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Persistence in cluster-cluster aggregation

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Persistence is considered in one-dimensional diffusion-limited cluster-cluster aggregation when the diffusion coefficient of a cluster depends on its size s as $D(s)\sim s^{\gamma}$. The probabilities that a site has been either empty or covered by a cluster all the time define the empty and filled site persistences. The cluster persistence gives the probability of a cluster remaining intact. The empty site and cluster persistences are universal whereas the filled site depends on the initial concentration. For $\gamma>0$ the universal persistences decay algebraically with the exponent $2/(2-\gamma)$. For the empty site case the exponent remains the same for $\gamma<0$ but the cluster persistence shows a stretched exponential behavior as it is related to the small s behavior of the cluster size distribution. The scaling of the intervals between persistent regions demonstrates the presence of two length scales: the one related to the distances between clusters and that between the persistent regions.

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I. INTRODUCTION

Persistence in dynamical systems is usually defined as the probability $P(t)$ that at a fixed point in space a fluctuating nonequilibrium field $\phi(x;t)$ does not change sign up to time t [1]; that is, the probability that $\mathrm{sgn}[\phi(x;t)-\langle\phi(x;t)\rangle]$ remains unchanged. It was originally introduced for a simple diffusion process [2,3] and since then has been considered in spin systems [4–12], reaction-diffusion systems [13–19], the voter model [20,21], and for interfaces [22–25]. Some ways of measuring it experimentally exist [26–30], as well as a few exact results [31–34].

In many systems persistence decays algebraically, $P(t)\sim t^{-\theta}$, with a nontrivial persistence exponent θ. The significance of the phenomenon stems from the fact that the exponent is not, in general, related to the usual static or dynamic exponents. This in turn implies that not necessarily all of the properties of a system are characterized by a single length scale.

The length scales may not be well separated, which causes problems if one studies the universality aspects of persistence. For example, in the diffusion-reaction model $A+A\rightarrow\varnothing$ the empty site persistence (probability that a site has not been visited by a particle) was first claimed to be nonuniversal [14–16]. Afterward the same authors argued for universality and claimed the poor separation of two length scales, the diffusive scale $\mathcal{L}_D\sim t^z$ and the persistence one $\mathcal{L}_p\sim t^{\theta}$, to be the origin of the confusion [17,18]. Such an effect was also suggested to be the reason for the poor scaling of the interval size distribution between the persistent regions in the one-dimensional q-state Potts model [4]. In neither of these two problems is the length scale separation evident. It is therefore worthwhile to look for a system where one could, without controversy, both verify the universality of persistence and at the same time explicitly demonstrate the effect of the presence of two length scales.

In this article we study persistence in an aggregation process. The particular one used is the one-dimensional diffusion-limited cluster-cluster aggregation (DLCA) model [35], with each cluster diffusing with a size-dependent diffusion coefficient, $D(s)\sim s^{\gamma}$ with $\gamma<2$. In DLCA one meets immediately the possibility of defining several persistent quantities, each of which describes a different aspect of the aggregation process. An important difference from, e.g., spin models (Ising, Potts) is the asymmetry between the clusters and the empty space.

The persistence probabilities considered in this work are (i) the probability of a cluster to remain unaggregated (cluster persistence) and (ii) the probability that a site has been empty (empty site persistence) and (iii) filled (filled site persistence) up to time t. Notice that all these are *local* quantities. It would be possible to start also from definitions that involve a global quantity like the average cluster size [36,37] but these would be harder to study numerically than (i)–(iii) above, which also have the pleasant aspect of being, possibly, experimentally relevant.

We first discuss the three persistence definitions with size-independent diffusion coefficients ($\gamma=0$) in order to clarify the universality of these quantities. The filled site persistence turns out to be nonuniversal in contrast to the two others. Thereafter we concentrate on the two universal ones and consider the influence of size-dependent diffusion ($\gamma\neq 0$). We discover that for $0<\gamma<2$ these decay algebraically with the same exponent but for $\gamma<0$ they are unrelated. Finally, we consider the distribution of persistent regions and intervals between them. As the empty site persistence exponent θ_E is twice the dynamic exponent z, the length scales $\mathcal{L}_D$ and $\mathcal{L}_p$ become well separated at late times. Before this, however, the effect of the presence of two length scales can be nicely demonstrated in the scaling of the interval size distributions.

This paper starts by introducing the model and describing the quantities of interest in Sec. II. In Sec. III each persistence probability is considered separately. Section IV discusses the scaling of the region and interval size distributions. The dependence of the persistent quantities on concentration and initial conditions is studied by simulations at the beginning of Sec. V. The end of that section shows the numerical results for the region and interval size distributions. Section VI concludes the paper.

II. MODEL AND QUANTITIES OF INTEREST

The DLCA model is here considered on a one-dimensional lattice of L sites with periodic boundary conditions. Initially the lattice is filled up to a concentration ϕ such that occupied lattice sites correspond to particles and sites connected via nearest neighbor occupancy belong to the same cluster. Each cluster performs a random walk, and when two clusters collide, they aggregate irreversibly together. The diffusion coefficient of a cluster of size s is given by $D(s)=D_1 s^{\gamma}$. D_1 sets the time scale and it is irrelevant regarding the dynamic scaling properties [38–41]. We concentrate on $\gamma<2$ for which the growth of the average cluster size is algebraic.

The persistence probabilities studied in this work are the following.

(1) Empty site persistence: the probability that an originally empty site has never been occupied by a cluster, $P_E(t)\sim t^{-\theta_E}$.

(2) Filled site persistence: the probability that a site originally covered by a cluster has been covered by it all the time, $P_F(t)\sim t^{-\theta_F}$.

(3) Cluster persistence: the probability that a cluster has not aggregated, $P_C(t)\sim t^{-\theta_C}$.

When the probabilities decay algebraically, one has the persistence exponents θ_E, θ_F, and θ_C. The cluster persistence differs from the other persistence definitions since it is *not* a quantity defined per a fixed site on the lattice but is a property associated with each cluster.

In special cases the DLCA model is closely related to other models, in one dimension. For example, when the clusters are considered to be pointlike particles with ''mass'' s, the DLCA model becomes equal to the reaction-diffusion model $A_s+A_{s'}\rightarrow A_{s+s'}$. For $\gamma=0$, i.e., when the diffusion is mass independent, this reduces to coalescing random walkers $A+A\rightarrow A$, which is exactly solvable [42,43]. This may further be connected to the zero temperature q-state Potts model in the limit $q\rightarrow\infty$, in which the empty site and cluster persistences have recently been studied [4,12].

The terminology and notation used are as follows. The word region is reserved for a bunch of consecutive persistent sites. The distances between regions, i.e., between two consecutive persistent sites, is called an interval. The word cluster has the obvious meaning. The number of clusters of size s per lattice site at time t is $n_s(t)$ with the normalization $\Sigma_s s n_s=1$. Region size is denoted by l and the number of regions of size l (per site) by $p_X(l;t)$. The subscript is the same as for the persistence probabilities and it refers to the persistence definition used: $X\in\{C,F,E\}$. When using the continuum description we use the symbol r instead of l. The letter k labels the interval sizes and the corresponding distribution function is $n_X(k;t)$. The number densities of persistent sites (the persistence probability) and nonpersistent ones are denoted by capital letters $P_X(t)$ and $N_X(t)$, respectively.

As an example, consider intervals between persistent empty sites and their distribution function $n_E(k;t)$. The corresponding distribution of persistent regions is $p_E(l;t)$. The number densities are obtained by summing

$$N_E(t)=\sum_{k=1}^{\infty} k n_E(k;t), \tag{1}$$

$$P_E(t)=\sum_{l=1}^{\infty} l p_E(l;t). \tag{2}$$

Obviously these two are related by the equation $P_E(t)=1-N_E(t)$. Similar formulas apply to other persistence definitions, too, except in the cluster persistence case Eq. (2) is replaced by

$$P_C(t)=\sum_{l=1}^{\infty} p_C(l;t) \tag{3}$$

and naturally $P_C(t)\neq 1-N_C(t)$.

III. PERSISTENCE PROBABILITIES

A. Empty site persistence

We start by giving a heuristic argument for obtaining the empty site persistence exponent for arbitrary γ. Since the clusters on both sides of a persistent empty region are independent, we are led to consider the maximum excursion of a single, diffusing cluster. The only complication is that for $\gamma\neq 0$ collisions will change its diffusivity.

As the average cluster size $S(t)\sim t^z$ with $z=1/(2-\gamma)$ [39,40], we take each cluster to have a *time-dependent* diffusion coefficient $D(t)=D_0 t^{\gamma z}\sim S(t)^{\gamma}$. The probability $P(x;t)$ of finding a cluster at position x at time t obeys a diffusion equation

$$\partial_t P(x;t)=D_0 t^{\gamma z}\partial_x^2 P(x;t). \tag{4}$$

A time transformation

$$T(t)=\frac{D_0}{\gamma z+1}t^{\gamma z+1} \tag{5}$$

reduces this to an ordinary diffusion equation with the diffusion constant $D=1$. The persistence of the empty space between particles diffusing with a constant D has recently been considered for $A+A\rightarrow A$ in [4] and we just quote the main results here.

In the long time limit the probability density $p_E(r;t|\ell)$ that a persistent empty region (originally of size ℓ) has size r at time T is given by

$$p_E(r;T|\ell)=\frac{\ell-r}{\pi T} \tag{6}$$

and the probability that a cluster survives up to time t is

$$P_E(t|\ell)=\frac{\ell^2}{2\pi T}=\frac{z}{\pi D_0}\frac{\ell^2}{t^{2z}}\sim t^{-2z}. \tag{7}$$

For a general initial length distribution of regions $P_E(\ell,0)$ the result will remain the same except ℓ^2 will be replaced by the average over the initial length distribution $\langle\ell^2\rangle$.

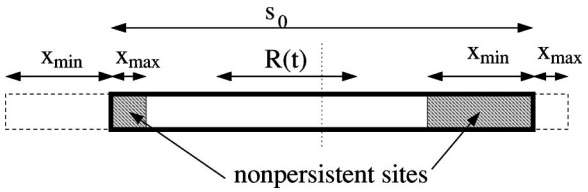

FIG. 1. The length of the nonpersistent filled part is equal to the span $R(t)=x_{\min}(t)+x_{\max}(t)$ of the middle point of a cluster. The length of the persistent region is $s_0-R(t)$. The cluster position at $t=0$ is denoted by a thick rectangle, whose middle position is marked by a dotted line. The dashed lines show the maximum excursions of the cluster.

These considerations show that the persistence exponent $\theta_E(\gamma)=2z=2/(2-\gamma)$. This agrees with the exact result $\theta_E(0)=1$ [4]. Furthermore, the result is independent of the initial spatial distribution or concentration. All these results are confirmed by simulations (see Secs. V A and V B).

B. Filled site persistence

The mechanism by which the filled sites become nonpersistent is different at low and high concentrations. At a low concentration a cluster contains typically only one persistent region which is usually destroyed before the cluster aggregates with its neighbor. At a high concentration a large cluster is created out of many aggregation events, contains an assembly of persistent regions, and only the regions near the edges of the cluster are affected by its motion.

In the low concentration limit we consider the persistence problem in a continuum, valid for clusters of initial length $S_0 \gg 1$. For $\phi \to 0$ the time required for a cluster to move its own length is much smaller than the time required to overcome the distance between clusters. Therefore one could assume that at low concentrations collisions between clusters do not matter. This is true only up to some crossover time, which diverges in the limit $\phi \to 0$. However, to obtain more insight we will first ignore the collisions.

When clusters do not collide, the persistent sites under different clusters are destroyed independently. A single diffusing cluster destroys persistent area at both ends. As Fig. 1 shows, the length of the nonpersistent filled part is equal to the span of the middle point. The span $R(t)$ of a random walk is defined as $R(t)=x_{\min}(t)+x_{\max}(t)$, where $x_{\min}(t)$ and $x_{\max}(t)$ are the maximum displacements in the negative and positive directions at time t, respectively.

The probability distribution for the span of an unlimited random walk is given by [44]

$$w(R;t)=\frac{8}{\sqrt{2\pi Dt}}\sum_{j=1}^{\infty}(-1)^{j+1}j^2\exp\left(-\frac{j^2R^2}{2Dt}\right). \quad (8)$$

In our case the maximum span is limited by the size of the cluster s and the probability distribution $p_F(r;t|s)$ that an interval of length r of a cluster initially of size s has survived up to time t is

$$p_F(r;t|s)=\frac{8}{\sqrt{2\pi Dt}}\sum_{j=1}^{\infty}(-1)^{j+1}j^2\exp\left(-\frac{j^2(s-r)^2}{2Dt}\right) \quad (9)$$

for $r \leqslant s$ and zero otherwise.

For the asymptotic long time behavior it is useful to transform Eq. (9) to a more tractable form as follows. Writing the sum in Eq. (9) in the form $\Sigma_{j=-\infty}^{\infty}G_j$ and applying the Poisson sum formula [45] $\Sigma_{j=-\infty}^{\infty}G_j=\Sigma_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}dx\,G(x)e^{2\pi imx}$ leads to

$$p_F(r;t|s)=\frac{8Dt}{(s-r)^3}\sum_{m=0}^{\infty}\left[\frac{\pi^2Dt(2m+1)^2}{(s-r)^2}-1\right]\times\exp\left(-\frac{\pi^2Dt(2m+1)^2}{2(s-r)^2}\right). \quad (10)$$

From Eq. (10), the probability of finding persistent sites inside the cluster decays exponentially at large times:

$$p_F(t|s)=\int_0^s dr\,p_F(r;t|s)=\sum_{m=0}^{\infty}\frac{8[s^2+\pi^2Dt(2m+1)]}{s^2\pi^2(2m+1)^2}\times\exp\left(-\frac{\pi^2Dt(2m+1)^2}{2s^2}\right) \quad (11)$$

$$\approx\frac{8Dt}{s^2}\exp\left(-\frac{\pi^2Dt}{2s^2}\right). \quad (12)$$

The persistence probability is obtained by integration

$$p_F(t)=\int_0^{\infty}ds\,p_F(t|s)n_s(0), \quad (13)$$

so that the decay will depend on the initial distribution. For example, for $n_s(0)=\delta(s-s_0)$ it is an exponential [Eq. (12)] and for $n_s(0)=2\exp[-s^2/(\pi s_0^2)]/\pi s_0$ we get, using the approximation (12), a stretched exponential

$$p_F(t)\approx\sqrt{\frac{128Dt}{\pi^3s_0^2}}\;e^{-\sqrt{2\pi Dt/s_0^2}}. \quad (14)$$

For the exponential initial distribution $n_s(0)=s_0^{-1}e^{-s/s_0}$, the application of the saddle point method gives

$$p_F(t)\approx\sqrt{\frac{128Dt}{3\pi s_0^2}}\exp\left[-\frac{3}{2}\left(\frac{\pi^2Dt}{s_0^2}\right)^{1/3}\right]. \quad (15)$$

All of these examples show a stretched exponential decay for the persistence probability. The stretching exponent de-

pends on the initial condition and is therefore nonuniversal. The long time behavior, in the approximation ignoring collisions, is governed by the ratio Dt/s_0^2, and the decay depends on concentration due to the factor s_0.

The approximate span distribution can also be used to calculate the mean size of persistent regions. First write

$$\langle r(t|s)\rangle = \int_0^s dr\, r p_F(r;t|s) = s - \langle R(t|s)\rangle \tag{16}$$

and use the normalized approximate form for the span distribution

$$\tilde{w}_N(R;t|s) = \begin{cases} \delta(R-s)[1-(1+\xi s^{-2})e^{-\xi/s^2}] + 2\xi^2 R^{-5} e^{-\xi/R^2}, & R \leqslant s, \\ 0, \quad R > s, \end{cases} \tag{17}$$

where $\xi = \pi^2 Dt/2$, to calculate $\langle R(t|s)\rangle$, from which

$$\langle r(t|s)\rangle \approx s \exp\left\{ -\frac{\pi^2 Dt}{2s^2} \right\}. \tag{18}$$

As an example, for the simplest case of a fixed initial size s_0 $[n_s(0) = \delta(s - s_0)]$, the mean length of surviving regions is

$$\langle s_{\mathrm{surv}}\rangle = \frac{\langle r(t|s_0)\rangle}{p_F(t|s_0)} \approx \frac{s_0^3}{8Dt} \sim t^{-1}. \tag{19}$$

Here again, the decay exponent depends on the initial distribution $n_s(0)$.

In the high concentration limit $\phi \to 1$, we adopt another mean-field type approach: we consider a deterministic model combined with scaling arguments. Let the average cluster and empty interval sizes initially be s_0 and d_0, respectively. The concentration $\phi = s_0/(s_0 + d_0) \approx 1$ for $s_0 \gg d_0$.

Now consider the doubling times $t_1, t_2, \ldots, t_i, \ldots$ at which the average cluster and interval sizes are $s_i = 2^i s_0$ and $d_i = 2^i d_0$, respectively. At each step n the doubled cluster is constructed as follows. First, $d_{n-1}/2$ sites (these do not have to persistent but they may be) from both ends of the cluster are made nonpersistent. Second, the resulting cluster is duplicated. The probability of finding persistent sites, $p(n)$, at step n decays as $p(n) \sim e^{-\alpha(\phi)n}$ for high enough values of ϕ. Since $s_n = 2^n s_0 \sim t^z$, it follows that $p(t) \sim t^{-\alpha(\phi) z/\ln 2}$.

According to this simple argument the filled site persistence probability decays algebraically for large enough concentrations. The persistent sites are swept by domain walls, which annihilate at aggregation. Since the probability to be touched by a domain wall depends on their density, the persistence exponent depends on concentration, implying nonuniversality. Simulations qualitatively agree with this behavior and furthermore show that the persistence probability decays algebraically for low concentrations, too. The reason for the deviation from the span argument lies in the approximation, which neglects the collisions between clusters. We return to this issue in a more detail in Sec. V A.

C. Cluster persistence

Cluster persistence is considered in detail elsewhere [46] but, for completeness, we briefly report the main results and their implications.

It turns out that the cluster persistence probability

$$P_C(t) \sim \begin{cases} \exp(-Ct^{\beta_S}), & \gamma < 0, \\ t^{-3/2}, & \gamma = 0, \\ t^{-2/(2-\gamma)}, & 0 < \gamma < 2, \end{cases} \tag{20}$$

where β_S fits well to the expression $\beta_S = 2/3(1-2z)$ and $C > 0$ is a constant. The discontinuity of the exponent as $\gamma \to 0^+$ can be understood in terms of a mean-field random walk analysis. The completely different behavior for $\gamma < 0$ and $\gamma \geqslant 0$ is related to the behavior of the cluster size distribution. It is known to scale as

$$n_s(t) = S(t)^{-2} f(s/S(t)), \tag{21}$$

where $S(t)$ is the average cluster size [35]. For small argument values $(x \to 0^+)$ the scaling function $f(x)$ decays as $x^{-\tau}$ for $0 \leqslant \gamma < 2$ and as $\exp(-x^{-|\mu|})$ for $\gamma < 0$ [38,41]. These results can be used to obtain a scaling relation between the exponents

$$\theta_C = (2-\tau)z. \tag{22}$$

This equation together with Eq. (20) allows one to determine τ for $0 \leqslant \gamma < 2$. That is, by solving for the cluster persistence one obtains the small size tail of the scaling function, the knowledge of which is of primary importance in aggregation systems. This connection may exist in other models and offer a way to approach the problem of computing the cluster size distribution.

IV. DISTRIBUTIONS OF PERSISTENT INTERVALS AND REGIONS

In the following, we concentrate only on the universal empty and cluster persistences. The interval size distributions between consecutive persistent sites are studied using the natural scaling assumption

$$n_X(k;t) = K_X(t)^{-\alpha} f_X(k/K_X(t)), \tag{23}$$

where $K_X(t)$ is the average interval size and X denotes either C or E. Inserting the scaling form (23) into Eq. (1) and replacing the sum by an integral results in

$$N_X(t) = K_X(t)^{2-\alpha} \int_0^\infty dy\, y f_X(y). \tag{24}$$

As $P_X(t) \to 0$ for $t \to \infty$ it follows from the relation $P_X(t) = 1 - N_X(t)$ that $N_X(t) \to 1$ as $t \to \infty$. The only way to keep the right hand side of Eq. (24) constant is to require $\alpha = 2$, as a direct consequence of mass conservation. Note that the above argument does not require the persistence probability $P_X(t)$ to decay algebraically.

Since the persistence exponents are larger than the dynamic exponent, $\theta_X > z$, the persistence length scale will be much larger than the diffusive one at large times. The persistent regions are well separated and get destroyed by uncorrelated processes since the correlations grow only as t^z. In the scaling limit the scaling functions are therefore simple exponentials

$$n_X(k;t) = K_X(t)^{-2} e^{-k/K_X(t)}. \tag{25}$$

Consider next persistent empty sites. At large times the region size distribution is given by Eq. (6) for a monodisperse initial condition. For other initial distributions $n_s(0)$ it is obtained as

$$p_E(r;T) = \int_0^\infty d\ell\, p_E(r;T|\ell) P_E(\ell,0) = \frac{\ell_0}{\pi T} e^{-r/\ell_0}, \tag{26}$$

where T is the rescaled time [Eq. (5)] and the last form corresponds to the initial distribution $P_E(\ell,0) = \ell_0^{-1} e^{-\ell/\ell_0}$. The dependence on the diffusion exponent enters only through the time scale $T \sim t^{2/(2-\gamma)}$. The spatial and time dependences in $p_E(r;t)$ are decoupled. From this it follows that the average size of the persistent empty regions, $L_E(t)$, is a constant at large times. For the monodisperse and exponential initial conditions we get $L_E^{\mathrm{mono}}(t) = P_E(T|\ell_0)^{-1} \int_0^{\ell_0} dr\, r p_E(r;T|\ell_0) = \ell_0/3$ and $L_E^{\exp}(t) = \int_0^\infty dr\, r p_E(r;T) / \int_0^\infty dr\, p_E(r;T) = \ell_0$, respectively. These both are independent of time. The simulations (see Sec. V D) confirm this.

V. SIMULATIONS

The simulations are done on a one-dimensional lattice with periodic boundary conditions with a standard algorithm [41]. In all the simulations $D_1 = 1$, the system sizes range from 5×10^4 to 1.5×10^6, and the data are averaged over 1000–50 000 realizations.

A. Dependence on concentration and initial conditions

We test the sensitivity of the persistence probabilities against concentration changes and two different initial conditions. The first initial condition used is random: each lattice site is filled with probability ϕ. The other one is deterministic and monodisperse with equidistant clusters of a given size

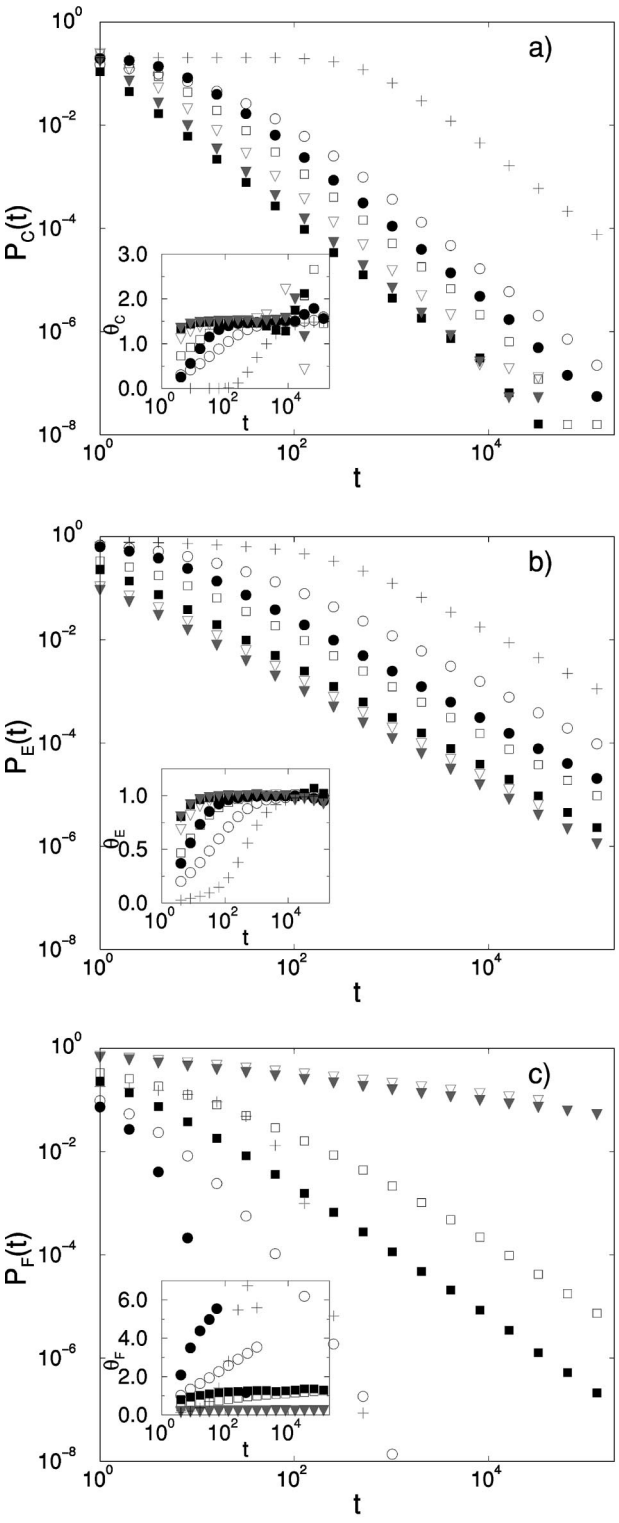

FIG. 2. The persistence probabilities (a) $P_C(t)$, (b) $P_E(t)$, and (c) $P_F(t)$ for size-independent diffusion coefficients ($\gamma = 0$) and for concentrations $\phi = 0.20$ (○), 0.50 (□), and 0.80 (∇). Results from simulations using random and monodisperse ($s_0 = 1$) initial conditions are denoted by open and filled symbols, respectively. Pluses (+) are obtained with monodisperse initial conditions for $\phi = 0.20$ and $s_0 = 10$. The insets show the running exponents.

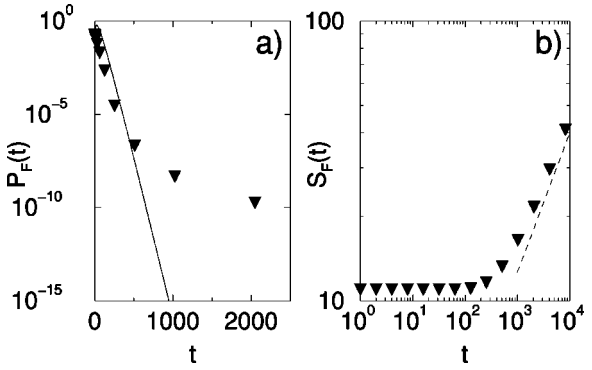

FIG. 3. (a) Filled site persistence probability and (b) average cluster size of those clusters that contain persistent sites for deterministic initial conditions with $\gamma=0$, $s_0=11$, and $\phi=0.2$. The solid line is given by Eq. (12) and the dashed line shows $t^{1/2}$ behavior.

s_0. We present data only for size-independent diffusion coefficients but we have checked that for $\gamma\neq 0$ the behavior is similar.

As Fig. 2 shows, a change in the initial condition does not have a significant effect on any of the persistences. A change in the concentration affects only the amplitudes of the cluster and empty site persistence distributions. The numerical estimates for the exponents obtained from the saturated part of the running exponents are $\theta_C(0)=1.48\pm0.03$ and $\theta_E(0)=1.00\pm0.02$. These are in excellent agreement with the exact results $\theta_C(0)=\frac{3}{2}$ [5,47] and $\theta_E(0)=1$ [4].

For the filled site persistence the distribution shows a transition from an algebraic decay to a relatively faster one when decreasing the concentration. The nonalgebraic decay seen in simulations is only a crossover behavior. The discrepancy between the analysis of Sec. III B and simulations is due to cluster aggregation. This occurs for times larger than the average collision time $t_{\mathrm{coll}}\sim 1/(D\phi^2)$, which indeed diverges for $\phi\to 0$. The large time persistence probability of filled sites is dominated by the clusters, that have collided with others. This is illustrated in Fig. 3, which shows both the persistence probability and the average size of those clusters that contain persistent sites. There is a clear crossover from the behavior given by the analysis of Sec. III B at $t\approx 300$ to the one for which the collisions are significant. After this crossover time, the clusters including persistent empty sites grow similarly to the other ones.

To summarize, the simulations support the universality of empty and cluster persistences and show without doubt that the filled site persistence is nonuniversal. Similar concentration-dependent behavior has been observed also for persistence of bubbles in soap froths [27].

B. Dependence on γ

All the dynamic scaling properties of the DLCA depend on γ and the same is true for the persistence probabilities as Fig. 4 shows. The empty site persistence probability decays algebraically for all values of $\gamma<2$. The analytical prediction $\theta_E=2z$ is compared to simulations in Fig. 5. The agreement is excellent. Hence, there are no nontrivial correlations and the clusters surrounding persistent empty sites grow like the others. The cluster persistence decays algebraically only for $\gamma\geqslant 0$ and faster than any power of t for $\gamma<0$ in accordance with Eq. (20).

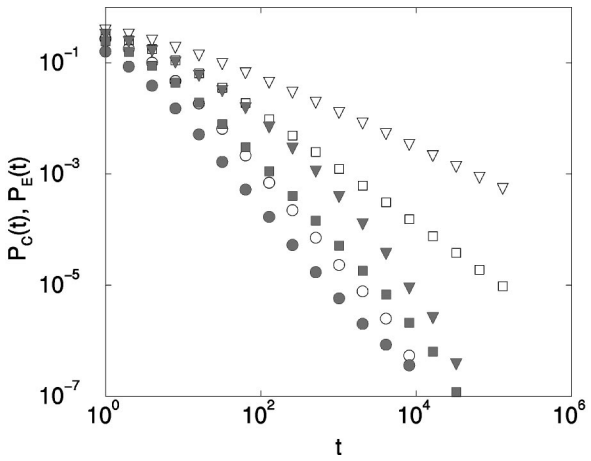

FIG. 4. The cluster and empty site persistence probabilities $P_C(t)$ (filled symbols) and $P_E(t)$ (empty symbols) for $\gamma=-1.0(\nabla)$, $0.0(\square)$, and $0.75(\bigcirc)$.

C. Intervals between consecutive persistent sites

In Sec. IV we argued that the size distribution of intervals between persistent quantities would scale according to $n_X(k;t)=K_X(t)^{-2}f_X(k/K_X(t))$ with a simple exponential scaling function [Eq. (25)]. When the corresponding persistence probability decays algebraically, $K_X(t)\sim A_X t^{\theta_X}$, with a nonuniversal amplitude $A_X(\phi)$, this can be presented also as $n_X(k;t)=t^{-2\theta_X}\tilde{f}_X(k/t^{\theta_X})$.

There is a difference between these two scaling forms: for the latter the scaling functions will not overlap each other for different concentrations due to the nonuniversal amplitude dependence. Therefore for clarity we show the scaling plots using this formulation. Furthermore, we prefer to show the scaling of the complement of the cumulative distribution

$$I_X(k;t)=\sum_{i\geqslant k} n_X(i;t). \tag{27}$$

It is easy to see from the scaling of $n_X(k;t)$ and Eq. (25) that this should scale as $I_X(k;t)=t^{-\theta_X}g_X(k/t^{\theta_X})$ with $g_X(x)=A_X^{-1}e^{-A_X^{-1}x}$.

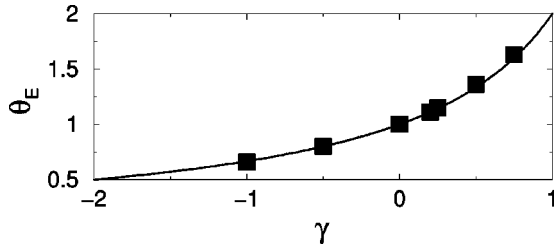

FIG. 5. Comparison of the numerically obtained empty site persistence exponent $\theta_E(\square)$ to the mean-field result $\theta_E=2z=2/(2-\gamma)$ (solid line) as a function of the diffusion exponent γ.

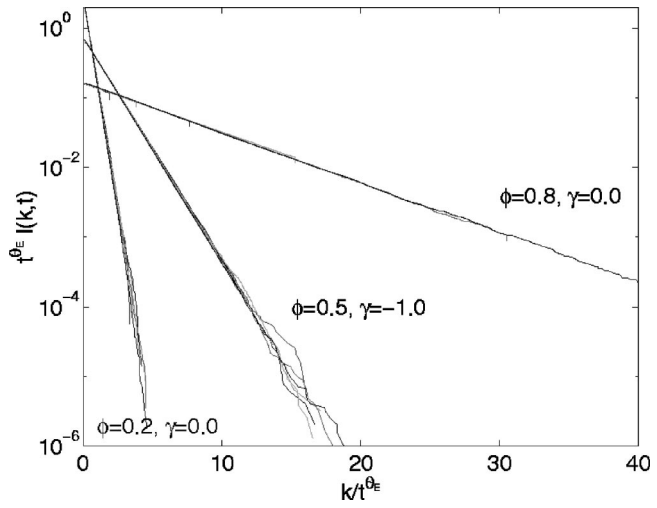

FIG. 6. Scaling plot for the complement of the cumulative distribution of intervals between two consecutive persistent empty sites at $t=2^9,\ldots,2^{14}$ for concentrations and diffusion exponents shown in the figure. The value used for the persistence exponent $\theta_E=2/(2-\gamma)$.

Figure 6 shows that the scaling works for the empty site persistence and that the scaling function is an exponential one indeed. The plots for the cluster persistence are similar (not shown). The scaling function is universal and all the curves in Fig. 6 would overlap each other if one plotted $K_E(t)I(k;t)$ as a function of $k/K_E(t)$. Note that the diffusion exponent γ has no influence on the scaling function.

Although the summation in Eq. (27) smooths the data, at the same time it loses information about the small k/t^{θ_E} behavior. This is illustrated in Fig. 7 where no summation is done. For $k \ll t^{\theta_E}$ the scaling does not work. The reason is the following. The scaling should work in the limit $k\to\infty$ and $t\to\infty$ with $y=k/t^{\theta_E}$ fixed. In particular, the condition $k \gg t^z$ should be satisfied for the two length scales of the problem to be well separated. Define now a time-dependent $y_c(t)$ so that the scaling works for $y>y_c(t)$. This quantity gets smaller at

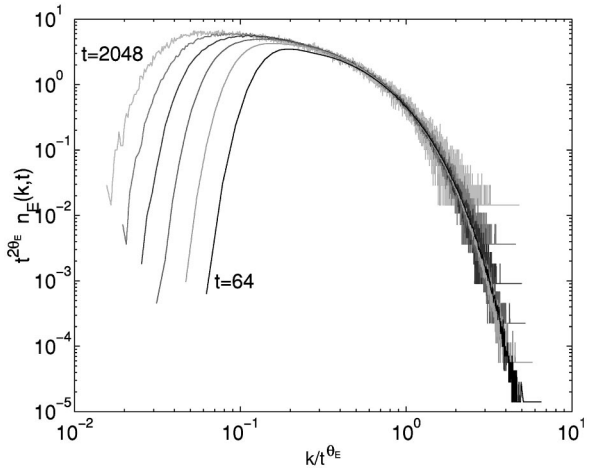

FIG. 7. Scaling plot for the distribution of intervals between two consecutive persistent empty sites at $t=2^6,\ldots,2^{11}$ for $\phi=0.2$ and $\gamma=0$.

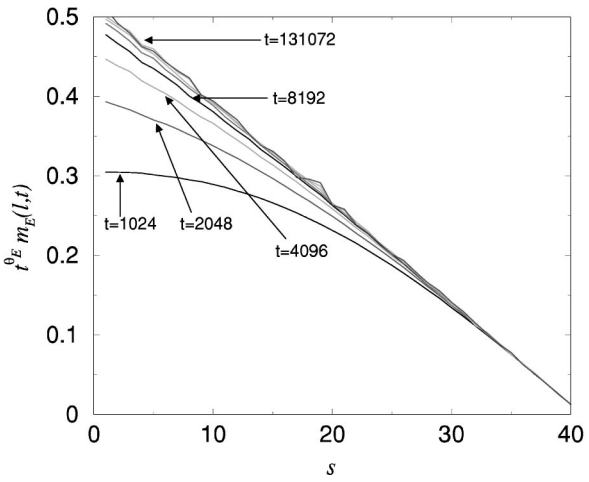

FIG. 8. The size distributions of persistent empty regions for $\gamma=0$ and ordered initial conditions with $s_0=10$ and $\phi=0.2$. The time instants range from $t=2^{10}$ to $t=2^{17}$.

the same rate at which the curves in Fig. 7 shift towards zero. Our estimate from the numerical data gives $y_c(t) \sim t^{-0.50\pm0.03}$, which is consistent with $y_c(t)\sim t^{-z}$. Thus the poor scaling in Fig. 7 for $k \ll t^{\theta_E}$ is just a manifestation of the finite time behavior with two competing length scales, $\mathcal{L}_D \sim t^z$ and $\mathcal{L}_p \sim t^{\theta_E}$. This effect vanishes in the scaling limit. A similar, although not as clear, violation of scaling induced for the same reason is seen in the q-state Potts model [4].

D. Persistent regions

Figures 8 and 9 show the empty region distributions for the initial cluster size distributions $n_s(0)=\delta(s-s_0)$ and $n_s(0)=s_0^{-1}\exp(-s/s_0)$. These confirm the analytical predictions of a linear [Eq. (6)] and an exponential [Eq. (26)] decay. Hence, the dependence on the diffusion exponent enters only through a multiplicative factor of $t^{-2/(2-\gamma)}$ in the distributions and the average region size approaches a constant at late times. In Fig. 8 the smallest times shown are not large

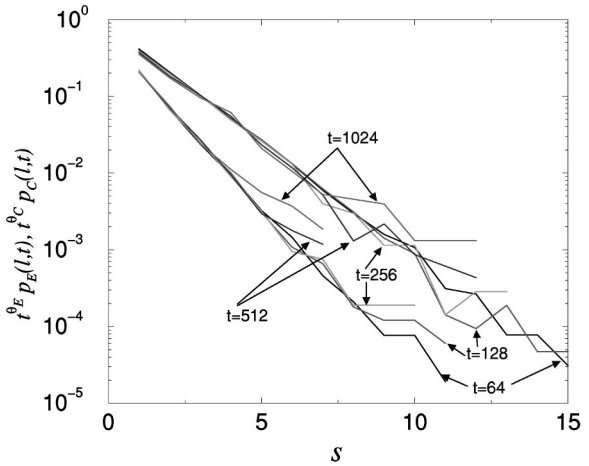

FIG. 9. The size distributions of persistent empty regions (upper curves) and clusters (lower curves) for random initial conditions and $\gamma=0.75$. The measurement times are shown in the figure.

enough for the analysis of Sec. III A to be valid but the tendency of the distribution to approach a straight line is clearly visible.

Naturally, the size distribution of persistent clusters remains unaltered for a monodisperse initial condition $n_s(0) = \delta_{s_0,s}$. The same is true also for size-independent diffusion coefficients ($\gamma = 0$) no matter what the initial distribution is. Therefore we present only the result for random initial conditions and for $\gamma = 0.75$ in Fig. 9. In this case also the scaling function is a pure exponential, i.e., it remains unaltered.

VI. CONCLUSIONS

In this paper we have considered persistence in an aggregation process, in the case of one-dimensional DLCA. The emphasis is on local properties: empty, filled, and cluster persistences together with the corresponding region and interval size distributions. We have shown that the three persistences are independent and each has its distinct scaling properties.

The perhaps most natural choice, the probability that a site has remained in the same state—filled or empty—is nonuniversal. The filled site persistence is responsible for this. The empty site persistence is universal. The difference in the dynamics of empty and filled sites in DLCA thus becomes apparent. The cluster persistence is a third independent quantity since it classifies clusters whereas the two other persistence definitions are considered at a fixed point in space.

To summarize, the universality of empty site persistence is supported both by mean-field continuum arguments and by simulations. The former leads to a relatively simple relation between the persistence exponent and the dynamic exponent, $\theta_E = 2z$, verified by simulations. This is one of the few examples [4] where the inequality $\theta > zd$, where d is the spatial dimension, is satisfied. The consequence of this is that the persistent empty regions do not have a fractal character. This is not true, for example, for the persistent regions in the Ising [10] or diffusion-annihilation [17] models. The fact that θ_E is notably larger than z makes the separation of the diffusive and persistence length scales clearly visible in the scaling of the interval size distribution.

The filled site persistence decays asymptotically algebraically for $\gamma < 2$ and for all concentrations: $P_F(t) \sim t^{-\theta_F}$. The persistence exponent θ_F depends on concentration and is therefore nonuniversal. At low concentrations the filled site persistence decays as a stretched exponential up to a concentration-dependent crossover time, after which the collisions between clusters become important and start to dominate the persistence behavior.

The cluster persistence probability is universal and decays algebraically for $0 \leqslant \gamma < 2$ and as a stretched exponential for $\gamma < 0$. For $0 < \gamma < 2$ the persistence exponent is given by $\theta_C(\gamma) = 2z = 2/(2-\gamma)$ and it is discontinuous as $\gamma \to 0^+$ since $\theta_C(0) = \frac{3}{2}$. All these results are in close connection with the scaling of the cluster size distribution, especially with the small x decay of the scaling function $f(x) \sim x^{-\tau}$. In fact, there is a scaling relation between the exponents $\theta_C = (2-\tau)z$. The scaling relation together with the result for θ_C offers a way to determine the small size tail of the cluster size distribution in DLCA.

It is worth emphasizing that the universal empty and cluster persistences decay with the same exponent $\theta_E = \theta_C = 2z = 2/(2-\gamma)$ for $0 < \gamma < 2$ but have nothing to do with each other for $\gamma < 0$. In fact, for $\gamma \geqslant 0$ we can write $P_C(t) \sim [P_E(t)]^{\Gamma}$, where Γ takes the values 3/2 and 1 for $\gamma = 0$ and $0 < \gamma < 2$, respectively. The same persistence exponents, i.e., $\Gamma = 1$, for positive γ are due to the fact that the clusters giving the dominant contribution to the cluster persistence are those that asymptotically become stationary. Any other value of Γ makes the interpretation more opaque. A similar relation, $P_C(t) \sim [P_E(t)]^{\Gamma}$, with a nontrivial Γ has recently been observed also for noninteracting random walkers [19] and for the Potts model [12] in one dimension. A challenge for the future is to understand the origin and limitations of this relationship.

In conclusion, we have presented a rather comprehensive study of various local persistence probabilities in the one-dimensional DLCA model. Our study is of interest also for the sake of practical realizations. Aggregation processes are plentiful, and all of the definitions—whether a point in space is occupied or not by a cluster, or whether clusters survive intact—might well be possible to measure experimentally. It is interesting also to note that, when the decay of a persistence probability is algebraic and universal, the exponent is always directly in some relation to the dynamical exponent z of the aggregation process. It is an obvious question to ask how the various quantities work out in higher dimensions.

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