amplitude and the phase angle of the outward flux, $G(\mu_a, \lambda_1, \lambda_2, \kappa)$ is the model for the noiseless observation, μ_a is a vector consisting of the discrete values of the absorption coefficient in the pixels and n is the additive measurement noise. The estimation is based on minimisation of the functional $\mathcal{F}$,

$$\mathcal{F}(\mu_a, \lambda_1, \lambda_2, \kappa) = \|y - G(\mu_a, \lambda_1, \lambda_2, \kappa)\|^2 + \rho \int_{\Omega_1+\Omega_2} |\mu_a|^2 d\Omega + \rho \int_{\Omega_1} \left(|\lambda_1|^2 + |\lambda_2|^2\right) d\Omega + \rho \int_{\Omega_2} |\kappa|^2 d\Omega, \quad (16)$$

where ρ is a parameter related to the variance of the estimated parameters. Note that both measurements y and parameters μ_a, λ_1, λ_2 and κ are rescaled by average value.

In practise, the minimisation is performed by two stage Gauss-Newton iteration. Firstly, four global values $(\mu_a, \lambda_1, \lambda_2, \kappa)$ are recovered, secondly the iteration is continued in the local pixel basis for μ_a. The Jacobian of the data with respect to the parameters $(\mu_a, \lambda_1, \lambda_2, \kappa)$ takes the following form

$$\mathcal{J}_{\mu_a,\lambda_1,\lambda_2,\kappa} = [\mathcal{J}_{\mu_a} | \mathcal{J}_{\lambda_1} | \mathcal{J}_{\lambda_2} | \mathcal{J}_{\kappa}], \quad (17)$$

where $\mathcal{J}_{\mu_a}$, $\mathcal{J}_{\lambda_1}$, $\mathcal{J}_{\lambda_2}$ and $\mathcal{J}_{\kappa}$ are the Jacobians for μ_a, λ_1, λ_2 and κ, respectively, and μ_a is firstly the global value and secondly the vector of local pixel values.

V. Results

In the numerical example, we consider an isotropic sphere of a radius of 2 cm, with an anisotropic band Ω_1 and two perturbations of the absorption coefficient μ_a of radii of 0.2 cm. We estimate the absorption coefficient μ_a, the eigenvalues (λ_1, λ_2) in Ω_1 and the scalar diffusion coefficient κ in the isotropic region Ω_2. Figure 2 (a) displays the geometry in this numerical example.

In the following reconstructions, the initial values are $\mu_a = 0.15$ cm $^{-1}$ and $\lambda_1 = \lambda_2 = \kappa = 0.0125$ cm. First, the location of the anisotropic band is assumed to be known. Figure 2 (b) displays the reconstruction of the absorption coefficient in this case. However, in general the structural information on anisotropy which is used as a basis of the image reconstruction may not be accurate. To investigate how severely inaccuracies in the geometry of the anisotropy disturb the reconstruction, two alternative locations for the anisotropic band were considered. Figures 2 (c) and (d) display the reconstructions of absorption coefficient with an increasing error in the location of the anisotropy. As the location used in the reconstruction moves further away from the true location of the band, the reconstruction deteriorates seriously.

Finally, figure 3 displays the recovery of the global values of λ_1, λ_2 and κ during iteration for all three cases.

VI. Conclusions

In this paper, we have presented a possible model for optical anisotropy with numerical examples of reconstructions for optical tomography. Based on the results presented, anisotropy should be properly modelled for the image reconstruction to be successful. In addition, even if the strength parameters can be reconstructed simultaneously with the absorption coefficient, also the structural information of the anisotropy should be relatively truthful. However, generally we cannot assume this information to be very accurate. Further studies concerning the uncertainties in the location and direction of anisotropy need to be performed.

Acknowledgements. This work was supported by the Academy of Finland, the Foundation of Technology, the Finnish Cultural Foundation and the Jenny and Antti Wihuri Foundation. The author Heino would like to thank Dr. J Nenonen, HUT, for supporting her work.

References

[1] Stephan Nickell, Marcus Herrmann, Matthias Essenpreis, Thomas J. Farrell, Uwe Krämer, and Michael S. Patterson "Anisotropy of light propagation in human skin" Phys. Med. Biol. **45**, 2873-2886 (2000).

[2] P.J. Basser, J. Matiello, and D. Le Bihan "MR Diffusion Tensor Spectroscopy and Imaging", Biophysical Journal, **66**, 259-267 (1994).

[3] S. Chandrasekhar *Radiative Transfer* (Oxford University Press, London, 1950)

[4] M. Schweiger, S. R. Arridge, M. Hiraoka, and D. T. Delpy "The finite element method for the propagation light in scattering media: Boundary and source conditions," Medical Physics **22**, 1779-1792 (1995).

[5] Jenni Heino and Erkki Somersalo "Estimation of optical absorption in anisotropic background", Inverse Problems **18**, 559-573 (2002). (in press)