Essays on novel methods and applications of portfolio optimization

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Abstract

This dissertation advances the methods and applications of portfolio optimization and consists of two parts. The first part focuses on developing new portfolio optimization methods using drawdown measures. The second part utilizes existing portfolio optimization methods in novel application areas.

The first method developed in the dissertation is designed to minimize portfolio drawdown duration, an important criterion institutional investors use. The drawdown duration measure has been discussed in several academic and white papers, but no models that allow for minimizing or controlling drawdown duration in a decision support model are present in the literature. Thus, we cover that research and practice gap and present a new family of portfolio selection models: minimizing maximum drawdown duration, average drawdown duration and tail drawdown duration. The model testing reveals that none of the traditional alternative methods achieve drawdown duration levels close to optimal ones. The second new method, drawdown stochastic dominance, allows for portfolio choice based on comparing random drawdown profiles of investments. Portfolio optimization models exist to minimize drawdown as a single value in the objective function with simplifying assumptions, but no methods account for the whole drawdown distribution. Therefore, we introduce a new variant of the drawdown measure, the corresponding optimization method, and the new dominance rule for portfolio selection in the spirit of second-degree stochastic dominance. Both new methods can readily utilize historical data because they are formulated based on scenarios with discrete returns.

The first novel application demonstrates how previously established portfolio optimization models and Monte Carlo simulation methods can be utilized to support portfolio choice in venture capital (VC) markets. The second application study investigates performance enhancements in mixed-asset portfolios (MAPs) by including direct real estate investments (DREIs) in the traditional stock and bond portfolios. Third-degree stochastic dominance (TSD) is applied in the analysis to account for different risk-attitudes. Both DREIs and VC investments may represent an attractive option for investors to diversify risks, but scant studies are available in the literature dedicated to portfolio optimization applications, specifically in these markets.

Keywords  finance, portfolio optimization, drawdown, mixed-integer linear programming, stochastic dominance, decision analysis, venture capital, stochastic optimization, real estate, mixed-asset portfolio

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Preface

It was my pleasure during my work on this dissertation to benefit significantly from collaborating with my colleagues at Aalto University, in the Department of Information and Service Management. I received advanced training, full support, and freedom to realize my research goals here. The multidisciplinary and flexible approach employed at Aalto University nurtures innovation and new ideas and I am happy to be a part of this journey.

First of all, I thank my academic advisors Professor Juuso Liesiö and Professor Tomi Seppälä without whose painstaking efforts I could not have produced this dissertation. Their invaluable comments and feedback continuously encouraged me to achieve higher quality standards in my work.

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Helsinki, June 29, 2023,

Andrei Vedernikov
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This thesis consists of an overview and of the following essays which are referred to in the text by their Roman numerals.


Author’s Contribution

Essay I: “Portfolio Models for Optimizing Drawdown Duration”

Andrei Vedernikov provided the original research idea and is the primary author in the essay. Andrei developed the formal definition of the drawdown duration measure and the first versions of the optimization models. Andrei was responsible for testing of the developed models, comparing the results and analyzing the risk-return trade-off in Python software.

Essay II: “Optimal Portfolio Choice Based on Drawdown Stochastic Dominance”

Andrei Vedernikov provided the original research idea and is the primary author in the essay. Andrei developed a novel variant of the drawdown measure based on current portfolio value, the drawdown dominance criterion for portfolio selection, and the corresponding optimization model. Andrei implemented the model using the MOSEK solver in Python software and conducted the computational tests.

Essay III: “Portfolio Optimization model for Supporting Venture Capital Decision-Making”

Andrei Vedernikov proposed the original research idea and is the primary author in the essay. Andrei conducted the literature review and developed the first formulation of the optimization problem with a conditional value-at-risk as an objective function. Andrei collected the venture capital data and conducted model testing with the Gurobi solver in Python software.
Author's Contribution

Essay IV: “Choice of a Mixed-Asset Portfolio Based on Third-Degree Stochastic Dominance”

Andrei Vedernikov is the sole author of this essay.
Abbreviations

**CVaR**  Conditional value-at-risk

**DREI**  Direct real estate investment

**D-SSD**  Drawdown second-degree stochastic dominance

**ETF**  Exchange-traded fund

**FSD**  First-degree stochastic dominance

**GARCH**  Generalized autoregressive conditional heteroskedasticity

**LP**  Linear programming

**MILP**  Mixed-integer linear programming

**MAP**  Mixed-asset portfolio

**NACE**  Nomenclature of Economic Activities

**RE**  Real estate

**REIT**  Real estate investment trusts

**PERE**  Private equity real estate

**SD**  Stochastic dominance

**SSD**  Second-degree stochastic dominance

**SIC**  Standard Industrial Classification

**TSD**  Third-degree stochastic dominance

**TDD**  Tail drawdown duration

**VaR**  Value-at-risk

**VC**  Venture capital
1. Introduction

Portfolio optimization in finance involves selecting a combination of investments to maximize or satisfy certain objectives. Typically, a trade-off is considered between a portfolio's expected return and the corresponding financial risk to maximize an investor's utility. The utility function measures the relative benefit for an investor based on portfolio performance and depends on individual risk and consumption attitudes that define the cost of risk and the value of additional return for that investor. Several constraints reflecting economic rationales are usually included in portfolio optimization, such as the maximum portfolio weight for a single investment and the maximum portfolio share in a particular region or industry.

Portfolio optimization for financial applications has received wide attention in the academic literature since the early works of Markowitz (1952) and Lintner (1965). Mathematical optimization and robust statistical methods are herewith routinely utilized in this field. For example, Sharpe (1967) stated that a linear programming (LP) specification would greatly improve the prospects of the practical application of portfolio optimization models. Young (1998) later offered a minimax portfolio selection rule with a minimum return taken as a measure of risk. Furthermore, tail-based portfolio optimization has gained popularity, particularly, with conditional value-at-risk (CVaR) by Rockafellar et al. (2000) and value-at-risk (VaR) by Benati and Rizzi (2007). The concept of robustness, adapted from mathematical statistics, was then employed by Lotfi and Zenios (2018) to develop VaR and CVaR optimization under general data ambiguity and by Desmettre et al. (2015) to study optimal strategies for worst-case scenarios with uncertain crashes. Other modern advances have included Li et al. (2010) and Liu and Zhang (2015), who used fuzzy programming to model asymmetric stock returns in a mean-variance-skewness optimization model. Thereafter, genetic algorithms MODE-GL, NSGA2, and SPEA2 likewise found applications for financial portfolio optimization, for example, Lwin et al. (2017).

Much research has also been recently conducted to develop decision sup-
port models for the selection of assets that are not publicly traded. The multiple application areas include R&D, maintenance planning, infrastructure management, and strategy building. For instance, Gutjahr (2011) proposed a portfolio selection model for optimizing investments in human resources projects that considered the evolution of employees’ competencies. Moreover, Kettunen et al. (2009) optimized a contract portfolio of an electricity retailer accounting for uncertain electricity prices and loads, using the CVaR risk measure. Robust portfolio modeling (RPM, Liesiö et al. 2008) supports project portfolio selection with multiple evaluation criteria in the presence of project interdependencies and incomplete preference information. Scenario-based approaches likewise have found use in portfolio optimization, for instance, in ecosystem strategy building (Vilkkumaa et al., 2018).

Nevertheless, difficulties have persisted in applications of portfolio optimization in several domains due to the absence of relevant robust statistical methodology or present market inefficiencies. For example, while existing portfolio optimization models have mainly focused on traditional performance measures (e.g., variance and CVaR), drawdown-based measures, popular among institutional investors, have been long overlooked in the literature. Clients often introduce contract stipulations with a fund manager both in terms of drawdown magnitude, e.g., a maximum of 20%, or drawdown duration, e.g., a maximum of one year (Alexander and Baptista, 2006). Violations of these conditions may lead to warnings and closure of accounts, regardless of the long-term portfolio performance (Chekhlov et al., 2005). However, no decision support models are available to control for specifically drawdown duration in portfolio optimization. In turn, Chekhlov et al. (2005) introduced portfolio choice models based on the drawdown magnitude with several applied simplifying assumptions. In particular, the drawdown in these models is calculated using the uncompounded rate of return, by which rates of return are aggregated over time as an arithmetic sum. This definition helps simplify optimization into a LP problem, but it contradicts some well-established approaches in practice and the academic literature on aggregating rates of returns across time by using log-returns and the log-sum.

Thus, the first part of the dissertation addresses some of these existing research gaps in portfolio theory and introduces two new portfolio optimization methods using drawdown measures. First, a family of methods is developed for portfolio optimization, specifically using the drawdown duration measure, based on a discrete scenario set. Drawdown duration measures the time elapsed since a portfolio obtained its latest maximum value and is one of the important criteria in active portfolio management. Specifically, three model variants are presented in the study: minimizing portfolio maximum drawdown duration, average drawdown duration, and tail drawdown duration. These models can readily utilize historical return
data and also be extended to a stochastic setup. Second, a portfolio optimization method based on the drawdown measure is constructed that accounts for the whole range of the portfolio drawdown distribution (i.e., all possible drawdown scenario realizations). The portfolio drawdown distribution can be calculated based either on empirical historical data or data generated by GARCH or copula models. The stochastic dominance theory can present suitable methodological choices for this case. The principles of second-degree stochastic dominance (SSD; Kuosmanen, 2004; Kopa and Post, 2015) are particularly utilized in this work. For this purpose, a new variant of the drawdown measure is introduced with demonstrated beneficial properties for portfolio optimization, which allows choosing a dominant portfolio based on comparing of random drawdown profiles of two portfolios.

The full potential of advanced portfolio optimization models has not been utilized in analysis and decision support in application areas outside the standard stock markets. For instance, real estate and venture capital (VC) are some domains where decisions on allocating capital must be made under uncertainty and risk. In particular, a primary challenge in applications of portfolio optimization in the VC markets is obtaining historical financial data on VC firms (Cumming et al., 2017). Unlike investments in stock markets, investments in VC markets are not openly traded, so only limited observable market prices are available. Furthermore, numerous market inefficiencies persist. For example, VC investments can not be split among market participants and or recovered if a decision-maker wants to abandon the investment project. Additional difficulties arise from the uncertain return of VC investments that can not be approximated using the normal distribution. Similar data-related issues concern direct real estate investments (DREIs) for which only limited historical data are available and the data are often incomparable across different sources. Additionally, special features of DREIs such as entry/exit barriers and sophisticated fee structure further hinder applications of robust statistical methods in these markets. Therefore, no financial portfolio optimization studies currently exist in the academic literature that include DREIs.

This dissertation’s second part applies previously established portfolio optimization tools in a novel setup for VC and real estate capital markets. The first such application utilizes CVaR-based portfolio optimization to construct and analyze efficient frontiers of VC investments. Herewith, a discrete scenario set of uncertain future VC values is generated in a bootstrapping procedure using historical VC data. In the other novel application, a third-degree stochastic dominance criterion (TSD; Post and Kopa, 2017) is applied to analyze performance enhancements in traditional stocks and bonds mixed-asset portfolios (MAPs) by including direct real estate investments (DREIs). Portfolio optimization based on the TSD criterion takes investors’ risk aversion and skewness preferences into
account and is, hereby, particularly useful for applications with non-normal return distributions that are present in real estate markets.

This dissertation consists of four original essays and this summary. Section 2 discusses the methodology used in the essays while Section 3 overviews the essays and their contributions. Section 4 discusses the practical value and future implications of the presented research. The last Section Essays includes four studies dedicated to portfolio optimization topics.
2. Methodological Background

2.1 Portfolio risk measures

Drawdown measures in portfolio optimization include drawdown magnitude, which determines the portfolio value drop from its preceding maximum value, and drawdown duration, which determines the time period over which the portfolio value is below its preceding maximum value. Drawdown magnitude measure was first introduced in the works of Grossman and Zhou (1993) and Cvitanic and Karatzas (1997) who presented models to support portfolio choice by minimizing portfolio maximum drawdown magnitude. Chekhlov et al. (2005) later proposed additional drawdown functionals, namely, average drawdown and conditional drawdown based on the uncompounded rate of return. The former model is used to find a portfolio with a minimum average of all drawdown values, while the latter is applied to find a portfolio with a minimum average of values in a worst percentile of a drawdown distribution. Zabarankin et al. (2014) utilized the conditional drawdown functional to develop a capital asset pricing model (CAPM) for hedge funds. Schuhmacher and Eling (2011) showed that performance measures based on conditional drawdown could be considered as good as the Sharpe ratio.

For instance, suppose there exists a portfolio $\lambda$ with portfolio values $v_0^\lambda, \ldots, v_T^\lambda$ on the horizon $\{0, \ldots, T\}$. Then, portfolio drawdown magnitude at a time point $t \in \{0, \ldots, T\}$ is usually defined as a relative decrease in portfolio value from the maximum portfolio value up to time point $t$:

$$\delta_t^\lambda = \frac{\max_{0 \leq t' \leq t}(v_{t'}^\lambda) - v_t^\lambda}{\max_{0 \leq t' \leq t}(v_{t'}^\lambda)}.$$  \hspace{1cm} (2.1)

In turn, the drawdown measure variant introduced in the second essay
is calculated as a proportion of the current portfolio value:

\[ \phi_t^\lambda = \frac{\max_{0 \leq t' \leq t} \{ v_{t'}^\lambda \} - v_t^\lambda}{v_t^\lambda}. \]  

(2.2)

Drawdown \( \phi_t^\lambda \) is shown to have advantageous properties for portfolio optimization as demonstrated in the second essay of this dissertation. The drawdown \( \phi_t^\lambda \) is related to drawdown \( \delta_t^\lambda \) simply as \( \phi_t^\lambda = \frac{\delta_t^\lambda}{1-\delta_t^\lambda} \). Another variant of the drawdown measure, introduced by Chekhlov et al. (2005), is defined as

\[ \epsilon_t^\lambda = \max_{0 \leq t' \leq t} \{ v_{t'}^\lambda \} - v_t^\lambda, \]  

(2.3)

which allows for formulating the portfolio optimization model as a LP problem.

Drawdown duration measures a portfolio's continuous time, i.e., has non-zero drawdown magnitude. As defined in the first essay of this dissertation, portfolio drawdown duration at a time point \( t \) is calculated as

\[ d_t^\lambda = t - \max\{\arg\max_{t' \in \{0,...,t\}} v_{t'}^\lambda\}. \]  

(2.4)

Rotundo and Navarra (2007) previously used the drawdown duration measure to analyze speculative bubbles in stock markets. Moreover, Challet (2017) derives a robust Sharpe ratio estimator based on the total duration of drawdowns in a related study.

According to equation (2.4) a portfolio is in a drawdown whenever it is below its previous peak value. As soon as the previous or new portfolio peak value is reached the drawdown duration "resets" to 0. In the alternative formulation, achieving the previous portfolio peak value does not "reset" drawdown duration; only achieving the new portfolio peak value does so:

\[ d_t^\lambda = t - \min\{\arg\max_{t' \in \{0,...,t\}} v_{t'}^\lambda\}. \]  

(2.5)

For example, consider a special case when portfolio value is constant over time and is always equal to 1 in \( t = 20 \) periods. According to the definition in equation (2.4) the drawdown duration is equal to 0 but according to the definition in equation (2.4) the drawdown duration is 20.

The CVaR risk measure (Rockafellar et al., 2000) is used as an objective function in the optimization problems of the third and the fourth essays. The CVaR fulfills the requirements of a coherent risk measure (Delbaen, 2000) and has multiple applications in finance and management science fields, e.g., Zhu and Fukushima (2009) and Kakouris and Rustem (2014). For a random return \( R \), its inverse cumulative probability function \( F^{-1}_R(\alpha) \) and risk level \( \alpha \) (e.g. 5\%), the CVaR is defined as the conditional expected value of the left tail of the distribution (Dentcheva and Ruszczynski, 2006):

\[ \text{CVaR}_\alpha[R] = \mathbb{E}[R | R \leq F^{-1}_R(\alpha)]. \]  

(2.6)
In addition, the semivariance risk measure is used as part of the TSD criteria in the fourth essay (see Section 2.3). The difference of the semivariance measure from the traditional variance measure is that it considers only values below the mean or certain benchmark value of a random return. Thus, semivariance specifically measures the magnitude of a return’s downside fluctuation (i.e., the downside risk).

### 2.2 Optimization methods

The optimization models in the first and the third essays are formulated as mixed-integer linear programming (MILP) problems. Some of the variables in a MILP problem are included as integers, while others are continuous variables. MILP originated in production planning (Bowman, 1956; Hanssmann and Hess, 1960); however, the methods are currently also popular in decision analysis (Griggs et al., 1997) and financial portfolio optimization (Benati and Rizzi, 2007). For example, Benati and Rizzi (2007) replaced variance in the Markovitz model (Markowitz, 1952) with the VaR measure, resulting in a MILP problem. The optimization problems in the first essay include auxiliary binary variables implying a portfolio’s so-called “drawdown status”. Hence, if a portfolio is in drawdown this variable takes a value of 1 while its value is 0 otherwise. At the same time, the binary variables in the third essay represent a decision to invest (value of 1) or not to invest (value of 0) in a particular VC project.

The optimization problems in the second essay involve exponential constraints, including exponential functions (i.e., $a b^{cx}$). In recent studies, optimization problems involving exponential constraints have been applied to multiple areas, including Serrano (2015) and Dahl and Andersen (2021). Finally, quadratic programming is employed to solve optimization problems in the fourth essay, a type of nonlinear programming where mathematical optimization problems include quadratic functions in the objective function and/or constraints (Nocedal and Wright, 2006). Quadratic programming has found multiple applications in several fields, including decision analysis and finance, notably in Markowitz (1952) where portfolio variance is minimized in the objective function.

Furthermore, efficient frontiers are constructed as a part of the analysis in the first, second and third essays. An efficient frontier represents a set of efficient portfolios offering the highest expected return for the lowest or a specific user-defined risk level. Efficient frontiers have gained popularity in finance for the analysis of risk-return trade-off, see, for example, Markowitz (1952); Elton et al. (1978); Bailey and Lopez de Prado (2012). Portfolios lying below the efficient frontier are not optimal because they do not provide an adequate level of return for the corresponding risk level. Thus, the first essay constructs efficient frontiers with a trade-off
between return and drawdown duration for stock portfolios. Moreover, the third essay builds efficient frontiers based on the return-CVaR trade-off for VC portfolios, while the fourth essay constructs the MAP efficient frontiers in combination with TSD criteria where the CVaR and variance are used as risk measures.

Various other methods have been utilized in the essays supporting portfolio optimization. For example, the first essay uses a Big-M approach (Rubin, 1990) for portfolio optimization, where some of the model constraints are "switched on" or "switched off" based on some of the binary variables' values. Variants of the Big-M method have been applied in many fields including operations research and management science (e.g., Bedekar et al. 2009; Baldomero-Naranjo et al. 2020). The third study uses a bootstrapping procedure, demonstrating how utilizing the bootstrapping procedure helps overcome some of the difficulties in applying portfolio optimization methods for VC markets. Finally, the fourth essay utilizes a desmoothing algorithm of real state indexes based on the analysis of autocorrelation (Geltner, 1993) to support portfolio optimization. When appraisal methods are used to construct real estate indexes, the resulting indexes are often smoothed and calculated partially based on previous index values, gradually adjusting to new market conditions. Therefore, the smoothing effect is removed to reflect changes only in the current period.

2.3 Stochastic dominance

The background for this dissertation’s second and fourth essays constitutes multiple studies in the stochastic dominance (SD) domain. SD criteria are a family of analytical tools to support decision-making under uncertainty. The non-parametric SD approach assumes no functional form of the return probability distribution. Therefore, it is particularly useful for applications when the conditions of market efficiency do not hold (i.e., non-normal distributions, asymmetrical dependencies, and fat tails). The portfolio choice based on SD criteria also does not require the specification of a utility function of investors and holds for several assumptions on investors’ risk preferences well-accepted by scholars and practitioners.

In particular, the SSD criterion relies on the premise that investors are risk-averse (Fishburn, 1964; Rothschild and Stiglitz, 1970), which has found multiple applications in stock portfolio optimization including the works of Kuosmanen (2004), Kopa and Post (2015), and Hodder et al. (2015). For example, suppose there exist two investment portfolios \( \lambda \) and \( \tau \) where portfolio \( \tau \) is designated as the benchmark portfolio with their respective random portfolio returns \( R^\lambda \) and \( R^\tau \). Based on the CVaR definition in equation (2.6), the portfolio \( \lambda \) dominates the benchmark portfolio \( \tau \) in terms of SSD criterion if its CVaR is lower than the CVaR of the benchmark for
all risk levels $\alpha$ (Dentcheva and Ruszczyński, 2006), i.e.,

$$\text{CVaR}_\alpha(R^\lambda) \geq \text{CVaR}_\alpha(R^\tau), \quad \forall \alpha \in [0,1]. \quad (2.7)$$

Note that the equation above also implies that the expected return of portfolio $\lambda$ will be at least as high as the expected return of the benchmark $\tau$, i.e., the portfolio expected return will represent the portfolio CVaR at $\alpha = 100\%$. Hence, the second essay utilizes the principles of SSD to create a new method in the SD family: drawdown stochastic dominance.

Later, addressing some drawbacks of portfolio optimization based on SSD, Post and Kopa (2017) developed a portfolio optimization method based on TSD, accounting for investors’ preference for negative return skewness preferences. Hence, portfolio choice based on SSD can lead to investor’s preference for negative return skewness, contradicting the existing theoretical and empirical findings. Menezes et al. (1980) and Ebert and Wiesen (2014) showed that most "prudent" investors seek positive skewness in returns (i.e., a preference for a higher probability of smaller losses compared to a low of probability of very high losses). Moreover, Post and Kopa (2017) demonstrated that applying TSD significantly improves the out-of-sample performance of efficient portfolios compared to mean-variance (MV) or SSD-based portfolio optimization. The fourth essay applies the TSD portfolio optimization to test market efficiency and build efficient frontiers of MAPs consisting of stocks, bonds and DREIs. Revisiting the earlier example above, for portfolios $\lambda$ and $\tau$ with their corresponding random returns $R^\lambda$ and $R^\tau$, a portfolio optimization problem based on the TSD criterion includes the following constraints:

$$P(R^\lambda \leq r)E((r - R^\lambda)^2|R^\lambda \leq r) \leq P(R^\tau \leq r)E((r - R^\tau)^2|R^\tau \leq r) \quad \forall \ r \in \mathbb{R},$$

$$E(R^\lambda) \geq E(R^\tau). \quad (2.8)$$

Thus, portfolio $\lambda$ dominates the benchmark portfolio $\tau$ in terms of TSD if its return second-order lower partial moment (semivariance) is lower than that of the benchmark portfolio (Post and Kopa, 2017; Gotoh and Konno, 2000). At the same time, the expected return of portfolio $\lambda$ is to be at least as high as the expected return of the benchmark portfolio $\tau$. 
3. Contributions of the Dissertation

The first essay introduces the drawdown duration measure and the corresponding family of optimization methods. Drawdown duration measures the time elapsed since a portfolio obtained its maximum value and is an essential criterion in active portfolio management for institutional investors. The essay develops models for minimizing the average, maximum, and tail drawdown duration, formulated as MILP problems. Applying the developed models to stocks' historical data reveals that none of the alternative methods suggested in the existing literature based on the variance, CVaR, or drawdown magnitude, can minimize drawdown duration effectively. In particular, the results indicate that these alternative methods yield drawdown duration levels at least twice the optimal levels achieved by the models developed in this essay. Moreover, the constructed efficient frontiers also show a clear trade-off between minimizing drawdown duration and maximizing expected returns.

The second essay develops a new optimization method for portfolio choice based on comparing the portfolios’ uncertain drawdown profiles: drawdown stochastic dominance. Portfolio drawdown measures the magnitude of the portfolio value drop from its preceding maximum value and is another essential criterion in active portfolio management. First, a new variant of the drawdown measure is introduced based on the current portfolio value, subsequently shown to have beneficial properties for portfolio optimization. Second, an optimization method is constructed to minimize the conditional $\alpha$ tail of a drawdown distribution similar to the CVaR. Third, a novel drawdown dominance criterion, D-SSD, is developed to compare the drawdown distributions of portfolios. In particular, a portfolio dominates another in the sense of D-SSD if the conditional drawdown of the former portfolio is less or equal to that of the latter for any confidence level. This method is denoted as D-SSD in the spirit of the SSD method. In-sample testing indicates that in most periods over a 30-year horizon, it is possible to find a stock portfolio with a significantly higher return and a dominant drawdown profile than those of the U.S. all-stock benchmark. The testing also shows that the excess return of the D-SSD dominant portfolio is highest in
the periods following major recessions.

The third essay develops a model for optimizing a VC portfolio that considers stochastic dependencies among the investment candidates. The proposed model uses a bootstrapping procedure to generate a set of scenarios capturing the uncertain future values of VC investments based on historical data. The resulting discrete state-space is then used in a MILP model identifying the Pareto frontier of portfolios with efficient risk-return characteristics. The CVaR is used as a risk measure, motivated by its advantageous properties for applications in the presence of non-normal return distributions of VC portfolios. The model’s functionality is demonstrated in an empirical setup using historical data on 2,889 VC investments from 2007–2018. The optimization results show that several heuristic solutions failed to provide risk-return characteristics close to those obtained in efficient portfolios.

The fourth essay analyzes improvements in efficient frontiers achieved by including DREIs in the traditional stocks and bonds (stocks-bonds) MAPs. In particular, three asset classes are considered in the essay: 49 stock industry portfolios, ten bond exchange-traded funds, and ten randomly chosen DREIs. The return CVaR-efficient frontiers are constructed where the efficient MAPs are TSD-dominant relative to stocks-only and stocks-bonds benchmark portfolios. The choice of the TSD criterion is motivated by its advantages for applications in the presence of market inefficiencies and non-normal return distributions characteristic in real estate markets. The optimization results show that introducing DREIs helps decrease risks while increasing the return of MAPs compared to stocks-only and stocks-bonds efficient portfolios. For example, the lowest risk TSD-dominant MAP contains \( \sim 61\% \) stocks, \( \sim 20.5\% \) bonds, and \( \sim 18.5\% \) DREIs.

The data on stock returns for the first, second, and partially the fourth article was collected from Kenneth French’s library, including daily returns of 49 industry portfolios and U.S. all-stock benchmark CRSP between January 1990 to December 2020, a historical horizon of 7,800 trading days. The third essay utilizes the VC data from the Bureau Van Dyke (BvD) databases. The BvD Zephyr database is specifically used to collect data on important attributes of VC deals, such as size, deal date, financing subtype, and industry segment. The data on the performance of VC investments (i.e., turnover and net income) is then collected from the BvD Orbis database. The collected VC historical data from Zephyr and Orbis databases cover 2002–2018. The bond data for the fourth essay is also collected from the Thomson Reuters Eikon database, and the DREIs data is collected from the Zillow research database, specifically the Zillow Home Value Index (ZHVI) and Zillow Rent Index (ZRI) series. The ZHVI is a smoothed, seasonally adjusted measure of the typical home value available for different housing types and U.S. regions, while the ZRI is a dollar-valued index representing a typical rent for a given U.S. region and market segment. Both ZHVI and
ZRI data are collected with monthly periodicity.

Optimization problems in all essays are solved in Python software using the CVXPY interface, an open-source Python-embedded modeling language for optimization problems of different types. CVXPY can use both open-source solvers and commercial solvers. The first, third, and fourth essays employ a Gurobi solver, while the second essay uses a MOSEK solver to solve optimization problems involving exponential constraints.

Therefore, this dissertation makes several contributions by developing new methodologies and providing new empirical applications of portfolio optimization methods. Table 3.1 provides a summary of the research objectives and the corresponding contributions of each of the essays.
## Table 3.1. Summary of Research Objectives and Contributions

<table>
<thead>
<tr>
<th>Essay</th>
<th>Research Objectives</th>
<th>Main contributions and findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Portfolio Models for Optimizing Drawdown Duration</td>
<td>To develop an optimization method to minimize portfolio drawdown duration, a measure in active portfolio management</td>
<td>The three model variants are developed to optimize drawdown duration: minimizing the average drawdown duration, the maximum drawdown duration, and the tail drawdown duration. All three models are formulated as MILP problems, solvable by standard commercial and open-source solvers. Empirical testing on historical return data shows that none of the existing alternative models can minimize drawdown duration effectively compared to developed models.</td>
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<td>II. Optimal Portfolio Choice Based on Drawdown Stochastic Dominance</td>
<td>To develop an optimization method for portfolio choice based on comparison of uncertain drawdown profiles of investment portfolios</td>
<td>The drawdown stochastic dominance criterion (D-SSD) is constructed to support portfolio choice in the spirit of SSD. A new variant of the drawdown measure is introduced based on the current portfolio value and an optimization problem involving exponential constraints is formulated utilizing the equivalence of SSD and the CVaR. The model testing shows that it is possible to find a D-SSD dominant portfolio of stocks with significantly higher return than return of the US all-stock benchmark.</td>
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<tr>
<td>III. Portfolio Optimization Model for Supporting Venture Capital Decision-Making</td>
<td>To develop a model to support portfolio optimization of a VC portfolio</td>
<td>An optimization model that accounts for stochastic dependencies is built specifically for VC investments. The discrete scenario set is modeled in the bootstrapping procedure, and the optimization problem is formulated as a MILP problem with CVaR as the objective function. The application of the model using real VC data shows that heuristic approaches fail to provide risk-return characteristics close to those of the efficient portfolios produced by the developed model.</td>
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<tr>
<td>IV. Choice of a Mixed-Asset Portfolio Based on Third-Degree Stochastic Dominance</td>
<td>Analyze if it is possible to improve the performance of MAPs both in terms of risk and return by including DREIs</td>
<td>Introducing DREIs helps decrease risk while increasing the returns of MAPs compared to stocks-only and stocks-bonds TSD-efficient portfolios. In particular, the lowest risk TSD dominant MAP includes 61% of stocks, 20.5% of bonds, and 18.5% of DREIs.</td>
</tr>
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4. Discussions and Future Research

4.1 Conclusion

The multiplicity of challenges in financial portfolio optimization remains relevant, as evidenced in practice and the academic literature. This dissertation explores and addresses some of the existing research gaps through its four essays. The first two essays develop new portfolio optimization methods based on drawdown measures: minimizing drawdown duration, formulated as a MILP, and drawdown stochastic dominance, formulated as an optimization problem with exponential constraints. Both drawdown measures are important criteria institutional investors use in active portfolio management. Thus, the developed methods build on and extend the existing theory in risk analysis and portfolio optimization while readily usable in real-life applications.

In the second part of the dissertation, portfolio optimization is applied in novel domains, specifically for optimizing VC investment portfolios and MAPs, including stocks, bonds, and DREIs. Both VC and DREI markets are known to have significant operating inefficiencies and high entry barriers for market participants. In addition, the observed non-normality of returns, limited data quality, and volumes available in these markets also complicate the application of the portfolio optimization tools. Thus, the essays of the second dissertation part demonstrate how some of these challenges can be addressed using the latest advancements in portfolio theory, bridging the gap between practice and knowledge in these domains.

4.2 Practical relevance

The drawdown magnitude and drawdown duration are important measures in active portfolio management by institutional investors, e.g., mu-
tual funds and insurance companies. A fund manager’s portfolio must often satisfy several criteria regarding the portfolio drawdown and/or portfolio drawdown duration (Chekhlov et al., 2005; Alexander and Baptista, 2006). These criteria may include both maximum acceptable drawdown magnitude (e.g., maximum of a 20% portfolio value drop) and maximum drawdown duration (e.g., no more than one year). The violations of these criteria may lead to warnings and account closures, regardless of the long-term portfolio performance (Chekhlov et al., 2005).

Therefore, the proposed family of methods to minimize drawdown duration can be useful for fund managers who apply them to decrease further portfolio risk and to comply with existing regulation. The models are formulated based on a discrete set of return scenarios and can readily utilize historical data or simulated data generated by a stochastic forecasting model such as GARCH. In turn, portfolio selection models based on drawdown magnitude are already available with certain simplifying assumptions. For example, the conditional drawdown models by Chekhlov et al. (2005) were based on the uncompounded rate of return. They minimized drawdown as a single statistic in the objective function, while Alexander and Baptista (2006) and Klass and Nowicki (2005) demonstrated that constraining a drawdown as a single statistic (i.e. maximum drawdown) in a portfolio optimization problem might lead to suboptimal solutions. The developed drawdown stochastic dominance method uses the compound rates of return and accounts for the whole range of portfolio drawdown distribution. Allowing for choosing a dominant portfolio based on comparing portfolios’ random drawdown profiles.

Applying portfolio optimization models specifically for VC investments has been historically difficult for several reasons, such as the lack of data and special attributes of VC investments (e.g., non-normal return distributions). Thus, the third essay demonstrates how VC investors can address these challenges by applying well-established methods: Monte Carlo simulation with bootstrap and portfolio optimization using the CVaR. The fourth essay is then dedicated to real estate, showing that DREIs can also be an efficient diversifier when added to traditional stocks and bonds MAPs.

4.3 Future research directions

The presented essays open several new, exciting areas for future research. One obvious step is extending the models to include constraints on the drawdown duration in other portfolio optimization problems (e.g., minimum variance and the minimum CVaR). In the current form, the models are built to minimize the drawdown duration in the objective function. Thus, the drawdown duration model can also be applied to analyze ex-
otic options, such as barrier and Asian options, whose values depend on drawdown magnitude and/or duration (Carr et al., 2011; Zhang et al., 2021). Both introduced drawdown duration and drawdown stochastic dominance models can also be combined with other established methods, such as the minimum variance and CVaR, to investigate if further enhancements in risk-return performance can be achieved by using a combination of these measures. Further empirical applications may include out-of-sample testing to analyze potential return improvements compared to existing alternative methods such as SSD or TSD. The VC optimization model presented in the third essay can be readily extended for a more complex setting, including flexible investment size, leveraged investing, and customized exit patterns. Furthermore, private equity and new application areas for portfolio optimization can be explored. Private equity investments carry special characteristics that make applying portfolio optimization challenging. In particular, private equity investments are usually held in the medium-term, between three and five years, linked to decisions concerning the strategy and operations of the acquired companies. Typically, a special type of financing is utilized in private equity investments (i.e., leveraged buyouts), so a private equity firm is organized as a fixed limited partnership. For this reason, private equity investments are not intended to be transferred or traded before an investment maturity, while early funds withdrawals are subject to approval by other partnership members. Moreover, new investments are usually permitted strictly within defined scope, for instance, only in a specific industry, size, and geographic location.


References


References


As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.
— Albert Einstein