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A GAME MODEL OF IRREVERSIBLE INVESTMENT UNDER UNCERTAINTY

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Most of the literature on real options considers the optimal decision of a firm in isolation from competitors. In reality, however, the actions of competing firms often affect each other's investment opportunities. We develop a game model where many firms compete for a single investment opportunity. When one of the firms triggers the investment the opportunity is completely lost for the other firms. The value of the project for the firms is assumed to follow a geometric Brownian motion. The model combines game theory and the theory of irreversible investment under uncertainty. We characterize the resulting Nash equilibrium under different assumptions on the information that the firms have about each other's valuations for the project. As an example, we present a case of building a telecommunications network.

Keywords: Real options; irreversible investment; game theory; uncertainty; telecommunication.

1. Introduction

In recent years, the literature on real options has improved considerably our understanding of the irreversible capital investment problems under uncertainty. This literature stresses the similarity between a financial call option and an opportunity to invest in a real asset. The investment problem can be seen as a typical valuation problem of an American option in which the theory of optimal stopping times [e.g., Karatzas and Shreve (1988)] is used. An excellent survey of the main theory is given in Dixit and Pindyck (1994). Important contributions include, e.g., McDonald and Siegel (1986), Pindyck (1988) and Dixit (1995). Another survey on different models is Trigeorgis (1996).

In a market with no large investors, the value of the real option is equal to the net present value of the investment after all costs plus the time value of the real option. The entry time is selected so that the value of the option is maximized. In other words, investment is made at such a moment when the time value is zero and net present value is strictly positive. However, in the case of large investors [see e.g., Keppo and Lu (1999)] the problem is much more complicated because we have to consider the impact of the investments on the net present values. This leads to an investment game between the firms.

Most of the models presented in the literature do not take into account the competitive aspects. This is often a serious limitation if the purpose of the modeling is to get more understanding on the functioning of some industry, but also if the purpose of the modeling is to help decision makers make better investment decisions. Del Sol and Ghemawat (1999) argue that typical option valuation models tend to recommend waiting too long before investing, because they fail to recognize the competition for a limited number of business opportunities.

The existing literature that takes competitive aspects into account can be divided into two classes. First, there are papers, which study the market equilibrium under the assumption of perfect competition and free entry. Lucas and Prescott (1971) showed the social optimality of the equilibrium in a discrete-time Markov chain model. Leahy (1993) discovered that the equilibrium entry time under free entry is the same as the optimal entry time of a myopic firm who ignores future entry by competitors. Baldursson and Karatzas (1996) generalize the result utilizing singular stochastic control theory. Grenadier (1999) enriches the analyses by including construction delays. An interesting application is developed in Tvedt (1999).

A second and more recent stream of literature takes the game theoretic approach. In these models, the number of firms is exogenous and therefore independent on the equilibrium result. Some of the models are set in a two period framework, e.g., Kulatilaka and Perotti (1998), which analyze the strategic interaction of investments that induce cost advantage over rivals. In continuous time, Williams (1993) and Baldursson (1998) are examples of models where firms can adjust capacity continuously. On the other hand, Grenadier (1996), Lambrect (1999) and Joaquin and Butler (1999) present models where competing firms have opportunities to invest in discrete investment projects and where the game is played on the timing of these investments.

This paper adds to this second stream of literature. We analyze a particular kind of setting, where the investment of a firm completely eliminates corresponding investment opportunities of the other firms. The agents must first pay the fixed sunk cost before the new investment can start. Thus, at the beginning of the optimization period the agents hold American options and, therefore, we employ the optimal stopping theory. However, the strategic effects complicate the situation. Even if the model itself is dynamic, the strategic interaction between the players results in a one-shot game in investment strategies. The equilibrium strategies of the firms depend crucially on the information that the firms possess about the valuation of the business opportunity by their competitors. We find the Nash equilibrium under different assumptions on the information structure. Under the assumption that firms do not know each other's valuation for the project, we propose that firms use simple assessments on the likelihood that their competitors will invest within the next "time instant." We characterize the equilibrium that results from this type of assessments.

As an example of the model we present a case of building a new telecommunications network. If we assume that the new network will not affect the business opportunities of the competitors, the optimal investment strategy follows from standard real option theory and can be derived separately for each firm. Telecommunications investments in the case of a single large agent are studied in Keppo (2000). Large investments of the kind usually reduce the investment opportunities of the competitors. In this paper we have the strongest possible assumption: the firms compete on a single opportunity and only one of them gets the project. We demonstrate how the assumptions on the symmetry properties of the firms and information they have on each other's valuations for the project strongly affect the moment when the investment is made.

The paper is structured as follows. In Sec. 2 we present the basic notation and main assumptions behind our model. In the next two sections we analyze the behavior of the players under different assumptions on the information that they have. In Sec. 3 it is assumed that the players have full information on their competitors' valuation of the investment project. This results in a very intuitive equilibrium. In Sec. 4 players are assumed to have no information on their competitors' values. The analysis then requires additional assumptions on their behavior. The existence of the resulting equilibrium is stated and its qualitative nature is discussed. The model, as presented in Secs. 2–4, is quite general. Therefore, in Sec. 5 we apply the model in a specific example of investing in a telecommunications network and finally Sec. 6 concludes.

2. Model and Assumptions

We consider a continuous time model in an infinite horizon. We assume that there are n competing firms (players), each of which has an opportunity to invest in a single discrete project. Firms have different valuations for the project and we assume that the value of the project for each firm follows a geometric Brownian motion. The firms monitor the evolution of the project value and based on this they are free to carry out the investment at any moment. Players understand that the investment is irreversible, i.e., they cannot get the money they sunk in it back in case the value process turns unfavorable. To simplify the matters, we also assume that the firms are risk-neutral.

Let us first consider a single firm in isolation from the others. Then the firm has a monopoly right to the project. The analysis in this case is standard, and can be found, e.g., in McDonald and Siegel (1986) and Dixit and Pindcyk (1994). The following assumption summarizes this setting, where the effect of the players' actions on each other is not yet considered.

Assumption 2.1. *As long as the competitors of firm $i \in \{1, \ldots, n\}$ are not considered, the expected present value of the project for i follows a geometric*

Brownian motion

$$dV_i = \alpha_i V_i dt + \sigma_i V_i dz_i \tag{2.1}$$

where α_i *and* σ_i *are constants and* z_i *is a standard Brownian motion.*

Since the value V_i evolves in time, we call it the project value process. If the firm i triggers the investment at time t, it receives the payoff $V_i(t)$ and gives up the fixed cost I_i. The investment is irreversible, the firms are risk-neutral, and the risk-free interest rate is constant r.

It is well known that if we do not take the competitors' actions into account, the optimal investment policy of firm $i \in \{1, \ldots, n\}$ is to invest when the project value V_i hits a certain trigger level for the first time. This trigger level, V_i^M (M for monopoly), is given by

$$V_i^M = \frac{\beta_i^M}{\beta_i^M - 1} I_i \,, \tag{2.2}$$

where

$$\beta_i^M = \frac{1}{2} - \frac{(r - \delta_i)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta_i)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}}$$

and $\delta_i = r - \alpha_i$ [see e.g., Dixit and Pindyck (1994)]. Since $\beta_i^M > 1$, the investment rule (2.2) says that the firm should not invest before the value of the project has exceeded I_i by a certain gap. This is a fundamental result of irreversible investment under uncertainty. The reason for this is that the opportunity to invest has a certain value, which the firm has to give up when investing. In other words, the firm loses the freedom between the two choices (invest and not to invest) when it carries out the irreversible investment. When this option value of the investment opportunity is added into the analyses, the firm is more reluctant to invest.

To finish the description of our setting we now introduce the interaction of the firms. We assume that the investment project under consideration is unique in the sense that only one of the firms can invest in it. In other words, the value of the project for firm i behaves as given by Assumption 2.1 as long as no one has invested in it, but as soon as one of the firms triggers the investment, it jumps to zero for all firms except the one who invests. This is explicitly written in the next assumption.

Assumption 2.2. *At time* t *when one of the firms triggers its investment, the investment opportunity is completely lost for all the other firms.*

Assumption 2.2 means that in fear of losing the project, firms have a strong incentive to invest before the others as long as their payoff from the investment is positive. Intuitively, it is clear that Assumption 2.2 forces the firms to invest earlier than in the monopoly solution given by Eq. (2.2). However, the optimal investment rules depend on the information structure of the model. In the following sections we analyze the equilibrium under two different assumptions for the information that firms have about each other's project values: in Sec. 3 we assume that the firms know exactly the current project values of their competitors, while in Sec. 4 we assume that this information is not available.

It should be emphasized here that we do not want to specify at this stage whether the project value processes are correlated with each other or not. As will be pointed out more clearly in the following sections, our model does not take this correlation into account explicitly. In the case of perfect information, the possible correlation effects do not have any effect on the Nash equilibrium since the strategies of the players are functions of the observable project value processes. On the other hand, in the case of no information on competitors' values, it is assumed that players cannot measure the possible correlations and, therefore, they have no means to calculate the exact probability distributions of their competitors' instantaneous project values. Instead, we assume that firms arrive at simple estimates of "hazard rates" for their competitors' investment behavior. In doing so the firms may be assumed to take implicitly the estimated correlation effects into account.

3. Equilibrium with Full Information on Competitors' Project Values

When we consider a single firm in isolation from others, the strategy as given by Eq. (2.2) is simply a trigger level for the value process. In other words, it is a decision rule defining when the firm invests as a function of the value of the process. When we consider a game setting with many firms competing for the same project, we have to define the strategies of the firms as functions of all information they have on the state of the game. Strategies of the firms are such decision rules that define the evolution of the game. Throughout this paper we restrict the analysis to deterministic or pure strategies, which means that the firms are not allowed to randomize their actions.

In this section we assume that each firm knows exactly the current project values of its competitors. This results in strategies, which are functions of project values of all players.

Definition 3.1. The strategy of player $i \in \{1, \ldots, n\}$ in the case of full information on competitors' project values is a mapping $\Gamma_i \colon \mathbf{R}_+^n \to \{0, 1\}$, where 0 means "do not invest" and 1 means "invest." In other words, given the current project values of all players the strategy of player $i \in \{1, \ldots, n\}$ tells whether she invests or not if no one has invested so far.

Given the strategies for all players, the game is played by letting the project values evolve according to (2.1) until for one of the firms, say firm i, $\Gamma_i(V_1, \ldots, V_n) = 1$ for the first time. Then the game ends and firm i gets payoff $V_i - I_i$ and others get zero. There is, however, a technical detail that has to be taken into account. It is possible that the strategies determine two or more players to invest simultaneously. It cannot be allowed, because the model setting rests on the assumption that only one of the firms gets the project. For this reason, we add the following assumption without any loss of insight.

Assumption 3.1. *If two or more firms try to invest simultaneously according to their strategies then the one with the highest value V_i gets the project. If there are two or more firms with the same project values who try to invest simultaneously, then the one who gets the project is drawn randomly using an even distribution.*

To start with, consider a special case where the value processes of all players are fully correlated and that $V_i = V_j$, $I_i = I_j$, $\forall\, i, j$. It turns out that in this symmetric situation Nash equilibrium strategies have to be symmetric, i.e., if one of the firms decides to invest at some moment, then all the others decide the same thing. In this case we can also express the strategies in an equivalent but simpler form by defining the strategy of firm i to be the lowest level of project value, where i is willing to invest. In a case of symmetric strategies, this trigger level is the same for all firms and we can denote it V^N (N for Nash). Now we can state the following proposition.

Proposition 3.1. *When $V_i = V$, $I_i = I$, $\forall\, i$, the unique Nash equilibrium is for all firms to adopt strategies*

$$\Gamma_i^N(V) = \begin{cases} 0, & if\ V < I \\ 1, & if\ V \geq I \end{cases} \tag{3.1}$$

or equivalently $V^N = I$.

Proof. The strategies given in the proposition define a Nash equilibrium, because the expected payoff for each firm is zero and it is easy to check that no firm can increase that by changing its strategy. If firm i, for example, increases its trigger level, then the payoff remains zero the only difference being that then the firm will surely not get the project. If firm i lowers its trigger level, then its payoff gets negative, because then she will surely get the project but its value is less than the investment cost. To show that the equilibrium is unique, assume first that $\Gamma_i^N(V) = 1$ for some $V < I$. Then the expected payoff for i is negative, and i can always improve its payoff by choosing $\Gamma_i^N(V) = 0$ for all $V < I$. On the other hand, assume that $V \geq I$ and no one has invested so far. Then, firm i can get a positive expected payoff by choosing $\Gamma_i^N(V) = 1$, whereas by choosing $\Gamma_i^N(V) = 0$, firm i gets payoff zero if at least one of the other firms have $\Gamma_j^N(V) = 1$. Therefore, it is clear that in equilibrium every firm must have $\Gamma_i^N(V) = 1$ when $V \geq I$. Therefore, the equilibrium given in (3.1) is unique. □

Notice that in the equilibrium firms are in fact indifferent between getting the project and losing it to a competitor. This means that competition completely eliminates expected profits. This is the result of their aggressive non-cooperative behavior. If the firms cannot cooperatively coordinate their actions, as we assume in this paper, no more than two firms are required to ruin all profits.

Next, let us assume that the value of the project and the investment cost are not necessarily the same for all firms. Then, one lucky firm can make positive profits if its project value crosses the level I_i before the others.

Proposition 3.2. *The unique Nash equilibrium strategies in the case of full information on competitors' project values are given by*

$$\Gamma_i^N(V_1,\dots,V_n) = \begin{cases} 0, & if\ V_i < I_i \\ 0, & if\ I_i \le V_i < V_i^M\ and\ V_j < I_j\ \forall j \neq i \\ 1, & if\ I_i \le V_i < V_i^M\ and\ V_j \ge I_j\ for\ some\ j \neq i \\ 1, & if\ V_i \ge V_i^M \end{cases}. \tag{3.2}$$

Proof. Equation (3.2) is got by combining the monopoly strategies V_i^M and Nash strategies V^N of Proposition 3.1. That the strategies indeed make a unique Nash equilibrium can be shown in the same way as in Proposition 3.1. □

Proposition 3.2 means that when for the first time one of the firms, say firm i, crosses the level where its expected payoff from investment gets positive ($V_i = I_i$), the firm does not have to invest yet if no other firm has reached the corresponding level. Firm i does not have to fear preemption, because it would be unprofitable for its competitors. However, when any other firm reaches the level where expected payoff from investment ceases to be negative, the preemption becomes possible and the non-cooperative competition forces the firm i to invest before this happens. According to Assumption 3.1 she can do this at the exact moment when the preemption becomes rational for a competitor. On the other hand, if firm i reaches the level V_i^M it invests in any case, because that is the level where it would invest if it had no competitors. Notice that the equilibrium given in Proposition 3.1 is a special case of Proposition 3.2.

The results given in this section are straightforward. A more complicated setting results when players do not know each other's value functions. This will be analyzed in the next section.

4. Equilibrium with No Information on Competitors' Project Values

In this section we give up the assumption of full information that was used in the previous section. We adopt a completely opposite view and assume that the firms do not have any information on each other's project values, not even on their initial values. As the firms cannot observe the project values of their competitors, their strategies are investment trigger levels V_i^*, which do not depend on V_j, where $j \neq i$.

Definition 4.1. The strategy of player $i \in \{1,\dots,n\}$ in case of no information on competitors' project values is a trigger level $V_i^* \in \mathbf{R}_+$. This strategy gives the lowest project value, where firm i invests.

Given the strategies for all players, the game is again played by letting the project values evolve according to (2.1) until for one of the firms, say firm i, $V_i = V_i^*$ for the first time. Then firm i invests and the game ends. Firm i gets payoff $V_i - I_i$ and others get zero.

The lower the trigger level is, the sooner it is reached. Therefore, the firms understand that with a given strategy, their competitors' strategies directly affect the likelihood that they will get the project, i.e., being the first one to reach their trigger level. The possible correlation effects between the processes are now more relevant. If firm i, for example, values the project at $V_i > I_i$, then in deciding whether to invest or not she has to estimate the likelihood that one of her competitors will invest within the next "time instant." If the processes are strongly correlated, then a high V_i implies a high probability for a high value for competitors, and this in turn would imply a high probability of investment by competitors.

The strict analysis of the equilibrium with given correlation coefficients between the value processes would be extremely difficult. Also, it seems perhaps unrealistic that firms would know exactly all correlation effects and would be able to arrive at correct probability distributions on competitors' values given their own project value. Instead, we take an approach where we assume a certain kind of assessment of the firms concerning their competitors' likelihood to invest. Namely, the firms are assumed to model their competitors' investment likelihood by a "hazard rate," which multiplied by the length of a very short time interval gives the probability of investment within this interval.

However, it would be simplistic to assume that firms just assume a fixed hazard rate of investment for their competitors. Then the firms would ignore the effect of their competitors' strategies on their own optimal strategies, and thus the situation could be modeled using a simple optimization model. Instead, we assume that the firms understand that the probability that a competitor j invests during the next time interval depends on its trigger level V_j^*. This is taken into account by assigning firms functions that reflect their estimates of their competitors' investment hazard rate as a function of the trigger levels. The lower the trigger strategy the sooner it is reached and thus the higher the hazard rate must be.

Of course, it would be possible that firms would model the hazard rates as functions of time. However, as we assume that firms do not know even the initial values of their competitors' value processes, it is more realistic to assume them to be constant in time. What we assume seems the most reasonable way for a firm to assess the hazard rates of its competitors' investments.

Summarizing the preceding discussion, we make the following assumption.

Assumption 4.1. *The firm $i \in \{1, \ldots, n\}$ models the hazard rate of investment by a competitor $j \in \{1, \ldots, n\} - \{i\}$ as a continuous, positive and decreasing function $\lambda^j(V_j^*)$ defined for all $V_j^* \geq 0$.*

It should be stressed that we do not assume that firms announce to each other their trigger levels. Instead we require that firms are able to assess the hazard rate as a function of this level. The equilibrium is then based on the assumption that firms act rationally and also except their competitors to behave rationally.

To start the analysis, let us consider a situation where the project of firm $i \in \{1, \ldots, n\}$ follows Assumption 2.1, but the firm faces a constant hazard rate λ of losing the whole investment opportunity. The project value then follows equation

$$dV_i = \alpha_i V_i dt + \sigma_i V_i dz_i - V_i dq \,, \tag{4.1}$$

where

$$dq = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases}$$

The optimal investment trigger is [see, e.g., Dixit and Pindyck (1994)]

$$V_i^* = \frac{\beta_i}{\beta_i - 1} I_i \,, \tag{4.2}$$

where

$$\beta_i = \frac{1}{2} - \frac{(r - \delta_i)}{\sigma^2} + \sqrt{\left[\frac{(r - \delta_i)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2(r + \lambda)}{\sigma^2}} \,.$$

By inspecting the formulae, it is easy to conclude that β_i is always greater than one and increasing in λ. Therefore, the investment trigger is continuous and decreasing in λ. This means that the higher the probability that during the next short time interval the whole project is lost, the lower the project value required to trigger the investment. That is, the firm does not "dare" to wait for as high payoff to invest when it may lose the project altogether before that level is reached.

To relate this result to our model, adopt the Assumption 2.2 that firm $i \in \{1, \ldots, n\}$ models the investment hazard rates for all firms $j \in \{1, \ldots, n\} - \{i\}$ as constants dependent on the trigger levels V_j^*. If all competitors of firm i have fixed strategies, then the hazard rate of losing the investment opportunity is $\lambda = \sum_{j \neq i} \lambda^j(V_j^*)$, and using this the optimal trigger level is given by Eq. (4.2).

However, we have to allow all firms to optimize their trigger levels with respect to their beliefs, which in turn affect others' trigger levels. This leads us to the Nash equilibrium for the trigger levels of all firms. The following proposition states that there is such an equilibrium.

Proposition 4.1. *Given Assumptions* 2.1, 2.2, *and* 4.1, *there exists a Nash equilibrium* $V_1^N, \ldots, V_n^N$ *for the trigger level strategies. The sufficient and necessary condition for* $V_1^N, \ldots, V_n^N$ *to be the equilibrium is that*

$$V_i^N = \frac{\beta_i^N}{\beta_i^N - 1} I_i \quad \forall\, i \in \{1, \ldots, n\} \,, \tag{4.3}$$

where

$$\beta_i^N = \frac{1}{2} - \frac{(r - \delta_i)}{\sigma^2} + \sqrt{\left[\frac{(r - \delta_i)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\left[r + \sum_{j \neq i} \lambda^j(V_j^N)\right]}{\sigma^2}} \,.$$

Proof. Equation (4.2) gives the form of the optimal strategy for i. Thus, Eq. (4.3) gives the optimal reaction of i to its competitors actions, and it must be satisfied for all i if all firms have simultaneously optimal reactions for each other's strategies. Therefore, it is a necessary condition for the equilibrium. Clearly, the condition is also sufficient, because if (4.3) is satisfied, then all firms have optimal responses to each other's strategies. The existence of the equilibrium is a result of Kakutani's Fixed Point Theorem [see e.g., Myerson (1997)]. □

Because β_i^N is a decreasing function of other players' trigger levels, the existence of these trigger levels lowers V_i^N from the level of V_i^M. Further, because this is the case for all agents, i.e., every agent is lowering the trigger level because of the competition, we realize that the equilibrium trigger level V_i^N of Proposition 4.1 is always between I_i and V_i^M.

5. Example

In this section we illustrate the model with a hypothetical example, which considers a telecommunications network investment decision on an infinite time horizon. The net present value of the project is derived from the capacity market price of the network.

Assume first that there is one company, who can invest in a single network project. We assume first that this project does not have any influence on the values of the firms' current businesses, so that the project can be evaluated independently. The network is assumed to cost 3$ millions. This is assumed to be a sunk cost. The network has capacity of 155 Mb per second and we assume that the variable cost is zero. Therefore, we assume that it is always optimal to sell the whole capacity to the market.

The capacity spot price under the new investment is assumed to be stochastic. We assume that the risk-free interest rate (annual, continuous compounding) is $r = 0.05$ and the market is risk-neutral, i.e., the market price of risk is equal to zero. We denote by P the spot price (\$/Mb) for one-month capacity usage, and assume that this follows a geometric Brownian motion of the form

$$dP = \alpha P dt + \sigma P dz\,. \tag{5.1}$$

The revenue flow in time unit (year) from selling the capacity is

$$R = 155 \text{ Mb} \cdot 12 \text{ months/year} \cdot P \text{ \$/Mb} = 1860 \cdot P \text{ \$/year}\,.$$

The expected present value of the project is then

$$V = E\left[\int_0^\infty R \cdot e^{-rt} dt\right] = \frac{R}{r-\alpha}\,,$$

which, being the price times a constant, also follows a geometric Brownian motion of the form

$$dV = \alpha V dt + \sigma V dz\,. \tag{5.2}$$

The expected downward drift of the capacity spot price is assumed to be 2 percent per year, i.e., $\alpha = -0.02$. The volatility is $\sigma = 0.2$, i.e., 20 percent per year. With these parameter values, the present value of the project in terms of the current capacity price is $V = 2.657 \cdot 10^4 \cdot P$ \$.

The optimal investment rule can now be easily calculated. By Eq. (2.2), we get for the monopoly investment trigger $V^M = 4.6 \cdot 10^6$ \$. Equivalently, we can express this rule in terms of capacity price. That is, the firm should invest when the capacity price hits the level $P^M = 4.6 \cdot 10^6/2.657 \cdot 10^4 = 173.1$ \$/Mb for the first time.

Assume next that the parameters of the model are kept unchanged, but now there are two firms, who can invest in the project. The firms are symmetric and both value the project at V. Now, the equilibrium investment strategies are given by Eq. (3.1), and this means that both of the firms want to invest when the project value reaches the level of the investment cost, that is $V = I = 3$\$ millions, or in terms of capacity spot price, when $P = 112.9$ \$/Mb. Only one of the firms will in fact invest, and the firms are indifferent about which of them gets the project. Due to the competitive pressure the network will be built earlier in the case of two firms than with one firm. The result will not change if the number of firms is increased from two.

Next, consider the case, where there are two asymmetric firms who can invest in the network, and assume that they know exactly each other's parameters. There are many reasons why the net present values of the investment project may be different for different firms. As an example, we assume that the difference is in the investment costs. Assume that the investment cost for the firm 1 is 3\$ millions as before, but for firm 2 this is 4\$ millions. Now, the equilibrium investment rules for the firms are given by Eq. (3.2). In this case, the result is that firm 1 invests when the value of the project is 4\$ millions, or when the capacity price is at $P = 150.5$ \$/Mb.

Finally, assume that there are n firms who have different value processes for the project and the firms do not know each other's values. In this case we assume that the value of the process cannot be directly derived from the capacity spot price. This kind of a situation may arise, for example, if the project has some influences on the other businesses the firms are involved in, and these firm specific values are dependent on some other underlying stochastic factors. Specifically, we assume that for each firm, the total value of the network project still follows a geometric Brownian motion, but the firms do not know each other's values or the correlation effects between the values. Assume that there are n symmetric firms, each of whom has its own value process, which follows $dV_i = \alpha_i V_i dt + \sigma_i V_i dz$. The parameters are as before: $\alpha_i = -0.02$, $\sigma_i = 0.2 \; \forall i \in \{1, \ldots, n\}$. Further, assume that firm $i \in \{1, \ldots, n\}$ estimates the hazard rate of the investment of a competitor $j \in \{1, \ldots n\} - \{i\}$ by using a function of the form $\lambda^j(V_j^*) = A(V_j^*)^{-\gamma}$, where $\gamma > 0$ and $A > 0$ are parameters. We assume in this example that $\gamma = 1$, and we compare several different values for A. Notice that this function has the property that as the trigger level V_j^* goes to zero, the likelihood to invest within the next

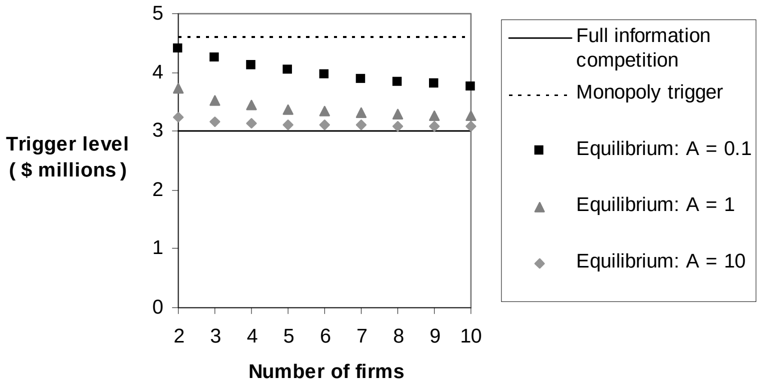

Fig. 1. The equilibrium investment trigger of symmetric firms as a function of their number. The full information competition and monopoly trigger levels are also shown in the figure.

short time interval approaches one, and as the trigger level goes to infinity, the likelihood to invest goes to zero. The parameter A reflects the firms' beliefs about the likelihood that a competitor hits its trigger level. The higher the value, the higher this likelihood is estimated.

The equilibrium investment strategies can now be calculated by solving the non-linear Eq. (4.3). This is done with values $A = 0.1$, $A = 1$, and $A = 10$, and with the number of players n ranging from 2 to 10. As the example is symmetric, all firms adopt the same strategy. Figure 1 shows this equilibrium trigger strategy with different parameter values. Also the monopoly strategy V^M and the symmetric full information strategy $V^N = I$ are shown in the figure. Notice that the equilibrium trigger level is lower the higher the parameter A and increasing the number of competing firms lowers the equilibrium trigger level. This is natural, since the tougher the competition, the sooner one of the firms should invest.

6. Conclusion

We presented a game theoretic model to study the competition for a single investment opportunity under uncertainty. It was assumed that for each firm, the value of the project follows a geometric Brownian motion. Further, it was assumed that when one of the firms invests in the project, the opportunity is completely lost for the other firms. We have illustrated our model with a telecommunications example.

It was shown that the assumption on the information that the competing firms have about each other's valuation for the project has an important effect on the equilibrium. If there are at least two symmetric firms with the same valuation for the project, then the competition completely eliminates all profits. When one of the firms invests in the project, she is in fact indifferent between investing and not investing. If, on the other hand, one of the firms has some advantage over the others, e.g., the investment cost is lower or the value of the project is higher for her than for the others, then in equilibrium that firm gets a strictly positive payoff.

If the firms do not know each other's valuation for the project, then an additional assumption is required to describe their expectations of their rivals' actions. We proposed that firms assess the likelihood that a given competitor invests within the next short time interval. In doing so, the firms understand that the higher the trigger level where the competitor requires the project value to be in order to invest, the lower the likelihood that the competitor in fact invests. We showed that under this assumption, the Nash equilibrium leads to investment triggers that are between the perfect information equilibrium level and the optimal monopoly level.

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