

# Discrete and Continuous Optimization Methods for Self-Organization in Small Cell Networks

Models and Algorithms

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Furqan Ahmed

# Discrete and Continuous Optimization Methods for Self-Organization in Small Cell Networks

Models and Algorithms

**Furqan Ahmed**

A doctoral dissertation completed for the degree of Doctor of Science (Technology) to be defended, with the permission of the Aalto University School of Electrical Engineering, at a public examination held at the lecture hall S4 of the school on 25 August 2016 at 12.

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Self-organization is discussed in terms of distributed computational methods and algorithms for resource allocation in cellular networks. In order to develop algorithms for different self-organization problems pertinent to small cell networks (SCN), a number of concepts from discrete and continuous optimization theory are employed. Self-organized resource allocation problems such as physical cell identifier (PCI) assignment and primary component carrier selection are formulated as discrete optimization problems. Distributed graph coloring and constraint satisfaction algorithms are used to solve these problems. The PCI assignment is also discussed for multi-operator heterogeneous networks. Furthermore, different variants of simulated annealing are proposed for solving a graph coloring formulation of the orthogonal resource allocation problem.

In the continuous optimization domain, a network utility maximization approach is considered for solving different resource allocation problems. Network synchronization is addressed using greedy and gradient search algorithms. Primal and dual decomposition are discussed for transmit power and scheduling weight optimizations, under a network-wide power constraint. Joint optimization over transmit powers and multi-user scheduling weights is considered in a multi-carrier SCN, for both maximum rate and proportional-fair rate utilities. This formulation is extended for multiple-input multiple-output (MIMO) SCNs, where apart from transmit powers and multi-user scheduling weights, the transmit precoders are also optimized, for a generic alpha-fair utility function. Optimization of network resources over multiple degrees of freedom is particularly effective in reducing mutual interference, leading to significant gains in network utility. Finally, an alternate formulation of transmit power allocation is considered, in which the network transmit power is minimized subject to the data rate constraints of users. Thus, network resource allocation algorithms inspired by optimization theory constitute an effective approach for self-organization in contemporary as well as future cellular networks.

**Keywords** Self-organization, optimization, small cell networks, resource allocation, graph coloring, network utility maximization, 4G, 5G**ISBN (printed)** 978-952-60-6939-5**ISBN (pdf)** 978-952-60-6938-8**ISSN-L** 1799-4934**ISSN (printed)** 1799-4934**ISSN (pdf)** 1799-4942**Location of publisher** Helsinki**Location of printing** Helsinki**Year** 2016**Pages** 177**urn** <http://urn.fi/URN:ISBN:978-952-60-6938-8>



# Preface

The research work for this doctoral thesis has been carried out during the years 2010-2015, at the Department of Communications and Networking (COMNET) of Aalto University. The work was funded by the Aalto University Graduate School fellowship, European Union (METIS and COHERENT projects), the Finnish Funding Agency for Technology and Innovation (NETS2020 and EWINE-D projects), the Academy of Finland (ROSECORN and TT5G projects), Nokia Siemens Networks, Ericsson Finland, Renesas Mobile, Elektrobit, and Nethawk.

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On a more personal note, special thanks to my beloved wife and daughter for the love and inspiration they gave me, especially towards the end of this thesis. My deepest appreciation and profound thanks go to my brother and sisters for their love and encouragement. Last but not least, no words can justly express my gratitude and appreciation towards my parents, for their unconditional love, dedication, and unwavering support through every step of my life. This thesis is dedicated to them.

Espoo, August 5, 2016,

Furqan Ahmed

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# List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

**I** F. Ahmed, O. Tirkkonen, M. Peltomäki, J.-M. Koljonen, C.-H. Yu, and M. Alava. Distributed Graph Coloring for Self-Organization in LTE Networks. *Journal of Electrical and Computer Engineering*, vol. 2010, Article ID 402831, 10 pages, 2010.

**II** F. Ahmed and O. Tirkkonen. Self Organized Physical Cell ID Assignment in Multi-operator Heterogeneous Networks. In *Proceedings of IEEE Vehicular Technology Conference Spring*, pages 1-5, May 2015.

**III** F. Ahmed and O. Tirkkonen. Simulated Annealing Variants for Self-organized Resource Allocation in Small Cell Networks. *Applied Soft Computing*, vol. 38, pages 762-770, Jan. 2016.

**IV** F. Ahmed and O. Tirkkonen. Topological Aspects of Greedy Self Organization. In *Proceedings of IEEE International Conference on Self-Adaptive and Self-Organizing Systems*, pages 31-39, Sep. 2014.

**V** F. Ahmed, O. Tirkkonen, A. Dowhuszko, and M. Juntti. Distributed Power Allocation in Cognitive Radio Networks under Network Power Constraint. In *Proceedings of IEEE International Conference on Cognitive Radio Oriented Wireless Networks and Communications*, pages 492-497, June 2014.

**VI** F. Ahmed, A. Dowhuszko, and O. Tirkkonen. Distributed Algorithm for Downlink Resource Allocation in Multicarrier Small Cell Networks. In *Proceedings of IEEE International Conference on Communications*, pages 6802-6808, June 2012.

**VII** F. Ahmed, A. Dowhuszko, and O. Tirkkonen. Distributed Downlink Resource Allocation for Multi-carrier MIMO Small Cell Networks. *Submitted to IEEE Transactions on Vehicular Technology*, 2016.

**VIII** F. Ahmed, A. Dowhuszko, O. Tirkkonen, and R. Berry. A Distributed Algorithm for Network Power Minimization in Multicarrier Systems. In *Proceedings of IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, pages 1914-1918, Sep. 2013.

# Author's Contribution

## **Publication I: “Distributed Graph Coloring for Self-Organization in LTE Networks”**

The author actively participated in planning and analyzing research ideas, algorithm development, and had the main responsibility in creating simulation results and the writing of the paper.

## **Publication II: “Self Organized Physical Cell ID Assignment in Multi-operator Heterogeneous Networks”**

The author actively participated in planning research ideas, and had the main responsibility in analysis, algorithm development, creation of simulation results, and the writing of the paper.

## **Publication III: “Simulated Annealing Variants for Self-organized Resource Allocation in Small Cell Networks”**

The author actively participated in planning research ideas, and had the main responsibility in analysis, algorithm development, creation of simulation results, and the writing of the paper.

## **Publication IV: “Topological Aspects of Greedy Self Organization”**

The author actively participated in planning and analyzing research ideas, algorithm development, and had the main responsibility in creating simulation results and the writing of the paper.

**Publication V: “Distributed Power Allocation in Cognitive Radio Networks under Network Power Constraint”**

The author actively participated in planning research ideas, and had the main responsibility in analysis, algorithm development, creation of simulation results, and the writing of the paper.

**Publication VI: “Distributed Algorithm for Downlink Resource Allocation in Multicarrier Small Cell Networks”**

The author actively participated in planning research ideas, and had the main responsibility in analysis, algorithm development, creation of simulation results, and the writing of the paper.

**Publication VII: “Distributed Downlink Resource Allocation for Multi-carrier MIMO Small Cell Networks”**

The author actively participated in planning research ideas, and had the main responsibility in analysis, algorithm development, creation of simulation results, and the writing of the paper.

**Publication VIII: “A Distributed Algorithm for Network Power Minimization in Multicarrier Systems”**

The author actively participated in planning research ideas, and had the main responsibility in analysis, algorithm development, creation of simulation results, and the writing of the paper.

# List of Symbols and Abbreviations

## Latin symbols

$B$	System bandwidth
$c$	Color currently in use at a vertex
$c'$	Randomly selected color
$\mathcal{C}$	Full range of the colors
$\mathcal{F}(\cdot)$	Number of conflicts
$\mathcal{I}$	Set of base stations (cells)
$\mathcal{K}$	Set of frequency carriers
$\mathcal{L}_i$	Set of users served by base station $i$
$n$	Iteration index
$\mathcal{N}_i$	Set of base stations (cells) in the neighborhood of a base station $i$
$p$	Exponent of geodesic distance norm
$\mathbf{p}$	Transmit powers of all base stations in a network
$\mathbf{P}$	Transmit powers of all base stations in a multi-carrier network
$p_c$	Probability of closed subscriber group
$p_i$	Transmit power of base station $i$
$\mathcal{P}_i$	Feasible set of transmit powers of base station $i$
$\mathbf{p}_i$	Transmit powers of base station $i$ in a multi-carrier network
$p_i^k$	Transmit power of base station $i$ on a carrier $k$
$P_{\max}$	Maximum transmit power of a base station
$P_{\min}$	Minimum transmit power of a base station
$P_{\text{net}}$	Total transmit power budget of a network
$\mathbf{Q}_i$	Normalized covariance matrices of base station $i$ in a multi-carrier network
$\mathbf{Q}_i^k$	Normalized covariance matrix of base station $i$ on a carrier $k$
$\mathbf{Q}$	Normalized covariance matrices of all base stations in multi-carrier network

**Latin symbols (cont.)**

$R_i$	Target data rate of a user in cell $i$
$r_l$	Achievable data rate of user $l$
$\mathcal{S}$	Feasible set of phases
$T(n)$	Current temperature
$T_0$	Initial temperature
$t_i[n]$	Time instant of an update in an iteration $n$
$u_l$	Utility of a user $l$
$U_i(\cdot)$	Utility of a base station (cell) $i$
$U_{\text{sum}}(\cdot)$	Network utility
$\mathbf{W}$	Scheduling weights of all users in a network
$\mathbf{w}_i$	Scheduling weights of all users in a cell $i$
$\mathcal{W}_i$	Feasible set of scheduling weights of a user in cell $i$
$\mathbf{W}_i$	Scheduling weights of all users in cell $i$ of a multi-carrier network
$w_l$	Scheduling weight of user $l$
$w_l^k$	Scheduling weight of user $l$ on a carrier $k$

**Greek symbols**

$\alpha$	Fairness parameter
$\beta^P$	Stepsize in transmit power optimization iterate
$\beta^Q$	Stepsize in transmit precoder optimization iterate
$\beta^W$	Stepsize in scheduling optimization iterate
$\gamma_l$	SINR of a user $l$
$\gamma_l^k$	(Normalized) SINR of a user $l$ on a carrier $k$
$\epsilon$	Stepsize in network synchronization iterate
$\lambda_j$	Lagrange multiplier
$\pi_{i,i}$	Power benefit
$\pi_{j,i}$	Power price
$\pi_{j,i}^k$	Power price on a carrier $k$
$\Pi_{j,i}^k$	Precoder price on a carrier $k$
$\rho_i^k$	Weighting factor comprising of prices on a carrier $k$
$\phi_i$	Synchronization variable of a base station $i$
$\Phi$	Synchronization variables of all base stations
$\psi$	Rate at which a dynamic network changes

**Abbreviations**

2G	Second Generation
3GPP	Third Generation Partnership Project
4G	Fourth Generation
5G	Fifth Generation
ABT	Asynchronous Backtracking
ANR	Automatic Neighbor Relation
AWC	Asynchronous Weak-Commitment
CAPEX	Capital Expenditure
CCO	Coverage and Capacity Optimization
CDF	Cumulative Distribution Function
CMP	Cooperative Global Reset Message-Passing
CSP	Constraint Satisfaction Problem
D2D	Device-to-Device
eICIC	Enhanced Inter-Cell Interference Coordination
ES	Energy Saving
ESM	Energy Saving Management
FAP	Frequency Allocation Problem
FM-IWF	Fixed-Margin Iterative Water-Filling
GSGP	Gauss-Seidel Gradient Projection
GSM	Global System for Mobile Communications
HetNet	Heterogeneous Network
ICIC	Inter-Cell Interference Coordination
IWF	Iterative Water-Filling
KKT	Karush-Kuhn-Tucker
LTE	Long Term Evolution
LTE-A	LTE-Advanced
MHO	Mobility Handover Optimization
MIMO	Multiple-Input Multiple-Output
MLB	Mobility Load Balancing
mmW	Millimeter-Wave
MRO	Mobility Robust Optimization
NCR	Non-Cooperative Local Reset
NFV	Network Function Virtualization
NGMN	Next Generation of Mobile Networks
NLGS	Non-Linear Gauss-Seidel
NUM	Network Utility Maximization



## Abbreviations

OPEX	Operating Expenditure
PCC	Primary Component Carrier
PCI	Physical Cell Identifier
PF	Proportional Fair
QoS	Quality of Service
RACH	Random Access Channel
SCN	Small Cell Network
SDLS-FP	Steepest Descent Local Search with Focused Plateau Moves
SDN	Software Defined Networking
SINR	Signal-to-Interference-plus-Noise power Ratio
SLS-FP	Stochastic Local Search with Focused Plateau Moves
SOCRATES	Self-Optimization and Self-Configuration in Wireless Networks
SON	Self-Organizing Network
WINNER	Wireless World Initiative New Radio
WLAN	Wireless Local Area Network
WSN	Wireless Sensor Network

# 1. Introduction

## 1.1 Motivation

The proliferation of wireless devices along with a plethora of popular mobile applications and services has led to an unrelenting demand for high data rates, seamless coverage, and ubiquitous connectivity. Recently, 5G cellular networks have been envisaged to meet these demands, not merely by an evolution of state-of-the-art, but by employing a host of potentially disruptive technologies, in conjunction with an evolved 4G [116]. To this end, vision for enabling  $1000\times$  more mobile data capacity by the year 2020 involves three key components: addition of more frequency bands, increased spectral efficiency, and dense deployments of small cell networks (SCNs) and heterogeneous networks (HetNets) [17]. These components individually involve a multitude of existing and upcoming techniques, which include cognitive radio networking, massive multiple-input multiple-output (MIMO), device-to-device (D2D) communication, green communications, and millimeter-wave (mmW) networking [30, 75, 147]. In order to provide higher aggregate data rates, these techniques primarily rely on an efficient utilization of resources across four main dimensions or degrees of freedom namely: frequency, space, time, and network topology. Optimization across these degrees of freedom is thus imperative to a system design aimed at achieving 5G objectives. For the automation and optimization of ultra-dense networks, self-organizing network (SON) paradigm is of paramount importance [29]. In fact, SON based techniques are considered to be one of the cornerstones in the success of contemporary cellular networks such as Long Term Evolution (LTE)/LTE-Advanced (LTE-A) [64], and will pave the way for seamlessly connecting billions of devices in future 5G cellular networks [112].

Apart from SCN and HetNet deployments, solutions based on the concept of cloud radio access networks (CRANs) have tremendous potential for 5G networks [129]. The CRAN comprises of a large number of low-cost radio remote heads, deployed randomly and connected to a base band unit through fronthaul links. As the underlying architecture is centralized, the operational efficiency can be enhanced by application of centralized SON [122]. Software defined networking (SDN) and network function virtualization (NFV) paradigms may further enhance the benefits of SON, through creation of suitable logical abstractions and application of SDN principles [59, 122, 125]. The key motivation for SON is the need to optimize the network resources, and automate the service provisioning. To this end, a number of alternatives have been proposed regarding the distribution of control, data, and computation among network elements [37, 61]. Centralized radio access network architectures, though becoming increasingly popular, are usually constrained to limited geographic areas. Multiple CRAN entities will be required to coexist and coordinate their actions in a distributed manner, through self-organizing mechanisms. Hybrid architecture is also considered as an option for 5G networks [37].

In general, the need for SON based resource allocation arises due to massive deployments of low-power nodes, which makes the configuration, optimization, and maintenance of the network difficult [18, 56]. For example, unplanned deployment of small cells, along with the disparity in transmit power of cells belonging to different layers in a HetNet may result in very high cross-layer interference to some users [16]. An effective mitigation of this interference is an important SON issue pertaining to HetNet deployments [97]. In future 5G networks, apart from throughput centric small cells deployments, novel scenarios such as massive machine communications, internet of things (IoT), and mission critical communications may also benefit from self-organization mechanisms, in maintaining quality of service (QoS) and connectivity in the network [67, 111]. Moreover, self-organization has always been an important component in the design of sensor and actor networks [50]. This is likely to continue in the 5G era as well, paving the way for IoT [22]. Other potential applications of SON include emerging scenarios such as multi-operator spectrum sharing, which involves resource allocation among non-cooperative networks. SON based approaches hinging on measurement based coordination are viable in such cases. Similar problems requiring non-cooperative

solutions may arise in unlicensed spectrum sharing scenarios, which are currently under consideration for both LTE/LTE-A and future 5G cellular networks [74].

Most importantly, SON is of key significance in reducing capital expenditure (CAPEX) and operational expenditure (OPEX) of cellular networks [47]. Automated networks possessing SON features do not require large workforce for ensuring seamless operation. Hence, SON can enable automated commissioning, installation, performance evaluation, and monitoring of large-scale networks, thereby reducing the expenditure involved in carrying out these processes manually [64]. Another major design consideration for future cellular networks with relevance to SON is energy efficiency. SON features are important for reducing the power expenditure as well as for minimizing environmental footprint. Enabling it at a network level may require the use of self-organizing energy management techniques [32, 46].

Motivated by the potential of SON in enhancing network performance by efficient resource allocation across multiple degrees of freedom [18, 70]; this thesis presents a network optimization approach towards the design of SON algorithms for resource allocation problems relevant to different SON use-cases of current LTE/LTE-A cellular networks, as well as the future 5G paradigm.

## 1.2 Scope of the thesis

This thesis focuses on the application of discrete and continuous optimization methods towards the design of SON algorithms, for different resource allocation problems pertinent to SCNs. We use the term SON in a broader context: it refers to engineering of self-organizing dynamics for resource allocation aimed at improving the performance of SCNs, by achieving a network level objective through simple local interactions among individual network nodes. Thus, SON algorithms discussed here are network optimization methods which adhere to the rudimentary self-organization principles. This summarizes the underlying philosophy we employ to design various SON algorithms for SCNs. Accordingly, the algorithms discussed here involve local or limited interactions between the cells, aimed at achieving a given network level goal (e.g. network sum-rate maximization or guaranteeing a target data rate to each user in the network). The local interactions between the cells may be non-cooperative (e.g. merely

based on user measurements in a cell) or fully cooperative (through dedicated message-passing between cells). The algorithms pertaining to these two cases can also be classified as *fully distributed* and *distributed*, respectively. The mathematical models and computational methods that we discuss under the rubric of SON algorithms, are general and applicable to wide variety of network optimization problems relevant to large-scale distributed networks, in which complete network state information is either infeasible or impossible to obtain at every network node. Consequently, nodes are forced to make local decisions with limited or even without cooperation with the neighboring nodes, which motivates the engineering of SON functionalities.

### 1.3 Contributions

The main contribution of this thesis is that it develops a mathematical approach for designing SON algorithms in a principled way. Both discrete and continuous optimization methods are considered for solving resource allocation problems related to SCNs. Discrete optimization methods are used for two well-known SON problems namely: physical cell identifier (PCI) assignment [7], and primary component carrier (PCC) selection [115]. Graph coloring models are used to solve these problems in a self-organized manner. The proposed algorithms are based on non-cooperative greedy local search heuristics, and cooperative constraint satisfaction algorithms including asynchronous backtracking (ABT) and asynchronous weak commitment search (AWC) [157]. Greedy local search algorithms are fast, and fully distributed in the sense that they do not involve any message-passing between the cells. This results in slight loss in performance in terms of convergence probability, when compared to constraint satisfaction algorithms which are complete, and rely on distributed message-passing to search for the optimal solution. The simulation scenario used for performance analysis consists of a dynamic pico-cell network deployed in a multi-story building, which is modeled using the Wireless World Initiative New Radio (WINNER) path-loss model [117]. Results indicate that algorithms based on greedy local search are highly effective in coloring interference graphs, thereby enabling self-organized PCI assignment and PCC configuration. PCI assignment problem is then considered in a multi-operator HetNet. In this case, multiple operators share the spectrum in the small cell layer. The underlying conflict-graph

is directed, as the interference couplings cannot be symmetrized through message-passing due to the lack of cooperation between operators. Accordingly, we employ local search SON algorithms based on a focused search metaheuristic [131], which do not involve any message-passing between the operators. The focused search principle allows local moves only to the cells with conflicts. For performance analysis, a dense HetNet with two operators having orthogonal spectrum for the macro-layer and shared spectrum for small cell layer is considered. The simulation results show that the proposed algorithms enable fast and efficient PCI assignment jointly across both operator networks. Furthermore, we develop a host of simulated annealing variants for generic orthogonal resource allocation problems, where a planar graph is used to model the SCN.

Next, we discuss SON algorithms for different resource allocation problems pertinent to third generation partnership project (3GPP) SON use-cases, which can be addressed through continuous network optimization methods such as network utility maximization (NUM). The NUM framework provides an impetus for a systematic design of self-organized resource allocation in SCNs, and it can enable both distributed and fully distributed SON algorithms. Distributed algorithms are cooperative in nature and involve an exchange of dedicated messages between the cells. Messages may simply communicate the transmission parameters of a given cell, or some more sophisticated parameter related to a NUM framework e.g. a *price* — a quantity defined to compute the loss in utility of a given cell, resulting from a change in another cell’s transmission parameters. The prices can be exchanged through dedicated message-passing between the cells over a backhaul link. Fully distributed algorithms can be obtained by simply setting the price terms equal to zero. An alternative approach towards fully distributed SON is motivated by *ad hoc* networking applications, where each cell broadcasts a network-level price over the air.

Starting with an introduction to the decomposition methods pertinent to distributed NUM, we discuss SON algorithms for network synchronization — an important self-organization problem for SCNs [14, 15]. Next, we develop SON algorithms for inter-cell interference coordination (ICIC) [97], which can be formalized as a resource allocation problem consisting of multiple degrees of freedom. The considered problem entails a cognitive radio setting, where a network constraint exists on the total transmit power a secondary network is allowed to distribute among the

base stations. The aim is the joint optimization over transmit powers and multi-user scheduling decisions (weights) in a single-carrier network. The scheduling weights represent the intra-cell orthogonal resource allocation, and can be understood as a special case of user priority weights considered in [76]. In addition, fairness considerations are incorporated through a proportional fair rate (PF-Rate) utility function. To solve the resulting NUM, primal and dual decomposition methods are used [118], which lead to an optimal solution. In this case, the information exchange parameter depends on the decomposition method used. In primal decomposition the prices are exchanged whereas dual decomposition involves an exchange of local primal variables as well. Next, we discuss a decomposition approach based on direct solution of Karush-Kuhn-Tucker (KKT) conditions [72], and apply it for joint optimization over transmit powers and multi-user scheduling weights in a multi-carrier SCN. Both Shannon rate (Max-Rate) and PF-Rate utility functions lead to an improvement in network sum data rate. The gains are less pronounced in the case of PF-Rate due to its fairness characteristics. This model is extended for MIMO SCNs, where apart from transmit powers and scheduling weights, transmit precoders are also optimized over multiple carriers in each cell. An  $\alpha$ -fair utility is considered, which incorporates both Max-Rate and PF-Rate utility functions [80, 108]. Existing work related to precoder optimization in MIMO networks includes [134], which considers a pricing based approach for two user MIMO channel. Moreover, a convex approximation based precoder design is proposed in [163], and a global optimization method is discussed in [95]. The maximization of mutual information over covariance matrices in the network is addressed in [154]. Power loading for multi-carrier MIMO networks is discussed in [124]. A joint transmitter receiver design for precoding and power loading is proposed in [45]. In [143], the focus is on joint bit and power loading for MIMO systems.

Finally, an alternate formulation of the multi-carrier transmit power allocation problem is considered, where the sum of transmit powers of all base stations is minimized subject to a rate constraint per user. This additional constraint ensures that all the users in each cell attain a given rate, while the expended transmit power in the whole network is minimized. The simulation results show a significant gain over the non-cooperative fixed-margin iterative water-filling (FM-IWF) approach [119].

We conclude that discrete and continuous optimization methods are viable for enabling self-organized solutions to resource allocation problems,

which often involve joint optimization over multiple degrees of freedom. Depending on the application, different types of utility functions and constraints can easily be incorporated in the NUM framework.

#### 1.4 Summary of the publications

This thesis consists of an overview and eight original publications. The first three publications discuss discrete optimization methods for the design of self-organizing algorithms. Publication I consists of graph coloring algorithms for self-organized resource allocation in LTE networks. Two well-known SON problems are discussed in this work — PCI assignment and PCC selection. Algorithms based on distributed local search and distributed constraint satisfaction are proposed for coloring the underlying conflict-graph, which models interference couplings between the cells. This work is extended in Publication II, where the PCI assignment problem is considered for a multi-operator HetNet with partially shared spectrum between the networks. There is no cooperation between the operators, thus the interference couplings are not symmetric, which leads to a directed conflict-graph. Publication III focuses on the problem in both static and dynamic scenarios, where self-organized orthogonal resource allocation is addressed using different variants of simulated annealing metaheuristic.

From Publication IV onwards, the focus is on continuous network optimization methods based on NUM framework for resource allocation in a SCN. Self-organizing algorithms for network synchronization are considered in Publication IV, where we consider two different types of updates namely best-response and gradient descent. In Publication V, transmit powers and scheduling weights are jointly optimized to maximize the network utility, under a network-wide total transmit power constraint. This formulation is motivated by cognitive radio networks, where the aim is to control the total interference emanating from the network. A PF-Rate utility function is considered in this case. Publication VI involves optimization over transmit powers as well as user scheduling weights in a multi-carrier SCN, for both Max-Rate and PF-Rate utilities. Publication VII extends this formulation in the MIMO direction, where the network utility is maximized over transmit powers, transmit precoders, and scheduling weights. Finally, Publication VIII focuses on a different formulation of the multi-carrier power allocation problem, where strict fair-



ness is incorporated as a separate constraint. Network transmit power is minimized over transmit power allocations of cells subject to a data rate constraint per user.

## 2. Self-Organizing Small Cell Networks

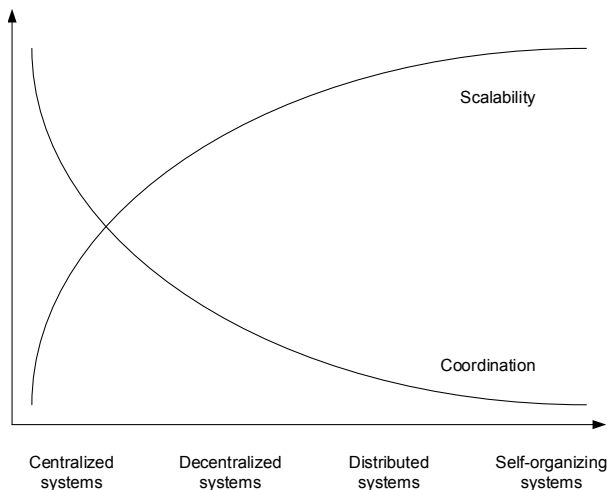
### 2.1 Self-organizing systems

Self-organization seem to have existed and evolved in nature since the very beginning. In early 1960's, Cameron and Yovits published the first collection of papers on the subject [35]. This was followed by a volume edited by Von Foerester and Zopf [164]. It included the original version of the famous paper by Ashby [21], which builds on the earlier contributions e.g. [19, 20], and generalizes the main principles that govern self-organizing systems. A number of notable works in different scientific and technical areas have led to the popularization of the term "self-organization" [34, 51, 62, 78, 82, 113].

A main feature of self-organizing systems is the lack of central control for coordinating the actions of agents in the system [34, 62]. The system is often distributed in the sense that agents acts as peers, and each agent in the system interacts only with a limited number of neighbors. These interactions collectively drive the system to an ordered state. Self-organizing systems are inherently scalable and the underlying principles are well-suited to highly complex and large-scale dynamic systems. This motivates the engineering of self-organizing mechanisms in practical systems [132]. Principles of self-organization have been successfully applied for solving complex problems in electrical engineering and computer science, especially in areas such as multi-agent systems [133, 139], complex systems [62], and emergent computation [44, 57]. Its popularity stems from the fact that it enables distributed solutions with limited communication, computation, and energy requirements for the system components. Self-organizing systems should involve local and limited coordination among system components. Another key feature that self-organizing

systems often exhibit is an emergent behavior [34, 152]. Emergence is defined as the system level (global) behavior pattern which arises as a result of meaningful local interactions between system components, and shows the capabilities of system beyond the capability of individual components [50]. A prime example of emergence in agent-based networks is the convergence of the firefly synchronization algorithm to a global optimum [106]. The system level or global behavior pattern refers to the structure and functionality at a higher level, which system components in a self-organizing system strive to achieve. Fully distributed systems which involve local interactions without dedicated message-passing between components may be termed as ideal self-organizing systems, especially if they exhibit emergent behavior. Emergence is not always considered as a necessary condition for self-organization, but rather an independent phenomenon [148]. It is particularly promising to combine both in the context of complex adaptive systems.

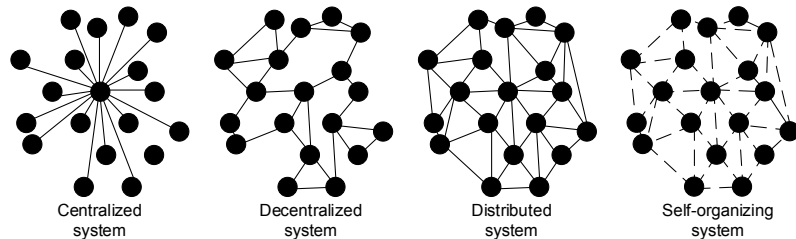
From a perspective of computer networks, a hierarchy of networking approaches can be identified, ranging from centralized to fully distributed self-organizing systems [25]. In Figure 2.1, the level of coordination and system scalability are illustrated along this axis of centralization, emphasizing that engineering of self-organization enhances the scalability of a network. Figure 2.2 shows pictorial representations of networks along



**Figure 2.1.** Local interactions and limited coordination leads to high scalability in self-organizing systems.

this axis. A decentralized network consists of a network of multiple centers, whereas in a distributed network, the nodes are peers, and local in-

teractions may be based on bilateral message-passing. Fully distributed networks with emergent behavior can be considered as an example of ideal self-organization, because there is no dedicated message-passing. In this case, local information exchange is based on the passive observation of changes in environment caused by the actions of neighbors, or on reception of generic multicast or broadcast signals from other nodes.



**Figure 2.2.** Classification of networks based on the level and type of coordination between the nodes.

The main characteristics of self-organizing systems that motivate the engineering of self-organization across a number of problems in different types of networks can be summarized as follows [50, 126, 146]:

### 2.1.1 Scalability

One of the foremost aspects of self-organizing systems is scalability, and it is of utmost importance in large-scale networks. Scalability ensures that the system can be scaled-up with manageable complexity. Thus, the system can be expanded without an unbounded increase in complexity. Scalability emanates from the self-organization principle itself. Due to the local nature of interactions between the system components, addition of new components results in limited additional complexity. This is especially important for dynamic systems, which evolve over time in terms of size and number of components. A design principle based on local interactions yields scalable solutions for a wide range of problems. Local interaction between system components can be formulated in many different ways, depending on the problem at hand. For ideal self-organization, the local interaction should be entirely passive, and based on mere observations of the local neighborhood. Ideal self-organization with entirely passive local interactions is common in nature, e.g. in bird flocking. However, when dealing with engineering of self-organization, e.g. in wireless networks, the local interactions may also be active and involve an exchange of information, or cooperation in the form of message-passing. In such cases, it

is desirable to keep the information exchange local, and at a minimum, to reduce the overhead resulting from the addition of new components. The interactions among the system components may be based on cooperation, but they should preferably be local. Long-ranged cooperation among components jeopardizes the scalability, as it may lead to an unmanageable increase in overhead information as the system grows. It is worth noting that in wireless networks, path-loss often maintains the scalability in that it is sufficient to exchange information only with the adjacent nodes.

### **2.1.2 Adaptability**

Due to a high degree of adaptability, self-organizing systems can effectively respond to changes in the environment and adjust operation parameters accordingly to reach equilibrium. This means that a system can re-organize in order to adapt to changes such as addition/removal of components in the system, or loss in system performance resulting from both external and internal factors. In this regard, agility is also important as the system should be able to react to the changes in a timely manner to ensure that the performance remains unaffected. Thus, adaptability and agility play an important role in guaranteeing rapid convergence to a desirable state.

### **2.1.3 Emergence**

A key property of self-organizing systems is that they possess emergent properties, i.e., the local interactions among system components lead to an emergence of desirable properties at the global level. All the system components contribute towards the desirable global behavior and are not capable of achieving it individually. Thus, the collective intelligence leads to an emergence of global behavior that transcends the capabilities of the individual components.

### **2.1.4 Robustness**

Self-organizing systems are adaptable and decentralized, which makes them particularly robust against failures. This is due to the fact that the system can adapt to changes in environment, and has the capability to reconfigure itself in the case of failure. The decentralized architecture is intrinsically robust, and leads to properties such as stability and graceful degradation. In order to enhance the robustness, the self-organizing sys-

tems are often designed with self-healing mechanisms to tackle failures in a systematic manner.

It should be noted that the formal definition of self-organization discussed here, and in most of the scientific literature is partially in a different context than the SON paradigm discussed for the mobile cellular networks by 3GPP and other standardization bodies. 3GPP standardization activities related to SON focus on the automation of network functions, using methods which may be fully self-organizing adhering to the above discussed definition, or based on more or less centralized approaches [64, 130]. This point is further elaborated in the following sections.

## 2.2 From self-organization to SON in mobile cellular networks

In wireless networks, the term “self-organization” has been used with different meanings in literature, and the concept has been applied to different types of networks in recent years: it has been used in the context of mobile *ad hoc* networks [73], peer-to-peer networks [63], multi-hop networks [107], wireless local area networks (WLANs) [79], and in wireless sensor networks (WSNs) [136]. Self-organizing methods have been studied extensively for networks that do not involve any central planning or coordination. Most popular examples include WSNs and *ad hoc* networks [50]. The key benefit of self-organization is that it leads to a distributed and scalable architecture [126, 146]. On the other hand, the design of cellular networks has traditionally been based on centralized planning. For example, 2G networks such as Global System for Mobile Communications (GSM) require central frequency planning and network dimensioning. However, the recent increase in popularity of high data rate wireless connectivity has led to a paradigm shift in the design of cellular networks. Among the emerging technologies, self-organized networking functionalities known as SON, have proven to be extremely useful, and are expected to play a key role in meeting demands of the future cellular networks [64, 70].

The research on self-organization in wireless networks covers a number of areas, which include complex systems [99], multi-agent systems [144], machine learning [27], distributed networks [43], and game theory [100]. Most practical applications of self-organization are related to advanced networks that are large-scale, diverse and highly connected, and thus

can be modeled as complex systems [50]. Self-organization can enhance the network performance by adding artificial intelligence and adaptability to network nodes, allowing them to react to the changes in the network topology and the state of the environment. The adaptive nature of such networks enables the utilization of the situational and local knowledge for achieving an optimal end-to-end performance. In particular, enabling the network nodes to do intelligent adaptations at an individual level, based on local information, guarantees a significant performance improvement over traditional networking technologies. It provides the ultimate solution to the problem of managing a large number of bandwidth intensive network nodes competing for scarce resources. Future generations of wireless networks are predicted to be much more complex than the present ones, comprising of billions of nodes. Different problems pertaining to such large-scale networks can be effectively handled by engineering the system using self-organizing principles.

Thus, self-organization is a broad research interest in the area of wireless networking. It spans not only the well-established areas of distributed and *ad hoc* networking, but also the SON in contemporary and future mobile cellular networks [13]. SON features can address the challenges of ensuring high data rates, improved coverage, scalability, efficient radio resource management, energy efficiency, reduced CAPEX/OPEX, fault tolerance, and quick recovery. The overall objective is network automation through approaches that are capable of adapting to varying channel conditions and QoS requirements of the users, which leads to better network performance, higher scalability, and improved robustness. Consequently, a number of SON use-cases have been considered by 3GPP in the standardization of LTE and LTE-A. These use-cases have been identified in different research projects and standardization activities. Notable examples include the use-cases proposed by Next Generation of Mobile Communications (NGMN) [54], Self-Optimization and Self-Configuration in Wireless Networks (SOCRATES) project [53], and 3GPP [1–9]. The SON use-cases related to network deployment and operation are usually classified into three main categories: self-configuration, self-optimization, and self-healing. In 3GPP Release 8, the following SON use-cases for self-configuration were introduced [1, 2, 53, 54]:

- Automatic inventory

- Automatic software download
- Automatic neighbor relation (ANR)
- Automatic PCI assignment

This was followed by an addition of the following self-optimization use-cases in 3GPP Release 9 [5, 7]:

- Mobility robustness/handover optimization (MRO/MHO)
- Random access channel (RACH) optimization
- Mobility load balancing (MLB) optimization
- Energy saving (ES)
- ICIC

The 3GPP Release 10 focused on enhancements of the existing use-cases such as enhanced ICIC (eICIC) and energy saving management (ESM), along with the addition of new self-healing functionalities [3, 4, 8, 9]:

- Coverage and capacity optimization (CCO)
- eICIC
- ESM
- Cell outage detection and compensation
- Minimization of drive testing (MDT)

In addition to CCO, MHO, ES, and MDT, in 3GPP Release 11 the following enhancement was considered [6]:

- Coordination between various SON functions



Similar concepts have been discussed in the further releases. Due to substantial overlap, it is possible to identify the main categories, as given in [13].

The aforementioned SON use-cases can be tackled using a variety of approaches, each having different requirements on computation and communication between cells. Distributed approaches may involve direct cooperation among the nodes through dedicated message-passing. On the other hand, fully distributed approaches that do not involve message-passing are also possible in some cases, and are more attractive from a practical standpoint. Such approaches are closer to the classical definition of ideal self-organization discussed earlier.

In 3GPP standardization activities of cellular networks, SON algorithms may be fully distributed that do not involve message-passing, distributed approaches with message-passing, or even centralized mechanisms of network operation [64]. Centralized SON means that all SON algorithms and functions are located in operations and management system, at a high hierarchical level. In distributed SON, all algorithms and functions are located at a relatively lower level, i.e., at evolved Node B level. Architecture comprising of combination of a set of SON functions located at different hierarchical levels is called hybrid SON [128]. Distributed algorithms with message-passing are important due to the fact that in 3GPP LTE/LTE-A radio access network architecture, message-passing between base stations is possible via the X2 interface. The main advantages of message-passing algorithms over the fully distributed ones are the improved network performance and the better convergence properties. Thus, according to the 3GPP standardization terminology, centralized, distributed, and fully distributed approaches come under the rubric of SON in mobile cellular networks.

### **2.3 Optimization models for self-organization in SCNs**

In SCNs, scalability is a key requirement, and is the main motivation for applying self-organization principles. Network parameters are uncertain due to random and unplanned deployment of large number of small cells. Moreover, users may have different QoS requirements. As a result, manual network control and maintenance becomes increasingly complex and costly, which paves the way for application of SON algorithms [18, 70].

The first step towards developing effective SON algorithms is to iden-

tify the SON problems from the SON use-cases, and classify them accordingly. A use-case may involve multiple problems, and it is also possible that the same or closely related problems are part of different use-cases. The problems may be modeled as static or dynamic, and the requirements on inter-cell coordination may vary accordingly. For example, in multi-vendor and multi-operator networks, complete coordination among all the cells may not be possible. The most popular classification of SON use-cases is according to their respective role in the network life cycle, i.e., self-configuration, self-optimization, and self-healing. Self-configuration entails configuring different network parameters. Therefore, the identification of configurable parameters in a mobile cellular network yields a partial classification of SON problems [130]. Problems related to self-optimization are numerous and more complicated in nature.

SON problems in mobile cellular networks can be modeled using an optimization framework, where the aim is to achieve a network level objective under various constraints. This approach has been considered for a number of SON problems including capacity enhancement [11], coverage optimization [69], dynamic frequency allocation [42], spectrum assignment [149], resource allocation for self-healing [88], and interference mitigation [26, 56, 90, 94, 96, 140, 161]. In most cases, distributed optimization methods are employed to design self-organizing algorithms for SCNs. Moreover, the time-scales involved depend on the nature of the problem. Time-scales for CCO and ESM use-cases may be in a range of few minutes to hours, whereas for MRO, channel-aware scheduling and interference mitigation, it would be milliseconds to seconds [55, 56]. A classification of SON based on time-scales is given in [13, 70]. In SCNs, different SON parameters may change at very different time-scales ranging from milliseconds/seconds to days/months [70].

In the following chapters, we discuss optimization algorithms for SON problems relevant to SCNs. In 3GPP standardization activities, both centralized and distributed SON have been considered. For the sake of clarity, we reserve the term SON for self-organization in the context of wireless 3GPP networks, with a more relaxed definition, including any automated network functionalities, even the centralized ones. However, our main aim is to strive for SON approaches that are close to the wider scientific meaning of “self-organization”, than centralized algorithms. Broadly speaking, the focus is on SON use-cases defined by NGMN, SOCRATES, and 3GPP. In particular, optimization algorithms for PCI and PCC selec-

tion, network synchronization, and ICIC are considered. The algorithms are distributed in nature, but may involve an exchange of information (in the form of message-passing) between the cells. Fully distributed algorithms that do not involve any dedicated message-passing are also discussed. These are much closer to the original concept of self-organization, but do not perform well in some cases.

The discussion in the following chapters focuses on how to formulate different use-cases as optimization problems, and what computational methods can be employed to solve them. To this end, concepts from both discrete and continuous optimization theory are used to develop a systematic approach towards the design of efficient SON algorithms. In the discrete or combinatorial optimization model, graph coloring formulation is discussed along with different coloring algorithms with self-organizing properties. For the continuous optimization case, NUM approach is considered, which focuses on the maximization of some performance metric of the network, with/without dedicated message-passing.

### 3. Discrete Optimization Methods for SON

A number of problems in the area of wireless networking can be modeled as discrete or combinatorial optimization problems. The system model comprises of a network cost function, formulated in terms of logical network variables, which may be optimized. Examples include topology control, routing optimization, QoS provisioning, and general resource allocation problems [38]. Most of the existing literature focuses on these problems in the context of *ad hoc* networks, satellite networks, and WSNs. Recently, the benefits of SON in cellular networks have paved the way for an application of both continuous and discrete optimization methods to resource allocation problems related to SON use-cases. To address these problems, different approaches have been considered in the literature; see e.g. [66]. A number of problems such as conflict-free channel selection (or frequency assignment), PCI assignment, PCC selection, optimal user association with base stations, and power allocation can be solved by discrete network optimization methods. Some SON problems that are important for SCNs have not been addressed before in other types of networks, and have not been discussed extensively using systematic optimization methods. These include PCI assignment, RACH optimization, and MLB optimization, to name a few. Among the methods that can be used for solving discrete network optimization problems pertinent to cellular networks, distributed constraint satisfaction [157] and graph coloring [145] are of notable importance.

A constraint satisfaction problem (CSP) arises when multiple autonomous agents controlling logical variables coexist in a system with constraints among their possible actions. In a CSP, values are assigned to a set of variables under a given set of constraints. The aim is to obtain a consistent assignment of values to variables, so that the constraints are met. A common example of such problems is the N-queens problem, where the

problem is to place  $N$  queens on a chess board in a way that they do not threaten each other. A wide range of problems in artificial intelligence can be formalized as CSPs, with applications in a diverse range of areas such as machine vision, belief maintenance, scheduling, and belief propagation, among others [86]. A subclass of CSPs can be reduced to graph coloring problems. In graph coloring problems, the aim is to assign colors to the vertices of a graph in non-conflicting manner, so that no two vertices connected by an edge have the same color. Graph coloring has an old and rich history in wireless systems, which dates back to the design of the first generation of cellular networks [24]. It has been studied extensively for the frequency assignment problem (FAP) in wireless networks. Recent applications of significant importance are in the area of WLANs [89,101]. Graph coloring usually entails a conflict-graph model of the network, where vertices represent the cells and the edges represent the interference couplings. The aim is to color the graph using a fixed number of colors, where each color represents a frequency. To construct a conflict-graph, the interference statistics from the interfering base stations are required, which can be used to create edges between the vertices. In order to color the graph for FAP, a number of approaches have been discussed in the literature [52]. However, to benefit from graph coloring based solutions for SON, distributed algorithms are required [83, 121].

Existing work on distributed graph coloring algorithms mainly focuses on finding colorings with  $\Delta + 1$  or  $O(\Delta)$  colors, where  $\Delta$  is the largest number of neighbors of any node. For these cases, fast converging distributed algorithms exist, and the convergence characteristics can usually be analyzed in closed form [85]. When the number of colors is smaller than  $\Delta$ , constraint satisfaction algorithms may be used. The constraint satisfaction algorithms such as ABT and AWC are complete algorithms, and are able to converge if a solution exists. Distributed stochastic algorithms, which fall under the category of local search, are not complete [162].

Algorithms based on local search metaheuristics can be used to solve CSPs and graph coloring instances [58]. In Publication I and Publication II, distributed versions of these algorithms are considered, as well as message-passing based distributed constraint satisfaction algorithms [157]. In both categories, the SON design principles are followed and the proposed algorithms are distributed in nature. In local search algorithms, the local interactions by the cells involve only the observation of the colors of neighboring cells. These local interactions among the cells guide the

actions taken by the individual cells. Thus, the local interactions between the cells result in a colored graph in a self-organized way, provided that the algorithm converges. The constraint satisfaction algorithms also involve local interactions, albeit with message-passing. Nevertheless, both local search and constraint satisfaction algorithms have self-organization properties, which make them suitable for application in self-organizing cellular networks. Publication III focuses on the orthogonal resource allocation problem in static and dynamic networks. Different variants of simulated annealing metaheuristic are proposed for graph coloring in static and dynamic self-organizing SCNs. Next, we provide an overview of key algorithms and results from Publication I, Publication II, and Publication III.

### 3.1 Local search algorithms

The local search principle involves local changes by individual agents to move in the configuration space from one point to another, satisfying constraints or avoiding conflicts locally. The search space is explored by making perturbations to the existing configuration, known as local moves. For graph coloring, a local move is the change of color by one vertex. Two states that are connected by a local move are neighbors in the configuration space. The configuration space is searched by a sequence of local moves taken by vertices. To this end, a cost function is defined, e.g. in terms of the number of conflicts. A local move that decreases the cost is known as a downhill move. For graph coloring, a local move that reduces the number of conflicts of a given vertex is a downhill move. One of the key features of the configuration space of colorings on undirected graphs is the existence of plateaus, i.e., neighboring states with the same number of conflicts. A greedy local search in which each vertex picks the best move gets trapped in the local minima, which are often located at plateaus. A main feature of local search algorithms is that they can move on the plateaus and avoid entrapment in local minima. In this context, a plateau move is a local move that keeps the number of conflicts unchanged. These moves are important for escaping from local minima on plateaus, and reaching a conflict-free state. Another effective strategy is to use simulated annealing, which involves occasional uphill moves, i.e., accepting local moves that increase the number of conflicts.

## 3.2 Complete distributed algorithms

From the perspective of SON, complete algorithms for distributed CSPs are of interest. Local search algorithms are not complete, in that they do not guarantee convergence. More involved message-passing algorithms can be created to guarantee completeness, i.e. returning a solution if it exists. Such algorithms have been discussed for generic distributed CSPs [157], and for distributed graph coloring [121]. A distributed CSP comprises of multiple variables and constraints distributed among nodes. Such problems are common in the areas of multi-agent systems and distributed networks. Applications of the concept include problems such as distributed resource allocation, distributed scheduling, and truth management systems [156]. The underlying principles are generic and can be applied to solve a broad range of problems related to self-organized resource allocation in wireless networks. This makes them important from a practical standpoint of designing SON algorithms for SCNs. Graph coloring is a typical example of CSP. Therefore, complete distributed algorithms provide an attractive alternative for distributed graph coloring, and can be considered for several SON use-cases. Each vertex of the graph can be considered as a variable, whereas the colors represent the values to be assigned under the constraints dictated by the adjacency matrix of the graph. A number of complete distributed algorithms can be applied for self-organized graph coloring. These include backtracking algorithms and iterative improvement algorithms. In what follows, two constraint satisfaction algorithms from [157], used for PCI selection in Publication I are briefly discussed.

### 3.2.1 Asynchronous backtracking

The asynchronous backtracking (ABT) algorithm is based on the backtracking principle, which entails the construction of a partial solution by an assignment of values to a subset of variables, such that all the constraints are met. The partial solution is then expanded to a full solution by adding more variables, one by one. When no value satisfies all the constraints of a given variable, the value of the last added variable to the partial solution is changed. This procedure is known as backtracking, and is a key principle of many constraint satisfaction algorithms. The ABT algorithm is a distributed and asynchronous version of the backtracking algorithm, in which agents interact via message-passing. In ABT, agents

communicate their current values using *ok* messages and new constraints using *nogood* messages. Each agent maintains a view of the values of other agents, which is known as *agentview*. There is a priority order of the agents which is determined by an alphabetical order of agent identifiers. An agent changes its value if it is not consistent with the assignments of agents having higher priorities. If there is no value that can be used to achieve consistency with the higher priority agents, a *nogood* message is generated and sent to the higher priority agents, which then change their value.

### 3.2.2 Asynchronous weak-commitment search

A downside of ABT is the fixed priority of agents, set by an alphabetical order. Thus, if the value selection of a high priority agent is bad, execution time increases notably because the lower priority agents have to perform an exhaustive search to reverse the bad decision. To tackle this issue, the asynchronous weak-commitment search (AWC) algorithm introduces the *min-conflict* heuristic to change the priorities leading to an avoidance of bad decisions. Thus, AWC dynamics enable reversal of a bad decision without resorting to an exhaustive search. To reflect the agent priorities, a parameter *priorityvalue* is used, which is communicated via *ok* messages.

## 3.3 PCI assignment and PCC selection in SCNs

The PCI configuration problem is an important self-configuration SON use-case for LTE/LTE-A, and has been discussed extensively from both technical and standardization perspectives [64]. Its potential is underscored by the fact that ultra-dense deployment scenarios are currently being considered for enabling high capacity in future networks. In such networks, self-organized assignment of cell identifiers will be of paramount importance for ensuring seamless handovers and efficient operation [141].

In Publication I, we have considered four distributed local search algorithms to color a conflict-graph. These algorithms can be classified according to two characteristics. The first classification is related to the type of price or interference coupling between cells. When real valued interference couplings are used, a real valued price may be considered between neighbors using the same PCI. In contrast, when binary conflicts are considered, the decisions are made on the basis of only the number



of conflicting neighbors, and not on the strength of the conflicts. In addition, algorithms are classified according to the number of alternatives tried by a cell, when it is its turn to update the PCI. One alternative is to select the PCI randomly. On the other hand, in multiple-try algorithms, a cell calculates the price for all PCIs, and randomly selects one of the PCIs with the lowest price. It is worth noting that the algorithms with binary price have an absorption feature, i.e., the cell does not escape a conflict-free state [12, 162]. In a global optimum or a colored state of a graph coloring problem, each cell sees a local optimum, i.e., there are no conflicts between any given pair of cells. Moreover, plateau moves are allowed to a given cell if it is in conflict, which means that if the price of the tested PCI (or with multiple-try, the lowest price of all PCIs) is the same as the price of the PCI being used, the cell changes the PCI. This enables the algorithm to breakout from a local optimum in search of a global optimum. The four local search algorithms can be summarized as follows:

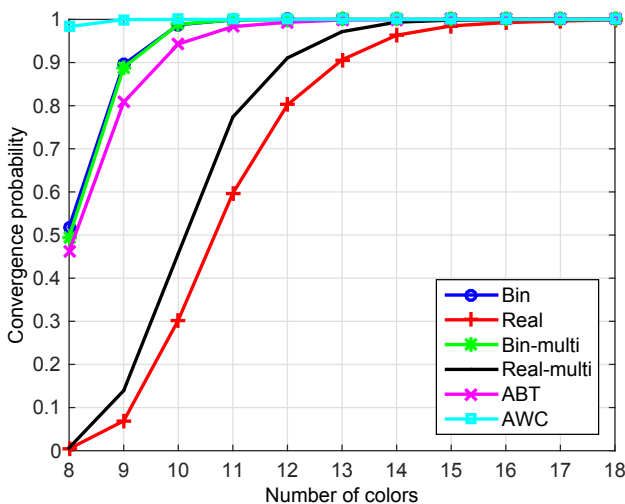
- Bin: binary price, random selection.
- Real: real price, random selection.
- Bin-multi: binary price, all candidates considered, random selection among the best candidates.
- Real-multi: real price, all candidates considered, random selection among the best candidates.

The real price algorithms with random and best candidate selection were discussed in [110] and [23], respectively. The binary multiple-try algorithm is an asynchronous version of distributed stochastic algorithm-D, which was proposed in [162]. For comparison, constraint satisfaction algorithms ABT and AWC are also considered.

The performance of these algorithms is analyzed in a pico-cell network deployed in four multi-story office buildings in a Manhattan grid, with propagation characteristics modeled according to the WINNER path-loss model [117]. The total number of base stations is 96, with 24 in each building (4 per floor). A dynamically growing network is considered where 86 base stations are initially switched-on before the algorithms are run.

The rest of the base stations are then added one-by-one, and the algorithms are re-run after each addition. Measurement capabilities of users are modeled through a synchronization threshold, which is a signal-to-interference-plus-noise power ratio (SINR) threshold under which a user is not able to synchronize to a base station. A base station considers the worst interference to any of its served users as the interference cost caused by that interferer. The interference caused by an interfering base station to a user is measured in terms of carrier-to-interference (C/I) power ratio measured when the user is synchronized to that base station. Simulation results are illustrated in Figure 3.1, in which all the algorithms are compared in terms of convergence probability for 500 random network instances, where the maximum iterations is 1000. It is clear from the results that binary price algorithms have a higher convergence probability compared to real price ones. This is due to the reason that binary algorithms can escape from local optima via plateau moves, and thus have a higher probability of finding a global optimum. Overall, the AWC outperforms all other algorithms in terms of convergence properties, as it is able to color the conflict-graph with 9 colors only. On the other hand, the performance of ABT is constrained by the limit on the maximum iterations. In ABT, the agents have fixed order and the solution is found by an exhaustive search through the backtracking principle, which makes the process inefficient in terms of convergence speed. AWC handles this problem through a dynamic change of priorities along with minimum-conflict heuristics. Nevertheless, ABT still has a higher convergence probability than the real price local search algorithms.

A closely related self-configuration problem, which can be mapped to a graph coloring problem, is PCC selection. In this problem, the interfering cells are configured with different component carriers to mitigate the mutual interference [115]. The aim is to configure the network efficiently using a small number of component carriers. For simplicity, it is assumed that there is no secondary usage of resources. Graph coloring enables an efficient selection of component carriers, and thus yields an improvement in SINRs experienced by the users. For simulations, single-try and multiple-try variants of the binary and real price algorithms are considered. The total number of component carriers to be distributed in the network is 5. Each base station randomly selects a PCC during the initialization phase and starts serving its users while acquiring handover measurements. This is followed by an execution of the distributed

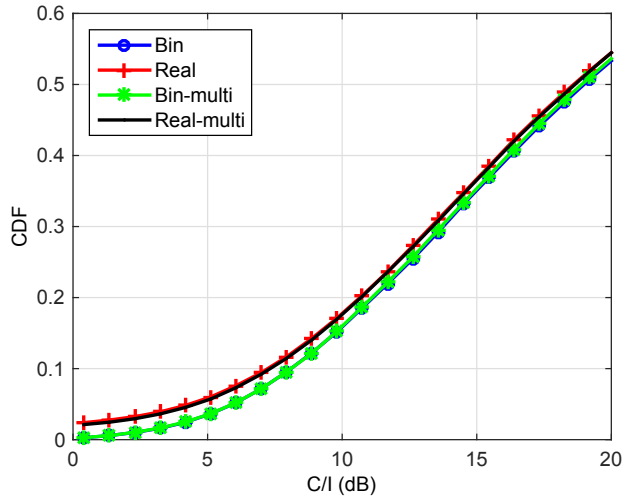


**Figure 3.1.** Comparison of graph coloring algorithms for PCI assignment.

graph coloring algorithm until convergence is reached, where the maximum number of iterations is fixed to 1000. Finally, the statistics related to the  $C/I$  ratios experienced by the users in the system are collected. A comparison of the resulting cumulative distribution functions (CDFs) is given in the Figure 3.2, which illustrates that binary price algorithms result in higher  $C/I$  ratios compared to their real price counterparts. The gain is especially more pronounced for the users in the low  $C/I$  region. It is worth noting that the local search algorithms discussed here do not involve any dedicated message-passing, except for the symmetrization of interference graph. The decisions by individual nodes are based on passive observation of the values of interfering cells. On the other hand, constraint satisfaction algorithms involve message-passing among the cells. However, the cooperative self-organizing framework employed by constraint satisfaction algorithms is more systematic, and is applicable to a vast variety of problems. The main advantage of distributed constraint satisfaction algorithms is completeness (i.e., the solution is found if it exists, or the algorithm terminates otherwise).

### 3.4 PCI assignment in multi-operator HetNets

In Publication II, the PCI assignment problem is considered for densely deployed HetNets, consisting of a multi-operator scenario with shared

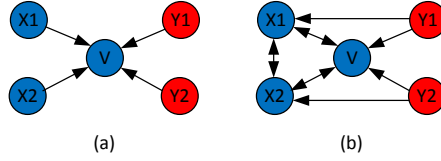


**Figure 3.2.** Comparison of graph coloring algorithms for PCC selection.

spectrum in the small cell layer. The aim is to achieve a valid PCI assignment, jointly for multiple operators. Each operator has a macro-layer and a small cell layer. The macro-layer spectrum allocation is orthogonal among the operators, whereas the small cell layers use dedicated spectrum, with inter-operator spectrum sharing enabled. Thus, the PCI resource is shared among small cells belonging to different operators. These PCIs are primarily used in handover signaling between the small cell and macro-cell layers. It is therefore important for an operator network to know which neighbors belong to the own-operator network. Moreover, conflict-freeness and confusion-freeness of PCIs must hold for all the cells sharing the spectrum.

It is assumed that there is no message-passing between the cells belonging to different operators. A direct implication of this assumption is that for different operators, the resulting interference couplings among base stations are not necessarily symmetric, which complicates the coloring problem and differentiates the problem addressed here from the single network case. This is illustrated in the interference graph shown in Figure 3.3 which delineates directed edges that arise when nodes belonging to the red and blue operators interact to create an interference graph. Here, the arrow points from the interferers to a given cell which is an interference victim. In Figure 3.3(a) cell  $V$  belonging to operator blue adds directed edges, based on measurements, from its own operator neighbors  $X_1, X_2$  as well as other-operator neighbors  $Y_1$  and  $Y_2$ . Then as

shown in Figure 3.3(b), it requests its own operator neighbors to add directed edges from its own operator and other-operator cells. Hence, there is no inter-operator message-passing.



**Figure 3.3.** Asymmetric edges in a multi-operator graph. © 2015 IEEE

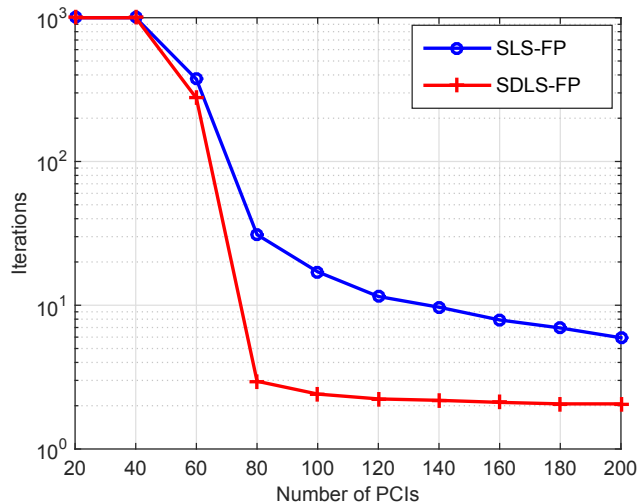
The following local search algorithms are considered for coloring the resulting directed-graph.

- Stochastic local search with focused plateau moves (SLS-FP): If vertex  $v$  using color  $c$  is in conflict, it randomly selects a color  $c' \in \mathcal{C} \setminus c$ , where  $\mathcal{C}$  is the full range of colors. It evaluates the number of conflicts  $\mathcal{F}(c')$  resulting from color  $c'$ , and starts using it if doing so either reduces the number of conflicts, or keeps them unchanged  $\mathcal{F}(c') \leq \mathcal{F}(c)$ .
- Steepest descent local search with focused plateau moves (SDLS-FP): If vertex  $v$  using color  $c$  is in conflict, it selects at random one of the best colors  $c' = \arg \min_{c \in \mathcal{C}} \mathcal{F}(c)$  among all possible alternatives in the first step.

In both cases, asynchronous and periodic updates are considered, where each vertex in the conflict-graph updates its color on its turn in every iteration. The principle of allowing the local moves only to the unsatisfied variables is known as focused search [12, 131]. Accordingly, these update rules involve focused plateau moves. This is an important step towards avoiding unnecessary reconfigurations of PCIs by the cells. Each cell decides its PCI by observing the PCIs of its neighbors which include its own network cells as well as the cells that belong to the other operators, but share the same spectrum.

For simulations, a HetNet comprising of two operators with overlapping coverage area and shared spectrum in the pico-layer is considered. The macro-layer of each operator entails a  $10 \times 10$  grid of hexagonal cells, where the grids of the operators are overlapping but not aligned. In each hexagonal cell, 10 small cells are dropped randomly and the number of users per cell is also fixed to 10. Thus, the total number of base stations in

the whole network is 2200, with 1100 base stations belonging to each operator. Each user is associated to either of the two operators, with equal probability. Both algorithms can successfully reach the fully colored state with a reasonable number of PCIs. The convergence with probability close to one is possible with 120 PCIs in the fairly dense HetNet scenario considered here. Moreover, due to the steepest descent principle, SDLS-FP outperforms SLS-FP in terms of the number of iterations required for convergence, as shown in Figure 3.4.



**Figure 3.4.** Comparison of graph coloring algorithms for PCI assignment in a multi-operator HetNet. © 2015 IEEE

### 3.5 Simulated annealing for self-organized orthogonal resource allocation

A graph coloring formulation of generic orthogonal resource allocation problem is considered in Publication III, where the network is modeled as a planar graph. The focus is on simulated annealing metaheuristic, which is essentially based on balancing exploration and exploitation, to search the configuration space effectively. Simulated annealing based methods have been applied to resource allocation and network optimization problems in a multitude of wireless networking scenarios such as WLANs [36], LTE-A [31], and cognitive radio networks [103]. Apart from plateau moves, a key feature of simulated annealing is uphill moves, which enables it to escape local minima in search of an optimal solu-

tion. Moreover, in simulated annealing, the probability of uphill moves is controlled by a noise parameter (temperature). A cooling schedule is defined (e.g. exponential, logarithmic), according to which the temperature is lowered. The main principle of simulated annealing can be understood by considering a vertex  $v$  using color  $c$ , with current number of conflicts given by  $\mathcal{F}(c)$ . The vertex may or may not be in conflict with neighbors while using resource  $c$ . On its turn to update, it picks a resource  $c' \in \mathcal{C} \setminus c$  randomly and evaluates its cost in terms of conflicts, given by the cost function  $\mathcal{F}(c')$ . If  $\Delta = \mathcal{F}(c') - \mathcal{F}(c) \leq 0$ , vertex  $v$  selects resource  $c'$ , otherwise it selects  $c'$  with the probability  $e^{-\Delta/T}$ . Thus, the higher is the cost of taking the uphill move, the lower is the probability of it being accepted. The temperature parameter  $T$  is reduced according to some cooling schedule such as  $T(n) = T_0 / \log_2(2 + n)$ , where  $n$  is the update time (iteration) and  $T_0$  is the initial temperature. The concept of focused search [12, 131], is employed to develop variants of simulated annealing, such as simulated annealing with focused uphill moves (SAFU), which is based on allowing uphill moves only to the cells that are in conflict.

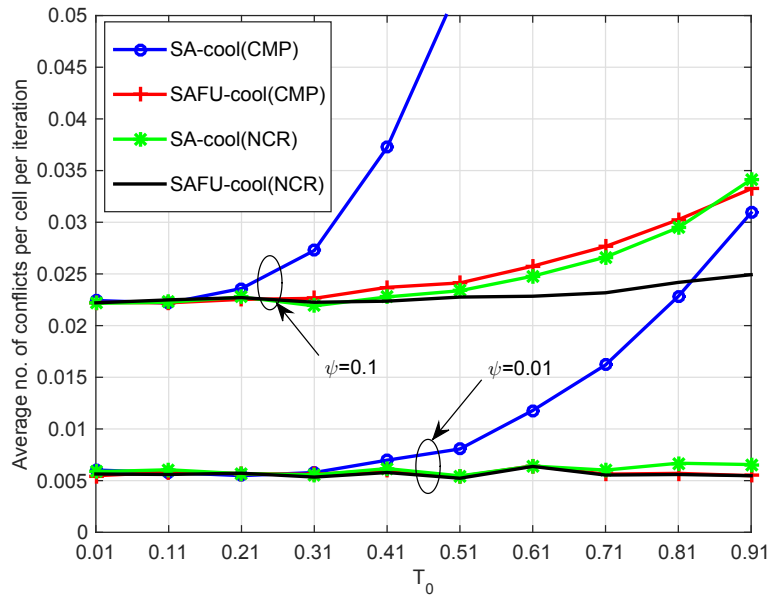
It is known that the performance of simulated annealing depends strongly on the initial value chosen for the temperature, and the cooling schedule. The optimal values are influenced by the characteristics of problem instances, and search strategy. Thus, we focus on the optimal temperatures, and analyze the performance of proposed methods in static and dynamic topologies. For static topologies, dedicated message-passing is not required between the cells, except for the symmetrization of conflict-graph. On the other hand, for dynamic topologies, a distributed temperature control protocol based on message-passing is developed. The simulated annealing algorithms that involve cooling are inherently complicated to implement in dynamic settings, especially in the distributed networks. In principle, all cells in the network must have the same temperature at all times. However, in a dynamic network, when a cell joins or leaves the network, the graph changes, and recoloring will be required to resolve the conflicts that might appear. Recoloring in this case should involve re-run of the algorithms in a distributed way, with limited or no cooperation between the cells. For enabling distributed temperature control in the network, we discuss the following cooperative and non-cooperative approaches.

- Cooperative global reset message-passing (CMP): When a new cell joins

the network, it starts with the initial temperature of  $T_0$ , regardless of the current network temperature. Any cell that detects a change in its neighbor relations, re-initializes its temperature to  $T_0$ , and sends a temperature reset message to all its neighbors. When a cell receives a temperature reset message from another cell, it resets its temperature to the current temperature of that cell, and sends a temperature reset message to its neighbors. This eventually leads to a network-wide consensus on a temperature.

- **Non-cooperative local reset (NCR):** There is no explicit cooperation between the cells in NCR, and no messages are generated. However, cells are capable of discovering new neighbors through measurements. A cell re-initializes its temperature to  $T_0$  only if it detects a change in its neighborhood. The underlying idea is to recolor only those parts of the graph which have changed. Because of the colored initial state, most of the graph would still be conflict-free. Therefore, the propagation of messages in the network to achieve a common temperature does not occur in this case, and cells may have different temperatures.

For simulations, a planar graph model of a SCN comprising of 100 cells is



**Figure 3.5.** Comparison of simulated annealing variants for coloring dynamic graphs.



considered. The maximum iterations is set to 1000, and the statistics are collected over 250 randomly generated network instances. A comparison of simulated annealing with cooling (SA-cool) and SAFU-cool is given in Figure 3.5. Here,  $\psi$  is the rate at which network changes, where  $\psi = 0.1$  corresponds to fast changes in network topology compared to  $\psi = 0.01$ , and therefore, results in an overall higher number of conflicts. It can be seen that CMP results in performance deterioration, when compared to NCR, as the temperature rise throughout the network leads to higher probability of uphill moves. A direct consequence of this is that cells with conflicts allow uphill moves with higher probability, whenever network topology changes. This is counter-productive, as the graph is mostly in a colored state already, and raising temperature leads to unnecessary uphill moves. Temperature control by NCR provides an effective alternative, where a limited number of cells raise their temperature (i.e., reset to  $T_0$ ) to enable uphill moves, and this happens in the region where a change in network occurs. The main idea here is to recolor only the sub-graph that has changed, due to the appearance or vanishing of a cell. Thus, in the colored parts of the graph, the temperature is zero, and there are no uphill moves.

### 3.6 Summary

The SON problems such as PCI assignment and PCC selection can be effectively modeled as graph coloring problems, which paves the way for application of different local search and constraint satisfaction algorithms. It should be noted that the performance of coloring algorithms as well as the structure of the underlying graph depends on the level of cooperation that exists among cells. Complete constraint satisfaction algorithms involving dedicated message-passing can avoid local minima, and outperform local search, provided that the number of iterations are sufficient. The same problem is considered in multi-operator networks, where a key requirement is that inter-operator message-passing is not possible. Consequently, the graph is asymmetric with directed edges between cells belonging to different operators, thereby complicating the coloring problem. The focused variants of distributed local search perform well in such cases. Moreover, metaheuristics that entail uphill moves such as simulated annealing and its different variants can be applied for coloring both static and dynamic graphs.

## 4. Continuous Optimization Methods for SON

Utility maximization is a popular approach used for solving network optimization problems in a number of areas such as operations research, economics, and finance. In communication networks, it was popularized by Kelly's work [80], and has since been applied for generic distributed resource allocation problems in different types of networks including *ad hoc* networks [40, 72], WSNs [33], and orthogonal frequency-division multiplexing (OFDM) based broadband networks [138]. Its popularity stems from the fact that it leads to algorithms that are simple, efficient, and distributed in nature. Thus, it is a model of choice for a systematic design of SON algorithms for current and future cellular systems, which comprise of a large number of small cells deployed randomly [105]. The NUM framework can be considered an invaluable tool for creating a clean-slate design of novel protocols, involving features such as on-the-fly optimization of network resources [68]. This sort of design approach will allow provisioning of elastic service for users with diverse QoS requirements, defined in terms of data rates and service availability [150]. This makes NUM highly relevant from the perspective of 5G protocol design, especially for enabling softwarization and NFV [39].

Different approaches from optimization theory can be applied to construct algorithms for distributed NUM [118]. In order to apply the NUM framework, each user in the network is assigned a utility function, which reflects its QoS or degree of satisfaction. The sum of utilities of individual users is often referred to as the network utility, which is maximized under a set of constraints. Examples of utility functions with applications in wireless networks include functions of the expected data rate, like the  $\alpha$ -fair rate utility function [80, 108]. A complicating factor that often arises in such problems is the existence of coupling among the utility functions of users, which leads to non-convexity of the resulting NUM problem. For

example, in the case of interference-limited wireless networks, the SINRs of the users are coupled due to mutual interference, which makes the problem challenging, especially in distributed settings.

Nevertheless, to enable an effective self-organization, the algorithm must not only be distributed but also involve limited communication overhead among the cells. Assuming that a communication interface like X2 exists between adjacent cells, we discuss self-organizing algorithms which involve an exchange of prices between the cells. Thus, the interactions among cells constitute an active exchange of information in the form of dedicated messages, which leads to the solution of NUM problem in a distributed manner. This systematic approach leads to (at-least) locally optimal SON algorithms, and enables higher gains compared to other approaches in which cells make decisions based solely on passive observations of broadcast messages made by other cells [10]. Typical SON use-cases that can be addressed using a NUM framework include ICIC, MLB, CCO, and ES. However, most of the existing work is focused on different NUM formulations of ICIC problems, such as self-organized fractional frequency reuse through distributed interference coordination [140], power control and scheduling for interference minimization [77], and heuristic fractional frequency reuse methods for interference avoidance and coordination [127]. Furthermore, interference mitigation aimed at distributed network power minimization and network capacity maximization is considered in [87]. The MLB and user-association problems are discussed in [137, 153], and the use of transmit beamforming for interference coordination is covered in [49]. A NUM based joint optimization of resources over multiple degrees of freedom for interference mitigation in multi-cell systems is considered in a number of publications including [160], Publication V, Publication VI, and Publication VII. The existing ICIC techniques are often categorized as time domain, frequency domain, and power control techniques [65, 71]. Therefore, an effective way to mitigate inter-cell interference is to optimize the allocated resources across all available degrees of freedom. To this end, we consider NUM problems involving joint optimization over multiple resources or degrees of freedom, where cells choose non-interfering modes of transmission via the allocation of resources in multiple dimensions (e.g. transmit powers over frequency carriers and multi-user scheduling weights). This reduces the mutual interference and leads to an increase in the network utility.

In order to solve the NUM problem in a distributed manner, a dis-

tributed version of the problem is formalized by decomposing the original problem into a number of subproblems, one per cell. The distributed version of the problem is then solved by a pricing exchange mechanism, where each cell solves its subproblem while taking into account the prices received from neighboring cells. Consequently, the individual cells are able to make intelligent decisions regarding the allocation of resources, which mitigates the mutual interference. The exact nature of the decomposition method depends on the structure of the problem, nature of the utility function, and the constraints. This is elaborated in the following discussion, which reflects on different approaches that can be used for designing NUM algorithms that are distributed in nature. The computational methods discussed here are general, and can be applied to different distributed resource allocation problems in wireless networks.

The first step towards the design of self-organizing algorithms using distributed NUM involves reformulation of the original NUM problem as a distributed problem. This can be achieved by decomposing the original problem into a number of subproblems, one per cell, which are then solved iteratively in a synchronous or asynchronous manner. In Publication IV, network synchronization problem is addressed, in which the NUM can be readily decomposed, and there are no constraints. Best-response and gradient algorithms are used to solve the resulting NUM. In Publication V there is a global constraint, and primal/dual decomposition with gradient search is used to decouple the global constraint. In Publication V, Publication VI, Publication VII, there are multiple optimization variables. In Publication VIII there are coupled constraints, whereas in Publication VI and Publication VII the constraints are local. These are addressed by distributed methods aiming to find KKT solutions based on greedy and gradient search.

#### **4.1 Consensus and synchronization problems**

Network synchronization is a fundamental issue in cellular systems, which has ramifications across a number of related radio resource management problems of prime importance for future wireless networks, e.g., flexible time division duplexing, coordinated multi-point transmission, and ICIC [14, 92, 165]. Hot spots comprising of dense deployment of SCNs are predominantly indoors, where the satellite signals are weak due to high penetration losses which makes satellite synchronization difficult.

Thus, distributed run-time network synchronization with minimum network overhead is perceived as a key enabler for indoor 5G small cell connectivity [109]. Frame synchronization in wireless networks is a prototypical self-organization problem. In a centralized realization, there are problems related to conveying timing messages from a central node to leaf nodes. However, on the algorithmic level, centralized network synchronization is trivial. Complexity in self-organizing solutions is therefore, strictly due to the requirement of engineered self-organization. Self-organizing algorithms for network synchronization are usually based on neighborhood interactions, in which each cell synchronizes to its neighbors by listening to their transmissions.

Network synchronization problem is considered in Publication IV, where the main aim of the base stations  $i \in \mathcal{I}$  is to agree on the value of the synchronization variable  $\mathbf{x}_i = \hat{\mathbf{x}}$ . For frame/event synchronization, the individual synchronization variables can be understood as points on a circle, represented as phases  $\Phi = [\phi_i \dots \phi_{|\mathcal{I}|}]$ , where  $\phi_i \in [0, 2\pi]$ . In order to apply NUM approach to the network synchronization problem, the first step is to define a utility function for each base station (or cell) as follows

$$U_i(\phi_i, \Phi_j) = \sum_{j \in \mathcal{N}_i} \|\phi_i - \phi_j\|_G^p, \quad (4.1)$$

where  $\mathcal{N}_i$  is the set of base stations in the neighborhood of base station  $i$ , with corresponding phases given by vector  $\Phi_j$ . Here  $p$  determines the size of the agreements that are taken into account among the synchronization variables, and  $\|\bullet\|_G$  is the geodesic distance given by

$$\|\phi_1 - \phi_2\|_G = \min(|\phi_1 - \phi_2|, 2\pi - |\phi_1 - \phi_2|). \quad (4.2)$$

The role of different norms in such utility functions was stressed in [158]. The geodesic norm is the timing difference, or the measure of frame asynchrony between a pair of base stations. The network utility is given by

$$U_{\text{sum}}(\Phi) = \sum_i U_i(\phi_i, \Phi_j). \quad (4.3)$$

Let us consider a greedy self-organizing approach for maximizing network utility, in which base stations update their phases at discrete time instants with the following best-response update

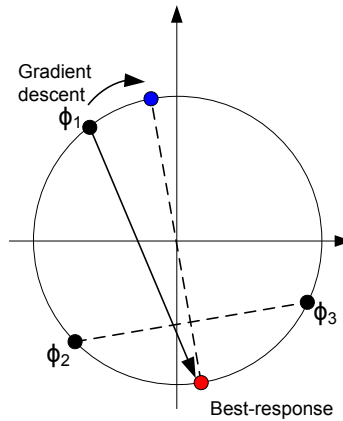
$$\phi_i^* = \arg \min_{\phi} U_i(\phi, \Phi_j). \quad (4.4)$$

Both synchronous and asynchronous updates are possible with this update rule. However, the synchronous rule may not converge due to oscillatory behaviors induced by the best-response dynamic, in which each point

on the circle representing a phase jumps to the other side as shown in Figure 4.1. In contrast to best-response, the following update rule is based on gradient descent principle, and will result in a small step  $\epsilon$  towards the solution

$$\phi_i^* = [\phi_i + \epsilon \nabla_{\phi_i} U_{\text{sum}}(\Phi)]_{\mathcal{S}}, \quad (4.5)$$

where  $[\bullet]_{\mathcal{S}}$  represents the projection of the result on the feasible set  $\mathcal{S} \triangleq [0, 2\pi]$ , and  $\nabla_{\phi_i} U_{\text{sum}}(\Phi)$  is the gradient with respect to  $\phi_i$ . It can be seen that both best-response and gradient descent updates are simple to implement and can enable fully distributed self-organized synchronization in SCNs. However, depending on the initial state of the network, both of these methods can get trapped in a local optimum, where although the network is not synchronized, no local moves can improve synchronicity. This phenomenon is a fundamental consequence of the interaction of the topology of the configuration space (a collection of circles in this case), and the topology of the communication graph of the network.



**Figure 4.1.** Different update rules for NUM based network synchronization. © 2014 IEEE

Due to the simplicity of the consensus problem, convergence properties can be investigated in general cases, where the configuration space is a more generic compact manifold than a circle. Interesting topological properties related to the mapping of the communication graph to the configuration space can be identified. In short, the homotopy classes of such mappings characterize the possible local minima. For example, if the communication graph is planar, and the configuration space is a sphere, a distributed algorithm can find global consensus, if the objective function is based on minimizing a 2-geodesic norm distance, or a higher geodesic norm.

## 4.2 Primal and dual decomposition

Primal and dual decompositions constitute an important class of decomposition methods that can enable distributed approaches for network optimization, and are applicable to a vast range of problems [118]. For convex problems, both methods perform well and achieve an optimal solution [142]. Selecting the appropriate decomposition method is an important step in the algorithm design procedure and depends on the problem structure [40]. In Publication V, an application of primal and dual decomposition for ICIC in a cognitive radio network setting is discussed.

### 4.2.1 Proportional fair-rate maximization in multi-user cognitive radio SCNs

Cognitive radio networking techniques are envisaged to play an important role in providing high capacity for 5G networks by enabling an efficient use of spectrum. Cognitive radio based technologies have applications in many paradigms related to future 5G networks, such as HetNets, D2D communication, and coexistence with other networks [93]. To unlock underutilized spectrum to be used for mobile broadband communication, a number of spectrum sharing scenarios based on primary, licensed, and unlicensed spectrum sharing models are currently under consideration. Among the licensed modes, the licensed shared access and authorized shared access approaches are highly promising for increasing the available spectrum under incumbent protection guarantees and regulatory control [74]. Incumbent protection comprises mechanisms involving geographical constraints on the usage of spectrum through a geo-location database. Thus, efficient use of radio resources to maximize the usage of the unlocked spectrum within incumbent protection constraints is important for SCNs. Moreover, the optimization of network-wide resources should be enabled in a self-organizing manner, without compromising the potential gains.

In Publication V, the focus is the maximization of network utility in SCNs, under a network-wide constraint on transmit powers. The maximization here involves optimization over two degrees of freedom — transmit powers and multi-user scheduling weights. Local and network level constraints in such NUM problems can be handled effectively by using primal and dual decomposition methods from optimization theory. The system model consists of a secondary user SCN comprising of a set of cells

(or base stations)  $\mathcal{I}$ . Each user  $l \in \mathcal{L}_i$  is assigned a utility function  $u_l$ , where  $\mathcal{L}_i$  is the set of users served by base station  $i$ .

The secondary user aspect is emphasized through a network level constraint  $\sum_{i \in \mathcal{I}} p_i \leq P_{\text{net}}$ , where  $p_i$  denotes the transmit power of base station  $i$  and  $P_{\text{net}}$  is the maximum sum transmit power that the network is allowed to use. For base station  $i$ , the local constraint set for the feasible transmit power is

$$\mathcal{P}_i = \left\{ p_i \in \mathbb{R}_+ : P_{\min} \leq p_i \leq P_{\max} \right\}. \quad (4.6)$$

Intra-cell scheduling of users served by base station  $i$  is reflected by the scheduling weights  $\mathbf{w}_i = \{w_l\}_{l \in \mathcal{L}_i}$ , where  $w_l$  is the fraction of orthogonal resources that base station  $i$  allocates to user  $l \in \mathcal{L}_i$ . It is assumed that each base station distributes all its resources among its users, thus the local constraint set for scheduling weight allocation can be expressed as

$$\mathcal{W}_i = \left\{ \mathbf{w}_i \in \mathbb{R}_+^{|\mathcal{L}_i|} : \sum_{l \in \mathcal{L}_i} w_l = 1, w_l \geq 0, l \in \mathcal{L}_i \right\}. \quad (4.7)$$

Here, we apply a PF-Rate utility function [80,108], which leads to network-wide fairness among all the users. The PF-Rate is the logarithm of the normalized Shannon rate

$$u_l = \log(w_l \log(1 + \gamma_l)), \quad (4.8)$$

where  $\gamma_l = \frac{p_i h_{i,l}}{I_l + N_0}$  is the SINR that user  $l$  (served by base station  $i$ ) experiences in downlink, and  $h_{i,l}$  is the channel power gain between base station  $i$  and user  $l$ , which is assumed to be frequency-flat. The interference power and the additive white Gaussian noise power experienced by user  $l$  are given by  $I_l = \sum_{j \neq i} p_j h_{j,l}$  and  $N_0$ , respectively. The utility of base station  $i$  is given by

$$U_i(\mathbf{p}, \mathbf{w}_i) = \sum_{l \in \mathcal{L}_i} u_l. \quad (4.9)$$

The aim is to find the transmit power and multi-user scheduling weight allocations of all base stations that maximizes the network utility under the total network power constraint  $P_{\text{net}}$ . To this end, the network level optimization problem can be formulated as

$$\begin{aligned} & \text{maximize} && \sum_{i \in \mathcal{I}} U_i(\mathbf{p}, \mathbf{w}_i) \\ & \mathbf{p}, \mathbf{W} && \\ & \text{subject to} && \sum_{i \in \mathcal{I}} p_i \leq P_{\text{net}}, \\ & && p_i \in \mathcal{P}_i, \mathbf{w}_i \in \mathcal{W}_i, \quad i \in \mathcal{I}, \end{aligned} \quad (4.10)$$



where  $\mathbf{W} = [\mathbf{w}_1 \dots \mathbf{w}_{|\mathcal{I}|}]$  comprises of scheduling weights of all the users in the SCN. This is a non-convex problem for generic  $U_i$ , as the SINRs of the receivers are coupled. However, it is convex for the PF-Rate utility considered here [98]. Note that in frequency-selective channels, this model would naturally generalize to the multi-channel version of the power allocation problem, which is non-convex [72]. Next, we apply the decomposition procedure to formulate a pricing algorithm for finding an optimal solution to (4.10) in a distributed way. We consider primal decomposition [118], for solving the optimization over  $\mathbf{p}$ . The  $|\mathcal{I}|$  scheduling weight optimization subproblems, one per base station, can be solved independently of  $\mathbf{p}$ . Thus, the optimization problem (4.10) can be solved using a two level optimization procedure. At a lower level, with  $\mathbf{p}$  fixed, the scheduling weight optimization subproblem in all cells  $i \in \mathcal{I}$ , becomes

$$\begin{aligned} & \text{maximize} && U_i(\mathbf{w}_i) \\ & && \mathbf{w}_i \\ & \text{subject to} && \mathbf{w}_i \in \mathcal{W}_i, \end{aligned} \tag{4.11}$$

which is convex and can be solved at each base station by assigning equal values to all  $w_l$ . For updating the variable  $\mathbf{p}$ , we have a master problem for all  $i \in \mathcal{I}$ , given by

$$\begin{aligned} & \text{maximize} && U_i(\mathbf{p}) \\ & && \mathbf{p} \\ & \text{subject to} && \sum_{i \in \mathcal{I}} p_i \leq P_{\text{net}}, \quad p_i \in \mathcal{P}_i. \end{aligned} \tag{4.12}$$

The power vector  $\mathbf{p}$  is the complicating variable that couples the cells both via the utility functions and the constraints. There is a network level constraint on the powers, which makes the problem more challenging than the conventional distributed power control problem [72]. In Publication V, it is erroneously stated that  $\mathbf{p}$  and  $\mathbf{W}$  are decoupled by using the decomposition. However, for a more generic utility function, the decomposition would indeed decompose  $\mathbf{W}$  from the master problem as well. In that case, the overall problem would not be convex [98].

We devise a pricing algorithm to solve the optimization over  $\mathbf{p}$  by an iterative descent method. To this end, the network utility can be expressed as

$$U_{\text{sum}}(\mathbf{p}, \mathbf{W}) = U_i(\mathbf{p}, \mathbf{w}_i) + \sum_{j \neq i} U_j(\mathbf{p}, \mathbf{w}_j). \tag{4.13}$$

Differentiating with respect to  $p_i$  we have

$$D_i = \frac{\partial U_{\text{sum}}(\mathbf{p}, \mathbf{W})}{\partial p_i} = \frac{\partial U_i(\mathbf{p}, \mathbf{w}_i)}{\partial p_i} + \sum_{j \neq i} \frac{\partial U_j(\mathbf{p}, \mathbf{w}_j)}{\partial p_i}. \tag{4.14}$$

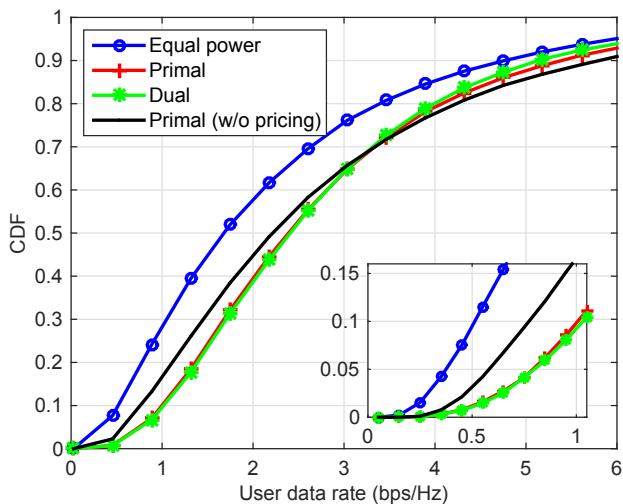
Let us define the following terms as power benefit and power price, respectively

$$\begin{aligned}\pi_{i,i} &= \frac{\partial U_i(\mathbf{p}, \mathbf{w}_i)}{\partial p_i} \quad i \in \mathcal{I}, \\ \pi_{j,i} &= \frac{\partial U_j(\mathbf{p}, \mathbf{w}_j)}{\partial p_i} \quad i, j \in \mathcal{I}, i \neq j.\end{aligned}\tag{4.15}$$

Note that the power price indicates the negative effect that an increase in transmit power of base station in cell  $i$  has on utility of cell  $j$ . By exchanging power prices, the cells know how much interference they are causing to each other, and can cooperatively maximize the total utility of the network over their respective transmit powers. In absence of temporal changes in the channels, the optimal scheduling weight allocation is the Round Robin allocation, in which all the users get an equal share of the resources. The optimization over  $\mathbf{p}$  is carried out using a distributed coordinate descent method, with asynchronous and periodic updates. In a given iteration  $n$ , each base station  $i$  performs the updates at a unique time instant  $t_i[n]$ . It receives power prices and  $D_j$  from the base stations  $j \neq i$ , and calculates its  $D_i$ . In the next step, it selects a base station  $j^*$  which can increase the network utility most. If  $D_j^* > D_i$ , it sends a power increase message to base station  $j^*$ , while reducing its own power so that the network power constraint is not violated.

It is also possible to apply the dual decomposition principle in this case, which leads to the same solution. However, the dual decomposition algorithm requires an extremely small stepsize, and therefore takes a substantially large number of iterations to converge. For performance analysis, a SCN network comprising of 12 base stations and 12 users deployed inside a multi-story WINNER office building is simulated [117]. A comparison of both algorithms in terms of user data rates is shown in the Figure 4.2. Here, the baseline case is the fixed power allocation, in which all the base stations have the same transmit power. The primal and dual decomposition algorithms reach the same solution, which is significantly better than the baseline solution. Moreover, the primal decomposition algorithm is also simulated without pricing (only with benefits), which results in a moderate reduction in performance gains.

In the cognitive radio literature, a total interference constraint has been widely considered [60,81,84,120]. The power constraint as such, is similar to a total interference constraint, and the same algorithms would apply for the power distribution problem, with suitable modifications. In [60], Max-Rate utility function is considered with power optimization. When



**Figure 4.2.** Comparison of primal and dual decomposition based algorithms in a cognitive radio SCN. © 2014 IEEE

addressing total interference, one needs additional channel state information between the secondary transmitters and primary receivers. On the algorithmic level, our approach differs from [81], in that we present a completely distributed solution to the primal problem under network-wide constraint on total transmit power. Moreover, we consider coupled utility functions, as well as coupled constraints, whereas in [84] only the (interference) constraints are coupled. A game theoretical approach is employed in [120], for a Max-Rate utility function.

### 4.3 Decomposition based on KKT conditions

A common method for solving constrained optimization problems is through the direct solution of so-called KKT conditions, which essentially are equations guaranteeing the vanishing of the gradient of the network utility. In the context of SON, we are interested in distributed solutions. The first step is to formulate the KKT conditions of the original NUM problem, followed by the formulation of distributed problem with the same KKT conditions. With the distributed formulation, the original problem is decomposed into a set of subproblems which are then solved in an iterative manner. The details are explained next, in the context of ICIC problems considered in Publication VI, Publication VII, and Publication VIII.

### 4.3.1 Rate and proportional fair-rate maximization in multi-user multi-carrier SCNs

Publication VI extends the previously discussed system model to a multi-carrier SCN model with multiple users per cell. The system bandwidth  $B$  is divided into  $|\mathcal{K}|$  equal-size carriers. It is assumed that the maximum transmit power  $P_{\max}$  of all base stations is the same, and can be distributed over  $\mathcal{K}$  carriers such that  $\sum_{k \in \mathcal{K}} p_i^k \leq P_{\max}$  for  $i \in \mathcal{I}$ , where  $0 \leq p_i^k \leq P_{\max}$  is the power that base station  $i$  uses on carrier  $k$ . Likewise, intra-cell scheduling decisions are per carrier  $k \in \mathcal{K}$ , and are reflected by the scheduling weights  $w_l^k$  that base station  $i$  allocates to each associated user  $l \in \mathcal{L}_i$ . It is assumed that each base station  $i$  distributes its resources of  $\mathcal{K}$  carriers among its active users, fulfilling  $\sum_{l \in \mathcal{L}_i} w_l^k = 1$  for  $k \in \mathcal{K}$ . The aim is to maximize the system utility which is the sum

$$U_{\text{sum}}(\mathbf{P}, \mathbf{W}) = \sum_{i \in \mathcal{I}} U_i(\mathbf{P}, \mathbf{W}_i), \quad (4.16)$$

of individual cell utilities. Here  $\mathbf{P}$  and  $\mathbf{W}$  comprise of the transmit powers, and scheduling weights of all base stations in the system, respectively, while

$$U_i(\mathbf{P}, \mathbf{W}_i) = \sum_{l \in \mathcal{L}_i} u_l \quad (4.17)$$

is the utility of cell  $i$ , which is the sum of the individual utilities of the users  $l \in \mathcal{L}_i$  served by base station  $i$ . Moreover,  $\mathbf{W}_i$  consists of scheduling weights of base station  $i$ . We consider the generic  $\alpha$ -fair rate utility

$$u_l(r_l) = \begin{cases} \frac{1}{1-\alpha} (r_l)^{1-\alpha} & \alpha \neq 1 \\ \log(r_l) & \alpha = 1 \end{cases}, \quad (4.18)$$

where the user rate  $r_l$  is simply taken as the sum of the Shannon rates of the user over  $\mathcal{K}$  carriers, with interference treated as Gaussian noise

$$r_l = \sum_{k \in \mathcal{K}} \frac{B}{|\mathcal{K}|} w_l^k \log(1 + \gamma_l^k). \quad (4.19)$$

Here  $\gamma_l^k$  denotes the SINR that user  $l$  experiences on carrier  $k$ . It can be seen that with an  $\alpha$ -fair formulation,  $\alpha = 0$  gives the rate maximizing (Max-Rate) utility function and  $\alpha = 1$  corresponds to the PF-Rate utility function. The idea is to maximize the network utility over transmit powers and scheduling weights of all the users in the network. To this

end, the NUM problem is given by:

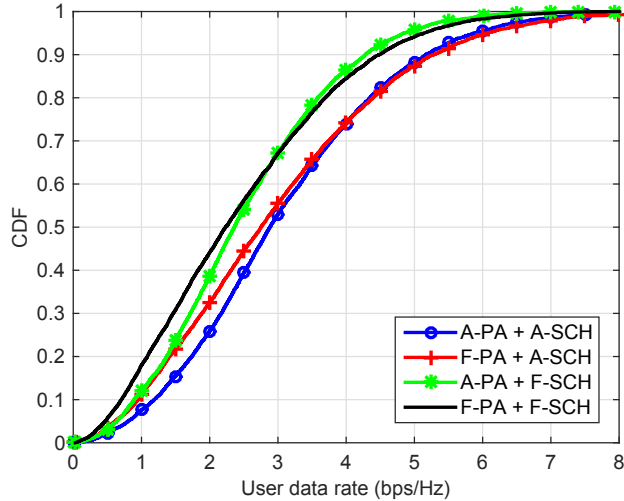
$$\begin{aligned}
 & \text{maximize} && \sum_{i \in \mathcal{I}} U_i(\mathbf{P}, \mathbf{W}_i) \\
 & && \mathbf{P}, \mathbf{W} \\
 & \text{subject to} && \sum_{k \in \mathcal{K}} p_i^k \leq P_{\max}, \quad p_i^k \geq 0, \\
 & && \sum_{l \in \mathcal{L}_i} w_l^k = 1, \quad w_l^k \geq 0.
 \end{aligned} \tag{4.20}$$

Let us define the term  $\pi_{j,i}^k = -\frac{\partial U_j(\mathbf{P}, \mathbf{W}_j)}{\partial p_i^k}$  as the power price on carrier  $k$  that cell  $j$  reports to cell  $i$ . Then, it can be seen that the KKT conditions of (4.20) are the same as the KKT conditions of the following distributed problem, which comprises of  $|\mathcal{I}|$  subproblems, one for each cell:

$$\begin{aligned}
 & \text{maximize} && U_i(\mathbf{p}_i, \mathbf{W}_i) - \sum_{k \in \mathcal{K}} p_i^k \left\{ \sum_{j \neq i} \pi_{j,i}^k \right\} \\
 & && \mathbf{p}_i, \mathbf{W}_i \\
 & \text{subject to} && \sum_{k \in \mathcal{K}} p_i^k \leq P_{\max}, \quad p_i^k \geq 0, \\
 & && \sum_{l \in \mathcal{L}_i} w_l^k = 1, \quad w_l^k \geq 0.
 \end{aligned} \tag{4.21}$$

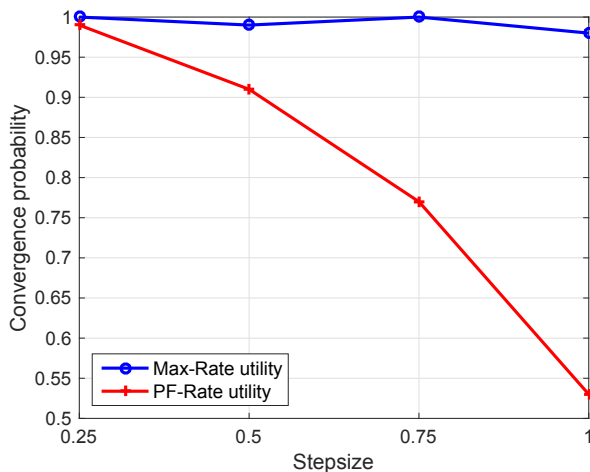
Each cell solves its subproblem and reports the updated prices to other cells. The updates are done in an asynchronous and periodic way, where a base station of a cell updates once in a given iteration at a unique time instant. In an iteration  $n$ , each base station  $i$  performs the updates at a unique time instant  $t_i[n]$ . First, a base station solves its own subproblem and calculates the power and scheduling weights. This is followed by a price update step, in which new prices are reported to other cells. Results shown in Figure 4.3, correspond to a SCN comprising of 4 base stations deployed indoors in a building with WINNER office characteristics [117]. Each base station is configured with a closed subscriber groups in its corresponding coverage area. In the coverage area of a base station, the probability of having a visiting user belonging to a different closed subscriber group is 20%. Four different cases related to power and scheduling weight optimization are compared for the PF-Rate utility. The CDFs of the experienced user data rates are plotted for different algorithms. In the baseline case, both power allocation (PA) and scheduling weights (SCH) are fixed, i.e. (F-PA + F-SCH). This is followed by adaptive power allocation combined with fixed scheduling weights (A-PA + F-SCH). A-PA yields a higher mean data rate per user, and especially improves the situation of the users in low SINR region. The CDFs for other combinations namely fixed power and adaptive scheduling (F-PA + A-SCH), and adaptive power and adaptive scheduling (A-PA + A-SCH) are also shown. It is worth noting that (F-PA + A-SCH) is the baseline solution provided by

LTE/LTE-A network in absence of ICIC functionality. Joint allocation of power and scheduling weights results in a substantial improvements in the data rates of users in low SINR regime. This is due to the fact that the PF-Rate utility enables network-wide fairness among the users in terms of data rates.



**Figure 4.3.** Comparison of KKT-based algorithms in terms of user data rates in a multi-user multi-carrier SCN. © 2012 IEEE

The stepsizes used to update the optimization variables power and scheduling weights are denoted by  $\beta^P$  and  $\beta^W$ , respectively. The impact of stepsizes on the probability of convergence is shown in Figure 4.4, for both Max-Rate and PF-Rate utility functions. In the case of Max-Rate utility, the algorithm converges with very high probability for all four stepsizes (0.25, 0.5, 0.75, 1), with  $\beta^P = \beta^W$ . It can be seen that the PF-Rate utility is more sensitive to the stepsize, with large stepsizes essentially leading to problems in convergence. The complementary curve in Figure 4.5 shows the average number of iterations (over converged network instances) against the stepsizes. Here, Max-Rate benefits from larger stepsizes in that iterations required for convergence are reduced, with no significant impact on convergence probability. However, this trend cannot be observed for the PF-Rate utility, where (on average) the iterations required for convergence do not differ significantly.

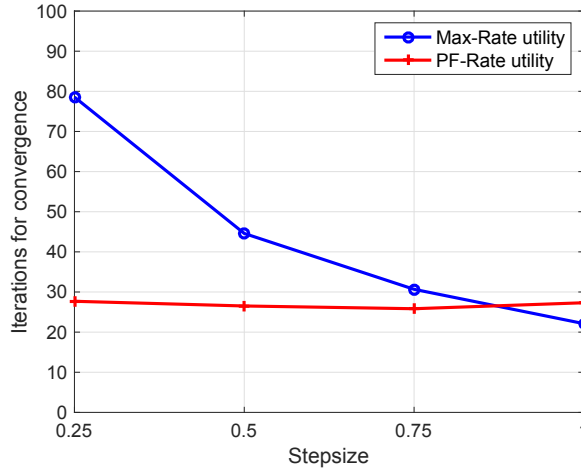


**Figure 4.4.** Impact of stepsize on the convergence probability.

### 4.3.2 Rate and proportional fair-rate maximization in multi-user multi-carrier MIMO SCNs

Publication VII extends the developed multi-user multi-carrier SCN model for MIMO systems, and aims for joint optimization over multiple resources in the downlink. This involves a joint solution of the following three related subproblems: 1) transmit power allocation, 2) transmit precoder allocation, and 3) multi-user scheduling weights allocation. To the best of our knowledge, the joint optimization over these three parameters has not been addressed in the literature before.

In order to incorporate transmit precoders in NUM problem formulation, the transmit covariance matrix of base station  $i$  on carrier  $k$  is split as  $p_i^k \mathbf{Q}_i^k$ , where  $p_i^k$  is the transmit power on carrier  $k$ , and  $\mathbf{Q}_i^k$  is the normalized covariance matrix which is positive semi-definite  $\mathbf{Q}_i^k \succeq 0$ , with  $\text{Tr}(\mathbf{Q}_i^k) = 1$ . The carrier specific covariance matrices of base station  $i$  can be stacked to a matrix  $\mathbf{Q}_i = [\mathbf{Q}_i^1 \dots \mathbf{Q}_i^k \dots \mathbf{Q}_i^{|\mathcal{K}|}]$ , and all covariance matrices in the network are stacked to a larger matrix  $\mathbf{Q}$ . The optimization over  $\mathbf{Q}$  is considered as a network-wide precoder design problem, which has to be solved jointly with optimizations over  $\mathbf{P}$  and  $\mathbf{W}$ . Thus, the utilities of cells are coupled through both  $\mathbf{P}$  and  $\mathbf{Q}$ . This means that the utility of cell  $i$  will be affected by any change in the power or the precoder allocation of a neighboring base station  $j \neq i$ . The utility of cell  $i$ , which is the



**Figure 4.5.** Impact of stepsize on the iterations required for convergence.

sum of utilities of the users served by base station  $i$ , can be expressed as

$$U_i(\mathbf{P}, \mathbf{Q}, \mathbf{W}_i) = \sum_{l \in \mathcal{L}_i} u_l, \quad (4.22)$$

where  $u_l$  is the utility of user  $l$  chosen as  $\alpha$ -fair rate utility. The achievable rate of user  $l$  in cell  $i$  is given by

$$r_l = \sum_{k \in \mathcal{K}} \frac{B}{|\mathcal{K}|} w_l^k \log \left( \det \left( \mathbf{I} + \mathbf{Z}_{i,l}^k \left( \mathbf{X}_{i,l}^k \right)^{-1} \right) \right), \quad (4.23)$$

where  $\mathbf{Z}_{i,l}^k$  is defined as

$$\mathbf{Z}_{i,l}^k = p_i^k \mathbf{H}_{i,l}^k \mathbf{Q}_i^k \left( \mathbf{H}_{i,l}^k \right)^H. \quad (4.24)$$

The noise-plus-interference covariance matrix is

$$\mathbf{X}_{i,l}^k = \left( \mathbf{R}_{n_{i,l}} + \sum_{j \neq i} p_j^k \mathbf{H}_{j,l}^k \mathbf{Q}_j^k \left( \mathbf{H}_{j,l}^k \right)^H \right), \quad (4.25)$$

where  $\mathbf{R}_{n_{i,l}}$  is the noise covariance, and  $\mathbf{H}_{j,l}^k$  is the channel matrix from base station  $j$  to user  $l$  on carrier  $k$ . For ease of presentation, we define

$$\mathbf{M}_{i,l}^k = \mathbf{I} + \mathbf{Z}_{i,l}^k \left( \mathbf{X}_{i,l}^k \right)^{-1}. \quad (4.26)$$



The NUM problem that we aim to solve is then given by

$$\begin{aligned}
 & \text{maximize } \sum_{i \in \mathcal{I}} U_i(\mathbf{P}, \mathbf{Q}, \mathbf{W}_i) \\
 & \mathbf{P} \quad \mathbf{Q} \quad \mathbf{W} \\
 & \text{subject to } \quad \text{Tr}(\mathbf{Q}_i^k) = 1, \quad i \in \mathcal{I}, k \in \mathcal{K}, \\
 & \quad \quad \quad \mathbf{Q}_i^k \succeq 0, \quad i \in \mathcal{I}, k \in \mathcal{K}, \\
 & \quad \quad \quad \sum_{l \in \mathcal{L}_i} w_l^k = 1, \quad i \in \mathcal{I}, k \in \mathcal{K}, \\
 & \quad \quad \quad \sum_{k \in \mathcal{K}} p_i^k \leq P_{\max}, \quad i \in \mathcal{I}, \\
 & \quad \quad \quad \sum_{k \in \mathcal{K}} p_i^k \geq P_{\min}, \quad i \in \mathcal{I}, \\
 & \quad \quad \quad p_i^k \geq 0, \quad i \in \mathcal{I}, k \in \mathcal{K}, \\
 & \quad \quad \quad w_l^k \geq 0, \quad i \in \mathcal{I}, k \in \mathcal{K}, l \in \mathcal{L}_i.
 \end{aligned} \tag{4.27}$$

The objective function is the sum of the individual utilities of all users served by the base stations in the network, and the optimization is carried out over transmit powers, transmit precoders, and scheduling weights. The sum of scheduling weights on each carrier equals 1, and the sum of base station power over carriers is constrained by a maximum and minimum transmit power limit, given by  $P_{\max}$  and  $P_{\min}$ , respectively. Also, there are non-negativity constraints on powers and scheduling weights. It is well-known that this is a non-convex problem in both transmit power and covariance matrices, and therefore difficult to solve, even in a centralized setting [72] [154]. Following the decomposition procedure detailed in [72], we formulate a distributed problem which has same KKT conditions as the network level optimization problem (4.27)

$$\begin{aligned}
 & \text{maximize } s_i \\
 & \mathbf{p}_i, \mathbf{Q}_i, \mathbf{W}_i \\
 & \text{subject to } \quad \text{Tr}(\mathbf{Q}_i^k) = 1, \quad k \in \mathcal{K}, \\
 & \quad \quad \quad \mathbf{Q}_i^k \succeq 0, \quad k \in \mathcal{K}, \\
 & \quad \quad \quad \sum_{l \in \mathcal{L}_i} w_l^k = 1, \quad k \in \mathcal{K}, \\
 & \quad \quad \quad \sum_{k \in \mathcal{K}} p_i^k \leq P_{\max}, \\
 & \quad \quad \quad \sum_{k \in \mathcal{K}} p_i^k \geq P_{\min}, \\
 & \quad \quad \quad p_i^k \geq 0, \quad k \in \mathcal{K}, \\
 & \quad \quad \quad w_l^k \geq 0, \quad k \in \mathcal{K}, \quad l \in \mathcal{L}_i,
 \end{aligned} \tag{4.28}$$

where  $s_i$  is a surplus function defined as

$$s_i = U_i - \sum_{k \in \mathcal{K}} p_i^k \left( \sum_{j \neq i} \pi_{j,i}^k \right) - \sum_{k \in \mathcal{K}} \text{Tr} \left( \mathbf{Q}_i^k \sum_{j \neq i} \mathbf{\Pi}_{j,i}^k \right). \tag{4.29}$$

Note that  $\pi_{j,i}^k$  and  $\mathbf{\Pi}_{j,i}^k$  are the prices related to the transmit power and transmit precoder optimization subproblems, respectively. The power

price  $\pi_{j,i}^k$  takes the following form

$$\begin{aligned} \pi_{j,i}^k &= -\frac{\partial U_j}{\partial p_i^k} \\ &= \sum_{l \in \mathcal{L}_j} r_l^{-\alpha} w_l^k \text{Tr} \left( \left( \mathbf{M}_{j,l}^k \right)^{-1} \mathbf{Z}_{j,l}^k \mathbf{V}_{j,l}^k \right), \quad j \neq i, \end{aligned} \quad (4.30)$$

where  $\mathbf{V}_{j,l}^k = \left( \mathbf{X}_{j,l}^k \right)^{-1} \mathbf{H}_{i,l}^k \mathbf{Q}_i^k \left( \mathbf{H}_{i,l}^k \right)^H \left( \mathbf{X}_{j,l}^k \right)^{-1}$ . Similarly, the precoder price  $\Pi_{j,i}^k$  is defined as

$$\begin{aligned} \Pi_{j,i}^k &= -\frac{\partial U_j}{\partial \mathbf{Q}_i^k} \\ &= \sum_{l \in \mathcal{L}_j} r_l^{-\alpha} w_l^k \tilde{\mathbf{V}}_{j,l}^k, \quad j \neq i, \end{aligned} \quad (4.31)$$

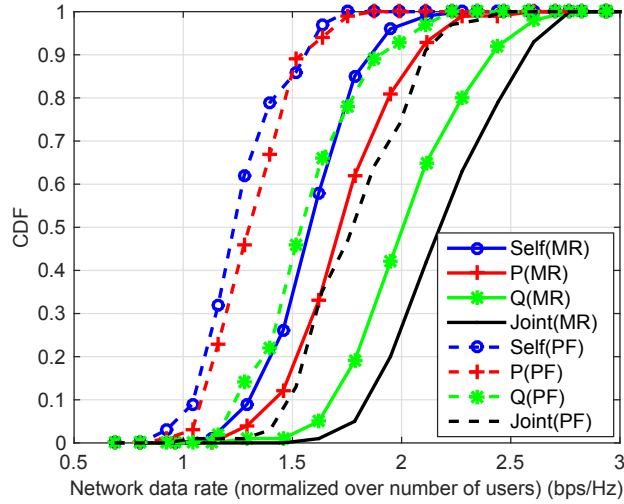
where  $\tilde{\mathbf{V}}_{j,l}^k = \left( \mathbf{H}_{i,l}^k \right)^H \left( \mathbf{X}_{j,l}^k + \mathbf{Z}_{j,l}^k \right)^{-1} \mathbf{Z}_{j,l}^k \left( \mathbf{X}_{j,l}^k \right)^{-1} \mathbf{H}_{i,l}^k$ . These terms can be explicitly calculated for Max-Rate ( $\alpha = 0$ ) and PF-Rate ( $\alpha = 1$ ) utilities under consideration. The power allocation and precoder allocation problems couple the decisions between cells, whereas the multi-user scheduling does not directly affect the interference experienced in neighboring cells, but it does affect the price of interference reported to the adjacent cells. Each base station  $i \in \mathcal{I}$  solves its individual subproblem, which involves joint optimization over  $\mathbf{p}_i$ ,  $\mathbf{Q}_i$ , and  $\mathbf{W}_i$ . Following the approach presented in [41], it is possible to separate the joint optimization for each base station  $i \in \mathcal{I}$ , so that the optimizations over  $\mathbf{p}_i$ ,  $\mathbf{Q}_i$ , and  $\mathbf{W}_i$  are carried out separately.

Different update rules can be used to solve individual subproblems at each base station. For example, the gradient of the total utility of the network can be calculated using the power and precoder prices that each base station receives from the rest of the base stations. Therefore, it is possible to formulate a Gauss-Seidel gradient projection (GSGP) algorithm for optimization of both transmit powers and precoders. In this section, we consider Non-Linear Gauss-Seidel (NLGS) algorithm, which is another generic approach used for solving the non-linear optimization problems in an iterative manner. It involves each base station taking a fixed step towards the solution of its individual optimization subproblems in every iteration, where the effect on other cells is taken into account by gradient based prices. For details on the background of concept see, e.g. [28], which discusses algorithms that are based on full non-linear information. We consider price exchange, where in a given iteration round, each base station updates its powers, precoders, and scheduling weights while taking into account the prices from other base stations. This is followed by an update of power and precoder prices.

For simulations, we consider a high interference scenario comprising of  $|\mathcal{I}| = 4$  small cell base stations and  $|\mathcal{L}| = 12$  users, uniformly distributed on the surface of a circular region with radius  $R = 15$  m. Furthermore, the base stations are distributed randomly for each network instance with a minimum inter-base station separation of  $d_{\min(\text{bs}-\text{bs})} = 5$  m. Similarly, the users are also deployed randomly with a minimum distance to the closest base station  $d_{\min(\text{ms}-\text{bs})} = 5$  m. The cell association of each user is based on received signal power from base station in the downlink. Under the assumption that all base stations have the same transmit power, each user is served by the base station from which it sees the lowest path-loss attenuation. The system bandwidth is  $B = 10$  MHz, and is divided equally into  $|\mathcal{K}| = 2$  non-contiguous carriers with center frequency 3.5 GHz. The antenna heights at base stations and users are  $h_{\text{bs}} = 10$  m and  $h_{\text{ms}} = 1.5$  m, respectively, and the number of transmit and receive antennas is  $N_T = N_R = 2$ . The distance dependent path-loss and shadow fading parameters are according to the urban micro-cell scenario specified in [102].

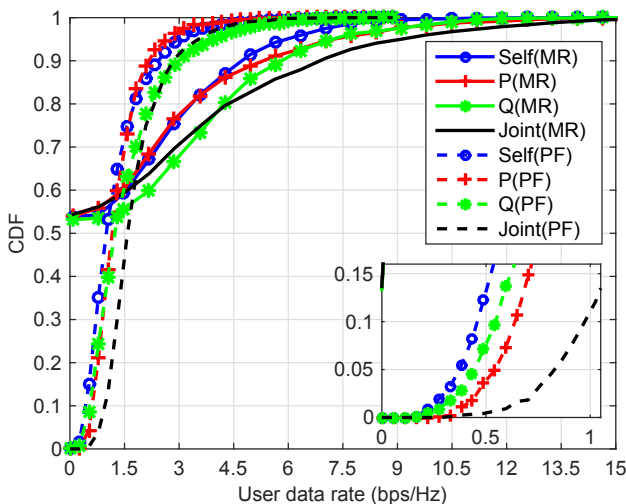
To analyze the performance, we consider NLGS over different combinations of inter-cell and intra-cell resource allocation strategies. The statistics are gathered over 100 network instances. For a given instance, each algorithm is run until convergence is achieved or a maximum number of iterations is reached. The stopping condition for convergence is that the change in total network utility is less than 0.1% in successive iterations. The maximum number of iterations is 250. However, with a proper stepsize for each resource, convergence can be easily observed in a much smaller number of iterations in all cases. The stepsizes for updating the optimization variables are set as  $\beta^P = \beta^Q = 0.7$ , and  $\beta^W = 1$ . This stepsize is chosen to obtain a balance between obtained convergence probability, and the number of iterations required for convergence.

A comparison of Max-Rate and PF-Rate utilities in terms of network data rate for the NLGS is given in Figure 4.6. The baseline case (Self) is the fully non-cooperative or selfish scheme in which no prices are exchanged, where Self-MR and Self-PF are the variants for Max-Rate (MR) and PF-Rate (PF) utilities, respectively. It can be seen that Self-MR significantly outperforms Self-PF in terms of network data rate. This is because Max-Rate utility function aims at maximizing the network data rate directly over all resources. On the other hand, PF-Rate maximizes the logarithmic function of rate to ensure fairness among the users at a network



**Figure 4.6.** Comparison of KKT-based algorithms in terms of network data rate in a multi-user multi-carrier MIMO SCN.

level, taking into account all allocations across all resources. Note that in Self-MR and Self-PF, no pricing information is exchanged; therefore, surplus maximization steps in NLGS algorithms are done with power and precoder prices set to zero. This enables us to quantify the gain achievable by the use of pricing for powers (P) and precoders (Q). First, partially cooperative pricing alternatives like P(MR) and Q(MR) are considered, in which the pricing is exchanged over one degree of freedom only. Thus, P(MR) means pricing is used for surplus maximization in the power allocation step of NLGS, while the surplus maximization for precoder allocation step is done without prices. Similarly, the converse case is Q(MR), in which cooperation or pricing exchange takes place over the precoder only, where the power allocation step is carried out without pricing. The fully cooperative case happens when the resources are optimized jointly (Joint), with prices exchanged over both power and precoders. All four variants are considered for both utility functions. Note that in all these cases, the scheduling allocation is always carried out without cooperation, as it is an intra-cell resource allocation problem and does not require any pricing exchange. In all cases, the MR variants outperform their PF counterparts. The net gain in terms of network data rate of the pricing (Joint) over the selfish case (Self) is around 40% for both utility functions. A complementary plot of user data rates is illustrated in Figure 4.7, where a significant gain can be observed for low-percentile users (highlighted in the inset) in



**Figure 4.7.** Comparison of KKT-based algorithms in terms of user data rates in a multi-user multi-carrier MIMO SCN.

the case of PF. Thus, the main advantage of PF is a high degree of fairness in terms of data rates of individual users. The difference is especially pronounced towards the lower end of the CDF, where MR variants exhibit a large outage.

### 4.3.3 Power minimization in multi-carrier SCNs

Publication VIII considers application of KKT based decomposition to an alternative formulation of the multi-carrier power allocation problem for ICIC. The aim is to minimize the total transmit power subject to a rate constraint per user. The authors of [119] address this problem using non-cooperative game theory, and in [114] a generalized approach is presented, where each transmitter minimizes its sum power over carriers, weighted by a fixed power price function that is different for each spectral portion of the transmit signal bandwidth and does not depend on the power allocation on other links. In [123], successive convex approximation is used for handling the constraints, and a primal decomposition method is employed to decouple the optimization among base stations in a multi-cell multi-user MIMO system.

The method we discuss here leads to a dynamic pricing algorithm for power minimization in the network, subject to a rate constraint for each user. For simplicity, it is assumed that there is a single user per cell. Thus,  $|\mathcal{L}_i| = 1$  and  $w_l^k = 1$ , for  $l \in \mathcal{L}_i$ . The aim is to find the power

allocation of all base stations over all carriers  $\mathbf{P} = [\mathbf{p}_1 \cdots \mathbf{p}_i \cdots \mathbf{p}_{|\mathcal{I}|}]$  with  $\mathbf{p}_i = [p_i^1 \cdots p_i^k \cdots p_i^{|\mathcal{K}|}]$ , such that each user is able to achieve the target rate  $R_i$ , while the sum power over all base stations is minimized. To this end, the NUM problem can be formulated as follows

$$\begin{aligned}
 & \underset{\mathbf{P}}{\text{minimize}} && \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} p_i^k \\
 & \text{subject to} && r_l \geq R_i, \\
 & && p_i^k \geq 0, \quad k \in \mathcal{K}, \quad l \in \mathcal{L}_i.
 \end{aligned} \tag{4.32}$$

This is a non-convex problem, thus finding a global optimum is prohibitively complex in practice, as it would require a centralized control and complete information regarding power allocations and channel power gains of all base station to user links.

We proceed with a distributed formulation, and define  $\pi_{j,i}^k$  as the power price from cell  $j$  to cell  $i$  on carrier  $k$  as

$$\pi_{j,i}^k = -\lambda_j \frac{\partial r_i^k}{\partial p_i^k} = \lambda_j \frac{p_j^k (\gamma_j^k)^2 h_{i,j}^k}{(1 + p_j^k \gamma_j^k) h_{j,j}^k} \quad l \in \mathcal{L}_j, \tag{4.33}$$

where  $\lambda_j$  is a Lagrange multiplier, and  $r_i^k$  denotes the fraction of the data rate that user  $j$  gets from carrier  $k$ . In this case, the SINR  $\gamma_j^k$  is normalized by  $p_j^k$  for notational convenience. The prices  $\pi_{j,i}^k$  reflect the marginal loss in data rate that cell  $j$  experiences as a result of increase in power on carrier  $k$  by base station  $i$ . Application of the decomposition procedure yields the following distributed problem with the same KKT conditions as (4.32)

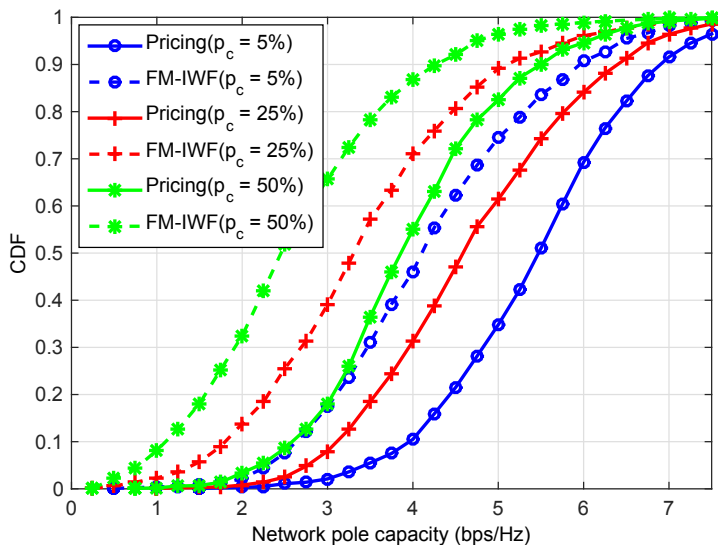
$$\begin{aligned}
 & \underset{\mathbf{P}_i}{\text{minimize}} && \sum_{k \in \mathcal{K}} \rho_i^k p_i^k \\
 & \text{subject to} && r_l = \sum_{k \in \mathcal{K}} \log(1 + p_i^k \gamma_i^k) \geq R_i, \\
 & && p_i^k \geq 0, \quad k \in \mathcal{K}, \quad l \in \mathcal{L}_i,
 \end{aligned} \tag{4.34}$$

where

$$\rho_i^k = 1 + \sum_{j \neq i} \pi_{j,i}^k \tag{4.35}$$

is a weighting coefficient comprising of power prices on carrier  $k$ .

In each cell, the base station tries to minimize the weighted sum of its powers, where the weights comprise of the prices reported by interfering cells. The weights reflect the strength of interference coupling among cells on different carriers. Thus, the larger the weight coefficient in a given carrier, the smaller the power that the base station should allocate to it, to minimize its impact on the achievable data rate of neighboring



**Figure 4.8.** Comparison of KKT-based power minimization algorithms in a multi-carrier SCN. © 2013 IEEE

cells. Note that if we set  $\rho_i^k = 1$  for all  $k \in \mathcal{K}$ , the optimization problem in (4.34) reduces to non-cooperative optimization of power per transmitter, which can be solved in a distributed way with the Fixed Margin IWF (FM-IWF) algorithm [119, 159], without any exchange of prices. The optimal response of cell  $i$  when the rest of the cells keep their power profile constant is similar to the well-known water-filling power allocation, but in this case it includes the weighting coefficients  $\rho_i^k$  that depend on the prices that neighboring cells report. Each base station updates its powers and prices on a unique time  $t_i[n]$ , where  $n$  is the iteration index. For the power update, each base station  $i$  takes into account the prices reported by neighboring cells first, constructing the power weighting coefficients per carrier and solving its individual optimization problem (4.34). This is followed by a price update step, where base station  $i$  computes the prices that corresponds to the updated power allocation profile  $\mathbf{p}_i$ , and sends this information to the base stations  $j \neq i$ .

For simulations, we consider a SCN scenario with  $|\mathcal{I}| = 4$  closed-access base stations, each serving a single user, deployed in a single-story building. The system bandwidth is equally divided into  $|\mathcal{K}| = 4$  carriers. Each base station is serving a primary coverage area, which consists of the section of the building that is closest to that base station. The users belonging to the closed subscriber groups are served by the correspond-

ing base station irrespective of their location. The location of the users is uniformly distributed on the primary coverage area of their own cell with probability  $1 - p_c$ , and uniformly distributed on the primary coverage area of the other cells with probability  $p_c$ . Note that by increasing  $p_c$ , the probability of high co-channel interference grows as well. Each cell in the system attempts to reach a given target data rate with minimum transmit power. The *network pole capacity* for a given network instance is defined as the maximum rate that an algorithm can provide to all users simultaneously [135]. In Figure 4.8, the CDFs of network pole capacities of the proposed algorithm and FM-IWF are compared for three different values of visiting probability  $p_c$ . The statistics are gathered over 1000 random network instances. For each instance, both algorithms are run with the given set of target rates and the largest feasible target rate is considered as the pole capacity for that instance (which may depend on initial conditions). The criteria for convergence is that in successive iterations, relative change in powers is less than 1% for all carriers in all cells, and the maximum difference between rates of individual users and the target rate is less than 1%. A rate is considered infeasible if the convergence conditions are not met within 500 iterations. The considered convergence criteria has an impact on the numerical results related to convergence. To reduce the number of iterations for convergence, these numbers may be relaxed. The comparison of CDFs given in Figure 4.8 clearly shows that the proposed pricing algorithm can attain higher target rate than FM-IWF. For instance, when  $p_c = 5\%$ , the median network pole capacity is of the order of 5.5 bps/Hz, as compared to 4.1 bps/Hz for FM-IWF. A similar trend ensues when  $p_c$  is set to higher values, but the network pole capacity drops as expected, because of higher cross-interference among cells.

#### 4.4 Summary

In this chapter, the focus is on continuous optimization methods that can be employed to solve a number of SON problems such as network synchronization and ICIC. Network utility maximization framework is used to design distributed pricing algorithms for these problems. The utility is a function of optimization variable, which reflects the satisfaction level or QoS of a user, and is chosen according to the network level objective. The core idea is to formulate a distributed version of the NUM problem, by decomposing the original problem into a number of subproblems, one per



base station, which are then solved distributively, in an iterative manner. When the utility functions are coupled, prices from other cells are taken into account, for solving the subproblems. A price reflects the effect of change in the resource allocation of a given cell, on the other-cell utility. Resource allocation algorithms based on price exchange mechanism lead to significant gains over selfish and non-cooperative schemes.

The concept of decomposition is introduced using network synchronization problem, in which there is no coupling among the utility functions. In such cases, exchange of prices is not required. In more complicated NUM problems, e.g. downlink resource allocation, the utility functions are coupled, and an exchange of prices results in significant gains over selfish and non-cooperative schemes. It is worth noting that NUM framework allows joint optimization over multiple resources. Exchange of prices over these multiple degrees of freedom results in further performance improvement. Furthermore, the selection of decomposition procedure, and the algorithm design often depends on the structure of the NUM problem.

## 5. Conclusions and Future work

### 5.1 Conclusions

An efficient utilization of network resources is pivotal for meeting the ever-increasing and diverse QoS demands of users, while minimizing the CAPEX/OPEX, in current as well as future mobile cellular networks. Enabling it in ultra-dense small cell and HetNet deployment scenarios necessitates the use of SON algorithms, which have proven highly successful in contemporary cellular networks such as LTE/LTE-A. In this thesis, the core idea is to develop a network optimization approach towards algorithm design for enabling self-organization in SCNs. Different mathematical models from discrete and continuous optimization theory have been studied to develop distributed algorithms for different SON problems, identified by industry and standardization bodies. In discrete optimization models, graph coloring algorithms based on local search and complete algorithms with message-passing can be effectively used to solve SON problems with constraints on logical degrees of freedom. Next, the focus is on the SON problems which can be modeled as continuous network optimization problems. For such problems, a NUM framework can be applied in conjunction with different decomposition methods which lead to distributed pricing algorithms. The pricing algorithms involving joint optimization over all available degrees of freedom in the network are not only useful for optimization of existing networks, but can enable design of new, flexible and optimizable protocols for future cellular networks. Exchange of dedicated information among cells results in substantially better performance and faster convergence, as compared to fully distributed solutions. When the constraints on the variables are local, direct distributed solution of KKT equations can be obtained, with limited

message-passing. When there are global constraints, more involved decomposition methods such as primal and dual decompositions have to be used.

We conclude that discrete network optimization models based on coloring undirected and directed conflict-graphs, and the continuous models such as NUM emerge as useful mathematical tools pertinent to self-organization problems in SCNs. The discussed approaches are effective in designing SON algorithms for a wide range of SON problems relevant to future mobile networks.

## 5.2 Future work

The proposed methods entails a generic framework that can be used to design network capable of meeting highly diverse requirements of 5G use-cases which includes massive broadband, massive machine communications, and mission critical communications. In this work, the main focus is on massive broadband, i.e. maximizing throughput or utility of the network through resource allocation over multiple degrees of freedom. The proposed NUM framework can also be extended to machine-type communications. Mission critical communications involves constraints related to network reliability, availability, and latency, and massive machine communications require a very high number of devices with stringent constraints in terms of cost and power. The developed first-principles approach based on NUM can easily be extended to incorporate such constraints. Thus, network can be designed as an elastic service optimizing overall resources jointly, and meeting the requirements of users. Some work in this direction has been presented in [150], for a generic network model. From a 5G perspective, the NUM framework is also useful for optimizing resource allocation in the context of NFV [39, 77], and SDN [91].

The core issue is how the continuous NUM problems or discrete network optimization problems are solved for enabling self-organization, especially for mission critical and massive machine 5G networks. In this regard, an interesting research direction is to generalize towards network gossiping which focuses on distributed communication and computation through network diffusion, subject to the cost of communication and the network connectivity constraints. Gossiping algorithms are a powerful tool for realizing emergent behavior, even with simple computation and limited communication overhead between nodes [48]. Minimizing the

data and information to be exchanged, and formulating the dissemination mechanism among energy-constrained nodes is also a relevant problem. Graph theory proves to be an invaluable tool in this case for modeling the communication and connectivity constraints among the nodes.

Another exciting and challenging avenue for future research is to characterize the convergence properties of self-organizing algorithms, especially when each network node has limited information regarding the complete network state.

Investigating the issues discussed above in dynamic network topologies is another prolific direction for future work. The idea would be to investigate the impact of random changes in the network topology (i.e. nodes joining and leaving the network), through the application of a stochastic NUM framework [155]. Distributed stochastic and robust optimization theory is of due importance in this regard, as many parameters in real networks are time-varying, unknown, or based on inaccurate measurements and estimates [151]. Furthermore, understanding trade-offs between the level of information exchange and performance metrics, such as robustness, speed of convergence, network throughput, reliability, availability and latency, is also of great significance from the perspective of an overall efficient network architecture [68, 104].

Finally, there are SON problems that can be modeled through both discrete and continuous network optimization models. Identifying these problems and analyzing the interplay between discrete and continuous optimization models, as well as associated theoretical and algorithmic issues is also of notable interest.



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# Errata

## Publication V

It is erroneously stated that  $p$  and  $W$  are decoupled by using the decomposition. However, for a more generic utility function, the decomposition would indeed decompose  $W$  from the master problem as well. In that case, the overall problem would not be convex [98].



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