Link to the original article: http://publish.aps.org/abstract/PRL/v83/p1171

Nondiffracting Bulk-Acoustic X waves in Crystals

J. Salo,[1] J. Fagerholm,[2] A. T. Friberg,[3] and M. M. Salomaa[1]

[1]*Materials Physics Laboratory, Helsinki University of Technology, P.O. Box 2200 (Technical Physics), FIN-02015 HUT, Finland*

[2]*Center for Scientific Computing (CSC), P.O. Box 405, FIN-02101 Espoo, Finland*

[3]*Department of Physpics II-Optics, Royal Institute of Technology, S-10044 Stockholm, Sweden*

(Received 18 February 1998)

The concept of nondiffracting waves is generalized to encompass bulk-acoustic waves within crystalline media. We introduce acoustic Bessel beams and generalized X waves for anisotropic elastic materials. Detailed numerical predictions for propagation-invariant bulk-acoustic beams of various orders, and also X pulses, are presented for experimental verification. The material parameters used have been chosen appropriately for quartz, the most important material for acoustic device applications.

PACS numbers: 62.30.+d, 41.20.Jb, 43.20.+g, 63.20.–e

Stratton first derived expressions for nondiffracting beams [1] and axicons were first employed to produce them [2]. However, these waves attracted wide interest only after Durnin and co-workers reported diffraction-free optical J_0 beams [3]. The nondiffracting beams remain diffraction-free on passage through all paraxial optical systems [4]. Bessel beams have distinct advantages in metrology, in particle confinement and acceleration [5], and in nonlinear optics [6] where they also show a self-reconstruction property [7]. In modern optics, nearly diffraction-free laser beams are generated by diffractive elements [8], in optical resonators [9], and, programmably, by spatial light modulators [10].

Within modern acoustics, nondiffracting or limited-diffraction beams have been produced with two-dimensional ultrasonic array transducers [11]. The first experimental measurements to confirm the existence of "directed-energy pulse trains" for ultrasonic waves in water were reported by Ziolkowski *et al.* [12]; recently, "acoustic bullets" were discussed by Stepanishen [13]. Lu and Greenleaf, in particular, found a novel class of acoustic nondiffracting X waves [14] which are exact solutions of the free-space scalar wave equation; they also realized X waves with finite-aperture ultrasonic transducers. We rederived nondiffracting X waves in the framework of the angular spectrum of plane waves [15]. X waves and other localized fields, such as focus wave modes, can be generated by finite dynamic apertures [16]. Recently, propagation-invariant localized X shaped light waves were demonstrated [17].

Owing to their depth of field, limited-diffraction beams have found important applications, e.g., within medical real-time imaging [18] and in optical microlithography [19]. To date, the design and synthesis of nondiffracting beams and localized-wave physics and engineering [20] have been performed only for free space [21] or for an isotropic medium [22]; this also applies to nondiffracting electromagnetic fields [23]. In this Letter we consider, for the first time, nondiffracting waves in an anisotropic medium [24].

Elastic wave motion in an anisotropic crystal is described by the equation of motion

$$\rho \frac{\partial^2 u_i(\vec{r},t)}{\partial t^2} = \sum_{j,k,l=1}^{3} c_{ijkl} \frac{\partial^2 u_k(\vec{r},t)}{\partial x_j \partial x_l}\,, \tag{1}$$

where $\vec{u}$ is the displacement of a volume element. The material properties are contained in the fourth-rank stiffness tensor c_{ijkl} [25]. Fourier transforming, $u_i(\vec{r},t) = (2\pi)^{-2} \int \tilde{u}_i(\vec{k},\omega) e^{i(\vec{k}\cdot\vec{r}-\omega t)}\, d\vec{k}\, d\omega$, each component obeys the Cristoffel equation

$$\sum_{k=1}^{3} \left(\beta_{ik} - \frac{\omega^2}{q^2} \delta_{ik} \right) \tilde{u}_k(\vec{k},\omega) = 0\,. \tag{2}$$

Here $\beta_{ik} = \sum_{j,l=1}^{3} c_{ijkl} a_j a_l / \rho$, the unit vector $\vec{a}$ defines the direction of wave propagation, and $\vec{k} = q\vec{a}$ is the wave vector. Equation (2) possesses nontrivial solutions provided $\det[\beta_{ik} - (\omega^2/q^2)\delta_{ik}] = 0$. This is a third-order polynomial for ω^2/q^2 having three real solutions.

For ordinary crystalline materials, these solutions are positive, resulting in three possible dispersion relations. Therefore, for a given direction $\vec{a}$ there exist three independent waves with respective phase velocities ω/q. Each corresponds to a plane wave in which the atoms oscillate along the direction given by the eigenvector, $\vec{\tilde{u}}(k_x, k_y, k_z, \omega)$, of the Cristoffel equation. This direction is referred to as the polarization of the wave; it depends both on the direction of wave propagation and on the choice of the dispersion relation. The polarizations of the three modes are orthogonal by construction. The dispersion relations are commonly expressed in terms of the slowness (inverse phase velocity) vector $\vec{s} = (q/\omega)\vec{a}$ which points in the direction of wave propagation; the corresponding plane waves are $\tilde{u}_i \exp[i\omega(\vec{s}\cdot\vec{r} - t)]$. Elastic media support one (quasi)longitudinal (L), one fast, and one slow (quasi)transverse (FT and ST) propagating modes, given by the three slowness surfaces.

A nondiffracting wave propagates invariant in shape. First consider nondiffracting beams localized in the

 © 1999 The American Physical Society

transverse direction only. Taking a wave of the form $\phi(x,y,z,t) = f(x,y)e^{ik_z(z-vt)}$, its Fourier transform is bound to a subset of the Fourier space for which $v = \omega/k_z$, i.e., $s_z = 1/v$. The function f is limited by the wave equation involved. Nondiffracting pulses, on the other hand, are localized wave packets propagating with a fixed velocity v. We look for waves of the form $\phi(x,y,z,t) = g(x,y,z - vt)$; the Fourier transform proves proportional to $\delta(k_z - \omega/v)$. This is again just the condition $s_z = 1/v$, but now the angular frequency ω is free. Thus nondiffracting pulses are polychromatic generalizations of nondiffracting beams.

Now consider nondiffracting elastic waves that propagate with a preassigned velocity v along z. We impose two restrictions on the Fourier transform of the wave: (i) all Fourier components lie on the three allowed slowness surfaces, and (ii) $k_z = \omega/v$; hence $s_z = 1/v$. If a solution satisfying these two conditions is monochromatic, it is a beam; otherwise it constitutes a propagating pulse. The properties of the nondiffracting wave solutions intimately depend on the intersection of the slowness surfaces and the plane $s_z = 1/v$. We may consider the most general nondiffracting wave by taking intersections of all the slowness surfaces with the plane where $s_z = 1/v$, but we may also include just one slowness curve at a time. Let us parametrize one of the slowness curves with $\theta \in [0, 2\pi]$, such that θ is proportional to the arc length measured from one fixed point on the curve. This is not the only possible parametrization but it is chosen as the most unbiased alternative. Moreover, other integration measures can always be incorporated into the arbitrary function $A(\omega,\theta)$. The nondiffracting wave is now represented as

$$u_i(x,y,z,t) = \int_{-\infty}^{\infty} d\omega \int_0^{2\pi} d\theta\, A(\omega,\theta)\tilde{u}_i(s_x,s_y,s_z) \times e^{i\omega[s_x(\theta)x + s_y(\theta)y + s_z z - t]}, \quad (3)$$

where $A(\omega,\theta)$ is an arbitrary function. If we omit the ω integration [or assume that $A(\omega,\theta) \propto \delta(\omega - \omega_0)$], the wave becomes a nondiffracting beam since it then reduces to the form $f(x,y)e^{i(k_z z - \omega_0 t)}$.

Heretofore, no further assumptions have been imposed on the wave solution, except for the nondiffraction requirement. Now choose a function $A(\omega,\theta)$ which is separable: $A(\omega,\theta) = \alpha(\omega)\beta(\theta)$, where $\beta(\theta) = \sum_{n=-\infty}^{\infty} b_n e^{in\theta}$; hence

$$u_i(x,y,z,t) = \int_{-\infty}^{\infty} d\omega\, \alpha(\omega) e^{i\omega(s_z z - t)} \times \sum_{n=-\infty}^{\infty} b_n F_{i,n}(\omega x/\omega', \omega y/\omega'), \quad (4)$$

where

$$F_{i,n}(\omega x/\omega', \omega y/\omega') = \int_0^{2\pi} d\theta\, e^{in\theta}\tilde{u}_i(s_x,s_y,s_z) \times e^{i[\omega' s_x(\theta)(\omega/\omega')x + \omega' s_y(\theta)(\omega/\omega')y]}. \quad (5)$$

Above, an arbitrary reference frequency ω' appears which merely serves as a computational tool: the nondiffracting wave needs to be calculated for one frequency ω' only, the other frequencies are obtained through scaling: $x \to (\omega/\omega')x$; $y \to (\omega/\omega')y$. The latter integral may be evaluated numerically; finally, the wave is given by Eq. (4) which expresses a nondiffracting beam of frequency ω as the sum of elementary beams, and the integrated wave is its polychromatic generalization. This procedure parallels the decomposition of nondiffracting scalar beams and pulses into different Bessel waves and corresponding X waves; see, e.g., Ref. [15].

It is difficult to proceed analytically for the general anisotropic material. Therefore, first consider the special case of isotropic longitudinal waves assumed to propagate with the phase velocity c (the speed of sound). The general nondiffracting beam [26] is

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = e^{i\omega(\cos\gamma x_3/c - t)} \int_0^{2\pi} \beta(\theta) \begin{pmatrix} \sin\gamma\cos\theta \\ \sin\gamma\sin\theta \\ \cos\gamma \end{pmatrix} e^{i\omega(\sin\gamma\cos\theta x_1/c + \sin\gamma\sin\theta x_2/c)}\, d\theta\,. \quad (6)$$

Here γ is an arbitrary parameter $\gamma \in [0, \pi/2]$ which defines the beam velocity $v = c/\cos\gamma$. Since $\cos\gamma \le 1$, the velocity v is necessarily supersonic. Transforming into cylindrical coordinates, $x = r\cos\eta, y = r\sin\eta$, using the Fourier series $\beta(\theta) = \sum_{n=-\infty}^{\infty} b_n e^{in\theta}$, and evaluating the integrals we find for the order $n = 0$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{n=0} = 2\pi \begin{pmatrix} i\sin\gamma\cos\eta J_1(r\kappa) \\ i\sin\gamma\sin\eta J_1(r\kappa) \\ \cos\gamma J_0(r\kappa) \end{pmatrix} e^{i(\omega/c)\cos\gamma(x_3 - vt)}, \quad (7)$$

and for the higher orders $n \neq 0$:

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{n\neq 0} = i^n 2\pi \begin{pmatrix} i\,\frac{\sin\gamma}{2}[e^{i\eta}J_{n+1}(r\kappa) - e^{-i\eta}J_{n-1}(r\kappa)] \\ \frac{\sin\gamma}{2}[e^{i\eta}J_{n+1}(r\kappa) + e^{-i\eta}J_{n-1}(r\kappa)] \\ \cos\gamma J_n(r\kappa) \end{pmatrix} e^{in\eta} e^{i(\omega/c)\cos\gamma(x_3 - vt)}, \quad (8)$$

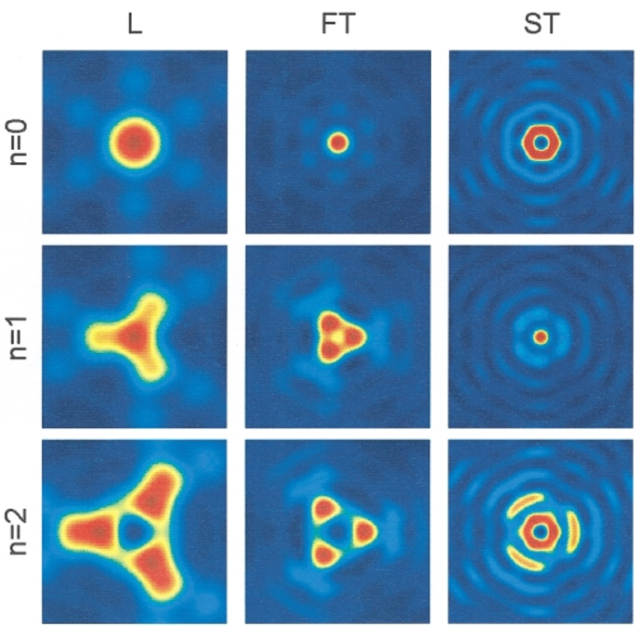

FIG. 1 (color). Nondiffracting acoustic beams in quartz. The left column shows the L mode for $n = 0, 1$, and 2, the middle one illustrates the FT mode, and the right column is the ST mode. Color shading denotes the time-averaged kinetic energy; each frame represents 100 μm $\times$ 100 μm area.

where $\kappa = (\omega/c)\sin\gamma$ is the transverse wave number. This shows that the different Bessel orders do not appear separately in the elastic waves, but rather couple to orders one lower and one higher. Consequently, the energy density of the wave need not display any zeros as in the scalar case [27] since the nodes of the different Bessel functions do not coincide.

By construction, a nondiffracting wave retains its transverse form and energy is effectively transported along the beam. However, the velocity of energy transport is not the velocity of the beam, but rather a velocity slower than that of sound. This is understandable, since otherwise a nondiffracting wave would lead to a supersonic (or superluminal) energy transport. This may be interpreted as follows: Energy is not transported only along the beam, but there is also a transverse energy flow towards the beam center and away from it. Therefore, the cross section of a nondiffracting beam is essentially a standing wave and the transverse energy flow vanishes. As the energy velocity is a vectorial sum of the velocity along the beam and the transverse velocity, with the latter canceling due to the standing-wave effect, the resultant velocity is slower than that of sound.

We evaluate nondiffracting beams and pulses numerically for quartz [25], one of the most important industrial materials for acoustic applications. The resulting nondiffracting beams of different orders, $n = 0, 1, 2$, constructed from the L, FT, and ST modes, respectively, are illustrated in Fig. 1. The threefold symmetry of the beams, reflecting the underlying trigonal symmetry of quartz, is evident. For the lowest-order ($n = 0$) modes, the L, FT, and ST nondiffracting beams each also displays mirror symmetry. For the L and FT modes, the $n = 0$ beams feature maximal intensity at the center. However, for the ST mode there occurs a dark spot in the middle. This is due to the u_1 and u_2 components being proportional to the function J_1, and u_3 averaging to zero owing to the complicated behavior of the ST mode polarization. For higher-order beams, symmetry is reduced to trigonal. Note that the ST mode features maximal intensity at the beam center for $n = 1$ and again a minimum for $n = 2$. The intensity in the higher-order L and FT mode beams is accumulated farther from the center, as for the X waves in free space, cf. Ref. [15].

Consider nondiffracting pulses in quartz. An X-shaped pulse is composed by evaluating the beam at the frequency $\omega' = 10^9$ s^{-1} and then constructing the beam corresponding to the frequency ω by scaling, cf. Eq. (4), and integrating over ω with weight $e^{-\eta\omega}$, where $\eta = 10^{-9}$ s. Figure 2 elucidates the 3D amplitude of the quasi-longitudinal lowest-order ($n = 0$) nondiffracting pulse. Note the distorted shape of X on projection into the yz plane, a consequence of crystal anisotropy and inversion symmetry of the X pulse. One may use either a planar [11,28] or a conical [29] piezoelectric transducer arrangement to excite the nondiffracting acoustic waves in crystals. Subsequently, they may be detected experimentally with acoustic wave-front imaging [30].

Nondiffracting optical and ultrasonic beams have novel and broad technical applications owing to their depth of field, ranging from optical microlithography [19] to medical real-time imaging [14] and, potentially, optical communication [31]. Nondiffracting bulk-acoustic X waves may be anticipated to have applications in high-quality crystal resonators and resonator filter devices within telecommunications. In the future, propagation-invariant nondiffracting electromagnetic waves and light pulses in

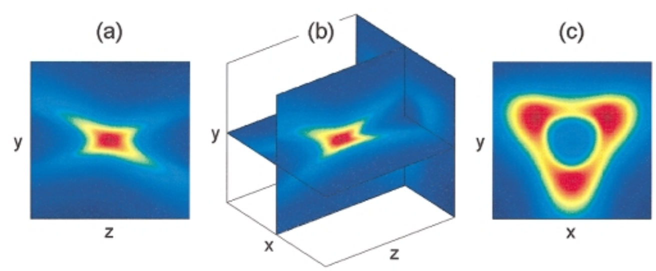

FIG. 2 (color). Nondiffracting quasilongitudinal fundamental ($n = 0$) acoustic pulse propagating along z with the velocity $v = 1.43 \times 10^4$ m/s in quartz. (a) Acoustic amplitude of the pulse in the yz plane at $x = 0$; note the reduced symmetry in the shape of X, owing to crystal anisotropy. (b) Acoustic amplitude of the pulse at a crosscut in the xz and yz planes. (c) Approach form of the acoustic pulse at $z = 70$ μm; the trailing form is a parity transform of that in (c). Width of each frame is 140 μm.

crystals are expected to become industrially increasingly important in connection with low-loss microwave electronics, in particular, miniaturized resonators and filters within mobile telecommunications.

We thank Julius Koskela and Tapani Makkonen for discussions. This work has been supported by Micronas Semiconductor SA, Bevaix, Switzerland, the NOKIA Group, Micronas, Inc., Espoo, Finland, and TEKES, Technology Development Centre, Finland.

[1] J.A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941), p. 356.
[2] L.M. Soroko, in *Progress in Optics XXVII*, edited by E. Wolf (North-Holland, Amsterdam, 1989), p. 109.
[3] J. Durnin, J.J. Miceli, Jr., and J.H. Eberly, Phys. Rev. Lett. **58**, 1499 (1987).
[4] A.T. Friberg, A. Vasara, and J. Turunen, Phys. Rev. A **43**, 7079 (1991).
[5] B. Hafizi, E. Esarey, and P. Sprangle, Phys. Rev. E **55**, 3539 (1997).
[6] T. Wulle and S. Herminghaus, Phys. Rev. Lett. **70**, 1401 (1993).
[7] S. Sogomonian, S. Klewitz, and S. Herminghaus, Opt. Commun. **139**, 313 (1997).
[8] A. Vasara, J. Turunen, and A.T. Friberg, J. Opt. Soc. Am. A **6**, 1748 (1989).
[9] A.J. Cox and D.C. Dibble, J. Opt. Soc. Am. A **9**, 282 (1992).
[10] J.A. Davis, E. Carcole, and D.M. Cottrell, Appl. Opt. **35**, 593 (1996); R. Liu, B.-Z. Dong, G.-Z. Yang, and B.-Y. Gu, J. Opt. Soc. Am. A **15**, 144 (1998).
[11] D.K. Hsu, F.J. Margetan, and D.O. Thompson, Appl. Phys. Lett. **55**, 2066 (1989).
[12] R.W. Ziolkowski, D.K. Lewis, and B.D. Cook, Phys. Rev. Lett. **62**, 147 (1989).
[13] P.R. Stepanishen, J. Acoust. Soc. Am. **103**, 1742 (1998).
[14] J.-Y. Lu and J.F. Greenleaf, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **39**, 19 (1992); **39**, 441 (1992); **41**, 724 (1994).
[15] J. Fagerholm, A.T. Friberg, J. Huttunen, D.P. Morgan, and M.M. Salomaa, Phys. Rev. E **54**, 4347 (1996).
[16] A.M. Shaarawi, J. Opt. Soc. Am. A **14**, 1804 (1997).
[17] P. Saari and K. Reivelt, Phys. Rev. Lett. **79**, 4135 (1997).
[18] J.-Y. Lu, T.K. Song, R.R. Kinnick, and J.F. Greenleaf, IEEE Trans. Med. Imaging **12**, 819 (1993).
[19] M. Erdélyi, Z.L. Horváth, G. Szabó, Zs. Bor, F.K. Tittel, J.R. Cavallaro, and M.C. Smayling, J. Vac. Sci. Technol. B **15**, 287 (1997).
[20] R.W. Ziolkowski, Phys. Rev. A **44**, 3960 (1991).
[21] J. Rosen, Opt. Lett. **19**, 369 (1994); A.T. Friberg, J. Fagerholm, and M.M. Salomaa, Opt. Commun. **136**, 207 (1997).
[22] J.-Y. Lu, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **44**, 181 (1997); P.R. Stepanishen and J. Sun, J. Acoust. Soc. Am. **102**, 3308 (1997).
[23] J. Bajer and R. Horák, Phys. Rev. E **54**, 3052 (1996); R. Horák, Z. Bouchal, and J. Bajer, Opt. Commun. **133**, 315 (1997).
[24] Preliminary results of this work are reported in J. Salo, J. Fagerholm, A.T. Friberg, and M.M. Salomaa, in *1997 IEEE Ultrasonics Symposium Proceedings* (IEEE, New York, 1997), p. 1761.
[25] Materials parameters are from M.A. Auld, *Acoustic Fields and Waves in Solids* (John Wiley, New York, 1973), Vol. 1.
[26] This is obtained from Eq. (3) taking $A(\omega,\theta) = \delta(\omega - \omega_0)\beta(\theta)$ and substituting wave vectors of constant length $\vec{k} = (\omega/c)[\sin\gamma\cos\theta, \sin\gamma\sin\theta, \cos\gamma]^T$ and polarizations collinear with the wave vector.
[27] Naturally, this strictly holds only for isotropic media, but this result serves to qualitatively help to understand also more complicated anisotropic cases which cannot be solved in closed form.
[28] P.R. Stepanishen and J. Sun, J. Acoust. Soc. Am. **102**, 3308 (1997).
[29] T. Koike, K. Yamada, and K. Nakamura, Jpn. J. Appl. Phys. **35**, 3184 (1996); Electron. Lett. **32**, 2304 (1996).
[30] M.R. Hauser, R.L. Weaver, and J.P. Wolfe, Phys. Rev. Lett. **68**, 2604 (1992); J.P. Wolfe, *Imaging Phonons—Acoustic Wave Propagation in Solids* (Cambridge University Press, Cambridge, 1998).
[31] J.-Y. Lu and S. He, Opt. Commun. **161**, 187 (1999).