Misbehavior in Nash Bargaining Solution Allocation

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Abstract—**Nash Bargaining Solution (NBS) has been broadly suggested as an efficient solution for problem of fair allocation of multiple resources, namely bandwidth allocation in datacenters.**

In spite of being thoroughly studied, and provably strategy-proof for most scenarios, NBS-based allocation methods lack research on strategic behavior of tenants in case of proportionality of resource demands, which is common in datacenter workloads.

We found that misbehavior is beneficial: by lying about bandwidth demands tenants can improve their allocations. We show that the sequence of selfish improvements leads to trivial demand vectors for all tenants. It essentially breaks sharing incentives which are very important for datacenter networks. We analytically prove that tenants can misbehave for 2 and 3 tenants cases.

We show that misbehavior is possible in one recently proposed NBS-based allocation system if demands proportionality is taken into account. Monte-Carlo simulations were done for $2-15$ tenants to show a misbehavior possibility and its impact on aggregated bandwidth.

We propose to use another game-theoretic approach to allocate bandwidth in case of proportional demands. That method performs significantly better on average than NBS after misbehavior.

I. Introduction

Recent studies on datacenter networking show that best-effort protocols harm tenants' performance [1]. Many modern network applications, e.g. Hadoop, utilize multiple resources at once. The performance of the whole application depends on each resource it uses: the stragglers increase application completion time and reduce quality of services (QoS). To deal with this problem network allocation methods were suggested. The problem can easily be widen to a general class of resource allocations, where overall performance or utility depends on the allocation of a less available resource.

Recent works suggest using game-theoretic approach to get required allocations of resources [2]. Classical approach to resource allocation is a Nash Bargaining Solution (NBS)[3]. Allocations of some flows are considered variables and optimization problem is constructed. Objective function is to maximize multiplication of all allocations subjected to bandwidth and demand constraints.

The same NBS allocation can be achieved in another way by using a pricing scheme or auction. In this approach each player with given budget can obtain share of each resource adjusting spending to acquire more desirable resources. All resources have some prices, which are adjusted according to demands. The repeating process of adjusting converges to Competitive Equilibrium from Equal Incomes (CEEI) which coincides with NBS in many pricing schemes. Different in nature pricing schemes produce the same allocation after market-clearing price is found. Moreover, one of the main conditions of a perfect market — the price-taking property, a property representing the absence of ability to manipulate the prices — is absent in many cases. Without the latter property resource management algorithms create space for player misbehavior. This often is forgotten during designing of new protocols for network allocations.

In this work we argue that application of game-theoretic and pricing mechanisms to bandwidth allocation in many cases does not satisfy the condition. Namely, then tenants' demands for bandwidth on different links are proportional. Players have the power to manipulate prices by increasing their demands on some resource which increases it's clearing price and forces other players to spend larger part of their budget on that resource. This decreases other players' share of another resource, which a manipulating player actually requires.

We construct a game for proportional network resources demands and prove that this manipulability breaks sharing incentives[1] for 2-player and 3-player game. For the 2-player game we produce an explicit solution, but 3-player game is more important for general case (we do not show an exact solution for 3-player game, but provide the proof for breaking of sharing incentives). To support our findings we simulate the game for 2-15 players competing for 2-15 links and show that misbehavior results in degradation of final allocation, where each player receives only $1/N$-th of each resource, where n is the number of players. This finding is similar to the tragedy of the commons – a known problem of game theory where selfish individuals increasing own short-term profit decrease the long-term profit of the whole system (and as a result of everyone). We compare our findings against dominant resource fairness (DRF) allocation method [4] as a strategy-proof baseline. We show that NBS allocation mechanisms produce good allocation result, however ability of each player to strategically manipulate demands degrades the allocations.

The rest of the paper is organized as follows. Section II provides motivation for the addressed problem. Section III formally defines compared allocation mechanisms. Section IV provides a proof of misbehavior for 2-player case. Section V presents a proof of misbehavior for the case of 3 players. In section VI we provide results of simulations and discuss impact of misbehavior on quality of resource allocation. Section VII reviews related work and section VIII concludes the

[1] A sharing incentive is an important property of multi-tenant or multi-user system. With this incentive the user can trade some of her rightful resources she does not need at the moment for another resource she could benefit from. The absence of sharing incentives among N users is similar to the fact that each of the users will receive only $\frac{1}{N}$ of each resource. The latter in many cases results in huge performance degradation of the system as well as individual users, as many resources remain under-utilized.

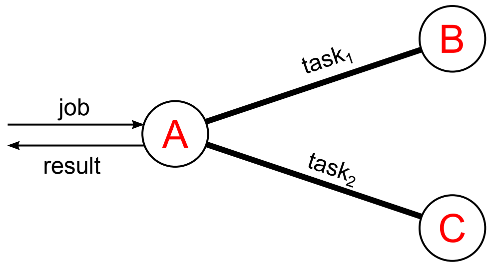

Fig. 1. Job model

paper.

II. Motivation

There are many examples where several resources are related. Players, requesting these resources, need all of them in some proportions. This comes from the fact that if a player misses some part of some resource then the whole process starts to lag behind, waiting for the straggler. In that case we say that demands are proportional. This is common scenario for any non-trivial multistage process. Especially we find such examples in modern datacenter and private cluster applications. We focus on that model in this paper.

Many applications, especially in datacenters, revolve around the notion of jobs. Each job requires bandwidth from several links, and these requirements are related: overall utility of an application is defined by the bottleneck link. Obvious examples of such applications are computation frameworks, such as Hadoop or Dryad. The job in these systems is not complete unless a transfer on a slowest link is finished. For clarity we give a small motivating example and provide a formal definition of a job model for network bandwidth allocation. This scatter-and-gather work-flow is common in datacenters.

An example (Fig. 1). Imagine we have a tenant which has three virtual machines (VMs) — **A**, **B** and **C** — designated for some *job*. The job is processed as following: VM **A** (*master node*) receives the job, splits it into two tasks, sends them to VMs **B** and **C** (*workers*) and collects the results. On a high level, the performance of the entire job is measured by the time VM **A** receives the last result. For simplicity assume that CPU time for tasks is the same for both workers or negligible in comparison to communication time.

We are interested in finding the amount of resource allocated to communication channels *A–B* and *A–C*. For a moment, imagine that on the channel *A–B* communication time is *10* ms and on the channel *A–C* it is *100* ms. Independently of the performance on the channel *A–B*, the entire task will not complete in time faster than 100 ms. In other words, this means that the performance of the task is determined by the slowest channel. Moreover, there is no benefit in utilizing the excess bandwidth on a non-bottleneck channel *A–B* at the job level, on another hand, the excess bandwidth may be given to or traded with another tenant.

The aforementioned problem can be solved if the tenant could explicitly proclaim how many bits per second she needs on channel *A–B* for every bit per second on channel *A–C*. This requirement leads us to the following definition of the job model.

Definition 1 (Job model)**.** *A set of communication tasks (job) of a tenant is defined by a set of virtual machines S and bandwidth requirements D for a set of links L over which the communication between these machines occurs. The job is said to be limited (after resource allocation) by the smallest ratio ($\frac{A_l}{D_l}$) between allocated (A_l) and demanded (D_l) bandwidth at some bottleneck link l (the slowest task in the set).*

In other words, this means that an increase in the allocated bandwidth $A_{l'}$ on some non-bottleneck link $l' \neq l$, will not improve the performance of the job, since equal proportionality between demanded and allocated bandwidths must be observed for all tasks (involved in the given job). This model leads to a natural definition of *utility* of a tenant — the smallest ratio between allocated and demanded bandwidth over all links in the network.

For simplicity, we limit ourselves to communication tasks. However, the limitation is not crucial. The model can be extended to a mixture of communication, computational and memory resource sharing. The latter requires some unified metric to make them comparable, for example, the completion time. Master node can send a task over a slow link to a fast machine, or over a fast link to a slow machine, the completion time defines what is preferred.

We consider that each tenant has only one job. Utility of each tenant is proportional to a minimal ratio of amount of allocated resource to amount of demanded among all resources. We call a set of tenant's demands for all resources — a *demand vector*. Because each tenant desires to get as much resources as possible, we are not interested in absolute values of tenants' demands. We have to consider only their relative values. Therefore, demand vectors of all tenants should be *normalized*. We suggest to divide all demands of a single tenant by the maximal demand of her. This results in all values of demand vectors to not exceed 1. As in DRF [4], we call the resource with the maximum demand *dominant resource* for each tenant, as it dominants her demands.

III. Allocation mechanisms

A. Pricing mechanism

We now consider a pricing mechanism to allocate several goods (link capacities, spectrum, virtual machine resources) to several players. For simplicity, we consider all resources to have equal capacity of 1 (we can always normalize it this way). A pricing mechanism is a set of prices $p \in R_+^m$, where m is a number of resources to be allocated, i.e. each good has a price associated with it. Each player tries to maximize her utility, while not exceeding the fixed budged of 1. Let x_i be utility of a player i and $d_i \in [0,1]^m$ be her demand vector (as discussed above, it is normalized).

Then, given set of prices, utility of a player i ($1 \leq i \leq n$)

will be defined by:

$$x_i = \frac{1}{\sum_{j=1}^{m} p_j d_{ij}}$$

It follows from the fact, that each player buys resources in proportion to her demands.

However, that computation does not account for limited resource stocks. At that point prices have to be adjusted according to market principles — lower the price if total demand is less than supply, and raise it if demand exceeds supply. More formally, total demand TD_j is defined by:

$$TD_j = \sum_{i=1}^{n} x_i d_{ij} = \sum_{i=1}^{n} \frac{d_{ij}}{\sum_{k=1}^{m} p_k d_{ik}}$$

Price p_j should be lowered, if $TD_j < 1$, and raised if $TD_j > 1$. Although this scheme does not converges in all possible scenarios, in a given model it always converges to Competitive Equilibrium from Equal Incomes (CEEI), which also corresponds to the Nash Bargaining Solution [5], [6].

B. Nash Bargaining Solution

NBS is defined by the following optimization problem:

$$\begin{gathered}\prod_{i=1}^{n} x_i \to max \\ \text{subject to} \\ \sum_{i=1}^{n} x_i d_{ij} \leq 1,\, j = 1..m \\ x_i \geq 0,\, i = 1..n\end{gathered} \tag{1}$$

Resulting allocation is unique and defined by demand vectors. It is Pareto efficient and envy free, i.e. it is an optimal allocation for given demand vectors. However, by adjusting their demand vectors players can manipulate allocations, increasing their utilities. For example, consider 2-tenant, 2-resource case with demand vectors as follows:

$$d_1 = \left(1, \tfrac{1}{2}\right), d_2 = \left(\tfrac{1}{3}, 1\right)$$

These demands will result in following utilities:

$$x_1 = \tfrac{4}{5},\, x_2 = \tfrac{3}{5}$$

However, if player 2 will increase her demand vector to $d_2' = \left(\frac{1}{2}, 1\right)$, utilities will be:

$$x_1 = \tfrac{2}{3},\, x_2 = \tfrac{2}{3}$$

Here the second player managed to increase her allocations by lying about her demands. Such misbehavior essentially removes sharing incentives, as shown in sections IV-VI.

C. Dominant Resource Fairness Allocation

DRF allocation is a strategy-proof allocation method there no misbehavior is possible by design. Main difference from NBS is that utilities of all players are forced to be equal. We denote this common utility as X. Then X is maximized subject to the same constraints as in NBS:

$$\begin{gathered}X \to max \\ \text{subject to} \\ \sum_{i=1}^{n} X d_{ij} \leq 1,\, j = 1..m \\ X \geq 0\end{gathered} \tag{2}$$

This optimization problem is trivial and can be solved explicitly:

$$X = \min_{j=1..m} \frac{1}{\sum_{i=1}^{n} d_{ij}}$$

DRF is obviously strategy-proof and no misbehavior is possible as allocations of all players are forced to be equal. Therefore any beneficial behavior of any tenant will always increase all allocations. However, equalization of all utilities restrain possible allocations space. Therefore DRF produces lower total utilization of resources.

IV. Two players case proof

First of all, let us consider a 2-player case (i.e. $n = 2$). 2-player game allows an explicit solution as well as intuitively easy to understand. Same idea will be utilized in a more general case. Without loss of generality, let us also set the number of resources to 2 ($m = 2$). The proof can be easily extended to multiple resources and two players. We reformulate problem (1) as follows:

$$\begin{gathered}\ln(x) + \ln(y) \to max \\ \text{subject to} \\ x + b \cdot y \leq 1, \\ a \cdot x + y \leq 1, \\ x \geq 0, y \geq 0,\end{gathered} \tag{3}$$

where demand vectors are $d_1 = (1, a)$, $d_2 = (b, 1)$, $0 < a < 1$, $0 < b < 1$. We are interested only in that case because sharing incentives are presented only here. If both players would want more of the first resource, allocation will not depend on their demands of the second resource at all. Therefore both players will put whole budget on the same bottlenecked resource leaving another one without attention.

Example of misbehavior can be seen on Figure 2. If optimal point is at the corner of feasible solutions set (grey area), player A can increase her demand for second resource. This will shift line bounding feasible set corresponding to he second resource to the left. Which then will shift intersection point to the right, increasing allocation of tenant A from x_1 to x_2. This procedure can be continued until intersection point will stop being an optimal solution.

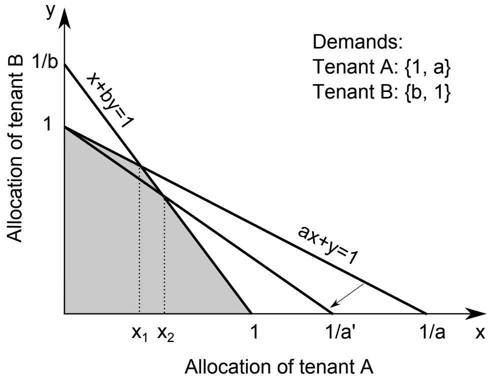

Fig. 2. Misbehavior example for 2-player case.

Because all constraints for variables x and y are linear we can apply the method of Lagrange multipliers and Karush-Kuhn-Tucker conditions [7]:

Theorem 1 (KKT conditions)**.** *There exist* $\lambda \geq 0$, $\mu \geq 0$ *such that:*

$$\begin{aligned} x &= \tfrac{1}{\lambda + a\cdot\mu} \\ y &= \tfrac{1}{b\cdot\lambda + \mu} \\ \lambda(x + b\cdot y - 1) &= 0 \\ \mu(a\cdot x + y - 1) &= 0 \end{aligned}$$

Proof. Let $\lambda \geq 0$, $\mu \geq 0$, $\alpha \geq 0$, and $\beta \geq 0$ denote Lagrange multipliers for 4 constraints from problem (3). Then, the Lagrangian of the problem is:

$$\begin{aligned} \mathcal{L}(x, y, \lambda, \mu, \alpha, \beta) = \ln(x) + \ln(y) \\ -\lambda(x + b\cdot y - 1) - \mu(a\cdot x + y - 1) + \alpha\cdot x + \beta\cdot y \end{aligned} \tag{4}$$

Giving the necessary and sufficient conditions for the Kuhn-Tucker conditions:

$$\begin{aligned} \nabla\mathcal{L}(x, y, \lambda, \mu, \alpha, \beta) = 0 \iff \\ \tfrac{1}{x} - \lambda - \mu\cdot a + \alpha = 0 \\ \tfrac{1}{y} - \lambda\cdot b - \mu + \beta = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} \lambda(x + b\cdot y - 1) = 0 \\ \mu(a\cdot x + y - 1) = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} x\cdot\alpha = 0 \\ y\cdot\beta = 0 \end{aligned} \tag{7}$$

Because it is obvious that $x > 0$ and $y > 0$, holds $\alpha = 0$ and $\beta = 0$. Thus we can omit equations (7), and find x and y from (5). □

Following theorem 1 there are 3 points satisfying necessary conditions:

$$\lambda \neq 0, \mu \neq 0 \iff \begin{cases} x + b\cdot y = 1 \\ a\cdot x + y = 1 \end{cases} \tag{8}$$

$$\lambda \neq 0, \mu = 0 \iff \begin{cases} x + b\cdot y = 1 \\ x = \frac{1}{\lambda} \\ y = \frac{1}{b\cdot\lambda} \\ a\cdot x + y < 1 \end{cases} \tag{9}$$

$$\lambda = 0, \mu \neq 0 \iff \begin{cases} a\cdot x + y = 1 \\ x = \frac{1}{a\cdot\mu} \\ y = \frac{1}{\mu} \\ x + b\cdot y < 1 \end{cases} \tag{10}$$

Equations (8)–(10) lead to following possible solutions:

$$x = \tfrac{1-b}{1-a\cdot b}, \quad y = \tfrac{1-a}{1-a\cdot b} \tag{11}$$

$$x = \tfrac{1}{2}, \quad y = \tfrac{1}{2\cdot b}, \quad 1 \leq b\cdot(2-a) \tag{12}$$

$$x = \tfrac{1}{2\cdot a}, \quad y = \tfrac{1}{2}, \quad 1 \leq a\cdot(2-b) \tag{13}$$

It is easy to see, that conditions (12) and (13) are mutually exclusive (see Figure 3). Note, that if either of points (12) and (13) is available, it is a global maximum. Consider $1 \leq b\cdot(2-a)$, therefore point $x = \frac{1}{2}, y = \frac{1}{2\cdot b}$ is available. Both points (12) and (11) are on the same line $x + b\cdot y = 1$ and point (12) is a maximum for a given objective function on that line. Therefore, it is a global maximum.

Theorem 2 (Misbehavior for 2 players)**.** *For each* $0 < a < 1$ *and* $0 < b < 1$ *it is possible to either increase* a*, or increase* b *in such a way, that new NBS defined by problem* (3) *will have larger utility* x *(after increasing* a*) or* y *(after increasing* b*).*

Proof. Consider parameters space in Figure 3. Two lines $b\cdot(2-a) = 1$ and $a\cdot(2-b) = 1$ separate that space into three zones:

Zone I: Only point (11) is available. Utilities of players are: $x = \frac{1-b}{1-a\cdot b}, \quad y = \frac{1-a}{1-a\cdot b}$. Note, that this zone doesn't include borders. Therefore it is possible for player 1 to increase a until $a\cdot(2-b) = 1$. Let that new demand value be a'. New allocation of player 1 will be:

$$\tfrac{1-b}{1-a'\cdot b} > \tfrac{1-b}{1-a\cdot b}$$

New allocation is larger than what player 1 had. Note, that after increase both possible points (11) and (13) coincide. Figure 3 shows that transition from point 1.

Zone II: Point (12) is available, therefore utilities of players are $x = \frac{1}{2}, \quad y = \frac{1}{2\cdot b}$. Similarly, player 1 can increase her demand for the second resource a until $a\cdot(2-b) = 1$. This transition corresponds to a horizontal shift from point 2 to point 3 in Figure 3). Player 1 will have new allocation:

$$\tfrac{1-b}{1-a'\cdot b} = \tfrac{1}{2\cdot a'} > \tfrac{1}{2}$$

Note, in that zone, player 2 could also increase her allocation $\frac{1}{2\cdot b}$ by decreasing demand b. But after that transition, system will enter zone I and both players will only increase their demands afterwards. Therefore, we can omit that possibility.

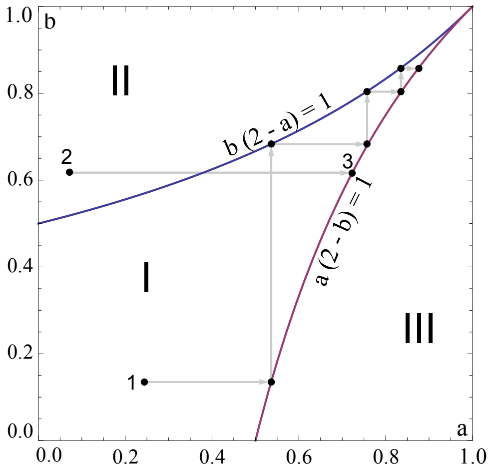

Fig. 3. Parameters space for 2-player case

Zone III: Point (13) is available, therefore utilities of players are $x = \frac{1}{2 \cdot a}, \quad y = \frac{1}{2}$. Player 2 can increase allocation the same way as player 1 can in zone II. This move will correspond to a vertical transition from point in zone III (see Figure 3). □

Following theorem 2 we can conclude that the only stable point is $a = 1$, $b = 1$. That corresponds to trivial demand vectors and results in each player getting same amount of each resource. This means absence of sharing incentives.

It is possible to prove a similar result in case of $m > 2$ resources. However, unlike the proof of theorem 2, one will have to consider additional cases, where the maximum point corresponds to intersection of several constraint lines at once. In that case players will have to increase several demands at once to shift a point of a global maximum in a desired way.

V. Three players case proof

In this section we consider 3-player case (i.e. $n = 3$, $m = 3$). We will also reformulate problem (1) as follows:

$$\begin{gathered} \ln(x) + \ln(y) + \ln(z) \to max \\ \text{subject to} \\ x + c \cdot y + e \cdot z \leq 1, \\ a \cdot x + y + f \cdot z \leq 1, \\ b \cdot x + d \cdot y + z \leq 1, \\ x \geq 0, y \geq 0, z \geq 0, \end{gathered} \tag{14}$$

where demand vectors are $d_1 = (1, a, b)$, $d_2 = (c, 1, d)$, and $d_3 = (e, f, 1)$, $0 < a, b, c, d, e, f < 1$. As in 2-player case, we are interested only in that case because sharing incentives are present only here. If two players would want more of a same resource, allocation will not depend on their demands of other resources at all, and problem could be reduced to a smaller one.

We do not provide an exact solution for that problem as we did for 2 players. It is possible to do so, however results will be too bulky. Regardless, we provide a proof of breakage of sharing incentives for 3 players. For the generality the case with 3 players is much more significant than 2-player case. While in 2-player case a player has a single opponent which she plays against and whose allocation should be decreased in order to increase own utility, in the 3-player case there is no such opponent, and a decrease in the first opponent's allocation could always result in an increase in the second's opponent allocation. The latter degrades own allocation.

Because all constraints for variables x, y and z are linear we can apply the method of Lagrange multipliers and Karush-Kuhn-Tucker conditions:

Theorem 3 (KKT conditions)**.** *There exist $\lambda \geq 0$, $\mu \geq 0$, and $\nu \geq 0$ such that:*

$$\begin{aligned} x &= \tfrac{1}{\lambda + a\cdot\mu + b\cdot\nu} \\ y &= \tfrac{1}{c\cdot\lambda + \mu + d\cdot\nu} \\ z &= \tfrac{1}{e\cdot\lambda + f\cdot\mu + \nu} \\ \lambda(x + c\cdot y + e\cdot z - 1) &= 0 \\ \mu(a\cdot x + y + f\cdot z - 1) &= 0 \\ \nu(b\cdot x + d\cdot y + z - 1) &= 0 \end{aligned}$$

Proof. Similar to the proof of Theorem 1 λ, μ and ν are Lagrange multipliers. Last equations are complementary slackness conditions. Again, as all utilities should be strictly positive, Lagrange multipliers for last constraints in problem (14) can be omitted. □

There are 3 types of points satisfying necessary conditions:

1) All Lagrange multipliers are non zero. Single point of this type is at the intersection of all three planes from constrains of problem (14):

$$x = \frac{\begin{vmatrix} 1 & c & e \\ 1 & 1 & f \\ 1 & d & 1 \end{vmatrix}}{\begin{vmatrix} 1 & c & e \\ a & 1 & f \\ b & d & 1 \end{vmatrix}}, y = \frac{\begin{vmatrix} 1 & 1 & e \\ a & 1 & f \\ b & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & c & e \\ a & 1 & f \\ b & d & 1 \end{vmatrix}}, z = \frac{\begin{vmatrix} 1 & c & 1 \\ a & 1 & 1 \\ b & d & 1 \end{vmatrix}}{\begin{vmatrix} 1 & c & e \\ a & 1 & f \\ b & d & 1 \end{vmatrix}} \tag{15}$$

2) Two Lagrange multipliers are non zero. There are 3 such points on the intersections of each 2 planes. These points can be considered, if they satisfy remaining linear condition. If any of such points is available, they yield a better value of the objective function than the point of the first type. That observation can be made as a point of the first type also lies on the same line in (x, y, z) space.
Consider $\lambda \neq 0$ and $\mu \neq 0$. Then we can describe a line of intersection of first 2 planes in a parametric form:

$$\begin{aligned} x &= \tfrac{1-c}{1-a\cdot c} + (c\cdot f - e)t \\ y &= \tfrac{1-a}{1-a\cdot c} + (a\cdot e - f)t \\ z &= (1 - a\cdot c)t \end{aligned} \tag{16}$$

More equations can be added from theorem 3 to find the optimal point on that line:

$$\begin{aligned} x &= \frac{1}{\lambda + a \cdot \mu} \\ y &= \frac{1}{c \cdot \lambda + \mu} \\ z &= \frac{1}{e \cdot \lambda + f \cdot \mu} \end{aligned} \tag{17}$$

3) Only one Lagrange multiplier is non-zero. There are 3 such points at each of 3 facets of a feasible set. From theorem 3 it is possible to conclude that these points are:

$$\begin{aligned} x &= \frac{1}{3} \\ y &= \frac{1}{3 \cdot c} \\ z &= \frac{1}{3 \cdot e} \end{aligned} \quad \text{or} \quad \begin{aligned} x &= \frac{1}{3 \cdot a} \\ y &= \frac{1}{3} \\ z &= \frac{1}{3 \cdot f} \end{aligned} \quad \text{or} \quad \begin{aligned} x &= \frac{1}{3 \cdot b} \\ y &= \frac{1}{3 \cdot d} \\ z &= \frac{1}{3} \end{aligned} \tag{18}$$

If any of points of type 3 is feasible, it is an optimal point. In fact these points are maximum points on whole corresponding planes. Adding more restrictions by intersecting these planes together will only reduce the objective function. That is why these points are better than any point of other type. Obviously only one point of that type could be feasible.

Theorem 4 (Misbehavior for 3 players)**.** *For each* $0 < a, b, c, d, e, f < 1$ *it is possible for some player to adjust her demands in such a way, that her utility defined by 14 will increase.*

Proof. Consider that global maximum is a point of type 1, as discussed above. Utilities of players are defined by (15). Consider utility of a first player x. Determinant in numerator does not depend on a nor b. However, denominator can be rewritten in a form: $\left|\begin{smallmatrix} 1 & f \\ d & 1 \end{smallmatrix}\right| + a \cdot \left|\begin{smallmatrix} d & 1 \\ c & e \end{smallmatrix}\right| + b \cdot \left|\begin{smallmatrix} c & e \\ 1 & f \end{smallmatrix}\right|$. If $d \cdot e - c < 0$ then by increasing a, player 1 could increase her utility. If $c \cdot f - e < 0$ then player 1 could increase b leading to decreased denominator, and thus, increasing utility. Note, that at least one of conditions above is true.

Similar argument is possible to show, that c can be increased if $f \cdot b - a < 0$, d can be increased if $a \cdot e - f < 0$, e — if $a \cdot d - b < 0$, and f — if $b \cdot c - d < 0$.

However, it is possible to increase utility only until any point of type 2 or 3 becomes feasible. Any further change will not affect maximum point in a previous manner, as it will be defined by another equations. At that time point of type 1 will coincide with point of another type. Because the function has only one maximum at a feasibility set, continuous change of demands will lead to continuous change of maximum point. Therefore, continuous increase of parameters will eventually make the point of type 1 coincide with a point of type 2 or 3.

We showed above, that each player can shift a point of type 1 into position with better utility until some point of other type will coincide with point of type 1. To show misbehavior possibility in all cases, we have to consider a situation, where that happens. Assume that the first point of type 2 coincides with point of type 1. Therefore all equations (15), (16), and (17) hold.

First, we will show that either $c \cdot f - e < 0$ or $a \cdot e - f < 0$ (these are conditions to allow increase of b or d for type 1 point). If opposite holds, then $\frac{e}{c} \leq f \leq a \cdot e$, which is impossible, as $c < 1$ and $a < 1$ and therefore $\frac{e}{c} > a \cdot e$. Thus it is either beneficial to increase b or increase d. Both actions will not affect equations (16) nor (17) and therefore will not shift point of type 2. But these actions will make that point unfeasible. Feasibility constraint for that point is: $x \cdot b + y \cdot d + z \leq 1$. That inequality is actually an equality for that point, as points of type 1 and type 2 coincide. Any increase in b or d will break that inequality and global maximum will be of type 1. Utility of a player who increased her demand will be larger after that increase, as from the previous case applies.

Suppose that a point of type 3 coincides with a point of type 1. Let it be the first point of type 3 defined by (18). Because point of type 3 is a maximum point for objective function and because it lies on intersections of any two planes (by definition of type 1) it is also a type 2 point for two lines on the first plane. Therefore, global maximum is simultaneously a point of type 1, point of type 3 and two points of type 2. Note, that all points of type 2 can't coincide as it will lead to all Lagrangian multipliers to be zero for that point, which is impossible.

Suppose again, that first a point of type 2 coincides with a point of type 1 and it is a global maximum. Similarly to previous reasoning, it is possible to increase either b or d, which will render all points of type 2 and 3 unfeasible and will increase the value of the objective function for a point of type 1, which will be a global maximum then.

Next, consider that the global maximum is reached at a point of type 2 (not coinciding with point of type 1). Suppose that it is a first point of type 2. Therefore equations (16) and (17) apply. As they do not depend on b nor d it is possible to increase these demands until $x \cdot b + y \cdot d + z = 1$. Then points of type 1 and 2 will coincide and reasoning above applies. AS shown above it is possible to further increase b or d for some player to increase her utility.

Lastly, consider that the global maximum is reached at a point of type 3. Let it be a point $(\frac{1}{3}, \frac{1}{3 \cdot c}, \frac{1}{3 \cdot e})$. It is possible for a third player to increase her utility by decreasing demand e. It can be done until point of type 3 coincides with point of type 2, as further decrease will render that point unfeasible and the global maximum will no longer be defined by same equations. Now, a second player can increase her utility by decreasing c. Again it can be done until point of type 3 coincides with another point of type 2. At that time the global maximum will be on intersections of two lines, which means it is on intersection of all planes and, in fact, is a point of type 1. Now, reasoning above applies and players can increase their allocation by increasing their demands.

Note, that in the last case, players also can increase a, b, d, f. Because it does not affect the global maximum point, it will not change players' utilities until point of type 3 will coincide with point of type 1. After that players could increase their utilities by further increasing their demands.

□

Now we will state the main result of this section:

Theorem 5 (Stable point)**.** *Optimal players' strategies converge to demand vectors* $(1, 1, 1)$ *for* 3*-player case.*

Proof. As shown in theorem 4, only in one case players benefit from decrease of their demands — then a point of type 3 is the global maximum, not coinciding with a point of type 1. But after optimal responses, game will enter and always stay at the first case — the maximum is a point of type 1. Another case, then point of type 2 is a global maximum leads to the same case after some increase of demands.

We must show now, that this process will converge to all demands equal to 1, not only some of them. Consider the maximum is at a point of type 1. We will show that it is always possible to increase the minimum demand. Suppose, w. l. o. g., that a is the smallest demand among all 6 parameters. Suppose it can't be increased. Therefore $c < e \cdot d$. If c can be increased, it is possible to make it so, that a can be increased. Suppose that c can't be increased either, therefore $a < f \cdot b$. We call that situation a *deadlock*. It is easy to see, that $a < b \Rightarrow a \cdot d < b$ and thus, e can be increased. Similarly d, e or f can be increased. We argue that deadlock is impossible, because if that state is reached, it is possible to reach a situation, where b, d, e, and f are arbitrary close to 1. It follows from the fact, that if the maximum is of type 1 we can increase the smallest demand among that 4 until we reach a border case then type 1 and type 2 coincide. After that we would make some of possible increases to make point of type 1 a maximum again. Then we could continue to increase the smallest of demands.

However, point of type 1 is not a maximum for these demands, as demand matrix is close to $\left(\begin{smallmatrix} 1 & c & 1 \\ a & 1 & 1 \\ 1 & 1 & 1 \end{smallmatrix}\right)$, and point of type 1 is arbitrary close to $(0, 0, 1)$, which is obviously not a maximum. But if deadlock were possible that state could be reached with maximum still at the point of type 1.

Because deadlock is impossible, the smallest demand always can be increased and thus all demands vectors converge to $(1, 1, 1)$, which is the only stable point. □

As in 2-player case, players have incentives to selfishly increase their demand, until all demands are equal and there are no sharing incentives at all.

VI. Simulations

In previous sections we have shown proofs of misbehavior for 2 and 3 player cases. Unfortunately, solution for a large number of tenants becomes more bulky. In order to verify the tendency as well as to measure exact performance we implemented a simulator that imitates behavior of individual players in the system with shared resources. To support our analytic results, we provide the simulation results for 2 to 15 players competing for 2 to 15 links.

We consider full bisection bandwidth datacenter topology as in Falloc [2]. N tenants are competing for M links. In this scenario there are M machines shared by all tenants and N machines dedicated to each tenant. Each tenant is gathering data from all shared machines to her dedicated machine. Therefore N links are shared by all tenants and M links are used by all flows of each tenant. M personal links have capacity 10 times more than shared N links. This is so tenants would actually compete for shared resources and not have bottlenecks on their personal links. Figure 4 shows sample topology and flows in the datacenter.

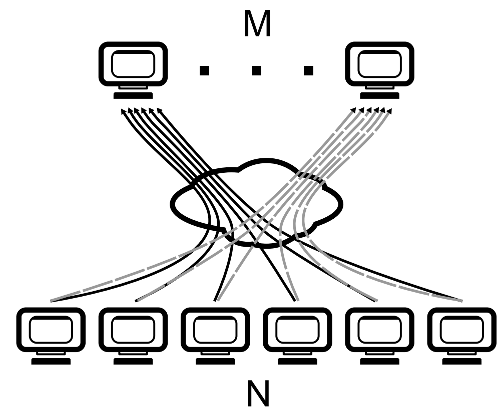

Fig. 4. Sample topology. Full-bisection bandwidth datacenter with $M + N$ machines. Flows of different tenants are marked with different styles.

Our simulator uses pricing mechanism to compute CEEI by simulating auction, which results in the same allocation as NBS (see section III-A). Each iteration some non-dominant demand of each tenant is increased randomly and new allocation is computed. If new allocation is not worse for some tenant her increased demands are fixed. Using Monte-Carlo method we randomly generated initial demands and simulated misbehavior. Simulation results show that all demand vectors converge to the same trivial demands, when each player requests the same amount of each resource. Each experiment is repeated 1000 times.

To investigate the impact of misbehavior we compared average aggregated bandwidth before and after misbehavior. Results are presented in Figure 5. In large datacenters misbehavior can reduce aggregated bandwidth by as much as 35%. We also compare NBS allocation with DRF [4]. We chose DRF as a baseline for comparison, as it is a strategy-proof mechanism to allocate multiple resources even in case of proportional demands. As a trade-off in DRF aggregated bandwidth is reduced, but not so much as in case of misbehavior for NBS, see Figure 6. DRF allocation produces aggregated bandwidth reduced only by less than 3.5% compared to 35% in case of NBS.

Impact of misbehavior grows quickly for small numbers of tenants/links. It is more sensitive to number of tenants than to number of links. Therefore systems with large number of players will suffer greater disadvantage. However even with as few as 2 tenants NBS misbehavior can reduce aggregated bandwidth by more than 10%.

DRF performs better when the number of links is increased. This is because bigger number links results in more constraints. Which makes set of feasible allocations more symmetrical. Therefore NBS optimal point happens to be close to DRF allocation. Degradation of aggregate bandwidth compared to original NBS allocation grows slowly in number of

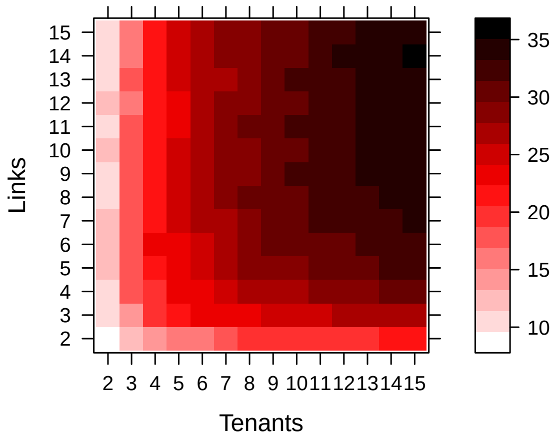

Fig. 5. Average reduction of aggregated bandwidth due to misbehavior in percentage.

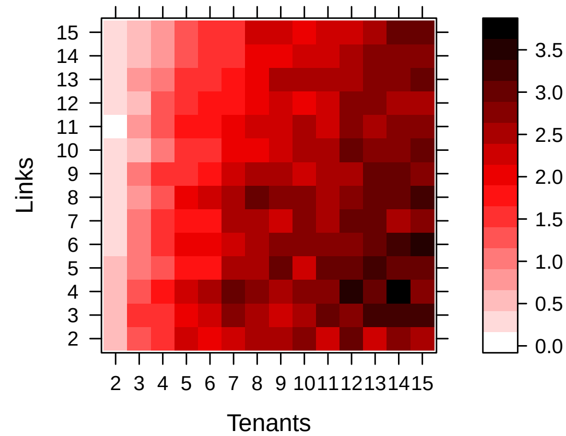

Fig. 6. Average reduction of aggregated bandwidth for DRF allocation compared to NBS allocation in percentage.

tenants. However, compared to original NBS, the performance deterioration is one order of magnitude less than misbehaving NBS compared to the same original NBS.

It can be concluded, that NBS allocation achieves better resources utilization than DRF. However, absence of sharing incentives and misbehavior lead to significant loss in performance. Therefore it is recommended to use DRF allocation over any NBS-based allocation scheme in case where demands are proportional.

VII. Related work

Recently, bandwidth allocation in data centers received much attention from researchers. Various reservation-based schemes including Oktopus [8], Gatekeeper [9] and SecondNet [10] were proposed for bandwidth guarantees without work conservation properties. Those often implement a hose model [11] where each user is connected by a virtual link with minimum guaranteed bandwidth.

FairCloud [12] analyzed the tradeoffs between payment proportionality, high utilization and minimum bandwidth guarantees. The authors proposed three allocation policies, including PS-L for proportional sharing on the link-level, PS-N on the network-level, and PS-P taking into account link proximity. While FairCloud policies can be efficiently implemented in switches, they lack the task concept which is necessary to capture the dependency among allocations on links forming a common path.

Several researchers suggested to utilize game theoretic approach to allocate bandwidth fairly. Yaiche *et al.* [13] used Nash Bargaining Solution (NBS) [3] to achieve fair allocation. Authors propose distributed method to calculate NBS. Although that method provides Pareto efficient allocation for given demand vectors, it does not account for ability of users to lie about desired demand vectors. Falloc [2] provides another method for finding NBS specifically for bandwidth allocations in datacenter. That work also presents prototype implementation of the suggested protocol using OpenFlow. That solution is also distributed.

Another method to fairly allocate resources, coming from microeconomic theory, is a Competitive Equilibrium from Equal Incomes. With CEEI each user initially gets the same amount of every resource and trades with other users in a perfectly competitive market. The outcome of CEEI corresponds to NBS. It was shown that any market-pricing scheme for multiple resources leads to CEEI allocation [5], [6]. It means that given an equal budget to multiple players and a market-driven prices, which are based on resource load, the players end up allocating their budget with the same scheme as CEEI allocation policy provides.

Another area of application of NBS and market-based allocation policies in networking is a Spectrum Market. Zhu *et al.* [14] proposes to use a market-based scheme, which leads to CEEI allocation, to fully utilize scarce spectrum resources. In a proposed multi-stage dynamic game optimal and collusion-free spectrum allocation is achieved. Kash *et al.* [15] present a scalable auction for spectrum sharing. Authors demonstrate their algorithm's ability to handle heterogeneous agent types involving different transmit powers and spectrum needs through extensive simulations. Niyato *et al.* [16] investigate three different pricing models for spectrum sharing in cognitive radio networks and provide solutions for these different pricing models. In that work, distributed algorithms are proposed to implement pricing models based on the theory of discrete-time linear control system. It is also shown that proposed algorithms converge to a stable solution.

Work [17] applies an auction based scheme for packet prioritization as a way to increase total utility among heterogeneous user base.

Many mentioned methods utilize a game theoretic approach to allocate resources in the network fairly. However, all proposed methods assume that users do not lie about their demands. We emphasize that such misbehavior is possible and in fact beneficial for users, leading to absence of sharing incentives for such allocation schemes.

VIII. Conclusion

Understanding properties of game-theoretic mechanisms for allocations is important for modern network protocols. While protocol designers suggest one or another allocation scheme for network resources, generally they forget that demands may be manipulable. In this work we show that NBS-based allocation is vulnerable to such manipulation in case of proportional demands.

We consider intelligent players which optimize amount of acquired resources and applications which are working on multiple resources at once. We show that no NBS-based allocation is strategy-proof in that case. Moreover intelligent players adjust their demands slightly increasing own profit. In the end, this iterative process results in an allocation without any sharing-incentive, where each player receives degraded performance, worse than what she gets before playing.

We prove these results for a 2-player game and a 3-player game. The 2-player game is an important in providing understanding of the misbehavior mechanism as well as exact solution. On another hand, 3-player game is important to show that third player is not breaking a degradation effect. While in 2-player game a player always has exactly one opponent which allocation she needs to reduce, in 3-player game the reduction of allocation of one player can increase allocation of another player. In this work we show that this is not happening. While we believe that we can prove this property in general case, the analysis becomes more complex.

To study impact of misbehavior on aggregated bandwidth in a datacenter we perform simulations for up-to 15 tenants and up-to 15 shared links. We found that demands manipulation severally deteriorates aggregated bandwidth. We also measure performance of another game-theoretic allocation method, namely DRF, and show that DRF allocation is preferable over NBS-based allocation for any number of tenants and shared links.

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