

## Erratum: Electromagnetic characterization of planar and bulk metamaterials: A theoretical study [Phys. Rev. B **82**, 165114 (2010)]

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Several errors were found in our paper. In this erratum we provide the corrected expressions for surface susceptibilities and reflection and transmission coefficients. Using the corrected formulas we reproduce the main results of the work and discuss the obtained quantitative differences.

In our paper we claimed that the factors 2 in front of the term  $\chi_{ES}^{zz}$  were missed in formulas (9) and (10) of Ref. 1. It was a wrong statement; actually these expressions are perfectly correct. We would like to apologize for our unjust criticism of the work in Ref. 1. Also, we made two errors [in formulas (5) and (6) of our manuscript]. We are grateful to the authors of Ref. 1, who pointed out some of our errors in their private correspondence. In this erratum we present the corrected expressions for both electric and magnetic surface susceptibilities and for both reflection and transmission coefficients of the metafilm we have derived after a thorough analysis. Notice that our corrected formulas still contain slight differences from those presented in Ref. 1 (i.e., formulas of Ref. 1 in our opinion contain some typos that we corrected below).

$$R_{TE}(\theta) = \frac{-(jk/2 \cos \theta)(\chi_{ES}^{yy} + \chi_{MS}^{zz} \sin^2 \theta - \chi_{MS}^{xx} \cos^2 \theta)}{1 + (jk/2 \cos \theta)(\chi_{ES}^{yy} + \chi_{MS}^{xx} \cos^2 \theta + \chi_{MS}^{zz} \sin^2 \theta) - (k/2)^2 \chi_{MS}^{xx} (\chi_{ES}^{yy} + \chi_{MS}^{zz} \sin^2 \theta)}, \quad (1)$$

$$T_{TE}(\theta) = \frac{1 + (k/2)^2 \chi_{MS}^{xx} (\chi_{ES}^{yy} + \chi_{MS}^{zz} \sin^2 \theta)}{1 + (jk/2 \cos \theta)(\chi_{ES}^{yy} + \chi_{MS}^{xx} \cos^2 \theta + \chi_{MS}^{zz} \sin^2 \theta) - (k/2)^2 \chi_{MS}^{xx} (\chi_{ES}^{yy} + \chi_{MS}^{zz} \sin^2 \theta)}, \quad (2)$$

$$R_{TM}(\theta) = \frac{-(jk/2 \cos \theta)(\chi_{ES}^{xx} \cos^2 \theta - \chi_{ES}^{zz} \sin^2 \theta - \chi_{MS}^{yy})}{1 + (jk/2 \cos \theta)(\chi_{MS}^{yy} + \chi_{ES}^{xx} \cos^2 \theta + \chi_{ES}^{zz} \sin^2 \theta) - (k/2)^2 \chi_{ES}^{xx} (\chi_{MS}^{yy} + \chi_{ES}^{zz} \sin^2 \theta)}, \quad (3)$$

$$T_{TM}(\theta) = \frac{1 + (k/2)^2 \chi_{MS}^{xx} (\chi_{MS}^{yy} + \chi_{ES}^{zz} \sin^2 \theta)}{1 + (jk/2 \cos \theta)(\chi_{MS}^{yy} + \chi_{ES}^{xx} \cos^2 \theta + \chi_{ES}^{zz} \sin^2 \theta) - (k/2)^2 \chi_{ES}^{xx} (\chi_{MS}^{yy} + \chi_{ES}^{zz} \sin^2 \theta)}, \quad (4)$$

$$\chi_{ES}^{xx} = \frac{2j R_{TM}(0) + T_{TM}(0) - 1}{k R_{TM}(0) + T_{TM}(0) + 1}, \quad \chi_{MS}^{yy} = \frac{2j R_{TM}(0) - T_{TM}(0) + 1}{k R_{TM}(0) - T_{TM}(0) - 1}, \quad (5)$$

$$\chi_{MS}^{xx} = \frac{2j R_{TE}(0) - T_{TE}(0) + 1}{k R_{TE}(0) - T_{TE}(0) - 1}, \quad \chi_{ES}^{yy} = \frac{2j R_{TE}(0) + T_{TE}(0) - 1}{k R_{TE}(0) + T_{TE}(0) + 1}, \quad (6)$$

$$\chi_{ES}^{zz} = -\frac{\chi_{MS}^{yy}}{\sin^2 \theta} + \frac{2j \cos \theta T_{TM}(\theta) - R_{TM}(\theta) - 1}{k \sin^2 \theta T_{TM}(\theta) - R_{TM}(\theta) + 1}, \quad \chi_{MS}^{zz} = -\frac{\chi_{ES}^{yy}}{\sin^2 \theta} + \frac{2j \cos \theta T_{TE}(\theta) + R_{TE}(\theta) - 1}{k \sin^2 \theta T_{TE}(\theta) + R_{TE}(\theta) + 1}. \quad (7)$$

Figure 1 shows the comparison between individual polarizabilities of inclusions obtained by the theory from Ref. 2 and those extracted from the reflection and transmission coefficients of an individual grid which are found using the formulas

$$\alpha_{ee}^{xx} = \frac{1}{\frac{1}{a^2 \chi_{ES}^{xx}} + \frac{1}{4sa^3} + \frac{jk^3}{6\pi}}, \quad (8)$$

$$\alpha_{mm}^{yy} = \frac{1}{\frac{1}{a^2 \chi_{MS}^{yy}} + \frac{1}{4sa^3} + \frac{jk^3}{6\pi}} \quad (9)$$

of the theoretical model. After corrections as above the agreement between these two models holds. However, the corrected reflection and transmission coefficients of the grid of effective nanorings shown in Fig. 2 reveal a certain disagreement, which was absent in our manuscript. Namely, sharply at the frequency of the electric resonance, the theoretical model of the metafilm gives an incorrect prediction for  $R$  and  $T$ . This disagreement can be explained by the significant shortening of the effective wavelength at this frequency due to a strong electric response of the medium. Due to this shortening the quasistatic model becomes inaccurate. It is interesting that our incorrect formulas did not give this disagreement, which is the only visible result of the present correction.

There was also a mistake in Eq. (18) of our paper. The corrected formula is

$$Z_w = \eta \frac{\gamma k + q}{\gamma q + k}. \quad (10)$$

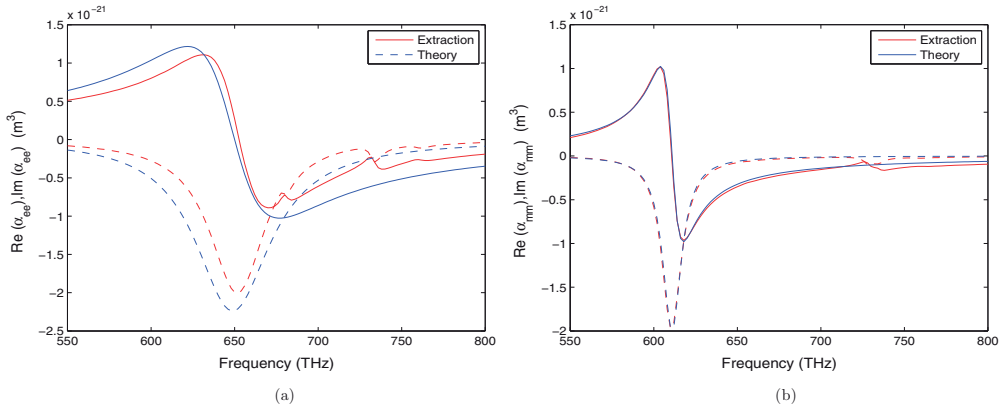


FIG. 1. (Color online) (a) Electric and (b) magnetic polarizability of an effective ring of four plasmonic spheres with  $d = 32$  nm,  $r = 38$  nm. Red curve: extracted from the reflection and transmission coefficients of an individual grid; blue curve: calculated using the theory from Ref. 2. Real and imaginary parts are shown by solid and dashed lines, respectively.

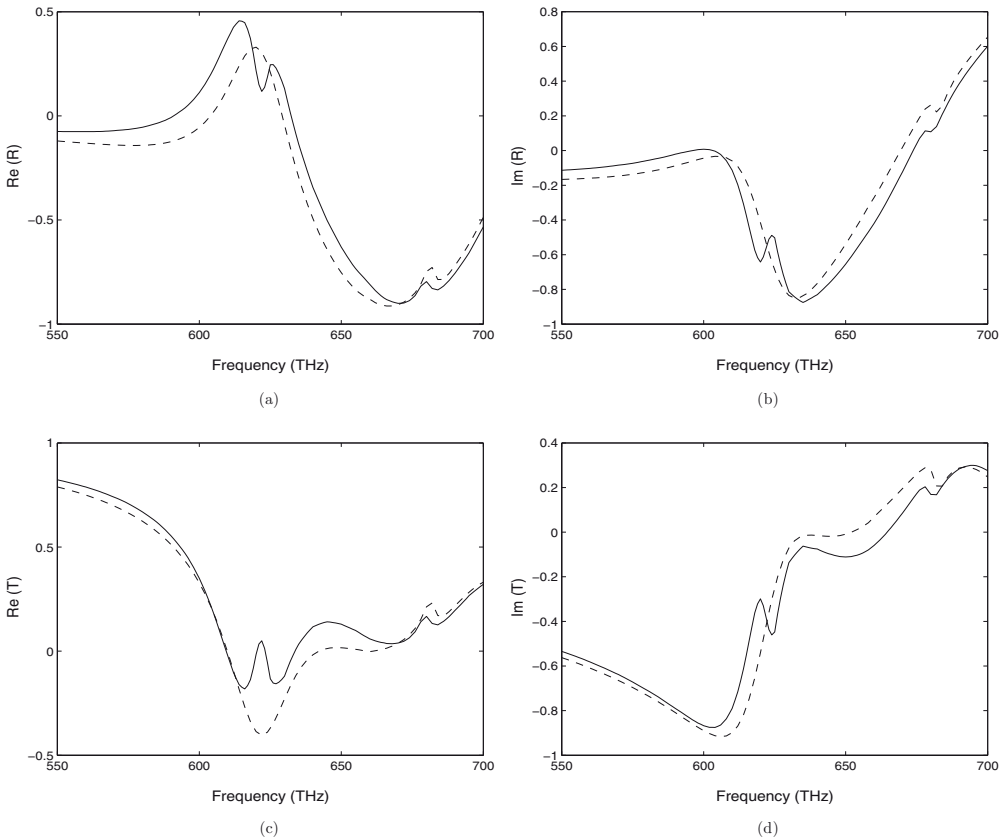


FIG. 2. Reflection and transmission coefficients of the grid of effective nanorings for the TM polarization, angle of incidence is  $30^\circ$ . Solid line: theoretical results calculated by the use of Eqs. (3) and (4); dashed line: results of full-wave simulation in Ansoft HFSS.

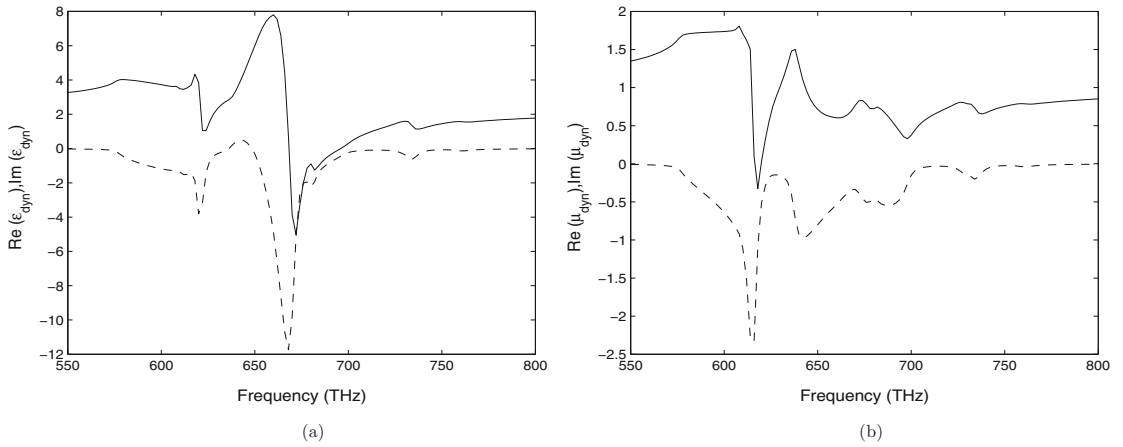


FIG. 3. Effective material parameters of the composite media of effective nanorings of silver spheres with parameters  $d = 32$  nm,  $r = 38$  nm,  $a = 96$  nm. (a) Effective permittivity obtained using the dynamic model. (b) Effective permeability obtained using the dynamic model. Real and imaginary parts of all parameters are depicted by solid and dashed lines, respectively.

In Fig. 3 one can see the dynamic material parameters extracted using the corrected expressions (5) and (10). One may see that the resonances of electric permittivity and magnetic permeability are coupled with one another. The amplitude of magnetic resonance is weaker than estimated in the paper, but still reaches negative values, so the main claim of our paper remains valid.

<sup>1</sup>C. L. Holloway, A. Dienstfrey, E. F. Kuester, J. F. O'Hara, A. K. Azad, and A. J. Taylor, *Metamaterials* **3**, 100 (2009).

<sup>2</sup>A. Alu, A. Salandrino, and N. Engheta, *Opt. Express* **14**, 1557 (2006).

# Corrigendum: Isotropic negative refractive index at near infrared

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In this corrigendum we amend an important omission in references, namely a paper [1] by Paniagua-Domínguez *et al*, which preceded our study on isotropic silver–silicon metamaterials. In this work, the authors proposed before us a similar structure for obtaining negative refractive index at near infrared frequencies. To our regret, we were not aware of this publication when submitting our paper. We came to the similar design independently and only recently became aware of previous work [1]. We fully recognize the priority of [1] concerning the design and regret our oversight, since their paper was published before we submitted ours. However, we also note significant differences between the two papers. We used different methods and came to slightly different design parameters that allow the negative refraction index at higher frequencies than in [1]. We obtained it at 877 nm, whereas in [1] the shortest operation wavelength was 1347 nm. Starting with the same Mie theory used in [1], we then provided its validation by an advanced retrieval procedure developed in the groups of Holloway and Kuester, whereas the authors of [1] used the traditional Nicholson–Ross–Weir method. Further, we made simulations of the refraction of a Gaussian beam on a metamaterial prism. This numerical demonstration of the negative refraction was absent in [1]. The analysis of the beam refraction and absorption in comparison with the predictions of the homogenization model (developed for infinite lattices) is, perhaps, the most important contribution of our work. In this way we have checked the accuracy of the bulk model avoiding the ambiguity which is inherent to the Nicholson–Ross–Weir retrieval procedure. Also, we have addressed the problem of scattering losses for an internally periodic but finite lattice. Therefore, we believe that our study

may be considered as a new development of [1] and not as its marginal extension.

Notice, that the main conclusions of the two papers are in good agreement. It is very important that both groups have shown the adequacy of the homogenization model for a cubic array of particles as large as  $\lambda/4$  ( $\lambda$  is the resonance wavelength) over the whole resonance band and at lower frequencies. Many authors do not believe that such optically sparse metamaterials can be effectively homogeneous.

Additionally, the authors of [1] have informed us of their further study [2] uploaded to arXiv. In this study they have simulated the negative refraction of a wave beam for a 2D analogue of the same Ag–Si metamaterial—a square lattice of core–shell cylinders. Since this 2D problem requires less memory than our 3D problem, the negative refraction in this reduced case can be inspected from the vicinity of the metamaterial prism to its far-field zone. It was done in [2] using the COMSOL software. In our paper we inspected the refracted beam only behind our prism of spheres: the capacities of the HFSS code did not allow us to observe it in the far zone. This shortcoming of our study made it not entirely convincing, and we are happy that the results of the new work [2] confirms our expectations, at least for the 2D variant.

## References

- [1] Paniagua-Domínguez R, Lopez-Tejeira F, Marques R and Sanchez-Gil J A 2011 *New J. Phys.* **13** 123017
- [2] Paniagua-Domínguez R, Abujetas D R and Sanchez-Gil J A 2012 arXiv:1210.8410v1, 29 Oct. 2012