

PAPER III

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Analysis of the terms  
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# **Time-averaged modeling of BFBs: Analysis of the terms in the momentum equations**

Timo Niemi\*, Juho Peltola, Sirpa Kallio  
VTT Technical Research Centre of Finland  
P.O.Box 1000, FI-02044 VTT, Finland

\*T: +358 20 778 5068; F: +358 20 722 7077; E: [timo.niemi@vtt.fi](mailto:timo.niemi@vtt.fi)

## **ABSTRACT**

Steady state CFD simulation would present an attractive alternative for the computationally demanding transient simulations in the case of large, industrial scale BFBs. In the present paper, the features and relative importance of the various terms appearing in steady state flow equations are analyzed based on a transient simulation. According to this analysis, the most important terms in the momentum equations are the gas-solid drag term and the solid volume fraction and gas pressure fluctuation correlation terms. The solid pressure and the Reynolds stress terms are also found to be important.

## **INTRODUCTION**

Computational fluid dynamics (CFD) combined with the kinetic theory of granular flow (KTGF) has proven to be a useful method for simulating gas-solid flows in bubbling fluidized beds (BFB). Due to the complex nature of the flow in BFBs, the simulations have been typically performed as time-dependent with small time-steps and fine meshes. This kind of an approach is computationally very time consuming and makes larger scale simulations of BFBs challenging.

With single phase flows and recently also with multiphase flows in circulating fluidized beds [1], steady-state CFD modelling approach with time-averaged flow equations has been used to greatly accelerate the simulations. Such an approach would be also attractive for flows in BFBs, but in order to use the time-averaged equations, valid closure models are required. Due to the different nature of the dense flow in BFBs, the relations developed for single phase or more dilute multiphase flows cannot be assumed to hold as such and further development is needed. This development can also help CFB steady-state

modelling, because the bottom region in some CFBs can resemble BFB flow conditions in which closures developed for BFBs would apply.

As a step towards time-averaged modelling of BFBs, in the present study a transient simulation of a lab-scale BFB is performed and time-averaged terms required for steady-state closures are computed. The features and the relative importance of the different terms are analysed and compared to those presented in the literature.

## NUMERICAL MODELS

### Transient and time-averaged equations

Following the notation by Taivassalo et al [1], the transient multiphase flow equations can be concisely written as

$$\frac{\partial \alpha_q \rho_q}{\partial t} + \frac{\partial \alpha_q \rho_q u_{q,k}}{\partial x_k} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial \alpha_q \rho_q u_{q,i}}{\partial t} + \frac{\partial \alpha_q \rho_q u_{q,k} u_{q,i}}{\partial x_k} = & -\alpha_q \frac{\partial p}{\partial x_i} + \frac{\partial \alpha_q (\tau_{q,ik} + \tau_{q,ik}^M)}{\partial x_k} \\ & + \alpha_q \rho_q g_i + (-1)^{(\delta_{qs}+1)} K_{gs} (u_{g,i} - u_{s,i}) - \frac{\partial p_q}{\partial x_i} \delta_{qs} \end{aligned} \quad (2)$$

where  $\alpha_q$  is the volume fraction,  $\rho_q$  density,  $u_q$  velocity,  $p$  gas pressure,  $p_q$  solid pressure,  $K_{gs}$  inter-phase momentum transfer coefficient,  $\delta_{qs}$  Kronecker delta,  $\tau_q$  the laminar stress, and  $\tau_q^M$  the local scale turbulent stress.

The steady-state flow equations can be obtained from the transient equations by averaging the equations over time. For the volume fraction and pressure terms the Reynold's averaging can be directly used. In Reynold's averaging the instantaneous flow variables are split into steady and fluctuating parts:

$$\phi = \bar{\phi} + \phi', \quad (3)$$

where  $\bar{\phi}$  represents the average value and  $\phi'$  is the fluctuating part. The average value over some time interval  $T$  is defined as

$$\bar{\phi} = \frac{1}{T} \int_T \phi dt \quad (4)$$

and for the fluctuating part the average vanishes

$$\bar{\phi}' = 0. \quad (5)$$

For the velocity the Favre averaging is used. The Favre average is defined as

$$\langle \phi \rangle = \frac{\overline{\alpha_q \phi}}{\overline{\alpha_q}} \quad (6)$$

By denoting the Favre-averaged velocity as  $U_{q,i} \equiv \langle u_{q,i} \rangle$ , the following averaged flow equations can be derived:

$$\frac{\partial \bar{\alpha}_q}{\partial t} + \frac{\partial \bar{\alpha}_q U_{q,k}}{\partial x_k} = 0 \quad (7)$$

$$\begin{aligned} \frac{\partial \bar{\alpha}_q \rho_q U_{q,i}}{\partial t} + \frac{\partial \bar{\alpha}_q \rho_q U_{q,k} U_{q,i}}{\partial x_k} = & \bar{\alpha}_q \rho_{qm} g_i - \bar{\alpha}_q \frac{\partial \bar{p}}{\partial x_i} - \alpha'_q \frac{\partial \bar{p}'}{\partial x_i} + \frac{\partial \bar{\alpha}_q \tau_{q,ik}}{\partial x_k} \\ & + \frac{\partial \bar{\alpha}_q \tau_{q,ik}^M}{\partial x_k} + (-1)^{(\delta_{qs}+1)} \overline{K_{gs}(u_{g,i} - u_{s,i})} - \frac{\partial \bar{p}_q}{\partial x_i} \delta_{qs} - \frac{\partial \rho_q \alpha_q u''_{q,k} u''_{q,i}}{\partial x_k} \end{aligned} \quad (8)$$

The terms on the right hand side in equation (8) are the gravitation, pressure, pressure fluctuation, laminar and turbulent stress, drag force, solid pressure and so-called Reynolds stress terms.

### Transient simulation parameters

To obtain time averaged terms, transient 3D simulation of a laboratory scale, cold pseudo-2D bubbling fluidized bed was performed. The height of the simulated geometry was 2 m, the width 90 cm and the thickness 1.5 cm. The air was brought to the bed through 9 separate 15x15 mm injectors and the average superficial gas velocity was 0.8 m/s. The simulated bed material had a mean particle size of 656  $\mu\text{m}$  and the particle density was 2480  $\text{kg/m}^3$ . Initially the bed was 60 cm high.

At the bottom region of the bed a uniform 5 mm sized 3D mesh was used. To save some computational time, coarser 10 and 20 mm mesh sizes were used above 1.0 and 1.5 meters, but the coarsening was not made in the smallest dimension. Also, during the simulation the bed mass stayed mostly in the fine mesh region, with only some occasional splashes reaching the 10 mm coarser part. All together the mesh had 115080 elements. According to our previous experiences and literature (see [2] and [3]), 5 mm mesh size has been found to give reasonably mesh independent results. Li et al. [3] observed in a 3D simulation of a similar pseudo-2D BFB that the results obtained with three and five elements in the depth direction were close to each other, which supports our selection of mesh spacing.

The simulation was performed with the commercial solver ANSYS Fluent v.14 using the kinetic theory of granular flow (KTFG). The various parameters and sub models for the different terms in the KTFG are presented in Table 1. Except for the interphasial drag term, for all the other terms the implementations provided by Fluent were used. For the drag term the standard Gidaspow [4] model was modified to use linear interpolation between the Ergun [5] and Wen-Yu [6] drag relations to avoid discontinuity. The linear modification is discussed more thoroughly by Leboireiro et al. [7] and Dahl [8]. In the present implementation the linear transformation between the Ergun and Wen-Yu models occurs in a solid

volume fraction range of 0.4-0.5. This range was chosen because it gives a smooth transition and is within the region recommended by Leboreiro et al.

At the boundaries the Johnson-Jackson [9] partial slip boundary condition was used with specularity and restitution coefficients of 0.1 and 0.9 respectively. To obtain sufficiently smooth average values, a time period of 120 s was simulated. The time step was 0.5 ms with first order temporal discretization and for the momentum and volume fraction equations the QUICK scheme was employed.

The time-averaged terms were computed using user defined functions (UDF) and data was collected at every time step. Unfortunately, since the exact way how the gradients are calculated in Fluent has not been provided by ANSYS, those terms in equation (8) that contained derivatives could not be accurately averaged in the first few cells near the boundaries of the geometry. Because the mesh had only 3 cells in the depth direction (z-axis), this problem affected the z-components of the derivative terms globally. For this reason, in the following analysis the z-components of the derivative terms are omitted. This has some effect at least on the stress terms, which should be likely larger due to side wall friction. However, in industrial scale 3D BFBs the side walls are further away and the wall effects may not be as large there. Therefore, the error caused by the omitted z-component may not be that significant if the term analysis is applied to larger scale. The same geometry was also simulated with a 2D mesh and the leading terms were same in both cases.

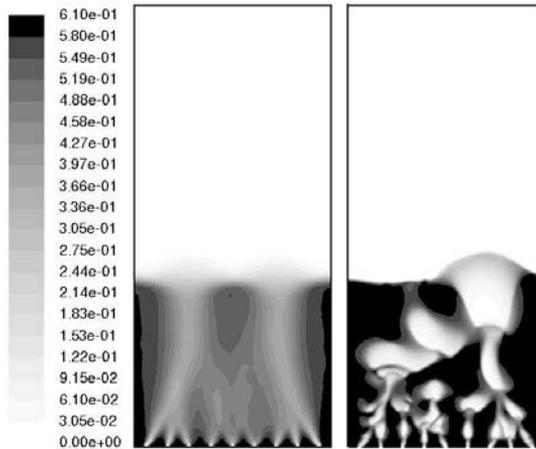
**Table 1:** The sub models and parameters for the transient simulation.

<b>Submodel or parameter</b>	<b>Model used</b>
Granular viscosity	Syamlal, et al. [10]
Granular bulk viscosity	Lun et al. [11]
Frictional viscosity	Schaeffer [12]
Frictional Pressure	Johnson et al. [9]
Granular conductivity	Syamlal, et al. [10]
Solids pressure	Lun et al. [11]
Radial Distribution	Ogawa et al. [13] (Lun et al. in Fluent)
Angle of internal friction	30°
Frictional and packing limits	0.58 and 0.61
Turbulence model	Standard k-epsilon, dispersed

## RESULTS

The average and instantaneous solid volume fraction fields from the transient simulation are presented in Figure 1. The simulated case was based on an experimental setup at Åbo Akademi University and it was possible to make qualitative comparison of the results. Overall the simulated flow field has a reasonable resemblance to the experiments, but there is a little bit more channelling in the simulated flow fields.

In Figures 2 and 3 the horizontal profiles of the terms appearing in the vertical (y-axis) and horizontal (x-axis) components of equation (8) are presented. As can be seen from Figure 2, the gas-solid drag term and gas pressure terms are clearly the largest terms in the vertical direction and together they compensate for most of the downward acceleration caused by gravitation. The drag term is relatively smooth through the whole width of the bed,



**Figure 1:** Average and instantaneous solid volume fraction fields.

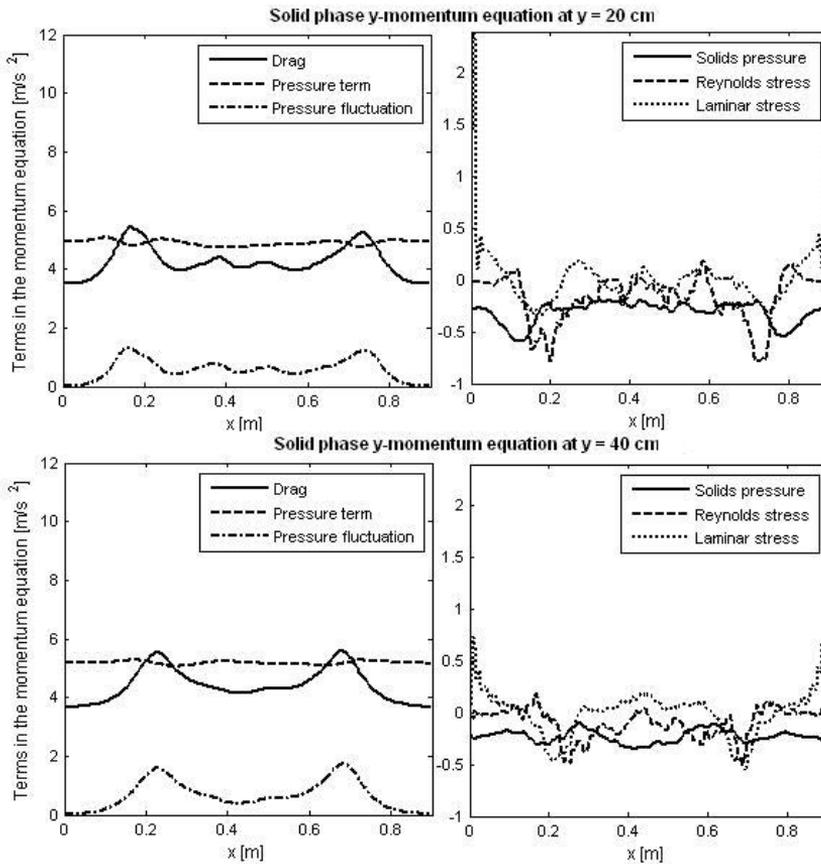
except within the two channels appearing in the flow field. Drag forces are also significant in the horizontal direction, but there they are of the same order as the other major terms.

Next largest term in the vertical direction is the term arising from the correlation between pressure gradient and solids volume fraction fluctuations. This finding supports the earlier, similar conclusions presented in the literature (see eg. De Wilde[14]). The shape of the fluctuation term is very similar to the drag term, but with a clearly smaller magnitude. In horizontal direction this term is small.

The solid pressure term is generally very important in dense flow conditions because it prevents unphysical packing of solid particles. When the particle volume fraction approaches the packing limit this term can grow very large. However, in a fully fluidized bed, as in the present case, the average contribution of this term is relatively small. In industrial scale BFBs there may be larger defluidized regions and within those areas this term can dominate. In the simulated bed there were quite dense regions at the very bottom between the individual air nozzles and there the solid pressure term was large. Also in the horizontal direction this term was significant near the side walls.

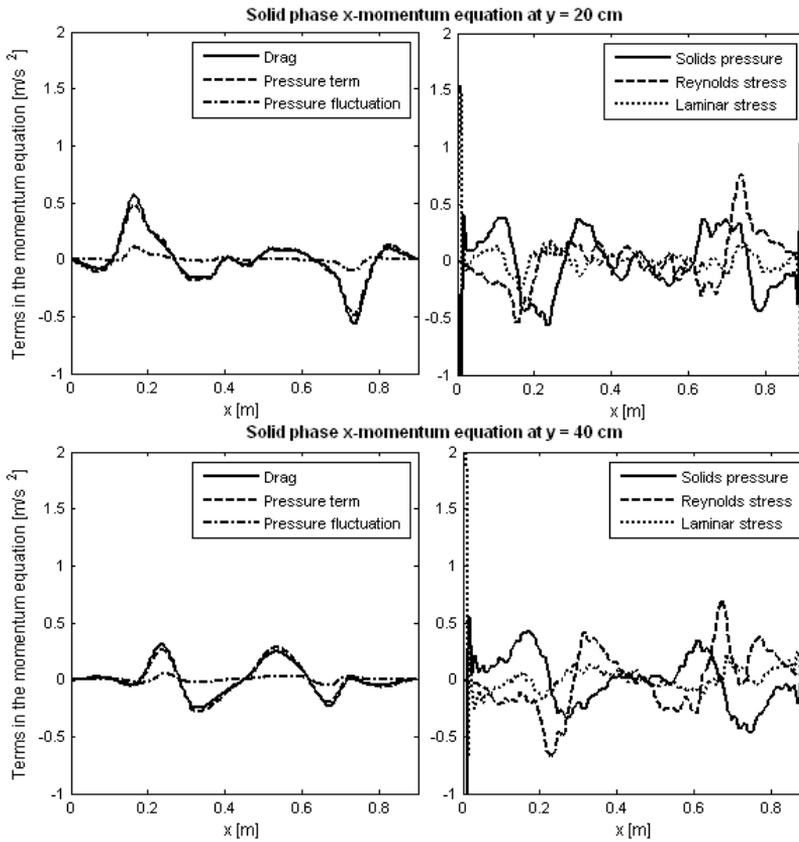
From CFBs it is known that, in addition to the drag force, the Reynolds stress terms are dominating both in the vertical and horizontal directions. [15,16] According to present analysis, the Reynolds stresses have a clearly noticeable contribution also here in the more dense flow. However, in this case its magnitude is more comparable to other terms and is for example clearly smaller than the pressure fluctuation term in the vertical direction. In the horizontal direction the Reynolds stress is also significant, but comparable to other terms.

The laminar stress is of the same order as the Reynolds stress, although it is likely that the laminar stress would be larger than what is shown here if the z-component would be included. Laminar stress is mostly concentrated on the dense side wall regions and near the channels. Again, as with the solid pressure term, near the packing limit the frictional contribution to the viscosity is significant and in those conditions the laminar stress can be a leading term. In CFBs laminar stress was found to be less significant. [15]



**Figure 2:** Horizontal profiles of the different terms appearing in the time-averaged vertical solid phase momentum equation for two different heights. The terms are divided with the average solid bulk density to express them as acceleration. Gravitation term (= const.  $-9.81 \text{ m/s}^2$ ) was left out from the figure.

Turbulent stress was overall insignificant compared to the other terms both in the vertical and horizontal directions. Convection terms were also quite small, but at the bottom of the bed and near the channels it had some impact.



**Figure 3:** Horizontal profiles of the different terms appearing in the time-averaged horizontal solid phase momentum equation for two different heights.

## CONCLUSIONS

Steady state CFD simulation would present an attractive alternative for the computationally demanding transient simulations in the case of large, industrial scale CFBs and BFBs. In the present paper, the features and relative importance of the various terms appearing in steady state flow equations were analyzed based on a transient simulation of a BFB. According to this analysis, the most important terms in the momentum equations are the gas-solid drag term and the solid volume fraction and gas pressure fluctuation correlation terms. The solid pressure and the Reynolds stress terms are also found to be important. Locally also the laminar stress terms that are of minor importance in CFB simulations can become significant in BFBs due to the larger frictional forces. In future more transient simulation data from different cases should be analyzed to gain further insight. Also it would be interesting to perform more thorough comparison of the averaged terms with those from CFB simulations and also with those from turbulent beds.

