

# Analytical model for melting in a semi-infinite PCM storage with an internal fin

P. Lamberg, K. Sirén

**Abstract** The most PCMs with high energy storage density have an unacceptably low heat conductivity and hence internal heat transfer enhancement techniques such as fins or other metal structures are required in latent heat thermal storage (LHTS) applications. Previous work has concentrated on numerical and experimental examination in determining the influence of the fins in melting phase change material. This paper presents a simplified analytical model based on a quasi-linear, transient, thin-fin equation which predicts the solid–liquid interface location and temperature distribution of the fin in the melting process with a constant imposed end-wall temperature. The analytical results are compared to the numerical results and they show good agreement. Due to the assumptions made in the model, the speed of the solid–liquid interface during the melting process is slightly too slow.

$k(x,t)$  source solution  
 $L$  length, m  
 $l(x,t)$  derived source solution  
 $Nu$  Nusselt's number,  $\frac{hS}{k}$   
 $q$  heat flow, W  
 $q'$  heat flow per unit length,  $W\ m^{-1}$   
 $q''$  heat flux,  $W\ m^{-2}$   
 $Ra$  Rayleigh's number,  $\frac{[g(T_f - T_m)\rho_l^2 c_{pl} S^3 \beta]}{\mu k_l}$   
 $S$  location of the phase change interface, m  
 $St$  Stefan number,  $\frac{c_{pl}(T_w - T_m)}{H}$   
 $T$  temperature, °C  
 $t$  time, s  
 $x$  distance in the  $x$ -direction, m  
 $y$  distance in the  $y$ -direction, m

## List of symbols

$A$   $\frac{k_f}{(\rho c_p)_f}$   
 $a$   $k_l(T_w - T_m)$   
 $B$   $\frac{h}{(\rho c)_f D}$   
 $b$   $\rho_l H$   
 $C$   $BT_m$   
 $c_p$  heat capacity,  $J\ kg^{-1}\ K^{-1}$   
 $D$  half thickness of the fin, m  
 $E'$  energy storage per unit length,  $W\ m^{-1}$   
 $E''$  energy storage per unit area,  $W\ m^{-2}$   
 $g$  acceleration of gravity,  $m\ s^{-2}$   
 $H$  latent heat of fusion,  $J\ kg^{-1}$   
 $h$  heat transfer coefficient,  $W\ m^{-2}\ K^{-1}$   
 $k$  heat conductivity,  $W\ m^{-1}\ K^{-1}$

## Greek symbols

$\alpha$  thermal diffusivity,  $m^2\ s^{-1}$   
 $\beta$  thermal expansion coefficient,  $^{\circ}C^{-1}$   
 $\mu$  dynamic viscosity,  $kg\ m^{-1}\ s^{-1}$   
 $\rho$  density,  $kg\ m^{-3}$   
 $\lambda$  root of the transcendental equation

## Subscripts and superscripts

$c$  convection  
 $cr$  critical  
 $f$  fin  
 $i$  initial  
 $l$  liquid  
 $m$  melting  
 $p$  phase change material  
 $s$  solid  
 $w$  wall

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## 1 Introduction

A latent heat storage system (LHTS) is preferable to sensible heat storage in applications with a small temperature swing because of its nearly isothermal storing mechanism and high storage density. During the last twenty years phase change materials (PCM) for storing energy have developed rapidly. Their thermal and physical properties such as long-term stability and durability have been improved a lot. However, phase change materials have some disadvantages which are not possible to solve with the development of the materials. In a large PCM storage the

solid-liquid phase transition suffers from the small thermal conductivity of the PCMs, especially in the liquid state (for paraffins  $k=0.1-0.2$  W/m<sup>2</sup> K and for salt hydrates  $k=0.4-0.6$  W/m<sup>2</sup>K) [1]. In a very small PCM storage, such as encapsulated PCM particles, the heat transfer surface is large compared to the mass of the PCM. Thus, no heat transfer problem exists inside the particle. The utilisation of LHTS is limited with large heat flux densities. Therefore, the internal enhancement of the heat transfer of PCM is essential when improving the energy efficiency of the latent heat storage system.

In general, the heat transfer problems in phase change materials are complicated for several reasons. First, there is the transient, non-linear phenomenon with a moving liquid-solid interface, generally referred to as a “moving boundary” problem. The liquid and solid state of a PCM have different kinds of material properties. Besides, the problem has to be handled in a 2 or 3-dimensionally to achieve an accurate enough solution. The only exact analytical solution for a 1-dimensional moving boundary problem called the Stefan problem was found by Neumann [2, 3]. The classical Stefan problem is a pure conduction problem with a constant end-wall temperature boundary condition. Some analytical approximations for moving boundary problems with different boundary conditions have been found such as the quasi-stationary approximation, perturbation methods, the Megerlin method and the Heat-balance-integral method [3]. However, buoyancy effects and the resulting natural convection motion in the liquid have been assumed to be negligible in these analytical solutions. Natural convection complicates the analysis of the moving boundary problem in the melting process. Experimental results have shown that natural convection during melting essentially improves the speed of melting and it has a big influence on internal heat transfer. Hence, it should be considered in any analysis of phase change systems [4].

It is possible to enhance the internal heat transfer of PCM with fins, metal honeycombs, metal matrices (wiremesh), lessing rings, high conductivity particles or graphite [5, 6]. Heat transfer in a phase change material with internal fins has been studied numerically and experimentally in wide range, but analytical solutions do not exist.

Al-Jandal [7] studied experimentally what effects the fin, metal honeycomb and copper matrix structure have on the total melting and solidification time. The results showed that the average thermal conductivity enhancement factors for melting are in the order of 3.3 and for solidification are in the order of 1.7 and natural convection has a significant effect on the acceleration of melting.

Humpries et al. [8] handled numerically a rectangular phase change housing, using straight fins as a heat transfer enhancement in a 2-dimensional grid. The data was generated over a range of realistic sizes, material properties and different kinds of thermal boundary conditions resulting in a design handbook for phase change energy storage.

Bugaje [9] made experiments on the use of methods for enhancing the thermal response of paraffin wax heat storage tubes with the incorporation of aluminium fins and star

structures. The conclusion was that internal fins performed much better than the star matrices reducing the loading time in the order of 2.2 and the cooling time in the order of 4.2.

Eftekhari et al. presented experimental data on the rate of the production of liquid as a function of time in a thermal storage system consisting of vertically arranged fins between heated and cooled surfaces [10]. Natural convection was observed to play an important role in melting the phase change material.

Henze et al. [11] introduced a simplified numerical model based on a quasi-linear, transient, thin fin equation which predicts the fraction of melted PCM, and the shape of the liquid-solid interface as a function of time in finite storage.

It is evident that fins and different kind of matrix structures enhance the internal heat transfer of a phase change material. In practice some kinds of heat transfer enhancement techniques have to be used in LHTS. To achieve the best economical and technical benefit of the storage, the loading and unloading time should be optimised. The geometry of the latent heat storage plays a very important role.

In engineering there is often no opportunity for complicated numerical computations when a pre-design of a storage is made. A fast analytical model saves time and effort. The design tools which are based on analytical models require less computational power and are easier to use in practice.

The objective of this paper is to examine the melting process in a semi-infinite PCM storage with a fin. The simplified analytical model based on a quasi-linear, transient, thin-fin equation is presented which predicts the solid-liquid interface location and temperature distribution of the fin.

## 2 Model

This work studies the melting process in a semi-infinite PCM storage with a thin fin (see Fig. 1). The storage is 2-dimensional and it is semi-infinite both in the  $x$ -direction ( $0 \leq x < \infty$ ) and  $y$ -direction ( $0 \leq y < \infty$ ) and the length of the fin approaches infinite. The end-wall with a constant temperature and the fin act as heat sources in the melting process.

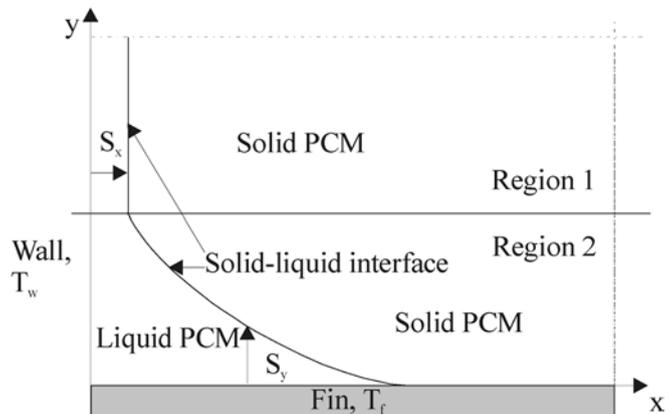


Fig. 1. Semi-infinite phase change material storage with a fin

In a storage the melting occurs in two different regions. The regions are shown in Fig. 1. In region 1, the only heat source is the constant temperature end-wall. Here the fin is not influencing the melting process. Heat transferred from the wall is first melting the phase change material by conduction and later by natural convection. With Rayleigh's number (see Eq. (17)) it is possible to determine when the dominant heat transfer mode turns from conduction to convection. When  $Ra \geq 10^3$ , the liquid PCM starts to flow up along the vertical hot wall surface and fall down along the cold solid-liquid interface causing natural convection [12].

In region 2, both the wall and the fin are transferring heat to the phase change material. There are three stages in the melting process: pure conduction from the constant temperature end-wall and the fin, conduction from the fin with some natural convection from the end-wall and finally, only natural convection from the fin. When  $Ra \geq 1708$ , natural convection becomes to dominate the heat transfer mode from the horizontal fin to the solid-liquid interface [12]. The fin tends to decrease natural convection from the end-wall due to the decreasing temperature gradient in the liquid [11]. After a short period the fin plays the most important role in the heat transfer in region 2.

## 2.1

### Assumptions

Due to the nonlinear, unsteady nature of the problem several assumptions have to be made to simplify the problem.

1. Initially the solid PCM and the fin are in the melting temperature of the phase change material  $T_m = T_s = T_f$ . Therefore, the heat conduction in a solid PCM is considered to be negligible.
2. The end-wall temperature  $T_w$  is kept constant and it is higher than the melting temperature of the phase change material  $T_m$ .
3. The temperature distribution of the thin fin is considered to be 1-dimensional in the x-direction.
4. The sensible heat of liquid PCM is assumed to be negligible. The latent heat of fusion is assumed to be the principal mode of energy storage.
5. In region 1 the heat is transferred from the wall to the solid-liquid interface 1-dimensionally in the x-direction. The main heat transfer mode is assumed to be conduction in liquid PCM. Natural convection is assumed to be negligible. With these assumptions, it is possible to find an analytical solution for the solid-liquid interface location in the x-direction.
6. In region 2 it is assumed that heat transfer is 1-dimensional from the fin to the solid-liquid interface in the y-direction because the fin plays the most important role in melting PCM in region 2. The main heat transfer mode is assumed to be natural convection in liquid PCM. Conduction is assumed to be negligible.
7. The physical properties for the phase change material and for the fin are assumed to be constant, because the temperature differences in phase change material storage are usually relatively small.

## 2.2

### Mathematical problem

The mathematical problem will also be handled in two parts. In region 1 the melting can be handled as 1-dimensional one-phase Stefan problem [2], which is the simplest explicitly solvable moving boundary problem with constant imposed end-wall temperature and constant thermophysical properties of the materials. The exact solution for the problem was found by Neumann in 1860 [4]. In a 1-phase Stefan problem the heat equation of a liquid phase change material  $T_1$  and heat equation for solid-liquid interface with initial and boundary conditions are defined as [3]:

$$\frac{\partial^2 T_1}{\partial x^2} = \frac{1}{\alpha_1} \frac{\partial T_1}{\partial t}, \quad t > 0 \quad (1)$$

$$(\rho H)_l \frac{\partial S_x(t)}{\partial t} = -k_l \frac{\partial T_1(S_x, t)}{\partial x}, \quad t > 0 \quad (2)$$

$$S_x(0) = 0 \quad (3)$$

$$T_1(S_x, t) = T_m \quad (4)$$

$$T_1(0, t) = T_w \quad (5)$$

where  $S_x(t)$  is the location of the solid-liquid interface in the x-direction as a function of time,  $\alpha_1$  thermal diffusivity of liquid PCM,  $\rho$  density,  $H$  latent heat of fusion and  $k_l$  conductivity of the liquid.

In region 2 all heat transfer is assumed to occur only in the y-direction. An arbitrary differential element  $dx$  is separated from the PCM storage to outline energy balances. The element is shown in Fig. 2. The energy balance of a differential element  $dx$  yields two equations, one for the fin and one for the PCM.

The energy balance for the fin is

$$E_f'' = q_x'' - q_{x+dx}'' - q_c'' \quad (6)$$

where  $E_f''$  denotes the rate of heat storage to the fin,  $q_x''$  the heat flux by conduction at position  $x$ , while  $q_{x+\Delta x}''$

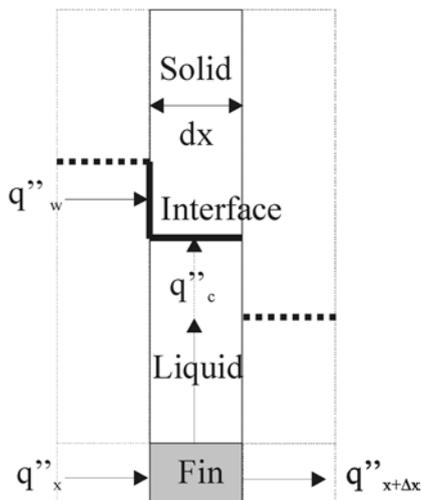


Fig. 2. Energy flows in the arbitrary differential element of the finned PCM storage

denotes heat flux by conduction at position  $x+dx$  and  $q''_c$  refers to the convective heat flux from the fin to the liquid-solid interface.

The rate equations are substituted into the energy balance Eq. (6) which can be rewritten with initial and boundary conditions as

$$(\rho c_p)_f D \frac{\partial T_f}{\partial t} = k_f D \frac{\partial^2 T_f}{\partial x^2} - h(T_f - T_m), \quad t > 0 \quad (7)$$

$$T_f(x, 0) = T_m \quad (8)$$

$$T_f(0, t) = T_w \quad (9)$$

$$T_f(\infty, t) = T_m \quad (10)$$

where  $T_f$  is the temperature of the fin,  $h$  the heat transfer coefficient from the fin to the solid-liquid interface,  $T_m$  the melting temperature of the PCM and  $D$  the half thickness of the fin.

The heat flows from the constant temperature end-wall and from the fin to the solid-liquid interface of the phase change material. The energy balance for the solid-liquid interface in the  $y$ -direction is the following:

$$E'_p = q'_w + q'_c \quad (11)$$

where  $E'_p$  is the rate of heat storage to the phase change material,  $q'_w$  is the rate of heat flow per unit length from the wall to the solid-liquid interface and  $q'_c$  the rate of heat flow per unit length from the fin to the solid-liquid interface. The stored heat due to melting in a  $dx$  wide element is

$$E'_p = (\rho H) \frac{\partial S_y}{\partial t} dx \quad (12)$$

where  $S_y$  is the distance from the fin to the solid-liquid interface in the  $y$ -direction. The heat transfer from the fin to the solid-liquid interface is assumed to take place by convection and it is defined as

$$q'_c = h(T_f - T_m)dx. \quad (13)$$

The rate of heat flow per unit length from the constant temperature end-wall to the solid-liquid interface  $q'_w$  is determined by pure conduction through the liquid in the  $x$ -direction:

$$q'_w = \frac{k_l}{x}(T_w - T_m) \frac{\partial S_y}{\partial x} dx \quad (14)$$

Eq. (14) is a first order approximation and it leads to inaccuracies in the model because natural convection is not taken into account. However, according to Henze et al. [11] heat source  $q'_w$  is dominant only during the very early stages of melting before sufficient heat energy has diffused through the fin to provide the amount of heat which later becomes the dominant source of the melting phase change material. Therefore, the error made is assumed to be small.

Now the Eqs. (12)–(14) are substituted into the energy balance Eq. (11) and it can be rewritten with an initial condition as

$$(\rho H)_l \frac{\partial S_y}{\partial t} = \frac{k_l}{x}(T_w - T_m) \frac{\partial S_y}{\partial x} + h(T_f - T_m), \quad t > 0 \quad (15)$$

$$S_y(x, 0) = 0 \quad (16)$$

Eqs. (1)–(16) are solved mathematically to achieve an analytical solution for the problem.

## 2.3

### The heat transfer coefficient

The heat transfer coefficient has to be defined before the equations can be solved. In the beginning the heat transfer from the fin occurs mainly by conduction. The fin and the melting interface form a hot and a cold surface and there is a liquid phase change material between these surfaces. In such a situation natural convection may occur because of density gradients in the liquid. The horizontal cavity heated from below has been considered by researchers. With Rayleigh's number it is possible to determine the main heat transfer mode in the cavity. Rayleigh's number may be defined as

$$Ra = \frac{g(T_f - T_m)\rho_l^2 c_{pl} S_y^3 \beta}{\mu k_l} \quad (17)$$

where  $\beta$  is the expansion coefficient,  $\mu$  dynamic viscosity and  $g$  the acceleration of gravity.

When Rayleigh's numbers are below a critical value  $Ra_{cr}=1708$ , buoyancy forces cannot overcome the resistance imposed by viscous forces and there is no advection within the cavity heated from below. In this case the heat transferred from the bottom (hot) to the top (cold) surface occurs exclusively by conduction [12]. Based on Eq. (17) it is possible to determine the critical thickness  $S_{cr}$  of the phase change material when the prime heat transfer mode turns from conduction to natural convection:

$$S_{cr} = \left( \frac{1708 \mu k_l}{\beta g (T_f - T_m) \rho_l^2 c_{pl}} \right)^{1/3}. \quad (18)$$

For example, the critical thickness of melted n-octadecane paraffin is not more than 1.8 mm at a temperature difference of  $T_w - T_m = 20$  °C. In the semi-infinite medium conduction occurs only a very short time in the beginning of the melting process. Therefore, the assumption that heat is transferred from the fin to the solid-liquid interface exclusively by natural convection makes not big error in the model.

Marshall [13, 14] has experimentally investigated the influence of natural convection on the interface of PCM in a rectangular store containing paraffin. Marshall's results for different paraffins and different boundary conditions have been defined as

$$Nu = 0,072 Ra^{1/3} \quad (19)$$

where  $Nu$  is Nusselt's number  $Nu = hS/k_l$ . In this study, the horizontal fin is not isothermal. Therefore, the heat transfer coefficient is the function of the fin temperature  $T_f$ . However, Eftekhari et al. [10] have concluded in their study that the effect of the temperature difference ( $T_f - T_m$ ) on the heat transfer coefficient is relatively small. To simplify the problem the temperature of the fin  $T_f$  is assumed to be constant when heat transfer coefficient is

determined. In that case the logical choice for fin temperature is the mean value of the constant wall temperature and PCM's melting temperature:

$$T_f = \frac{T_w + T_m}{2}. \quad (20)$$

Thus, the heat transfer coefficient is possible to solve from Eqs. (19) and (20) and it is

$$h = 0,072 \left[ \frac{\left[ g \left( \frac{T_w - T_m}{2} \right) \rho_1^2 c_{pl} k_1^2 \beta \right]}{\mu} \right]^{1/3}. \quad (21)$$

Thus, the value of the heat transfer coefficient  $h$  is independent of position or time. It depends on the material properties of the PCM and the constant end-wall temperature. At small  $x$  values Eq. (21) underestimates the heat transfer coefficient and at large  $x$  values it overestimates the heat transfer coefficient.

### 3 Solution

#### 3.1 Region 1

In region 1 the interface melts only in the  $x$ -direction. The Stefan problem Eqs. (1)–(5) has a well-known analytical solution solved by Neumann [3]. The location of the solid-liquid interface in each time step can be solved from Eq. (22):

$$S_x(t) = 2\lambda\sqrt{\alpha_1 t} \quad (22)$$

where  $\lambda$  is a root of the transcendental equation

$$\lambda e^{\lambda^2} \operatorname{erf}(\lambda) = \frac{St_1}{\sqrt{\pi}} = \frac{c_{pl}(T_w - T_m)}{H\sqrt{\pi}}. \quad (23)$$

#### 3.2 Region 2

In region 2, the temperature distribution for the fin is solved. First Eqs. (7)–(10) for the fin will be simplified by using the following notation:

$$A = \frac{k_f}{(\rho c_p)_f} \quad (24)$$

$$B = \frac{h}{(\rho c_p)_f D} \quad (25)$$

$$C = BT_m. \quad (26)$$

Eq. (7) takes the form

$$\frac{\partial T_f}{\partial t} = A \frac{\partial^2 T_f}{\partial x^2} - BT_f - C, \quad t > 0 \quad (27)$$

A new variable  $u(x,t)$  is defined as

$$T_f(x,t) = \frac{u(x,t)}{e^{Bt}} + \frac{C}{B}, \quad t > 0 \quad (28)$$

Eq. (28) is inserted into Eqs. (27) and (8)–(10). The energy balance for the fin with the initial and boundary conditions can be rewritten as

$$\frac{\partial u}{\partial t} = A \frac{\partial^2 u}{\partial x^2}, \quad t > 0 \quad (29)$$

$$u(x, 0) = T_m - \frac{C}{B} = u_i(x, 0) \quad (30)$$

$$u(0, t) = e^{Bt} \left( T_w - \frac{C}{B} \right) = u_w(0, t). \quad (31)$$

The problem is a parabolic partial differential initial boundary value problem. The general solution for Eqs. (29)–(31) is [15]:

$$u(x,t) = \int_0^\infty [k(x-y,t) - k(x+y,t)] u_i(y, 0) dy + \int_0^t l(x,t-y) u_w(0, y) dy \quad (32)$$

where  $k(x,t)$  is a source solution

$$k(x,t) = \frac{e^{-x^2/4At}}{\sqrt{4A\pi t}} \quad (33)$$

and  $l(x,t)$  derived source solution

$$l(x,t) = \frac{x}{t} k(x,t). \quad (34)$$

When Eqs. (30)–(31) and (33)–(34) are substituted into Eq. (32) one gets

$$u(x,t) = (T_w - T_m) \left\{ e^{Bt - \sqrt{\frac{Bx^2}{A}}} - \frac{1}{2} e^{Bt - \sqrt{\frac{Bx^2}{A}}} \left( 1 - e^{2\sqrt{\frac{Bx^2}{A}}} + \operatorname{erf} \left[ \frac{x}{2\sqrt{At}} - \frac{\sqrt{Bx^2 t}}{x} \right] + e^{2\sqrt{\frac{Bx^2}{A}}} \operatorname{erf} \left( \frac{x}{2\sqrt{At}} + \frac{\sqrt{Bx^2 t}}{x} \right) \right) \right\} \quad (35)$$

Eq. (35) is inserted into Eq. (28) and finally the solution for the temperature distribution of the fin is

$$T_f(x,t) = \frac{(T_w - T_m) \left\{ e^{Bt - \sqrt{\frac{Bx^2}{A}}} - \frac{1}{2} e^{Bt - \sqrt{\frac{Bx^2}{A}}} \left( 1 - e^{2\sqrt{\frac{Bx^2}{A}}} + \operatorname{erf} \left[ \frac{x}{2\sqrt{At}} - \frac{\sqrt{Bx^2 t}}{x} \right] + e^{2\sqrt{\frac{Bx^2}{A}}} \operatorname{erf} \left( \frac{x}{2\sqrt{At}} + \frac{\sqrt{Bx^2 t}}{x} \right) \right) \right\}}{e^{Bt}} + \frac{C}{B}. \quad (36)$$

The energy balance for the PCM interface location in the  $y$ -direction (Eq. 15) is a first order partial differential equation. It can be rewritten as

$$-\frac{a}{x} \frac{\partial S_y}{\partial x} + b \frac{\partial S_y}{\partial t} = h(T_f - T_m), \quad t > 0 \quad (37)$$

$$S_y(x, 0) = 0 \quad (38)$$

where

$$a = k_1(T_w - T_m) \quad (39)$$

$$b = \rho_1 H. \quad (40)$$

The heat transfer coefficient can be determined with Eq. (21). Eqs. (37) and (38) can be solved with the method of characteristics for quasilinear equations [16]. The solution of the fin temperature distribution Eq. (36) is placed into Eq. (37) which gives for the interface location  $S_y$  in the  $y$ -direction

$$S_y(x,t) = h(T_f - T_m)x \left( \frac{-bx + \sqrt{(bx)^2 + 2abt}}{ab} \right). \quad (41)$$

The method of characteristics for quasilinear equations has some limitations when solving partial differential equations and in this case the solution is not valid when  $x$  approaches zero. However, when  $x$  approaches zero the Neumann solution  $S_x$  for interface location (Eq. (22)–(23)) is valid instead of Eq. (41).

In conclusion, the analytical solution for the heat transfer problem consists of two parts:

1. the Neumann solution for the solid–liquid interface  $S_x$  in the  $x$ -direction in region 1,
2. the derived analytical solution for solid–liquid interface location  $S_y$  in the  $y$ -direction and the temperature distribution of the fin  $T_f$  in the  $x$ -direction in region 2.

## 4 Results

### 4.1 The temperature distribution of the fin

The derived analytical solution for the temperature distribution of the fin is compared to the numerical solution to verify the accuracy of the analytical solution. A test case is chosen. In the test case the physical properties of laboratory grade pure n-octadecane paraffin have been used as initial values, because it has a relatively narrow melting region,  $T_m = 28$  °C. The fin is assumed to be aluminium. The physical properties of the fin and the phase change material are shown in Table 1 [11].

The temperature difference between the temperature of the wall and melting temperature of the PCM is set to be 20 °C. Thus, the wall temperature is 48 °C in the calculations. Initially, the fin and the PCM are in the melting temperature of the paraffin, 28 °C. The half thickness of the fin is assumed to be  $D=1$  mm.

**Table 1.** Physical properties of the aluminium fin and n-octadecane paraffin

Property	Fin	PCM (l)
Density ( $\rho$ ) kg m <sup>-3</sup>	2713	777
Heat conductivity ( $k$ ) Wm <sup>-1</sup> K <sup>-1</sup>	180	0.149
Heat capacity ( $c_p$ ) J kg <sup>-1</sup> K <sup>-1</sup>	960	2660
Latent heat of fusion ( $H$ ) J kg <sup>-1</sup>	–	241360
Melting temperature ( $T_m$ ) °C	–	28
Viscosity ( $m$ ) kg m <sup>-1</sup> s <sup>-1</sup>	–	0.00385
Thermal expansion coeff.( $b$ ) K <sup>-1</sup>	–	0.001

The equations of the fin temperature distribution with initial and boundary conditions Eqs. (7)–(10) are solved numerically. The numerical calculation is carried out with a program called FEMLAB which is a simulation package that solves systems of coupled non-linear partial differential equations and linear partial differential equations through the finite element method in one, two and three dimensions [17]. The derived analytical solution can be calculated from Eq. (36). The derived analytical and numerical results for the temperature distribution of the fin are shown in Fig. 3 when  $t=3600$  s.

The derived analytical results and the numerical results are very close to each other. Fig. 4 shows the temperature distribution of the fin with five different time steps.

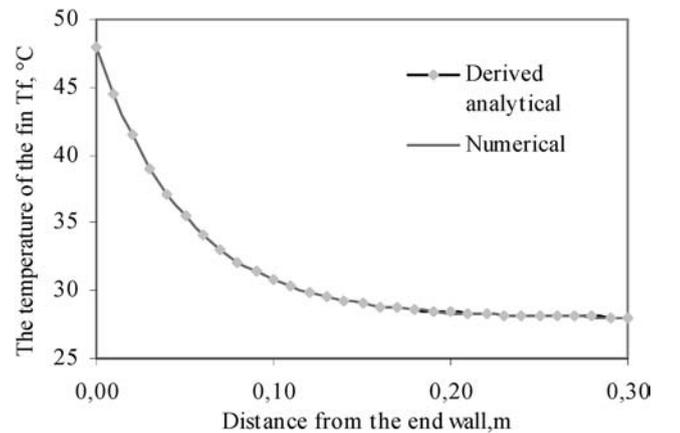
It can be seen that at these initial values the temperature of the fin stops changing at  $t=720$  s. It happens quite quickly because of the constant heat transfer coefficient value.

The thickness of the fin  $D$  has a big influence on the temperature distribution of the fin. Fig. 5 shows the temperature distribution of the fin at different thicknesses of the fin when  $t=3600$  s.

As was expected, the temperature of the fin increases when the thickness of the fin increases.

### 4.2 The location of solid–liquid interface

The Neumann solution for the solid–liquid interface location in region 1 is an exact solution and it has been proved in many references [2, 3, 4]. The derived analytical solution for the interface location should also be validated.



**Fig. 3.** The comparison of analytical and numerical results for the temperature distribution of the fin when  $t=3600$  s in a semi-infinite n-octadecane storage,  $T_w - T_m = 20$  °C

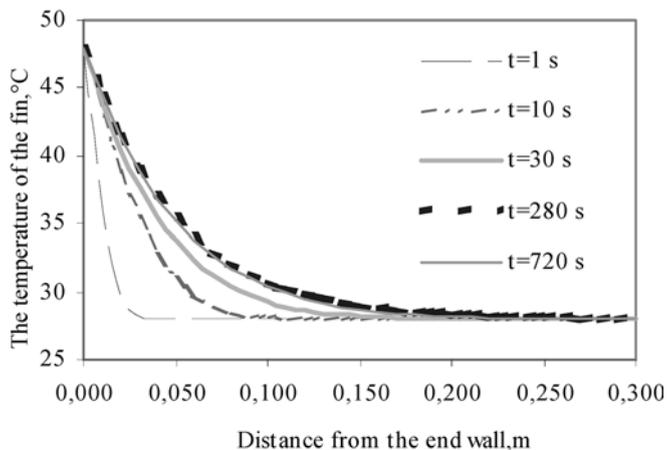


Fig. 4. The x-directional temperature distribution of the fin at different time steps in a semi-infinite n-octadecane storage,  $T_w - T_m = 20$  °C

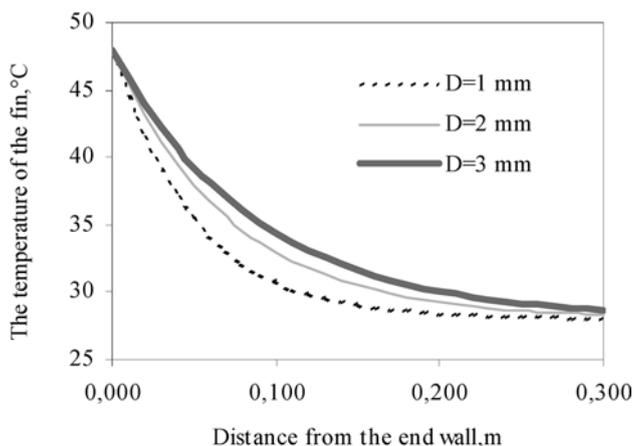


Fig. 5. The x-directional temperature distribution of the fin when  $t=3600$  s with different fin thicknesses in a semi-infinite n-octadecane storage,  $T_w - T_m = 20$  °C

This is done by comparing the derived analytical results to numerical results. The derived analytical results can be calculated with Eq. (41) and the numerical results with Femlab from Eqs. (15) and (16).

Fig. 6 shows the results for the solid-liquid interface location when  $t=3600$  s.

The numerical solution for the problem gives the same result as the derived analytical solution, except near the wall ( $x$  approaches zero). The values are not accurate near the wall because of the form of Eq. (37). For this reason, the Neumann solution Eqs. (22) and (23) is used for the interface location  $S_x$  in region 1 and the derived analytical solution Eq. (41) for the interface location  $S_y$  in region 2. Fig. 7 shows the location of both interfaces when  $t=280$  s.

The Neumann solution underestimates the speed of  $S_x$  because heat is assumed to transfer only by conduction in the  $x$ -direction. In reality, natural convection assists the melting of the phase change material and the real solid-liquid interface is ahead of the interface achieved with the Neumann solution.

The analytical solution  $S_y$  is not valid at small  $x$  values. However, when time increases, the error made decreases.

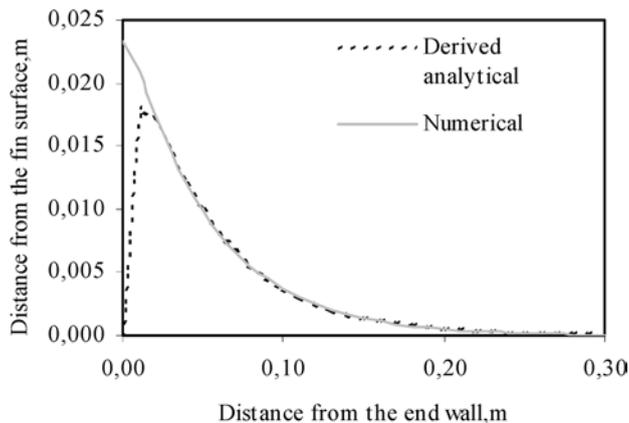


Fig. 6. The comparison of analytical and numerical results for a solid-liquid interface location when  $t=3600$  in semi-infinite n-octadecane storage,  $T_w - T_m = 20$  °C

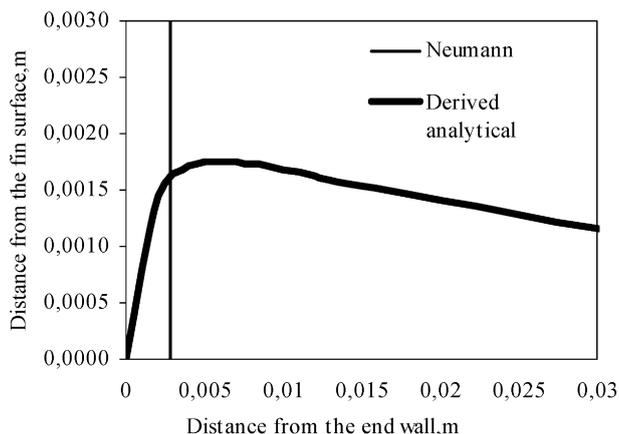


Fig. 7. The location of the solid-liquid interface when  $t=280$  s in a semi-infinite n-octadecane storage,  $T_w - T_m = 20$  °C

This can be seen from Fig. 8 which shows the solid-liquid interface location after  $t=7200$  s.

The moving speed of the solid-liquid interface achieved with the analytical solution is also underestimated because natural convection is taken into account only in the  $y$ -direction. However, the effect is negligible when the influences of other assumptions are also taken into account.

## 5 Comparison between basic heat transfer modes

As we know there are three stages in the melting process near the fin: first, pure conduction from the end-wall and the fin, then conduction from the fin with some natural convection from the end-wall and finally, only natural convection from the fin. It was assumed in the model that the main heat transfer mode from the fin to the solid-liquid interface was natural convection. Thus, it is important to study whether the model is behaving logically. In order to analyse the effects of the main heat transfer modes, the following computational analysis is carried out. The material properties and initial values used in the computation are the same as in the previous subsection (See Table 1).

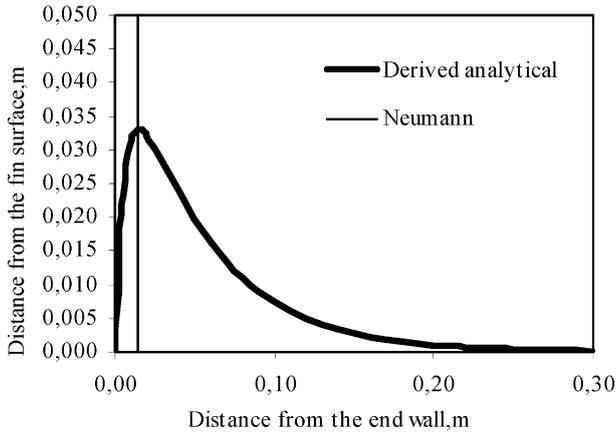


Fig. 8. The location of the solid-liquid interface when  $t=7200$  s in a semi-infinite n-octadecane storage,  $T_w-T_m=20$  °C

### Case 1. Conduction+natural convection from the fin

In the analytical model, the heat transfer from the fin to the solid-liquid interface is assumed to occur only by natural convection. To study the error made with this assumption, conduction in the early stage is taken into account. Eqs. (7)–(16) are solved numerically. The heat transfer coefficient is determined with Rayleigh's number in the following way:

$$Nu = \frac{hS}{k_l} = 1, \quad Ra \leq 1708 \quad (42)$$

$$Nu = \frac{hS_y}{k_l} = 0,072Ra^{1/3} \\ = 0,072 \left[ \frac{[g(T_f - T_m)\rho_l^2 c_{pl} S_y^3 \beta]}{\mu k_l} \right]^{1/3}, \quad Ra > 1708 \quad (43)$$

The solid-liquid interface location in case 1 is shown in Fig. 9 when  $t=3600$  s.

### Case 2. Pure conduction from the fin

The heat transfer by pure conduction from the fin to the solid-liquid interface is also investigated to find out how the solid-liquid interface behaves in the pure conduction case when natural convection is not taken into account. The energy balance for the fin is defined with initial and boundary conditions as

$$(\rho c_p)_f D \frac{\partial T_f}{\partial t} = k_f D \frac{\partial^2 T_f}{\partial x^2} - \frac{k_l}{S_y} (T_f - T_m), \quad t > 0 \quad (44)$$

$$T_f(x, 0) = T_m \quad (45)$$

$$T_f(0, t) = T_w \quad (46)$$

$$T_f(\infty, t) = T_m \quad (47)$$

and the energy balance for the solid-liquid interface as

$$(\rho H)_l \frac{\partial S_y}{\partial t} = \frac{k_l}{x} (T_w - T_m) \frac{\partial S_y}{\partial x} + \frac{k_l}{S_y} (T_f - T_m), \quad t > 0 \quad (48)$$

$$S_y(x, 0) = 0. \quad (49)$$

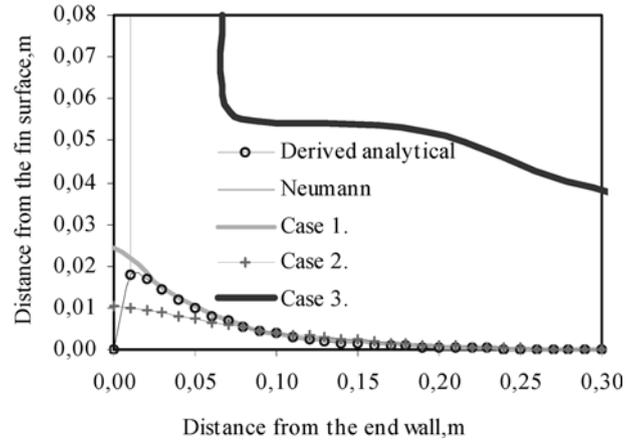


Fig. 9. The results of case studies 1-3 and the analytical solution when  $t=3600$  s in a semi-infinite n-octadecane storage,  $T_w-T_m=20$  °C

Case 2 is solved numerically and the solid-liquid interface location when  $t=3600$  s is shown in Fig. 9.

### Case 3. Solid material with a fin without phase change

The temperature distribution in the solid semi-infinite storage with a fin is examined. Heat transfers by pure conduction in the solid material. The location of the initial temperature interface in which the temperature of the solid material differs from its initial temperature is determined. The speed of the solid-liquid interface calculated with the derived analytical model should be much slower than the speed of the initial temperature interface calculated in this case because there is no phase change in the solid material. The solid material is assumed to have the same physical properties as the liquid phase change material  $\alpha_l = \alpha_s$  (see Table 1). The equations for the fin are the following:

$$\left( \frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) = \frac{1}{\alpha_f} \frac{\partial T_f}{\partial t}, \quad t > 0 \quad (50)$$

$$T_f(x, y, 0) = T_m \quad (51)$$

$$T_f(0, y, t) = T_w \quad (52)$$

$$T_f(x, 0, t) = T_s(x, 0, t) \quad (53)$$

and for the material

$$\left( \frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} \right) = \frac{1}{\alpha_l} \frac{\partial T_s}{\partial t}, \quad t > 0 \quad (54)$$

$$T_s(x, y, 0) = T_m \quad (55)$$

$$T_s(0, y, t) = T_w \quad (56)$$

$$T_s(x, 0, t) = T_f(x, 0, t). \quad (57)$$

The initial temperature interface location in case 3 is shown in Fig. 9 when  $t=3600$  s. The results of the analytical solutions including the Neumann solution and the derived analytical solution are also shown in Fig. 9.

In case 1, where heat transfers from the fin to the solid-liquid interface by conduction when  $Ra \leq 1708$  and with natural convection when  $Ra > 1708$ , the solid-liquid interface  $S_y$  is slightly ahead of the interface of the analytical solution. With these initial values the depth of the critical layer  $S_{cr}$  is only 1.83 mm for n-octadecane. Below this value the prime heat transfer mode is conduction. At small  $S_y$  values the heat transfer coefficient  $h = k_l/S_y$  is relatively large. That is the reason for the solid-liquid interface location being slightly ahead of the derived analytical solution. The assumption that the prime heat transfer mode is natural convection is justified. The error made is small.

In case 2, where heat transfers from the fin to the solid-liquid interface by pure conduction, the interface location  $S_y$  is much behind the derived analytical solution. Natural convection enhances heat transfer and accelerates melting. Therefore, it should be taken into consideration. Otherwise the model considerably underestimates the solid-liquid interface location.

In case 3, the heat from the fin and the constant temperature end-wall is conducted inside the material without phase change. It is obvious that the initial temperature interface in which the material's temperature differs from its initial temperature is situated substantially ahead of the solid-liquid interface of the derived analytical solution because there is no effect of latent heat of fusion.

## 6 Conclusions

This paper presents a simplified analytical model based on a quasi-linear, transient, thin-fin equation which predicts the solid-liquid interface location and temperature distribution of the fin in a melting process with a constant imposed end-wall temperature in a semi-infinite PCM storage. The analytical solution consists of the well-known Neumann solution which predicts the solid-liquid interface location  $S_x$  in the x-direction as well as the derived analytical solution which predicts the temperature distribution of the fin  $T_f$  in the x-direction and the solid-liquid interface location  $S_y$  in the y-direction. Some simplifying assumptions are made to make it possible to find an analytical solution.

The analytical solution is compared with numerical results using same initial and boundary values and material properties. The analytical model is also compared with three different cases to predict the influences of the basic heat transfer modes on the results. The following conclusions are made:

1. The Neumann solution is an exact solution. However, it underestimates the location of the interface  $S_x$  because the heat is assumed to transfer only by conduction in the x-direction and natural convection is assumed to be negligible.
2. The derived analytical solution for the solid-liquid interface location gives satisfactory results, except when  $x$  approaches zero the solution for the solid-liquid interface location  $S_y$  is not valid because of the

method used in solving the energy balance of the interface. The derived analytical solution also underestimates the location of the solid-liquid interface because the heat is assumed to transfer only by natural convection in the y-direction and conduction is assumed to be negligible. However, the assumption that the prime heat transfer mode from the fin to the solid-liquid interface is natural convection is justified. Conduction in the beginning of the process has a very small effect on the liquid-solid interface location. Despite all assumptions the derived analytical solution gives fairly satisfactory results for the solid-liquid interface location.

3. The derived analytical solution for the temperature distribution of the fin gives satisfactory results. The heat transfer coefficient from the fin to the solid-liquid interface is assumed to be constant. In reality it is a function of the temperature difference  $T_f - T_m$ . At small  $x$  values the value of the heat transfer coefficient is too low and at large  $x$  values it is too high. Thus the solid-liquid interface location is also underestimated at small  $x$ -values and overestimated at large  $x$  values. However, the error made is small.
4. The model is not suitable for the solidification process because the prime heat transfer mode in solidification is conduction, not natural convection. A similar analysis should be carried out for the solidification process.

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