Abstract—This paper reviews state-of-the-art loss-minimizing control strategies for synchronous reluctance motors. Methods can be categorized as loss-model controllers (LMCs) and search controllers (SCs). For LMCs, different loss models and the corresponding optimal solutions are summarized. The effects of the core losses and magnetic saturation on the optimal stator current are investigated; magnetic saturation is a more important factor than the core losses. For SCs, different search algorithms are presented and compared. The SCs are evaluated based on their convergence speed, parameter sensitivity, accuracy, and the torque ripple caused by the search process.

Index Terms—Core losses, drive, efficiency optimization, energy, loss minimization, magnetic saturation, synchronous reluctance motor.

I. INTRODUCTION

The synchronous reluctance motor (SyRM) is known to be an attractive alternative to other AC machines in variable speed drives [1], [2]. An optimal selection of the stator current vector can minimize the total electrical losses in the motor. The loss-minimizing control of the induction motor suffers from slow dynamic response due to the relatively large rotor time constant [3]. On the other hand, the dynamic performance of the SyRM is not significantly deteriorated due to the loss minimization, since the stator flux linkage is proportional to the stator current.

At lower speeds, the core losses are negligible compared to the copper losses. Therefore, maximum torque-per-ampere (MTPA) methods, which minimize the copper losses, are popular [4]. If magnetic saturation is omitted, the 45-degree current angle (with respect to the d axis) yields the MTPA operation. However, magnetic saturation and parameter variations usually lead to an optimal current angle larger than 45 degree [5], [6]. At higher speeds, the proportion of the core losses increases, and they have to be taken into account as well in order to minimize the total electrical losses of the motor.

Loss-minimizing control methods can be categorized as loss-model controllers (LMCs) and search controllers (SCs). The LMCs apply a loss function, which depends on motor parameters and knowledge of the operating point. The loss-minimizing solution for the angle of the stator-current vector is obtained by minimizing the loss function. Analytical solution can be found, if the loss function is simple enough. However, taking the core losses and magnetic saturation into account leads typically to a very complicated loss function, and the optimal angle has to be determined by numerical searching (either online or off-line).

The SCs adjust the control variable (typically the d-axis current) online based on the feedback from the input power measurement. The SCs are insensitive to model inaccuracies and parameter variations. If the input power of the converter is measured and used as a feedback signal, the converter losses are automatically taken into account. However, typical disadvantages of the SCs are the slow convergence speed and unexpected losses and torque disturbances due to the adjustments of the control variable during the search process.

Much research in the field of energy-efficiency control of SyRMs has been conducted, but no survey on this topic has been available. This paper attempts to review relevant publications in this area and summarize their key results. The SyRM model is briefly described in Section II. Different loss models and the corresponding optimal solutions of various LMCs are discussed in Section III. The effects of the core losses and magnetic saturation on the optimal stator-current angle are analyzed in Section IV. In Section V, SCs are evaluated based on their convergence speed, torque disturbances, and stability of the search algorithms. Finally, Section VI concludes the paper.

II. SYRM MODEL

The d-axis of the rotating coordinate system is defined as the direction of the maximum inductance. Real space vectors will be used in the model. For example, the stator-current vector is $\mathbf{i}_s = [i_{sd}, i_{sq}]^T$, where $i_{sd}$ and $i_{sq}$ are the components of the vector and the matrix transpose is marked with the superscript T. The magnitude is denoted by

$$i_s = \sqrt{i_{sd}^2 + i_{sq}^2} \quad (1)$$

Per-unit quantities will be used.
The dynamic equivalent circuit of the SyRM in rotor coordinates is illustrated in Fig. 1. The stator-voltage equation is

\[
\frac{d\psi_s}{dt} = u_s - R_s i_s - \omega_m J \psi_s
\]  

where \( \psi_s \) is the stator-flux vector, \( u_s \) the stator-voltage vector, \( R_s \) the stator resistance, \( \omega_m \) the electrical angular speed of the rotor, and the orthogonal rotation matrix \( J = [0 \ 1] \).

The core-loss resistance is \( R_c \), and the current \( i_c = i_s - i_m \) flows through it. The magnetizing current \( i_m \) is a nonlinear function of the flux due to magnetic saturation (cf. e.g. [7] and references therein):

\[
i_m = i_m(\psi_s) = \left[ \psi_{sd}/L_d(\psi_{sd}, \psi_{sq}) \psi_{sq}/L_q(\psi_{sd}, \psi_{sq}) \right] (3)
\]

where the d- and q-axis inductance functions are denoted by \( L_d \) and \( L_q \), respectively.\(^1\)

In steady state, the power balance of the SyRM model is given by

\[
P_{in} = P_{Cu} + P_{Fe} + T_e \omega_m
\]

where the power fed to the stator is \( P_{in} = u_s^T i_s \) and the electromagnetic torque is

\[
T_e = i_m^T J \psi_s = i_{mq}(\psi_{sd}) - i_{md}(\psi_{sq})
\]

The copper losses are \( P_{Cu} = R_s i_s^2 \) and the core losses are \( P_{Fe} = R_c i_c^2 \).

In steady state, the stator core losses can be modeled as

\[
P_{Fe} = k_h |\omega_m| \psi_s^2 + k_e \omega_m^2 \psi_s^2
\]

where \( k_h \) and \( k_e \) are the hysteresis and eddy-current coefficients, respectively. The first term in (6) corresponds to the hysteresis losses and second term corresponds to the eddy-current losses. The core-loss resistance corresponding to (6) becomes

\[
R_c = \frac{1}{k_h/|\omega_m| + k_e}
\]

It can be seen that the core-loss resistance \( R_c \) is constant if the hysteresis losses are omitted \( (k_h = 0) \).

III. LOSS-MODEL CONTROLLERS

For a given torque and speed, LMCs aim to minimize the input power \( P_{in} \) given in (4), or, equivalently, to minimize the losses \( P_{loss} = P_{Cu} + P_{Fe} \). The loss function is formulated based on the motor model. The accuracy and the complexity of the solution depend significantly on the modeling assumptions.

\(^1\)Alternatively, the saturation can be modeled as

\[
\psi_s = \psi_s(i_m) = \left[ L_d(i_{md}, i_{mq}) i_{md} \right] \left[ L_q(i_{md}, i_{mq}) i_{mq} \right]
\]

Both forms will be used in the rest of the paper.

A. Constant \( R_c \), \( L_d \), and \( L_q \)

In [8], [9], a simple LMC was proposed. The constant core-loss resistance \( R_c \) and constant inductances \( L_d \) and \( L_q \) have been assumed. The sum of the copper losses and the core losses is given by

\[
P_{loss} = P_{Cu} + P_{Fe} = \left[ R_s + (R_s + R_c) \omega_m^2 L_d^2 \right]_{md}^2 + \left[ R_s + (R_s + R_c) \omega_m^2 L_q^2 \right]_{mq}^2 + \frac{2R_s}{R_c} \omega_m (L_d - L_q) i_{md} i_{mq}
\]

From this loss function, the optimal ratio of the current components can be solved as

\[
\zeta_{opt} = \frac{i_{mq,opt}}{i_{md,opt}} = \sqrt{\frac{R_s R_d^2 + (R_s + R_c) \omega_m^2 L_d^2}{R_s R_q^2 + (R_s + R_c) \omega_m^2 L_q^2}}
\]

which results in \( \zeta_{opt} > 1 \) since \( L_d > L_q \). Therefore, the optimal current angle is slightly larger than 45 degree at all speeds and torques. Since constant inductances have been assumed, (9) does not depend on the torque. In order to simplify the implementation, \( i_{sq,opt}/i_{sd,opt} = \zeta_{opt} \) may be assumed. In [10], an approximation of (9) avoiding taking the square root is proposed.

In [11], the solution (9) was implemented, and the core-loss resistance and the inductances are estimated by extended Kalman filter. Since the parameter estimation needs time to converge to actual machine parameters, the loss-minimizing method works well in steady state but not in dynamic loss minimization.

Feedback linearization is applied in [12] to minimize the losses. This method can also be categorized as a LMC. The operating points are assumed to be unsaturated and the saturation effects are not considered. The power losses are minimum if the torque curve and power-loss curve are tangential. Hence, the loss-minimizing points can be solved from

\[
\|\nabla T_e(i_{sd}, i_{sq})\| \cdot \|\nabla P_{loss}(i_{sd}, i_{sq})\| \cdot \sin \theta = 0
\]

where \( \nabla T_e \) is the gradient of electromagnetic-torque curve, \( \nabla P_{loss} \) is the gradient of the loss curve, and \( \theta \) is the angle between \( \nabla T_e(i_{sd}, i_{sq}) \) and \( \nabla P_{loss}(i_{sd}, i_{sq}) \).

Neural networks (NNs) and fuzzy logic have been used in the LMCs. In [13], a NN is used as an adaptive model of the SyRM. The NN is trained online, and the input is the torque reference and outputs are the d-q axes currents references. The problem in the method is that the NN only maps the relationship between torque and optimal current without taking the rotation speed into account. However, based on (9), the optimal current depends on the rotation speed. Therefore, the NN should also have the rotation speed as an input to identify the optimal d-q axes currents. The LMC proposed in
where $k$ is the loss-minimizing current $i$ in the core-loss resistance $R_c$ function in [17] and [18]. The total loss function considering $T$ in the literature [7]. For example, a power function model is significantly affects the optimal current angle.

B. Effect of the Hysteresis Losses

In [16], the core losses are modeled according to (6). The loss function is the sum of the core losses and copper losses. The optimal d-axis current is expressed as

$$i_{sd,\text{opt}} = \left( \frac{\omega_m^2 L_q^2 + R_s R_c}{\omega_m^2 L_q^2 + R_s R_c} \left( \frac{T_e}{L_d - L_q} \right) \right)^{1/4}$$

where the core-loss resistance $R_c$ is the function (7). The stray-load losses are taken into account in the total loss function in [17] and [18]. The total loss function considering the hysteresis, eddy-current, and stray-load losses is given by

$$P_{\text{loss}} = a(i_{sd}^2 + i_{sq}^2) + b \left( \frac{L_d}{L_q} \right)^2 i_{sd}^2 + i_{sq}^2$$

where

$$a = R_s + k_s \omega_m^2, \quad b = k_c \omega_m^2 \frac{L_q^2}{L_d}$$

where $k_s$, $k_c$, and $\beta$ are the parameters of the core-loss model. The loss-minimizing current $i_{sd,\text{opt}}$ is given as

$$i_{sd,\text{opt}} = i_{sq} \sqrt{\frac{a + b}{a + b (L_d/L_q)^2}}$$

The solution (14) is not directly applied but it is rewritten as

$$i_{sd,\text{opt}} = i_{sq} \sqrt{\frac{1 + \omega_m^2 T_1^2}{1 + \omega_m^2 T_2^2}}$$

where $T_1$ and $T_2$ are combined functions of the motor parameters. $T_1$ and $T_2$ are identified experimentally by input power measurements.

C. Effect of Magnetic Saturation

For LMCs, magnetic saturation is one of the key issues to be considered. However, many loss-minimizing methods in the literature omit the saturation effects and use constant inductions for simplicity. However, the variation of the inductions significantly affects the optimal current angle.

The saturation characteristics of the d-axis and q-axis inductions are different because the d-axis is iron dominating and q-axis is air dominating. Hence, the variation of $L_d$ is more severe. If high accuracy is not required, it may be sufficient to model the saturation effect of the d-axis induction only.

Various explicit magnetic saturation models can be found in the literature [7]. For example, a power function model is given by

$$L_d(\psi_d) = \frac{L_{d0}}{1 + (\alpha|\psi_d|)^n}$$

Acrtangent functions, piecewise functions, and polynomial functions have also been used to model the saturation effects.

More precise models of saturation take also the cross coupling between the two axes into account. For example, the model given in [19] is

$$L_{d0}(i_{md}, i_{mq}) = L_{d0}(i_{md}) - L_{d1}(i_{md}) L_{q2}(i_{mq})$$
$$L_{q0}(i_{md}, i_{mq}) = L_{q0}(i_{mq}) - L_{q1}(i_{mq}) L_{d2}(i_{md})$$

where $L_{d0}$, $L_{d1}$, $L_{d2}$, $L_{q0}$, $L_{q1}$, and $L_{q2}$ are all expressed as rational functions. As an example, the function

$$L_{d0}(i_{md}) = A + \frac{B}{i_{md}^4 + C i_{md}^2 + D}$$

where $A$, $B$, $C$, and $D$ are constant parameters.

The saturation curves are nonlinear functions, which make the loss function much more complicated and difficult to derive an analytical solution for the loss-minimizing current. However, numerical methods can be applied to find the optimal points of the loss function.

There are only a few LMCs considering both core losses and cross-magnetic saturation. In [20], the inductions are measured and fitted to (17). The LMC is based on a repetition function by which the optimal current reference is searched numerically. Based on the results, this seems to be a good solution for high-accuracy loss minimization (if the parameter identification of the model does not cause problems).

IV. PARAMETER SENSITIVITY OF LMCs

In [21], parameter sensitivity of the LMC corresponding to (9) is studied. The sensitivity functions are defined as differentiation for each motor parameters:

$$\sigma_{Ld} = \frac{\partial \zeta_{\text{opt}}}{\partial L_d}, \quad \sigma_{Lq} = \frac{\partial \zeta_{\text{opt}}}{\partial L_q}$$
$$\sigma_{R_s} = \frac{\partial \zeta_{\text{opt}}}{\partial R_s}, \quad \sigma_{R_c} = \frac{\partial \zeta_{\text{opt}}}{\partial R_c}$$

Fig. 2 shows the parameter sensitivities calculated according to (19) using the original motor parameters in [21]. It can be seen that the sensitivities of the parameters have a relationship of $\sigma_{Ld} > \sigma_{Lq} > \sigma_{R_s} > \sigma_{R_c}$, which means that the variations of the inductions have more significant influence on the optimal current angle. Furthermore, the variation of $L_{d0}$ is much larger than the variation of $L_{q0}$.

Fig. 3 shows calculated loss curves of a transverse-laminated 6.7-kW four-pole SyRM with and without the core-loss and saturation models. The rated values of the SyRM are: speed 3175 r/min; frequency 105.8 Hz; line-to-line rms voltage 370 V; rms current 15.5 A; and torque 20.1 Nm. In order to model the inductions as a function of the current, the measured inductions, model in (17). The core losses are modeled as a constant $R_c$ which is identified by no-load measurement. For simplicity, the average value $R_c = 2.24$ p.u. is used, despite the variation in the actual $R_c$.

From the results shown in Fig. 3, it is obvious that the core losses increase the total losses but do not affect the optimal current significantly. On the other hand, the saturation effects have more significant influence on the optimal current.
Fig. 2. Parameter sensitivity of loss minimization. Results are obtained using the parameters of the motor in [21]. (a) Parameter sensitivity of the inductances; (b) Parameter sensitivity of the stator resistance and the core-loss resistance.

![Graph](image1)

![Graph](image2)

(a)

(b)

Table I

<table>
<thead>
<tr>
<th>Phenomena included in loss functions of LMCs</th>
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<tr>
<td>Publications</td>
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<td>[6], [8]–[12], [14]–[16]</td>
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The LMCs are categorized in Table I according to phenomena included in loss functions. It can be seen that most of the research has been focused on the effects of the core losses.

V. SEARCH CONTROLLERS

The SCs can be classified into three categories based on their search algorithms: discrete search, continuous search, and signal-injection method. This classification is illustrated in Fig. 4.

![Diagram](image3)

A. DISCRETE SEARCH

In discrete search methods, the control variable is changed in steps. Based on the measured (or estimated) input power, new values of the control variable are selected, and the search method iteratively approaches to the optimal point using a minimum search algorithm.

In [16], the d-axis current reference is decreased five steps and then increased ten steps and the input power is measured at each step. Then, the SC adjusts the current reference to the level where the input power has the minimum value and repeats this kind of search procedure. The current reference is changed up and down for the purpose of tracking the minimum input power operating point. This method is simple and easy to apply, but it is inefficient. For example, if the disparity between the optimal current and the actual current is $10\Delta i_d$ ($\Delta i_d$ is the current step), the method needs 30 steps (10 down and 20 up) to reach the new optimal value. If the current step is large, it will cause torque disturbances. Furthermore, the d-axis current still keeps changing up and down in steady state, where the speed and load are constant. This variation around

![Diagram](image4)

Fig. 4. Categories of SCs.
the optimal current will also cause torque ripple and additional losses.

In [23], the SC is implemented using the Fibonacci search algorithm. The Fibonacci search is known as an efficient search method for the minimum of a unimodal function in an interval. The Fibonacci search algorithm can precisely find the minimum point. However, it searches the whole current range, which means that very high and low currents are also searched. This causes torque ripple and high losses during the search procedure. However, for the loss-minimizing control, an approximate optimal point can be determined by a rough model (with or without the core losses and saturation model). The loss function is known to be a concave function with only one minimum point. Therefore, in order to make the search faster and reduce the torque ripple, initial values can be strategically selected for this search algorithm.

The SC proposed in [24] applies adaptive fuzzy logic for the first part of the search and the golden-section search method in the second part. The efficiency optimization strategy using fuzzy logic can speed up the search process. The fuzzy logic search changes the current in the direction in which the input power is reduced. When the input power starts to increase, the search method switches to the golden-section search, which can find the minimum point. Comparing with the Fibonacci search in [23], the hybrid search method with fuzzy logic can speed up the search procedure and reduce the steps needed.

In [25], the SC is implemented using the sequential quadratic interpolations. This method iteratively approaches to the optimal point assuming that the loss function is quadratic. Experimental results show that the method converges to the optimal flux with only a few iterations.

In [26], Steady state power losses are minimized by using a fuzzy logic search control system and the LMC is used during transient states. The fuzzy logic search control combines two fuzzy logic controllers: one decreases the d-axis current and the other increases the d-axis current in steps.

B. Continuous Search

Continuous search methods increase or decrease the control variable in a continuous manner instead of stepwise changes. The input power measurements are taken in a time window. The control variable is changed to the direction in which the input power is reduced. When the input power starts to increasing or the derivative of input power with respect to the control variable is zero, the optimal point has been found.

In [27], the control variable is the d-axis current, which follows a ramp trajectory. The difference of the input power is measured at the beginning and at the end of the moving window. In this manner, torque disturbances become smoother than in discrete search methods.

In [28], the loss minimization algorithm consists of three steps: 1) detect transient or steady state by the speed error; 2) determine the direction towards the loss-minimizing point; 3) change the control variable to the loss-minimizing direction and determine the minimum-loss condition. The flux is changed incrementally (or decrementally) in an exponential manner.

C. Signal Injection

The SC based on signal injection was proposed in [29], where the current angle $\theta$ yielding zero input power variation in steady state is searched for. In other words, the loss-minimizing criterion is $\partial P_{in}/\partial \theta = 0$, where $P_{in}$ is the input power. A high-frequency excitation signal is superimposed on the current angle. The response in the input power caused by the injected signal is monitored, and the loss-minimizing current angle is tracked based on this response. The convergence to the optimal current angle takes a few seconds.

VI. Conclusion

This paper reviewed loss-minimizing control methods of the SyRM. The LMCs are faster and have smaller disturbances in the torque than the SCs. The disadvantage of the LMCs is their dependence on motor parameters. The core losses and magnetic saturation are the most important phenomena to be considered in the LMCs. Based on parameter sensitivity analysis, it is especially important to take magnetic saturation into account in the LMCs.

The SCs minimize the input power based on the real-time power measurement. Hence, the loss model is not needed, and errors due to the parameter variations and modeling inaccuracies are avoided. In this review, the SCs were divided into three categories based on the searching algorithms: discrete search, continuous search, and signal-injection based methods. Typical disadvantages of the SCs are that the search process causes torque ripple and the convergence speed is not sufficient for dynamic applications.

References


