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Cooper pair coherence in a superfluid Fermi gas of atoms

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Abstract

We study the coherence properties of a trapped two-component gas of fermionic atoms below the BCS critical temperature. We propose an optical method to investigate the Cooper pair coherence across different regions of the superfluid. Near-resonant laser light is used to induce transitions between the two coupled hyperfine states. The beam is split so that it probes two spatially separate regions of the gas. Absorption of the light in this interferometric scheme depends on the Cooper pair coherence between the two regions.

1. Introduction

The coherence properties of clouds of trapped bosonic atoms have already been the focus of several interesting experiments. It has been convincingly demonstrated that the single-component Bose–Einstein condensate is coherent on the time-scale of most experiments [1], while for two-state fluids the intra- and inter-component coherence survives on remarkably long time-scales despite external manipulations which displace the centres of mass of the two fluids spatially with respect to each other [2].

In the case of trapped two-state Fermi gases with negative scattering length, the existence of a BCS transition has been predicted [3]. By now, intensive experimental work on trapping and cooling fermionic atoms [4] with lower and lower fractions of Fermi temperature being reported and the success in trapping mixtures of bosons and fermions as well as two states of the same fermionic atom allow most of the researchers to be optimistic that the BCS critical temperature will soon be reached.

The main candidates for atoms to undergo the BCS transition are ^{40}K and ^{6}Li [4]. For ^{6}Li, the states $|5\rangle$ and $|6\rangle$ as defined in [5] have an anomalously large negative s-wave scattering length and thus a relatively high critical temperature. They also have a small

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enough decay rate in the region of mechanical stability, which gives the system a long enough lifetime to be experimentally interesting [5]. It is also assumed that the densities of the two components are close enough so that below the critical temperature one has the usual s-wave BCS coupling between the two states. We will denote the two states involved in pairing by $|\downarrow\rangle$ and $|\uparrow\rangle$. The BCS transition is connected with the appearance of an order parameter [6]

$$\Delta(\vec{r}) = -\frac{4\pi\hbar^2 a}{m}\langle\hat{\psi}_\uparrow(\vec{r})\hat{\psi}_\downarrow(\vec{r})\rangle.$$

The ultraviolet divergences associated with using a contact interaction pseudopotential can be dealt with in the case of metallic superconductors by introducing a cut-off at the Debye energy, and in the case of trapped atoms by other renormalization schemes, such as pseudopotentials regularized with the operator $\partial_r[r\times]$ [7]. The properties of the order parameter of cold fermionic gases in the superfluid phase have been studied in the regime close to the critical temperature, where it satisfies a Ginzburg–Landau equation [8].

In this paper we study the spatial coherence and pairing properties of the BCS paired two-state Fermi gas at low temperatures, when the GL treatment is not applicable but instead one needs to use the Bogoliubov–de Gennes formalism [6]. We propose a method for checking experimentally the coherence of Cooper pairs and measuring the Cooper pair size. The method is based on breaking some of the Cooper pairs by driving transitions between the states $|\uparrow\rangle$ and $|\downarrow\rangle$ with nearly resonant light. The laser beam is split and the two resulting beams are focused on two spatially separated regions of the condensate, after which they are merged together as in a typical interferometry experiment. If both the laser and the condensate coherence are preserved between the two regions, interference should have an effect on the absorption of the light. We show that this is indeed the case and the existence of interference contributions for a given beam separation shows that the gas has coherence on that length scale.

Interaction of the superfluid gas with laser light has been considered previously as a method to probe the BCS transition of the gas [9, 10]. In these proposals the paired states were coupled to some unpaired states of the atoms, and in the proposals [9] the light was mainly thought of as being off-resonant. Here we consider laser-induced transitions directly between the two states that are paired. The light is nearly resonant and assumed to be absorbed in the pair breaking process. Moreover, the interferometric configuration in our proposal allows one to probe coherences, not only the existence of the superconducting gap.

We first describe the proposed method in section 2, and then derive the absorption rate for a general laser geometry in section 3. The absorption rate and the effect of coherences in the interferometric set-up are considered in section 4. The appendix discusses coherence length scales in the BSC Green's functions.

2. The idea

The basic idea of coupling the two paired states using a laser is depicted in figure 1. The two states $|\uparrow\rangle$ and $|\downarrow\rangle$ are typically hyperfine ground states, thus one would actually use a Raman transition to couple them. In the following we will, however, consider just laser light with one Rabi frequency and detuning—in the case of a Raman transition these can be understood as effective quantities. Likewise, figure 1 also shows only one laser driving the transition.

One may make an educated guess that the laser has to have a finite detuning δ to drive the transition. This is because one has to break the Cooper pair, i.e. to provide extra energy at

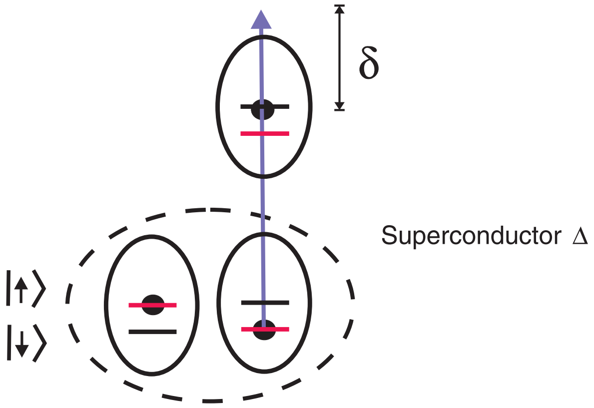

Figure 1. Laser coupling between the two paired states. Atoms in two different hyperfine states $|\uparrow\rangle$ and $|\downarrow\rangle$ have a negative s-wave scattering length and form a Cooper pair associated with the appearance of the superconductor order parameter (gap) Δ. The laser couples the states $|\uparrow\rangle$ and $|\downarrow\rangle$, the frequency of the laser is detuned from the atomic transition frequency by an amount δ. In order for transfer of the atom from one state to the other to occur the Cooper pair has to be broken and the atoms become excitations in the superfluid. Note that in a real experiment a Raman scheme would be used instead of a single laser beam.

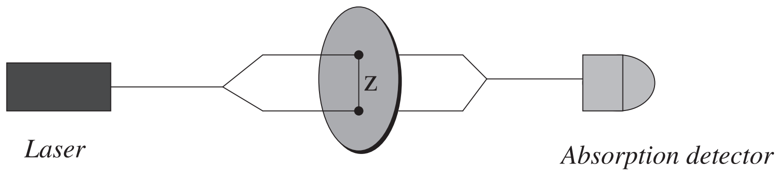

Figure 2. Schematic picture of the interferometric set-up for probing coherences. The initial laser beam is split coherently and focused on two spatial regions of the gas with separation z. After the beams have passed through the gas they are recombined again and the amount of absorption is measured.

least of twice the gap energy in order to transfer an atom from, say, state $|\downarrow\rangle$ to state $|\uparrow\rangle$, which makes both of the pairing partners have the same internal state and thus become excitations in the superfluid. Our calculations show that this extra amount of energy is indeed required.

The scheme for probing spatial coherence is shown in figure 2. The laser beam is split and focused into two regions with separation z. After this they have to be recombined, or detected non-selectively—this prevents us from gaining information on which of the beam's light was absorbed. This lack of information is essential for any interference phenomena to occur. Below we calculate, first for a general laser configuration, the transfer rate of atoms from one state to another, which corresponds directly to the rate of absorption. Then we demonstrate the idea by considering a specific interferometric configuration.

3. The absorption rate—general case

In the rotating-wave approximation the interaction of the laser light with the matter fields can be described by a time-independent Hamiltonian in which the detuning δ plays the role of an externally imposed difference in the chemical potential of the two states. The total Hamiltonian then becomes $\hat{H} = \hat{\tilde{H}} + \hat{H}_{\rm T}$, where

$$\hat{\tilde{H}} = \hat{H}_{\rm BCS} + \left(\mu + \frac{\delta}{2}\right) \int {\rm d}\vec{r}\, \hat{\psi}_{\uparrow}^{\dagger}(\vec{r})\hat{\psi}_{\uparrow}(\vec{r}) + \left(\mu - \frac{\delta}{2}\right) \int {\rm d}\vec{r}\, \hat{\psi}_{\downarrow}^{\dagger}(\vec{r})\hat{\psi}_{\downarrow}(\vec{r}). \quad (1)$$

Here μ is the chemical potential of the Fermi gas before the laser was turned on, which is the same for both components in order to allow standard BCS pairing. The Hamiltonian $\hat{H}_{\rm BCS}$ is the BCS approximation of the matter Hamiltonian with the chemical potential included [6]. The transfer Hamiltonian is given by

$$\hat{H}_{\rm T} = \int {\rm d}\vec{r}\, \Omega(\vec{r})\hat{\psi}_{\uparrow}^{\dagger}(\vec{r})\hat{\psi}_{\downarrow}(\vec{r}) + \Omega^*(\vec{r})\hat{\psi}_{\downarrow}^{\dagger}(\vec{r})\hat{\psi}_{\uparrow}(\vec{r}) \quad (2)$$

with $\Omega(\vec{r})$ characterizing the local strength of the matter–field interaction. We consider that at the time $-\infty$ the system was in its BCS ground state, $\Psi_{\rm BCS}$. Then the laser field has been turned on. The intensity of the electromagnetic field is small enough for the transfer Hamiltonian $\hat{H}_{\rm T}$ to be just a perturbation on $\hat{H}_{\rm BCS}$.

The main observable of interest is the rate of transferred atoms from, say, state $\downarrow$ to state $\uparrow$. This also corresponds directly to the absorption of the light. It is defined as

$$I_{\uparrow} = \frac{\partial}{\partial t} \int {\rm d}\vec{r}\, \langle \Psi(t)|\hat{\psi}_{\uparrow}^{\dagger}(\vec{r})\hat{\psi}_{\uparrow}(\vec{r})|\Psi(t)\rangle \quad (3)$$

and can be further evaluated with the help of the Schrödinger equation

$$ {\rm i}\hbar \frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

as

$$I_{\uparrow} = {\rm i} \int {\rm d}\vec{r}\, \langle \Psi(t)|\Omega^*(\vec{r})\hat{\psi}_{\downarrow}^{\dagger}(\vec{r})\hat{\psi}_{\uparrow}(\vec{r}) - \Omega(\vec{r})\hat{\psi}_{\uparrow}^{\dagger}(\vec{r})\hat{\psi}_{\downarrow}(\vec{r})|\Psi(t)\rangle. \quad (4)$$

In the following we call $I_{\uparrow}$ the current in analogy to metallic superconductors where the flux of electrons out of the superconductor constitutes the electrical current.

We introduce an interaction representation with respect to $\hat{\tilde{H}}$ and use linear response theory with respect to $\hat{H}_{\rm T}$ [11]. Validity of the linear response theory requires that the laser intensity is small and the transfer of atoms can be treated as a perturbation. This also implies that the number of atoms transferred from one state to another is not so large that it would severely imbalance the chemical potentials of the two species of atoms and thus break the BCS state. In order to simplify the notation we denote

$$\hat{O}(t) = {\rm e}^{{\rm i}\hat{H}_{\rm BCS}t}\, \hat{O}{\rm e}^{-{\rm i}\hat{H}_{\rm BCS}t} \quad (5)$$

where $\hat{O}$ is any operator. The current becomes

$$I_{\uparrow} = \int {\rm d}\vec{r} \int_{-\infty}^{t} \langle \Psi_{\rm BCS}|\big[\hat{j}(\vec{r},t), \hat{H}_{\rm T}(t')\big]|\Psi_{\rm BCS}\rangle. \quad (6)$$

Here, for example,

$$\hat{H}_{\rm T}(t) = \int {\rm d}\vec{r}\, \Omega^*(\vec{r})\,{\rm e}^{-{\rm i}\delta t}\hat{\psi}_{\downarrow}^{\dagger}(\vec{r},t)\hat{\psi}_{\uparrow}(\vec{r},t) + \Omega(\vec{r})\,{\rm e}^{{\rm i}\delta t}\hat{\psi}_{\uparrow}^{\dagger}(\vec{r},t)\hat{\psi}_{\downarrow}(\vec{r},t)$$

$$\hat{j}(\vec{r},t) = \Omega^*(\vec{r})\,{\rm e}^{-{\rm i}\delta t}\hat{\psi}_{\downarrow}^{\dagger}(\vec{r},t)\hat{\psi}_{\uparrow}(\vec{r},t) - \Omega(\vec{r})\,{\rm e}^{{\rm i}\delta t}\hat{\psi}_{\uparrow}^{\dagger}(\vec{r},t)\hat{\psi}_{\downarrow}(\vec{r},t)$$

are the transfer Hamiltonian and the current operator in the representation (5). We introduce the operator $\hat{A}(\vec{r},t) = \Omega^*(\vec{r})\hat{\psi}_\downarrow^\dagger(\vec{r},t)\hat{\psi}_\uparrow(\vec{r},t)$; using this the current of atoms to the $\uparrow$ state becomes

$$I_\uparrow = \int_{-\infty}^{t} \mathrm{d}t' \left\{ \mathrm{e}^{\mathrm{i}\delta(t'-t)} \int \mathrm{d}\vec{r} \int \mathrm{d}\vec{r}\,' \langle \Psi_{\mathrm{BCS}} | \big[\hat{A}(\vec{r},t), \hat{A}^\dagger(\vec{r}\,',t') \big] | \Psi_{\mathrm{BCS}} \rangle \right. \tag{7}$$

$$\left. - \mathrm{e}^{\mathrm{i}\delta(t-t')} \int \mathrm{d}\vec{r} \int \mathrm{d}\vec{r}\,' \langle \Psi_{\mathrm{BCS}} | \big[\hat{A}^\dagger(\vec{r},t), \hat{A}(\vec{r}\,',t') \big] | \Psi_{\mathrm{BCS}} \rangle \right\}. \tag{8}$$

This expression can be calculated by introducing a Matsubara correlation function

$$X(\mathrm{i}\omega) = -\int_0^\beta \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} \int \mathrm{d}\vec{r} \int \mathrm{d}\vec{r}\,' \langle \mathcal{T}_\tau \hat{A}(\vec{r},\tau)\hat{A}^\dagger(\vec{r}\,',0)\rangle \tag{9}$$

which gives the rate of transfer by $I_\uparrow = -2\,\mathrm{Im}\big[X_{\mathrm{ret}}(-\delta,\vec{r})\big]$ with the retarded correlation function defined by $X_{\mathrm{ret}}(\omega) \overset{\mathrm{i}\omega\to\omega+\mathrm{i}\epsilon}{=} X(\mathrm{i}\omega)$. In the spirit of Wick's theorem, we split the Matsubara function into a part which contains only the superconductor Green's function $\mathcal{G}$ and a part which contains only the anomalous superconductor Green's function $\mathcal{F}$: $X = X_{\mathcal{F}} + X_{\mathcal{G}}$,

$$X_{\mathcal{G}}(\mathrm{i}\omega) = \int_0^\beta \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} \int \mathrm{d}\vec{r} \int \mathrm{d}\vec{r}\,' \,\Omega^*(\vec{r})\Omega(\vec{r}\,')\mathcal{G}(\vec{r}\,',0;\vec{r},\tau)\mathcal{G}(\vec{r},\tau;\vec{r}\,',0) \tag{10}$$

$$X_{\mathcal{F}}(\mathrm{i}\omega) = \int_0^\beta \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} \int \mathrm{d}\vec{r} \int \mathrm{d}\vec{r}\,' \,\Omega^*(\vec{r})\Omega(\vec{r}\,')\mathcal{F}^\dagger(\vec{r},\tau;\vec{r}\,',0)\mathcal{F}(\vec{r},\tau;\vec{r}\,',0). \tag{11}$$

After summation over Matsubara frequencies, and choosing zero temperature and $\delta > 0$, we obtain $I_\uparrow = I_{\uparrow,\mathcal{G}} + I_{\uparrow,\mathcal{F}}$ with

$$I_{\uparrow,\mathcal{G}} = -2\pi \sum_{n,m} \left| \int \mathrm{d}\vec{r}\, \Omega(\vec{r}) v_n(\vec{r}) u_m(\vec{r}) \right|^2 \delta(\epsilon_n + \epsilon_m - \delta) \tag{12}$$

$$I_{\uparrow,\mathcal{F}} = 2\pi \sum_{n,m} \int \mathrm{d}\vec{r}\, \mathrm{d}\vec{r}\,' \,\Omega^*(\vec{r})\Omega(\vec{r}\,') u_n^*(\vec{r}) u_m(\vec{r}\,') v_m^*(\vec{r}) v_n(\vec{r}\,') \delta(\epsilon_n + \epsilon_m - \delta). \tag{13}$$

Here the triplet (u_n, v_n); ϵ_n is a solution of the (non-uniform) Bogoliubov–de Gennes equations [6]

$$\epsilon_n u_n(\vec{r}) = \left[\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r}) + U(\vec{r}) - \mu \right] u_n(\vec{r}) + \Delta(\vec{r}) v_n(\vec{r}) \tag{14}$$

$$-\epsilon_n v_n(\vec{r}) = \left[\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r}) + U(\vec{r}) - \mu \right] v_n(\vec{r}) - \Delta^*(\vec{r}) u_n(\vec{r}). \tag{15}$$

Here $V(\vec{r})$ is the external potential and

$$U(\vec{r}) = \frac{4\pi\hbar^2 a}{m} \langle \hat{\psi}_\uparrow^\dagger(\vec{r})\hat{\psi}_\uparrow(\vec{r})\rangle = \frac{4\pi\hbar^2 a}{m} \langle \hat{\psi}_\downarrow^\dagger(\vec{r})\hat{\psi}_\downarrow(\vec{r})\rangle$$

is the Hartree field. Given the potential $V(\vec{r})$ one can solve the Bogoliubov–de Gennes equations and insert the results into (12) and (13).

From equations (12) and (13) one can immediately see that given a non-trivial spatial dependence of the laser profile $\Omega(\vec{r})$, the BCS coherence, i.e., the spatial coherence of $v_n(\vec{r})u_m(\vec{r})$ on different length scales, will have an influence on the current. In the following section we demonstrate this by a simple example.

4. The absorption rate—interferometric case

We consider the set-up of figure 2, where the laser beam is split and focused in the z-direction but homogeneous in the other two directions, that is $\Omega(\vec{r}) = \Omega\,[\delta(z - z_1) + \delta(z - z_2)]$. This gives a stronger z dependence than focusing into two points, and is probably also simpler experimentally.

In order to derive instructive results in closed form we consider the case of a uniform gas. This corresponds to working in a region of space much smaller than the oscillator length of the trap, or having a shallow trap where confinement effects are small. Solutions of the Bogoliubov–de Gennes equations are given in this case by

$$u_k(\vec{r}) = u_k \mathrm{e}^{\mathrm{i}k\vec{r}} \qquad u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) \tag{16}$$

$$v_k(\vec{r}) = v_k \mathrm{e}^{\mathrm{i}k\vec{r}} \qquad v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) \tag{17}$$

$$E_k = \sqrt{\xi_k^2 + \Delta^2} \qquad \xi_k = \frac{\hbar^2 k^2}{2m} - \mu. \tag{18}$$

Cylindrical coordinates and momenta are used and the sums are transformed into integrals.

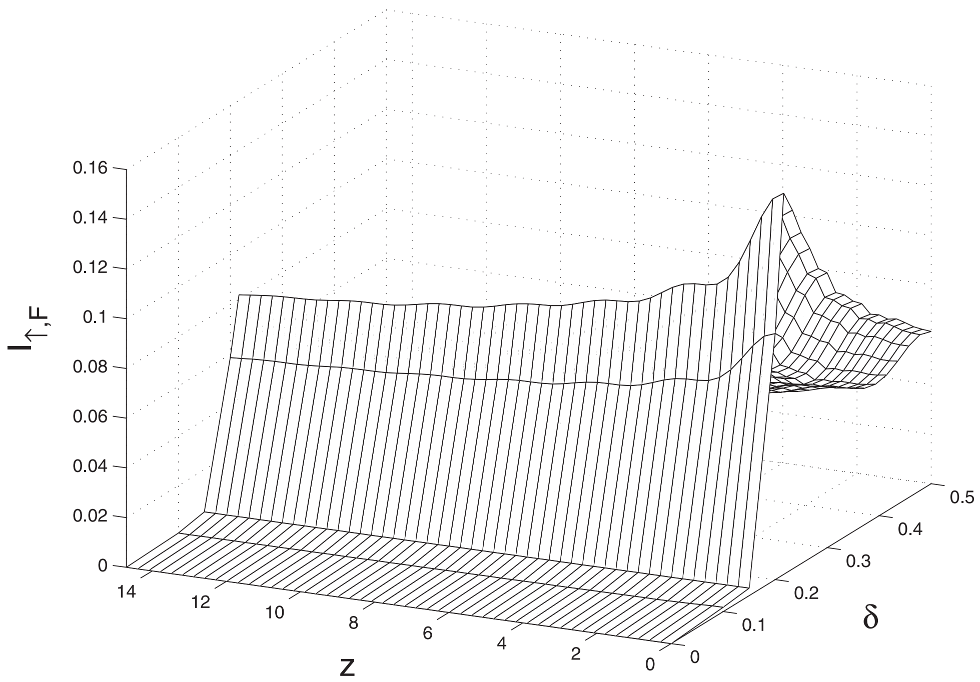

Figure 3. The current $I_{\uparrow,\mathcal{F}}$ as a function of the separation z and the detuning δ. The current oscillates with z, with a period of $\sim 2 \sim x_{\mathrm{CP}}/3$. The current is peaked around $\delta = 0.2 = 2\Delta$. Here the Fermi energy $\mu = 1$ and the gap $\Delta = 0.1$. All the variables are dimensionless. For instance, for the particle number $N = 10^5$ and the trap frequency $\Omega = 2\pi \times 150$ Hz the scales would be $[z] \sim 0.3\ \mu$m and $[\delta] \sim 75$ kHz.

The currents (12) and (13) become

$$I_{\uparrow,\mathcal{G}} = -\frac{|\Omega|^2}{2}\int u_k^2 v_q^2 \left[1+\cos(k_z+q_z)z\right]\delta(E_q+E_k-\delta) \tag{19}$$

$$I_{\uparrow,\mathcal{F}} = \frac{|\Omega|^2}{2}\int u_k v_k u_q v_q \left[1+\cos(k_z+q_z)z\right]\delta(E_q+E_k-\delta). \tag{20}$$

Here

$$k=\sqrt{\rho^2+k_z^2} \qquad q=\sqrt{\rho^2+q_z^2} \qquad z=z_1-z_2$$

and the integral symbol denotes

$$\int \equiv \int_0^\infty \mathrm{d}\rho\,\rho \int_0^\infty \mathrm{d}k_z \int_0^\infty \mathrm{d}q_z.$$

One can investigate how the absence of coherence affects the results by attaching to $u_k(\vec{r})$ and $v_k(\vec{r})$ phase factors which describe random space- and time-dependent fluctuations and by averaging over them. This makes the cosine-dependent term in $I_{\uparrow,\mathcal{G}}$, as well as the whole current $I_{\uparrow,\mathcal{F}}$, disappear. Thus one can vary z and see whether the current also varies, if the current becomes constant for large enough z one knows that oscillating terms are absent, i.e. coherence is not preserved on that length scale.

The currents $I_{\uparrow,\mathcal{G}}$ and $I_{\uparrow,\mathcal{F}}$ are presented in figures 3 and 4. In the numerics the δ-functions in (19) and (20) have been replaced by Lorentzians of width 0.01 to describe the finite laser linewidth. It is interesting to compare the variation of the currents as a function of z to the typical length scale of Cooper pairing, the Cooper pair size x_{CP}. The pair size

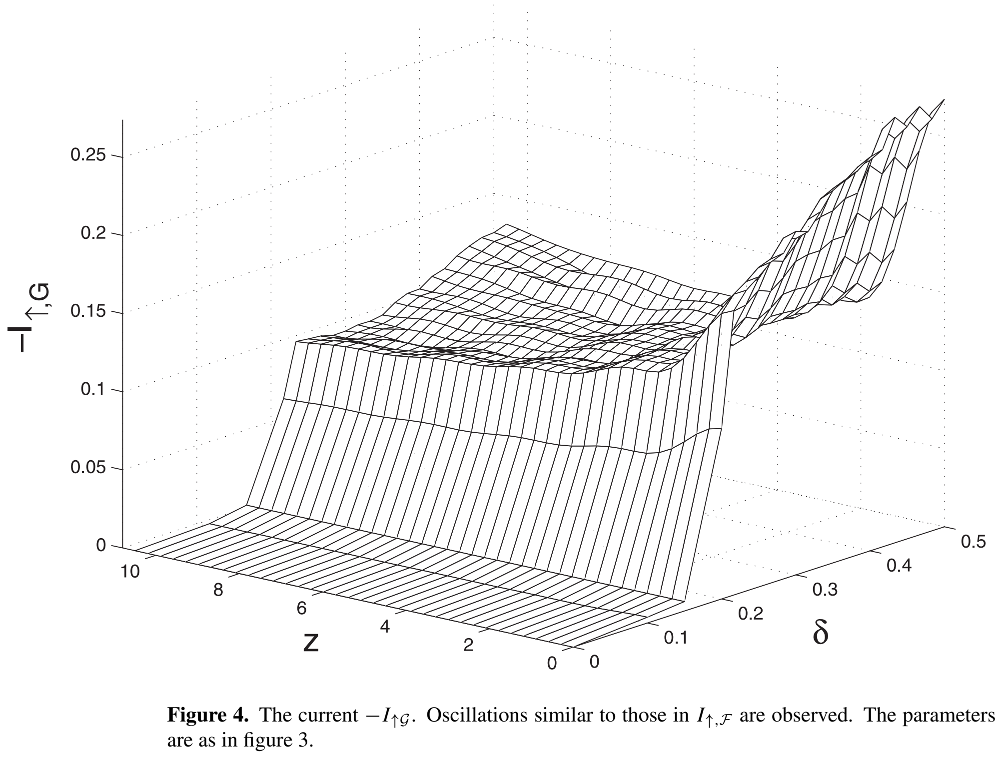

Figure 4. The current $-I_{\uparrow\mathcal{G}}$. Oscillations similar to those in $I_{\uparrow,\mathcal{F}}$ are observed. The parameters are as in figure 3.

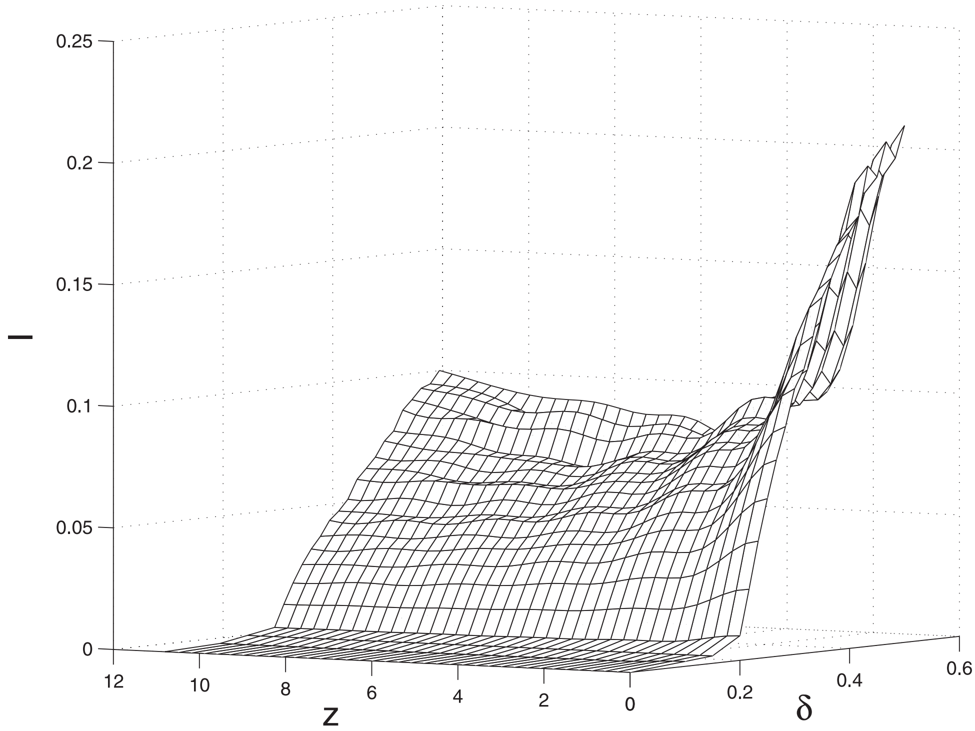

Figure 5. The total current $I = |I_{\uparrow\mathcal{G}} + I_{\uparrow,\mathcal{F}}|$. Oscillations are observed as in figures 3 and 4.

can be estimated simply by the momentum–coordinate uncertainty relation, the uncertainty in momentum being determined by the corresponding uncertainty in energy (the gap). This gives $x_{\rm CP} = (1/\Delta)\sqrt{\mu/2m}$. In the case of figures 3 and 4, $x_{\rm CP} \sim 7$. One observes oscillations occurring with approximately the period $\Delta z \sim 2$, which is of the same order of magnitude as $x_{\rm CP}$. For $z > x_{\rm CP}$ the current gradually becomes constant. The total current $I_{\uparrow,\mathcal{G}} + I_{\uparrow,\mathcal{F}}$ is shown in figure 5. Exactly at $\delta = 2\Delta$ the currents $I_{\uparrow,\mathcal{G}}$ and $I_{\uparrow,\mathcal{F}}$ cancel each other (this would be different in a non-homogeneous case) but for larger values of detuning the current is non-zero. The oscillations are visible.

Another length scale involved is the Fermi length $x_{\rm F}$ given by $x_{\rm F} = 1/\sqrt{2m\mu}$. The relationship to the Cooper pair size is typically $x_{\rm CP} \gg x_{\rm F}$, in the case of figures 3 and 4 $x_{\rm F} \sim 0.7 = 0.1x_{\rm CP}$. In physical units, $x_{\rm CP} \sim 0.3\ \mu$m and $x_{\rm F} \sim 0.03\ \mu$m in a system with particle number $N \sim 10^7$ and trap frequency $\Omega = 2\pi \times 150$ Hz (and the gap $\Delta = 0.1E_{\rm F}$). If $N \sim 10^5$, the length scales are $x_{\rm CP} \sim 2$ and $x_{\rm F} \sim 0.2\ \mu$m. Thus for realistic experiments, the Fermi length will always be below the diffraction limit of light and cannot be resolved. This is the case for $x_{\rm CP}$ too for very high particle numbers and/or trap frequencies. However, for moderate densities, which is expected to be the situation in the first experiments, the Cooper pair size may be of the order of several micrometres which can be resolved by light, in principle.

Note that the detuning δ has greater than twice the gap to produce any current at all (there is some current below 2Δ because the δ-functions in (19) and (20) were replaced by Lorenzians). Thus, whether or not there is BCS pairing (non-zero or zero gap) makes a significant difference to the result, which means that the method can also be used to detect the onset of the phase transition, cf [9, 10, 12, 13]. For certain types of perturbations the

Cooper paired atomic Fermi gas also has below-gap excitations. A density perturbation term of the form $U(\vec{r},t)[\hat{\psi}_\uparrow^\dagger(\vec{r})\hat{\psi}_\uparrow(\vec{r}) + \hat{\psi}_\downarrow^\dagger(\vec{r})\hat{\psi}_\downarrow(\vec{r})]$ leads to the appearance of a Bogoliubov–Anderson phonon [12]. In our case the laser detuning gives a similar term in the Hamiltonian, $\delta/2[\hat{\psi}_\uparrow^\dagger(\vec{r})\hat{\psi}_\uparrow(\vec{r}) - \hat{\psi}_\downarrow^\dagger(\vec{r})\hat{\psi}_\downarrow(\vec{r})]$ (assuming that the laser is turned on and confined to a spatial region, one may think of δ as having a space and time dependence). The minus sign, however, leads to cancellation of certain terms in the response calculation and to the absence of the usual Bogoliubov–Anderson phonon. This will be discussed in detail in another publication [14].

5. Conclusions

For standard metallic superconductors the coherence properties are usually investigated by creating interfaces with normal metals or with insulators. Optical manipulation of cold alkali gases offers the unique opportunity of creating such interfaces at any point in space and of controlling at will the transfer Hamiltonian across the interface. We described a method for testing BCS coherence in a system of atomic fermions cooled below the critical temperature by driving laser-induced transitions between the two paired hyperfine states in different regions of the space. We presented numerical and analytical evidence for the feasibility of the procedure. We show that the photon absorption rate changes when the distance between the transition regions is of the order of magnitude of the BCS correlation length. Absence of coherence would make the rate independent of the distance. The method can also serve for the detection of the onset of a BCS transition, because the absorption peak is shifted to the detuning of twice the gap energy.

Acknowledgments

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Appendix

The currents $I_{\uparrow,\mathcal{G}}$ and $I_{\uparrow,\mathcal{F}}$ are convolutions of pairs of correlation functions, thus their direct connection with x_{CP} and x_{F} is not transparent. In this appendix we discuss the dependence of the BCS correlation functions on these length scales. It gives an insight into the physics of Cooper pair coherence and to the more complicated length scale dependence of $I_{\uparrow,\mathcal{G}}$ and $I_{\uparrow,\mathcal{F}}$.

Green's functions for the superconductor are defined as

$$\mathcal{F}(\vec{r},\tau;\vec{r}',\tau') = -\langle \mathcal{T}_\tau[\hat{\psi}_\uparrow(\vec{r},\tau)\hat{\psi}_\downarrow(\vec{r},\tau')]\rangle \tag{A1}$$

$$\mathcal{F}^\dagger(\vec{r},\tau;\vec{r}',\tau') = -\langle \mathcal{T}_\tau[\hat{\psi}_\uparrow^\dagger(\vec{r},\tau)\hat{\psi}_\downarrow^\dagger(\vec{r},\tau')]\rangle \tag{A2}$$

$$\mathcal{G}(\vec{r},\tau;\vec{r}',\tau') = -\langle \mathcal{T}_\tau[\hat{\psi}_\uparrow(\vec{r},\tau)\hat{\psi}_\uparrow^\dagger(\vec{r},\tau')]\rangle. \tag{A3}$$

In the uniform case and at zero temperature we obtain for the zero-time correlation functions

$$\mathcal{G}(\vec{r},0;\vec{r}',0) = \frac{m}{4\pi^2 r}\int_{-\mu}^{\infty} \mathrm{d}\xi\,\sin\left[\sqrt{2m(\mu+\xi)}r\right]\left(1-\frac{\xi}{\sqrt{\xi^2+\Delta^2}}\right) \tag{A4}$$

$$\mathcal{F}(\vec{r},0;\vec{r}',0) = \frac{m}{4\pi^2 r}\int_{-\mu}^{\infty} \mathrm{d}\xi\,\sin\left[\sqrt{2m(\mu+\xi)}r\right]\frac{\Delta}{\sqrt{\xi^2+\Delta^2}} \tag{A5}$$

where $r = |\vec{r} - \vec{r}'|$. Information about any possible coherence length scales such as $x_{\rm CP}$ or $x_{\rm F} = 1/\sqrt{2m\mu}$ is hidden within these equations. Note that for a normal system (zero gap) the Fermi correlation has quite an explicit dependence on $x_{\rm F}$:

$$\mathcal{G}(\vec{r}, 0; \vec{r}', 0) \stackrel{\Delta=0}{=} \frac{1}{2\pi^2 x_{\rm F}} \left(\frac{x_{\rm F}^3}{r^3} \sin \frac{r}{x_{\rm F}} - \frac{x_{\rm F}^2}{r^2} \cos \frac{r}{x_{\rm F}} \right). \tag{A6}$$

The anomalous correlation function $\mathcal{F}$ is non-vanishing only when the gap is finite, reflecting the instability of the Fermi sea and the appearance of fermionic pairing. For $\mathcal{F}$ one can actually derive an approximate formula which shows the dependence on $x_{\rm CP}$ or $x_{\rm F}$: for the usual situation $\Delta \ll \mu$, and one can expand

$$\sqrt{2m(\mu + \xi)r} \approx \frac{r}{x_{\rm F}} + \frac{1}{2} \frac{r}{x_{\rm CP}} \frac{\xi}{\Delta}$$

which gives

$$\mathcal{F}(\vec{r}, 0; \vec{r}', 0) = \frac{m\Delta}{2\pi^2 r} \sin\left(\frac{r}{x_{\rm F}}\right) K_0 \left(\frac{r}{2x_{\rm CP}}\right). \tag{A7}$$

Here K_0 is the zero-order modified Bessel function which for a real argument a admits the integral representation [15]

$$K_0(a) = \int_0^\infty \frac{\cos(ax)\,{\rm d}x}{\sqrt{1 + x^2}}.$$

For example, using the series expansion of K_0 we find that at r small compared with $x_{\rm CP}$ the anomalous correlation function decreases with r as $\ln(r/x_{\rm CP})/r$,

$$\mathcal{F}(\vec{r}, 0; \vec{r}', 0) = -\frac{m\Delta \sin(r/x_{\rm F})}{2\pi^2 r} \left(\ln \frac{r}{4x_{\rm CP}} + 0.577\,215 \right). \tag{A8}$$

In conclusion, the spatial behaviour of the normal and anomalous Green's functions is governed by two parameters, $x_{\rm CP}$ and $x_{\rm F}$. However, since $x_{\rm F}$ is typically much smaller than the width of the laser beams we expect a dependence only on $x_{\rm CP}$ in the result of a measurement.

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