Applications of hybrid single-electron turnstiles: To current standards and beyond

Marco Marin Suárez
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Abstract

Hybrid single-electron transistors formed of a normal-metal island and superconducting leads (SINIS) have been used for generating single-electron currents. These single-electron turnstiles (ST) are voltage biased while a periodical voltage modulation is applied to its gate electrode. Accuracy in current generation in this device is limited, however, investigation of the physics behind its operation has allowed applications beyond metrological purposes. For example, they serve as a probe of the concentration of superconducting excitations in its leads.

In this thesis, these applications are used first by probing extraction of superconducting excitations generated in the leads of the device by its bare operation. These quasiparticles (QPs) are unpaired electrons in the superconducting condensate of Cooper-pairs. The extraction is done by voltage biased Josephson junctions which share one lead with the SINIS ST and have other with higher energy gap. A reduction of one order of magnitude in QP density is observed by using the deviation of the generated current as an accurate probe.

Then, an extension of the SINIS ST applications is presented. By driving this device with a signal of frequency $f$, two QPs are created in the leads close to the energy gap $\Delta$ so that a power $2\Delta f$ is generated in total. This enables to envision the development of the SINIS ST into a standard for the unit of power. It is also shown that such power generation is possible even in the absence of net particle current at zero bias. Furthermore, an analysis about the ultimate possible accuracy of power generation in a simplified version of this device is presented in this thesis. It is seen that errors increase with increasing operation frequency, tunnel resistance, temperature and presence of sub-gap states. Additionally, it is shown that detection efficiency of QP energy can be >99% at typical cryogenic temperatures.

Following this, it is shown that by injecting an extra modulation to the source electrode of the transistor, different driving trajectories can be drawn in its stability diagram. With this, a new driving method with twice the frequency applied to the drain-source bias compared to the one applied to the gate is proposed. By doing so, tunneling events occurring against the biased direction are suppressed. These tunneling events lower the current below the expected outcome. Accuracy of current generated by SINIS ST is increased by one order of magnitude using the new driving method. Furthermore, it is shown that a similar driving method can be used for generating single-electron currents at zero-average bias, which had not been investigated until now in SINIS STs.

Keywords single-electron turnstile, quasiparticle extraction, frequency-to-power conversion, bias modulation
This thesis contains a summary of (most of) the scientific research carried out during five years in Pico group led by Prof. Jukka Pekola at the Department of Applied Physics, Aalto University. These works are not only my making but were also done jointly with several co-authors which are duly acknowledged in each of the summarized papers. However, I'm not only indebted to them for my scientific development. Many other people have contributed to it by discussing, chatting and sharing ideas and laughs during these wonderful years. Now that I am away (but not far) from the university, I have come to appreciate much more the times spent in my PhD.

I would like to start by thanking and recognizing the work of my supervisor, Jukka Pekola. He invited me to join his group as a PhD student after giving me the opportunity to prove myself during an internship, turning me into an experimentalist. For this and for opening my eyes to the world of low-temperature and mesoscopic device physics I will be eternally thankful. Jukka was (hope he will continue to be) always open for consideration of new ideas from students. He always showed that to me by never saying discouraging words. Thanks also for saving me from funding-related worries. I will never forget the early mornings greetings and discussions.

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required an extra amount of resilience and I thank him for that.

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I learned that collaboration in science might not come from the start and still be completely crucial for the development of projects. This was tacitly taught to me by Dr. Dmitry Golubev and Dr. Bayan Karimi. Thank you both for providing important insight to the works contained here. Apart from physics, it was always interesting discussing with Dima about almost any topic as he always brought some unknown facts to the table. Thanks also to Bayan for bringing joy, loudness and chocolate, those are crucial ingredients in any laboratory. M.Sc. Tuomas Pyhäranta also contributed greatly by bringing his knowledge on numerical methods. External collaborations were of great importance, specially from Prof. Yuri Pashkin of Lancaster University. Thank you for bringing interesting proposals and believing in me to develop them. I am particularly indebted to Yuri for working with me hand in hand during my first “independent” measurements.

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Espoo, May 9, 2023,

Marco Marín Suárez
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This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.


Author’s contributions

Publication I: “Active Quasiparticle Suppression in a Non-Equilibrium Superconductor”

M.M.-S. performed simulations, analyzed the data, and performed the resistivity measurements. J.T.P. fabricated the devices and performed the rest of the experiments. The calculation of the injected power was done by J.P.P and M.M.-S. The manuscript was written by M.M.-S. with important input from J.T.P. and J.P.P. M.M.-S. and J.T.P. contributed equally to the research.

Publication II: “An electron turnstile for frequency-to-power conversion”

M.M.-S. made part of the fabrication, carried out the measurements, performed simulations and analyzed the data with important input from J.P.P. and D.S.G. Most parts of the devices were fabricated by J.T.P who also prepared the measurement setup. D.S.G. and J.P.P. estimated the heat losses along the system. The idea was conceived by M.M.-S. and J.P.P. The manuscript was prepared by M.M.-S. with important input from J.P.P., J.T.P. and D.S.G.

Publication III: “Ultimate accuracy of frequency to power conversion by single-electron injection”

J.P.P. proposed the problem and together with M.M.-S. conceived the idea of the paper. J.P.P developed the analytical model of both power injection and heat transport with input from M.M.-S. and B.K. M.M.-S. made the numerical calculations on power injection. T.P. developed and made the
numerical calculations on heat transport with important input from J.P.P. and M.M.-S. J.P.P. prepared the manuscript with input from all the other authors. M.M.-S. prepared the supplementary material.

**Publication IV: “Suppression of back-tunnelling events in hybrid single-electron turnstiles by source-drain bias modulation”**

M.M.-S. fabricated the samples with important help from Yu.A.P. and J.T.P. Yu.A.P. and M.M.-S. conceived the main idea. J.T.P. and M.M.-S. prepared the measurement setup. M.M.-S. measured the device, analyzed the data and made the calculations. All the authors discussed the results. M.M.-S. prepared the manuscript with important input from all the other authors.

**Publication V: “Zero-average bias bidirectional single-electron current generation in a hybrid turnstile”**

M.M.-S. fabricated the sample with important help from Yu.A.P. and J.T.P. M.M.-S. conceived the main idea. Yu.A.P. and M.M.-S. proposed the driving protocols. J.T.P. and M.M.-S. prepared the measurement setup. M.M.-S. measured the device and analyzed the data. All the authors discussed the results. Yu.A.P. and J.P.P. supervised the research at all its stages. M.M.-S. prepared the manuscript with important input from all the other authors.
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Abbreviations

**ST** Single-electron turnstile

**SINIS** Superconductor-insulator-normal metal-insulator-superconductor

**NIS** Normal metal-insulator-superconductor

**DOS** Density of states

**QP** Quasiparticle

**FPC** Frequency-to-power conversion

**SI** International system of units

**AC** Alternate current

**DC** Direct current
Symbols

e  Electron’s charge

h  Planck’s constant

$k_B$  Boltzmann’s constant

$\Delta$  Energy gap in superconducting QP DOS

$I$  Current

$E_c$  Charging energy

$R_T$  Normal-state resistance of tunnel junction

$\eta$  Dynes parameter

$n_g$  Normalized charge induced to the island

$A_g$  Amplitude of AC signal applied to gate

$V_b$  Bias voltage

$V_g$  Gate voltage

$f$  Signal frequency

$\delta \epsilon$  Energy cost of tunneling one electron through NIS junction

$\Gamma$  Tunneling rate
1. Introduction

"... *I wish to give an account of some investigations which have led to the conclusion that the carriers of negative electricity are bodies, which *I have called corpuscles, having a mass very much smaller than that of the atom of any known element, and are of the same character from whatever source the negative electricity may be derived.*"

– Joseph John Thomson from his Nobel Prize Award Address, 1906

Manipulation of individual charge quanta, electrons, is nowadays a task that can be done in mesoscopic devices which are easily controllable. Single electronics [1] has allowed control over these particles so that they can be emitted at known rates [2–4] or even counted one by one [5–8]. Among the devices that allow such control is the SINIS single-electron turnstile (ST) which is a special operation regime of a SINIS single-electron transistor. A SINIS single-electron transistor consists of two thin insulating tunnel barriers that separate a metallic nanoscopic island from two superconducting leads, this island is in turn capacitively coupled to a gate electrode. In the ST operation, single-electron currents can be generated [9]. Generation of single-electron currents not only requires ability to emit electrons one-by-one but also to do it at a constant rate several times. The SINIS device can be synchronized with a periodic voltage waveform of frequency \( f \) applied to the gate electrode to produce a current given by \( I = e f \). This means that it is possible to controllably emit one electron every \( 1/f \) seconds, or analogously \( f \) electrons every second. This particular operation regime of this particular device has several applications and is the central topic of this thesis.

The most immediate and most important of these applications is a metrological standard for the ampere. Following the revision of the most used system of units, the SI (système international d’unités, international system of units), in terms of constants of Nature which was enforced in 2019 [10], it became necessary to realize its units. A realization of a unit is an experiment that generates a quantity dimensionally equal to it and that is consistent with its definition [11] as the current emitted by the SINIS ST
is consistent with the definition of the ampere in terms of constants of Nature. In the frame of this revision, such constants are fixed by definition and without uncertainty rendering useless previous definitions in terms of experiments [12]. Therefore, the single-electron transport implementation as achieved in the SINIS ST given by a frequency-to-current relation mediated by $e$ is an elegant approach of realizing the ampere. Accuracies in the currents generated by this method with respect to the expected outcome show promise to improve and meet metrological requirements [11].

Other applications arise from the composition of this device, particularly from the fact that it is made out of two superconducting leads. A superconductor is a material that passes electrical current with no dissipation carried by paired electrons called Cooper-pairs [13]. Typically, these pairs are protected from breaking by an energy barrier known as the energy gap $\Delta$. When they break, quasiparticles (QPs) are formed and the performance of devices depending on superconducting coherence is compromised [14–17]. Creation of QPs in its leads is inherent to the working principle of SINIS STs, however excess of these also have a negative impact on the desired performance. It is this sensitivity to excess of superconducting excitations that led to the use of SINIS STs for sensing the density of them in its leads [18]. In the same sense, the device has also been used for studying the dynamics of events that lead to errors in single-electron transmission [19–21]. Furthermore, it also has been employed as a micro-cooler of its island [9, 22]. These applications and the standard operation of the SINIS device as a ST has been the object of study of previous PhD theses in Pico group [23–25].

In this thesis, the applications of the SINIS ST are exploited and expanded. Although these are mainly directed towards the use of this device as a standard, the intention of this thesis is directed towards explaining the physics that allows these applications and not to a metrological assessment of its performance. The introductory part of the thesis starts with a presentation of a general background in Chapter 2 which introduces concepts, facts and theoretical methods common to Chapters 4–6 and are crucial for its understanding. The phenomenon of superconductivity and some of its properties are presented together with an introduction to the concept of superconducting gap and QPs. Then, the basic concepts and facts of single electronics are shown with some historical outlook. Here, concepts such as gate-induced charge of the island and charging energy $E_c$ are first presented. Also, a mathematical description of the dynamics of single-electron tunneling is discussed. This provides one of the fundamental principles for describing the behavior of the SINIS single-electron transistor, namely the tunneling rate of single electrons between normal-metals and superconductors. The chapter also presents the main dynamics of the SINIS single-electron transistors and a brief literature revision of single-electron current sources other than the SINIS ST. Most importantly,
the mathematical model based on a Markovian master equation [26, 27] used for understanding the results of the five publications constituting this thesis is presented in Chapter 2 together with the numerical methods used to make calculations. Specific techniques or background that are not shared by all the presented publications are not presented here and rather left for the chapter corresponding to that particular publication.

In Chapter 3 all the methods used for carrying out the experimental research presented in this thesis are explained. These include processes for fabricating the measured samples and, specifically, tunnel junctions. The experiments explained in this thesis were made at cryogenic temperatures, hence the working principles of the used fridges and the fridges themselves are described in this chapter. Also, the constitution of the used measurement setup and the electrical wiring are presented. Some experimental measurements and certain processes followed, such as the estimation of device parameters, are outlined.

After this, the thesis is devoted to presenting the results of each of the publications that fundament it. Each of the following chapters contain a brief literature review. First, the results of Pub. I are presented in Chapter 4. In this publication a method for actively suppressing the concentration of QPs by biasing Josephson junctions is inspected [28]. This method is in contrast with approaches used earlier based on passive suppression [16, 17]. A simplified model of the employed Josephson junctions for QP extraction is used to present how this method can achieve QP extraction. In this work, the SINIS ST is employed as a controllable injector of QPs but also as a sensor of its density in the leads and acts as an appropriate evaluation tool of the extraction method. This is done by inspecting the generated current and comparing it with theoretical calculations. To enrich the analysis, a heat transport model describing the resulting density of excitations in the superconductors is discussed allowing to better understand the mechanisms behind QP relaxation. From the results of Pub. I it is concluded that the density of excitations can be lowered by one order of magnitude and that SINIS STs could be used for sensing non-coherent transport in Josephson junctions.

Chapter 5 presents a totally new application of the SINIS ST. Given its novelty, this chapter is considered as the central part of the thesis. This application consists in a realization of the watt, the SI unit of power, which works via single QP injection to the superconducting leads of the SINIS device. Hence, a brief introduction to the revision of the SI is done, also some recent developments of power standards are presented. Very similarly to single-electron current generation, the power produced in this new application should be $\sim 2\Delta f$ in what we call frequency-to-power conversion (FPC). This chapter describes the findings of Pubs. II and III. Inspired by the phenomenological description of FPC by QP injection, Pub. II shows a proof-of-concept demonstration of this new metrological
application in a SINIS ST and its features and operation regimes are described. It is seen that this application behaves much like the current emitted by the device with the important difference that it can also work at zero voltage bias, in other words in the absence of net particle current. Importantly, measurement deviations from the expected power outcome are observed in this first demonstration which lead to the development of Pub III. In this publication, the errors incurred in energy injection during the operation of a driven NIS junction [29] are analysed and simple mathematical expressions describing deviations from $2\Delta f$ are given. The main message from Pub. III is the fact that errors in power injection increase with the $2/3$rd power of the product between the resistance of the tunnel junction and the driving frequency. Additionally, errors also increase linearly with temperature. Another important contribution of this publication is the justification of the geometry used in Pub. II for measuring QP energies and its contribution to errors in FPC measurements which are less than 1% below $\sim 120$ mK. These are estimated with aid of a heat transport model, which is described in the chapter. Some auxiliary but important concepts are also presented, such as the use of SINIS structures as thermometers [30, 31].

Chapter 6 presents another innovation in the operation of the SINIS ST. This is, the introduction of an extra modulation to the source electrode of the transistor. The results of implementing this are the topic of Pubs. IV and V. First, it is presented how such extra modulation can be used to create new driving trajectories in the stability diagram of the SINIS transistor. Then, the main results of Pub. IV are presented; these are achieved by setting a frequency twice as large as that injected to the gate electrode to the source modulation. Remarkably, this new approach for driving a SINIS ST suppresses tunneling events that occur against the bias direction and hence reduce the generated current below $e_f$. This is a change of perspective with respect to what has been done so far for suppressing error events in this device [3]. To contextualize this, a brief presentation of typical error events and respective suppression methods is done in this chapter. After this, a more detailed description of the causes of tunneling events against the bias is done. Then, evidence of suppression of such errors is presented. Thanks to the used Markovian method it is clearly shown that the measured results are a product of suppression of tunneling against the bias. The results of Pub. V are presented towards the end of the chapter. There, it is shown that current can be generated with zero-average bias voltage with aid of a source modulation. This single-electron current can be flipped by just a phase shift, much like was done in the first metallic single-electron pumps [32].

Chapter 7 gathers the conclusions of Pubs. I–V and of this thesis and finalizes with a personal reflection about the interaction of the author with the research.
2. Background

The objective of the present chapter is to give an introduction to some central topics. First, the concept of superconductivity is introduced along with the Bardeen, Cooper and Schrieffer theory and how the spectrum of superconducting QPs arises from it. In Section 2.2 some historical development of single electronics is given but most importantly the experimental conditions for feasible condensed matter observations are given. Sections 2.3 and 2.4 present the SINIS single-electron transistor and some of its most important characteristics in DC regime which also have an impact on the results presented in this thesis. Furthermore a mathematical description of the device dynamics is presented. The SINIS ST and its characteristics are introduced in Section 2.5 along with a brief literature overview of metrological single-electron sources and finally a mathematical model of the system.

2.1 Superconductivity

A superconductor is a material through which current can flow without resistance and hence with no energy dissipation, furthermore it is a perfect diamagnetic material and no magnetic field can penetrate its bulk [13]. Already in 1911 Heike Kamerligh Onnes observed the first effect by noticing that the resistance of mercury vanished below certain temperature [33]. Later, in 1933, Fritz Walther Meissner and Robert Ochsenfeld observed the second phenomenon [34]. Attempts to understand these phenomena and describe them in terms of condensed matter constituents were first made by the London brothers [35], and Ginzburg and Landau [36] with limited but important success. In 1957 Bardeen, Cooper and Schrieffer (BCS), and Bogolyubov separately proposed what is considered, until today, the most successful theoretical description of conventional superconductivity [37, 38]. Under these descriptions, it is understood that a superconductor has a quantum ground state different from that at room temperature. For this to happen, the material goes through a phase transition when it
reaches a critical temperature $T_c$. When this occurs a symmetry in the material is broken, just like spatial symmetry is broken in a liquid-solid phase transition. Such a symmetry refers to the electronic quantum phase, which is usually random and isomorphic to the $U(1)$ group of symmetry. In a superconductor this quantity assumes a precise value within a characteristic distance, therefore breaking the symmetry and giving rise to collective quantum coherence. It is this symmetry breaking that enables current to flow without dissipation. Such flow is the state with the lowest energy that supports long range quantum coherence. Since this is the ground state, it is at equilibrium and hence entropy is constant, thus no energy is dissipated [39]. BCS theory describes the charge carriers in a superconductor as pairs of electrons, known as Cooper pairs, bounded by a phonon field. From this theory many facts can be deduced. In the following, some important remarks and results of the BCS theory are described.

2.1.1 BCS theory

Cooper pairs are the core of the BCS theory [37]. In 1956 Leon Cooper formulated [40] that a pair of electrons interacting attractively, for example through screening effects in a lattice carried by phonons, as is the case of classical superconductors [41, 42], forms a bound state against which the Fermi sea is unstable. Bound states will keep forming until the condensate product of this is at equilibrium. Such a condensate might undergo the desired phase transition and adopt a common coherent phase, with the already mentioned consequences. However, BCS theory is more a model about the excitations from the superconducting ground state than about this latter configuration. In order to achieve such a description, the following mean-field Hamiltonian is proposed

$$H_{BCS} = \sum_{k\sigma} \xi_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_k \Delta_k c_{-k\downarrow}^\dagger c_{k\uparrow} + \Delta_k^* c_{k\uparrow}^\dagger c_{-k\downarrow} - \Delta_k \langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \rangle. \tag{2.1}$$

Where the $c_{k\sigma}$ are Fermion operators that annihilate states with wave vector $k$ and spin $\sigma$, $c_{-k\sigma}$ annihilates a hole with wave vector $k$; furthermore $\uparrow (\downarrow)$ refers to spin up (down). The number $\Delta_k$ quantifies the pair coupling strength and is closely related to the phase.

The diagonalization of $H_{BCS}$ can be achieved by performing a Bogolyubov transformation [38, 43] as

$$\gamma_{k\uparrow} = u_k c_{k\uparrow}^\dagger + v_k c_{-k\downarrow}$$

$$\gamma_{-k\downarrow} = u_k c_{-k\downarrow} + v_k c_{k\uparrow}. \tag{2.2}$$

The new operators are known as Bogolyubov operators and satisfy the Fermi statistics. In Eqs. (2.2) $v$ and $u$ are known as coherence factors. Furthermore, notice that these Bogolyubov operators have an electron and hole component, hence they cannot be referred to as either of these
Figure 2.1. Results of the BCS theory. (a) Energy spectrum of superconducting excitations corresponding to Eq. (2.4) around the Fermi sphere, that is, \(|k| \sim k_F\). Notice that the minimum energy for excitations is \(\Delta\). (b) Density of quasiparticle states from Eq. (2.5), no states exist for \(E < \Delta\) corresponding to panel (a).

except for special cases. Instead, the excitations created (or annihilated) by the operator \(\gamma^\dagger_k \gamma_k\) are known as superconducting quasiparticles, Bogolyubov quasiparticles or simply quasiparticles (QPs). Even if the latter is a broad term that extends to many other fields of condensed matter physics, we stick to it for the remaining of this thesis.

With Eqs. (2.2), Eq. (2.1) turns into

\[
H_{\text{BCS}} = \sum_{k\sigma} E_k \gamma^\dagger_k \gamma_k + E_{\text{cond}},
\]

where \(E_{\text{cond}}\) represents the energy of the unperturbed condensate and \(E_k\) is the energy of the QP with wave number \(k\) so that

\[
E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}.
\]

It turns out that the quantity \(\Delta_k\) is isotropic and positive so that its modulus is constant and can be denoted simply by \(\Delta\) in Eq. (2.4), whose value depends on temperature. The spectrum of Eq. (2.4) is plotted in Fig. 2.1(a) for QPs near the Fermi sphere. The DOS corresponding to this energy spectrum is

\[
N_S(E) = \frac{N(0)}{\sqrt{E^2 - \Delta^2}},
\]

where \(N(0)\) is the DOS at the Fermi level in the normal state. Equation (2.5) is valid for \(E > \Delta\) and \(N_S(E) = 0\) for \(E \leq \Delta\).

As can be seen from the QP spectrum (Eq. (2.4)) depicted in Fig 2.1(a), there is a minimum energy \(\Delta\) at which QPs exist. In other words, in order to create an excitation, a minimum energy needs to be invested. This is best understood by plotting the DOS (Eq. 2.5) as done in Fig. 2.1(b), notice how there is a discontinuity around \(\Delta\) and that no QP states exist below this energy while just above it the number of available states tends to infinity. For this reason, \(\Delta\) is called the superconducting energy gap or, in short, the gap. This gap is of utmost importance for the present thesis since it is one of the enabling conditions for the proper functioning of the devices investigated here. In particular, this quantity is crucial for the results of Pubs. II and III.
2.2 Single-electron tunneling

The discrete nature of the electrical charge was established first by J. J. Thomson [44] and its charge first measured by Robert Millikan in 1910 [45]. Here, Millikan estimated an electron charge of $\sim 1.631 \times 10^{-19} \text{C}$, with only a remarkable 1.83% deviation from the now fixed value of $e \approx 1.602 \times 10^{-19} \text{C}$ [10, 12]. This experiment is the first documented single-electron transfer although in an uncontrolled fashion. It was not until the late 1980’s that advances in fabrication allowed to carry out solid-state experiments in which single elementary charges could be manipulated [46, 47]. However, the mechanisms behind these phenomena and the conditions for experimental observation were already described before that [26, 48–52]. Underlying this description is the Coulomb interaction of single electrons in mesoscopic metallic elements, henceforth referred to as island. This interaction is accurately quantified by the charging energy [47, 53]

$$E_c = \frac{e^2}{2C}. \quad (2.6)$$

Where $C$ is the capacitance of the island. Eq. (2.6) quantifies the energy required for adding or removing an electron from the island. In order to observe single-electron events the charging energy should set the energy scale of the system. The characteristics are then restricted by this condition. First, the charging energy must exceed the thermal energy available for electrons, that is, $E_c \gg k_B T$ where $T$ is the temperature of the electron system. Therefore, it is required the island to be small (so that $C$ is small) and also the system surroundings to be at low temperature, for example in this thesis all the measurements are done below 300 mK. Second, it is necessary that single-electron states can exist during a sufficiently long time. For this, it is necessary to have an electron reservoir weakly coupled to the island such that the resistance joining them holds $R_T \gg R_K$ where $R_K = h^2/e \approx 26 \text{k}\Omega$ is the resistance “quantum” [3]. This is usually achieved by separating a nanometric island from a large metallic lead by a thin insulating barrier, what is commonly known as a tunnel junction, which also provides an additional capacitance to the system adding the necessity of having a small contact area [27, 47]. In these systems single-electron tunneling is a quantum effect in the sense that transfer of electrons occurs via quantum mechanical tunneling.

2.2.1 Single-electron transistor

To see how the condition of high charging energy allows for single-electron tunneling we focus on how this property can suppress current flow through the island. This suppression of the current is commonly known as the Coulomb blockade and constitutes a fundamental concept for single-electron
transport [27, 47, 54] and its further applications. The origin of the blockade can be easily explained by simple energy considerations. For this, consider two equal tunnel junctions connected in series so that the total resistance is $R_T$, with a small island in the middle which is capacitively coupled, via a capacitance $C_g$, to a gate electrode, see Fig. 2.2(a). This device is known as the normal metal-insulator-normal metal-insulator-normal metal (NININ) single-electron transistor [54]. Here, the device is symmetrically biased with total voltage $V_b$ between the leads and temperature is assumed sufficiently low. A voltage $V_g$ applied to the gate induces a charge $en_g = C_g V_g$ to the island. In order to have a current with one electron flowing through the system the particle should have some residual kinetic energy after tunneling through both junctions which is possible only if

$$e |V_b| - \left[ \varepsilon_+ (0) - \varepsilon_- (1) \right] > 0.$$  \hspace{1cm} (2.7)

Here, $\varepsilon_+ (0)$ and $\varepsilon_- (1)$ are the island chemical potential when a single electron tunnels into the island and it is discharged and when it tunnels out of the island when it is charged with one electron, respectively. This quantity is given by

$$\varepsilon_\pm (n) = E_c (n + 1 - n_g)^2 - E_c (n - n_g)^2 = \pm 2E_c (n - n_g \pm 0.5),$$  \hspace{1cm} (2.8)

where the plus (minus) sign refers to events adding (subtracting) a single electron to the island and $n$ is the initial number of electrons (charge state) in the island.

Then, from Eq. (2.7) we can see that

$$e |V_b| - 2E_c (-n_g + 0.5) - 2E_c (1 - n_g - 0.5) = e |V_b| + 4E_c n_g - 2E_c > 0. \hspace{1cm} (2.9)$$

Notice that when $n_g = 0$ the condition for current flow transforms into $e |V_b| > 2E_c$, therefore in the interval $e |V_b| \leq 2E_c$ the current is suppressed.
Background

and the Coulomb blockade arises. On the other hand, when \( n_g = 0.5 \) then \( e |V_b| > 0 \) and current flows at every bias voltage value. Intermediate values of \( n_g \) suppress current for biases below values between zero and \( 2E_c/e \). Figure 2.2(b) shows the calculated current \( (I) \) of the considered system against bias voltage for several values of \( n_g \). Notice the different intervals within which current is suppressed. Within the blockaded region in the gate closed state \( (n_g = 0) \) conduction can be tuned from Coulomb blockade, that is, ideally zero current, and current carried by single-electron tunneling by varying \( V_g \). In this case, current is carried by electrons that tunnel one-by-one into the island and then are discharged in a similar way. This ability to tune the conductance of the island with the gate voltage is known as Coulomb oscillation which is periodic in \( V_g \), this period is one elementary charge, that is \( e/C_g \).

Here, the idea that a charging energy dominant over other available energies enables single-electron tunneling has been introduced. This gives way to the idea of Coulomb blockade, a basic idea in the development of this dissertation, and also to the notion of Coulomb oscillations, which underlies all the results shown in Publications I to V. Furthermore, we have presented a basic system, the NININ single-electron transistor (N stands for normal-metal and I for insulator). This type of devices were widely used in the early days of single-electron experiments as single-electron current sources, such as the so-called turnstiles [55] and pumps [32]. Both devices consist of arrays of metallic islands.

2.2.2 Tunneling rates

The conditions enabling single-electron tunneling between two metals through a thin insulating barrier have been shown. Since tunneling is a quantum mechanical process it is valid to ask for the rates at which this process happens. For guidance, a general picture is presented first.

A system of two metals joined by a tunnel junction can be modelled through the Hamiltonian [56, 57].

\[
H = \sum_{k\sigma} \xi_{k\sigma} b_{k\sigma}^\dagger b_{k\sigma} + \sum_{q\sigma} \varepsilon_{q\sigma} c_{q\sigma}^\dagger c_{q\sigma} + H_T. \tag{2.10}
\]

Where, \( b \) and \( c \) are the annihilation operators of both metals, respectively; \( \xi_{k\sigma} \) and \( \varepsilon_{q\sigma} \) are the energies of the states with wave number \( k \) and \( q \) and spin \( \sigma \), respectively. Additionally

\[
H_T = \sum_{kq\sigma} T_{qk} b_{k\sigma}^\dagger c_{q\sigma} + \text{h.c.}, \tag{2.11}
\]

is the tunneling Hamiltonian representing the creation of an excited state in one metal while annihilating one in the other and its reverse process. This Hamiltonian includes the amplitude \( T_{qk} \) of tunneling from state \( k \) to
state \( q \), which contains information about the barrier transparency, given in terms of \( R_T \), and the DOS of each metal. To consider Eq. (2.11) as a perturbation to the Hamiltonian in Eq. (2.10) we introduce the condition \( R_T \gg R_K \) once again. This ensures that we can apply Fermi’s Golden Rule to obtain transition rates from a global state \(|i⟩\) to \(|f⟩\). In \(|i⟩\) there is an occupied state in one metal and an empty one in the other. Conversely, in \(|f⟩\) the previously occupied state in the first metal is now empty and the unoccupied state in the other metal has been filled. Hence

\[
\Gamma_{i→f} = \frac{2\pi}{\hbar} |⟨f|H_T|i⟩|^2 \delta (E_i - E_f).
\] (2.12)

Where, \( E_i \) is the energy of the initial state and correspondingly for \( E_f \) and the final state.

Tunneling can, \textit{a priori}, occur between any pair of states. The observable rate is the total one, and hence Eq. (2.12) needs to be summed over every pair of states \( i \) and \( f \). In the case of a junction biased at voltage \( V \)

\[
\Gamma (eV) = \sum_{i,f} \Gamma_{i→f} = \frac{2\pi}{\hbar} \sum_{i,f} |⟨f|H_T|i⟩|^2 \delta (E_i + eV - E_f).
\] (2.13)

After some arduous but edifying algebra and collecting the resulting constant terms into a phenomenological transmission coefficient which turns out to be equal to \( R_T \), we get

\[
\Gamma (eV) = \frac{1}{e^2 R_T} \int dE \, n_1 (E) \, n_2 (E - eV) \, f_1 (E) \, [1 - f_2 (E - eV)].
\] (2.14)

Here, \( n_l \) is the DOS normalized to the value at the Fermi energy in the metal \( l \) and \( f_l \) is the Fermi-Dirac distribution function given by

\[
f_l (E) = \frac{1}{1 + e^{E/k_B T_l}},
\] (2.15)

here \( T_l \) is the temperature of metal \( l \).

### 2.3 The SINIS single-electron transistor

Ubiquitous to Publications I to V is the SINIS single-electron transistor. In this device, the leads of the one described in Subsection 2.2.1 are replaced by superconductors. Apart from the applications in this thesis, this geometry has been used in the past for electronic cooling [22, 58, 59] and thermometry [31] as detailed in Chapter 5. To have single-electron tunneling in normal metal-insulator-superconductor (NIS) junctions, one needs to create an excitation in the superconductor whose nature depends on its energy [60]. As we already saw in Subsection 2.1.1, an energy at least equal to the superconducting gap \( \Delta \) has to be supplied for this to happen. Hence, the condition of Eq. (2.9) transforms into

\[
e |V_b| + 4E_c n_g - 2E_c > 2\Delta,
\] (2.16)
since we need to create two quasiparticles, one in each lead. As a result, the blockaded regime extends, in fact, when \( n_g = 0 \), Eq. (2.16) becomes \( e|V_b| > 2 \Delta + 2E_c \). Perhaps more interestingly, for \( n_g = 0.5 \) we get \( e|V_b| > 2 \Delta \), therefore the SINIS transistor shows a non-negligible interval of \( V_b \) within which current is suppressed for all gate voltages. Typical current-voltage characteristics are depicted in Fig. 2.3(a), the blue vertical lines represent data from a real device, which are taken by fixing a voltage bias and sweeping the gate voltage through several periods. Notice how a wide region with negligible current appears, this is more obvious in panel (b) where a colormap of the same data is presented as function of normalized bias and gate voltages. There, the blue region indicates a zone where current is negligible and the charge state of the island is ideally fixed, therefore we call it a stability region. This region is continuous along \( n_g \), hence independent of \( V_g \). The stability region is composed of many individual diamond-shaped zones, called Coulomb diamonds, which overlap. The overlap is clearly shown in Fig. 2.3(c) with a more schematic depiction of the stability diagram. Importantly, these diamonds are clearly delimited by lines given by

\[
\Delta = \varepsilon(n) \pm eV_{b,L/R} \equiv \delta\varepsilon_{L/R}^\pm,
\]

which physically describe the energy thresholds for single-electron tunneling events between the left (L) or right (R) lead and the island. Here, \( \varepsilon(n) \pm \) are given in Eq. (2.8), and for a device biased from left to right \( V_{b,L} = \kappa_L V_b \) and \( V_{b,R} = -\kappa_R V_b \). Each diamond is bounded by four lines given by Eq. (2.17) with a common value of \( n \), the initial island charge state, suggesting that it is possible to ascribe a charge state to each diamond

**Figure 2.3.** Characteristics of a SINIS single-electron transistor. (a) Current-voltage characteristics of the SINIS transistor, notice how current is negligible for \(|eV_b| < 2\Delta\), while between \( 2\Delta \) and \( 2\Delta + 2E_c \) it is so only for certain gate voltages. Data from experiments similar to those conducted in Pub. II are shown in blue. The vertical position of each dot is determined by \( n_g \). Red lines are simulated maximum and minimum current by using the Markovian model and the methods outlined in Section 2.4. (b) Colormap of data from panel (a) showing the current dependence on gate-induced charge. Notice the continuous blue zone where a negligible current flows. (c) Schematic depiction of the stability diagram of panel (b) revealing its diamond-like distribution. Only two diamonds are shown, for zero and one electrons in the island. The diamonds are bounded by the tunneling thresholds described by Eq. (2.17). Blue (red) corresponds to right (left) junction and solid (dashed) lines correspond to \(+(-)\) events.
as done in Fig. 2.3(c) for the $n = 0, 1$ states. Therefore, it is reasonable to assume that as long as the applied gate and bias voltages are kept within one diamond the number of electrons in the island is $n$. This is a central feature for the use of this device as an electron turnstile. Clearly, $n$ cannot quantify the true number of electrons in the island but a fixed excess with respect to a constant charge background.

### 2.3.1 Tunneling rates in NIS junctions

The focus here is on the single-electron tunneling rates through NIS junctions. First notice that in the low potential difference limit ($eV \ll E_F$) the normal-metal DOS is constant. On the other hand, the DOS of the superconductor is given by Eq. (2.5), since we consider tunneling of single-electrons, that is, QP creation. Finally, we can consider $\delta \epsilon_{L/R}^\pm$ from Eq. (2.17) as the energy provided to the tunneling electron. In the context of SINIS transistors, tunneling changes the island charge by $\pm 1$, therefore the tunneling rate through a NIS junction in this context can be expressed as

$$\Gamma_{n \to n \pm 1}^{L/R} \left( \delta \epsilon_{L/R}^\pm \right) = \frac{1}{e^2 R_{L/R}} \int dE \, n_S \left( E - \delta \epsilon_{L/R}^\pm \right) f_N \left( E \right) \left[ 1 - f_S \left( E - \delta \epsilon_{L/R}^\pm \right) \right].$$

(2.18)

Here $n_S$ is the BCS DOS normalized to its value at the Fermi level, and $S$ and $N$ refer to superconductor and normal-metal, respectively. This expression is sufficiently general to contain both $N \to S$ and $S \to N$ tunneling processes, which are translated as $n \to n - 1$ and $n \to n + 1$, respectively. Additionally, Eq. (2.18) takes into account through which junction of the transistor the tunneling event is taking place, either left (L) or right (R).

Besides single-electron tunneling beyond the thresholds (Eq. (2.17)), two important deviations from ideal BCS tunneling appear in the SINIS transistor. In this section we only mention them and later, in Chapter 6 when the results of Pubs. IV and V are introduced, we will discuss these more in-depth in the context of single-electron turnstiles. First, electrons can tunnel even if $\delta \epsilon_{L/R}^\pm < \Delta$, this is known as leakage. In order to model these processes, the BCS DOS is replaced by one representing formally a non-vanishing presence of states within the gap as

$$n_S^D \left( E \right) = \Re \left[ \frac{E/\Delta + i \eta}{\sqrt{(E/\Delta + i \eta)^2 - 1}} \right],$$

(2.19)

with an extra phenomenological parameter $\eta$ called the Dynes parameter [61, 62]. The bigger the $\eta$, the larger the sub-gap leak. Physical origins of this leakage are further discussed later in this chapter and in Chapter 6.
The second relevant process is the tunneling of two electrons to or from the island via Andreev reflection [13, 63], this is, the reflection of an electron in a NS interface as a hole or vice-versa and the transmission of a Cooper pair. These transmission events in NIS junctions occur with a rate [20]

\[
\Gamma_{n \rightarrow n \pm 2} \left( \delta \epsilon_{L/R}^{\pm 2} \right) = \frac{\hbar \Delta^2}{16 \pi e^2 R_{L/R}^2 N} \int dE f_N \left( E + \delta \epsilon_{L/R}^{\pm 2}/2 \right) f_N \left( -E + \delta \epsilon_{L/R}^{\pm 2}/2 \right) \times \left| a \left( E + E_c - i \xi/2 \right) + a \left( -E + E_c - i \xi/2 \right) \right|^2.
\]

Where, \( N \) is the number of conducting channels in the junction, \( \xi \) is the energy of the intermediate state, which has a finite lifetime, furthermore

\[
a \left( x \right) = \frac{1}{\sqrt{x^2 - \Delta^2}} \ln \left( \frac{\Delta - x + \sqrt{x^2 - \Delta^2}}{\Delta - x - \sqrt{x^2 - \Delta^2}} \right).
\]

The energy cost of two electron tunneling is

\[
\delta \epsilon_{L/R}^{\pm 2} = \pm 4E_c \left( n - n_g \pm 1 \right) \pm 2eV_{b,L/R}.
\]
state at time \( t \), \( p(n, t) \). Such a probability satisfies \([24, 26, 27, 64]\)
\[
\frac{d}{dt} p(n, t) = \sum_{n' \neq n} \gamma_{nn'} p(n', t) - \gamma_{nn} p(n, t).
\] (2.23)

Here \( \gamma_{nn'} \) is the total rate for transitioning between the charge state \( n \) and \( n' \), given by \( \gamma_{nn'} = \Gamma_{n\rightarrow n'}^{L} + \Gamma_{n\rightarrow n'}^{R} \). For our case, \( \Gamma_{n\rightarrow n'}^{L/R} \) is given by Eq. (2.18) or Eq. (2.20), depending on the nature of the tunneling event. The dynamics of the system is described completely by Eq. (2.23), although it does not give direct information about transport. In the following, we show how to calculate these quantities for two important regimes, namely under constant biasing conditions, and later in the turnstile regime.

### 2.4 DC response of SINIS single-electron transistors

In this section, we focus on the DC characteristics of a SINIS single-electron transistor. First a method for calculating current through the device is presented followed by some comments on how the parameters of the device affect the current-voltage characteristics.

When calculating the device response to DC bias and gate voltages Eq. (2.23) needs to be solved for steady state, that is, \( dp(n, t)/dt = 0 \). Such a state must exist since the excitations to the system are constant, hence generating a constant current. To achieve this solution, we put Eq. (2.23) in matrix form
\[
A p = 0,
\] (2.24)
where \( p_n = p(n) \) (\( p \) is constant in the present context), \( A_{nn} = -\sum_{n' \neq n} \gamma_{nn'} \) and \( A_{nn'} = \gamma_{nn'} \) for \( n \neq n' \). The matrix \( A \) is readily recognized as a transition matrix of a classical Markov chain. It is enough, then, to find the null space of \( A \) in order to find \( p \). Notice that here we have not mentioned the dimension of \( p \), that is, the number of considered charge states. This number is chosen so that as many states as possible are taken into account, always bearing in mind the complexity of the calculations. Usually, taking between three to five states into account is enough for a good reproduction of experiments. Then, the current through the single-electron transistor \( I \) can be calculated as
\[
I = b \cdot p, \quad \text{where}
\]
\[
b_n = e \left( \Gamma_{n\rightarrow n+1}^{L} - \Gamma_{n\rightarrow n-1}^{L} \right) + 2e \left( \Gamma_{n\rightarrow n+2}^{L} - \Gamma_{n\rightarrow n-2}^{L} \right).
\] (2.25)

This calculation has one extra caveat, heating in the normal-metal island. Owing to its size, overheating of this region due to power dissipation is non-negligible \([59]\) and in order to properly model the system it has to be taken into account. To do this, one needs to calculate the total power transferred to the island in steady-state. This task is readily done
using a similar Markovian approach as applied earlier [22, 59]. For this purpose, we propose a vector \( q \) such that 
\[
q_{n \rightarrow n+1} = Q_{n \rightarrow n+1}^N + Q_{n \rightarrow n-1}^N,
\]
where
\[
Q_{n \rightarrow n+1}^N = Q_{n \rightarrow n+1}^N + Q_{n \rightarrow n+1}^L,
\]
and
\[
Q_{n \rightarrow n-1}^N = Q_{n \rightarrow n-1}^N + Q_{n \rightarrow n-1}^R.
\]

This is the power transferred by the tunneled electron. Also, \( \delta \epsilon_{L/R} \) is the related energy cost of single-electron tunneling defined by Eq. (2.17). Then the total power transferred to the island is
\[
\dot{Q} = q \cdot p.
\]

This power is dissipated by electron-phonon interaction [65, 66] creating a non-equilibrium steady-state in which the phonon temperature \( T_0 \) (which is taken as the temperature of the bath [67]) differs from the temperature of the island electron system \( T_N \). These temperatures depend on the power dissipated as [66]
\[
\dot{Q}_{e-ph} = V \Sigma (T_N^5 - T_0^5),
\]
where \( V \) is the volume of the island and \( \Sigma \) is the electron-phonon coupling constant of the normal-metal. This constant has been experimentally measured in Pub. II to be \( \approx 8.4 \times 10^9 \text{WK}^{-5} \text{m}^{-3} \) for copper, which is the material used as normal-metal across all publications. However, measurements done in copper structures larger than those studied here have established \( \Sigma \approx 2 \times 10^9 \text{WK}^{-5} \text{m}^{-3} \) [31, 59, 68].

An additional power transfer due to Andreev reflection is considered in the form of Joule heat, that is, \( \dot{Q}_A = \langle I_A \rangle V_b \) [69], where \( I_A \) is the current due only to Andreev events and \( V_b \) is the applied bias voltage. Extra heating gives as a result smearing of the current blockades since they cause a non-negligible tail of the thermal distribution of the electrons in the island. Finally, the power balance in the island is given by
\[
\dot{Q}_{e-ph} = \dot{Q} + \dot{Q}_A.
\]

At this point, it is important to bear in mind that the transition rates depend also on \( T_N \). Hence, this quantity has to be solved self-consistently to satisfy Eqs. (2.24) and (2.29) simultaneously. Once \( T_N \) is determined with sufficient accuracy (for the purpose of the works in this thesis we set a tolerance of 0.01 mK), Eq. (2.24) can be solved for \( p \) and the current can be calculated. Figure 2.3(a) shows results of these calculations as red solid lines. Only the maximum and minimum currents are depicted, notice that they agree with the experimental data (blue dots).

We are now equipped to understand how each of the parameters that characterize a SINIS single-electron transistor influence its response to an
applied DC drain-source and gate voltage. In the first place, the extension of the blockaded region is evidently affected by the charging energy ($E_c$) and the superconducting gap of the leads ($\Delta$). From Fig. 2.3(c) it is clear that the extension of the blockaded region in the gate closed state is given by $4(\Delta + E_c) / e$ in the $V_b$ space. On the other hand, in the gate open state this extension is $4\Delta / e$. Another important parameter is the total normal-state tunnel resistance $R_T$ which is the series combination between the two tunnel junction resistances. The asymptotic current at $e|V_b| \gg 2(E_c + \Delta)$ follows an ohmic behavior set by this resistance.

There are two parameters of importance, the ratio between junction resistances $r = R_L / R_R$ and the Dynes parameter of Eq. (2.19). The first one accounts for the symmetry of the device, usually this parameter $r$ can also account for the ratio between the junction capacitances as $C_L / C_R \approx 1 / r$. For the transistor in Fig. 2.3 $r \sim 1$, hence the junctions are almost identical. Non-identical junctions cause a larger gate modulation for $e|V_b| > 2(E_c + \Delta)$ and tilt the Coulomb diamonds in the stability diagram, see Figs. 2.4(a) and (b) for data of the extreme case of a device with $r \sim 0.01$. One important consequence is the dependence of the gate open position on the bias voltage such that

$$n_g^{\text{open}} = \frac{r - 1}{4E_c(r + 1)}V_b + 0.5.$$ \hspace{1cm} (2.30)

In fact, $(r - 1) / (4E_c(r + 1))$ is the slope of every gate state in the stability diagram. Notice that for identical junctions $r = 1$, thus the gate state has a constant charge induced for different biases, as expected. One last annotation, for a symmetric structure, the maximum and minimum DC current correspond to the open and closed gate states, respectively. However, for an asymmetric structure the maximum and minimum current
do not correspond to a single gate state.

Another important characteristic of the SINIS hybrid single-electron transistor is the Dynes parameter, which here is used to model current leakage at $eV_b < 2\Delta$. This parameter groups effects coming from sub-gap states \[61, 62\] and interactions between the electromagnetic environment and tunneling electrons \[19, 56, 70–72\]. In DC operation the influence of this parameter appears as an increase in the sub-gap current. Compare the various maximum currents in DC regime depicted in Fig. 2.4(c) for several values of the Dynes parameter.

One final important variable to take into account is the density of QPs in the vicinity of the tunnel junctions. The density of these unpaired electrons ($n_{\text{QP}}$) in the leads is modelled as a non-zero temperature $T_S$ owing to its energy distribution given by the Fermi-Dirac function Eq. (2.15), so that

$$n_{\text{QP}} = \int dE \, N_S(E) \, f_S(E).$$  \hspace{1cm} (2.31)

Here $N_S$ is given by Eq. (2.5). Even though we consider excitations, the works presented here are within the low density limit, meaning that $k_B T \ll \Delta$ and consequently $f_S(E) \approx e^{-E/k_B T}$ in the relevant energy range. Thus

$$n_{\text{QP}} = N(0) \, \sqrt{2\pi k_B T_S} e^{-\Delta/k_B T_S}. \hspace{1cm} (2.32)$$

Of importance for the results of Pub. I is the DOS at the Fermi level for normal-state aluminum $N(0) = 1.45 \times 10^{17} \text{ m}^{-3} \text{ J}^{-1}$ \[73\]. Furthermore, Eq. (2.32) is central for the results of Pub. I. Non-zero QP density near the tunnel junctions of the SINIS device enhances the leak current below the gap \[18, 74–78\].

### 2.5 The SINIS single-electron turnstile

Now that the dynamics of the SINIS single-electron transistor under pure DC stimuli has been clarified, let us discuss a perhaps more interesting regime of operation of this same device, the SINIS single-electron turnstile (ST). The main feature behind this operation is the injection of a periodic voltage signal into the gate electrode added to a non-zero bias, see Fig. 2.5(a). Key to this operation is the continuity of the stability zone as seen in Figs. 2.3(b) and 2.4(b) and hence current is suppressed for any $n_g$ for $V_b < 2\Delta$. This operation can be parametrized in the stability diagram of Fig. 2.5(b) as a horizontal line, as shown, so that the island chemical potential roams along it. When this line crosses the tunneling thresholds described by Eq. (2.17) and depicted in Fig. 2.5(b) then a tunneling event is energetically favored. In the turnstile operation the path is repeated back and forth so that a threshold for a tunneling into the island through one junction is crossed, and after that, it crosses another threshold for a
tunneling event out of the island through the other junction. At the end of the driving cycle we have transferred a single electron from one lead to the other. If this cycle is repeated $f$ times in a second an average current $I = ef$ appears, this behavior was first observed in 2008 [9] sparking a renewed interest for an all-metallic proposal of current standards. This device is the founding block of all the works within this dissertation.

A more physical picture of the working principle of the turnstile is depicted in Fig. 2.5(c). At some point within the driving cycle, the Fermi level of the island charged with one electron is aligned to the gap edge of the lead with the lowest Fermi level, say the left one, provoking a tunneling and creating a QP there. It is in this moment when one of the thresholds of Eq. (2.17) is crossed. Later in the period, the Fermi level of the island aligns with the gap edge of the other lead, since the island is discharged an electron tunnels from that lead into the island, creating a QP there. As mentioned, a current $I = ef$ is created when this cycle is repeated $f$ times per second.

As pointed out in Chapter 1, the SINIS ST is candidate for a current standard, in fact, single-electron transport, as achieved in this device, has been regarded as one of the principal proposals for realizing the redefined ampere [11]. However, many other proposals have also accomplished single-electron transport, for reviews of these see Refs. 3 and 4. Let us anyway highlight some important demonstrations. Among which stand out turnstiles composed of several biased junctions between small normal-metallic islands and driven by a single gate [55]. The metallic single-electron pump is a similar multi-island fully normal device demonstrated around the same time, with the difference that this one did not need a bias voltage because each island was coupled to its own gate electrode which were driven with appropriate phase-shifts with respect to each other [32, 79]. The most outstanding example of single-electron pumps are those based on semiconducting quantum dots which are usually obtained by using confining gates in a two dimensional electron gas (2DEG) and contacted with two electron reservoirs, these devices have been operated at gigahertz driving frequencies [80, 81] and achieved accuracies of around $10^{-7}$ at 600 MHz [82]. Classically, the operation of the device consists of a plunger gate that tunes the dot level and a loading and discharging gates [4, 83, 84]. Currently, the SINIS device lags behind in terms of accuracy and operation speed compared to these quantum-dot realizations [85]. Recently, single-electron transport was demonstrated at high frequencies (around 30 GHz) caused by phase slips in a superconducting nanowire [86].
Background

Figure 2.5. The SINIS single-electron turnstile. (a) Simple schematic of the device setup. Blue is superconductor and red is normal-metal. The variables are described in the text. (b) Stability diagram showing only charge states with one and zero electrons, the thick double headed arrow shows the parametrization of the turnstile operation. It is evident that this protocol crosses the tunneling thresholds at some non-zero pp amplitude $A_g$. (c) Sketch of the operation of a SINIS ST. The chemical potential of the island (center) is modulated by the signal. At some point during the period it is charged with one electron (yellow dot), its Fermi level aligns with the gap edge of the lead with the lowest Fermi level and a the electron tunnels into it. Later, the Fermi level of the island aligns with the gap edge of the other lead and an electron tunnels into the normal-metal. Finally, the system resets and the cycle is repeated. (d) Current produced by a SINIS ST against pp gate amplitude for different bias voltages $V_{0b} = 160, 200, 240, 280$ and $320\,\mu V$ from right to left with $n_{0g}$ at the degeneracy point. Data are from the device measured in Pub. II but for $f = 30\,\text{MHz}$. The red dashed lines depict the ideal current given by Eq. (2.34) for these conditions. Inset: close-up of the zone close to $I = e f$, black solid lines are simulations based on the method outlined in Subsection 2.5.2.

2.5.1 Characteristics

The operation of the SINIS ST is given by

$$V_b = V_{0b}$$

$$n_g = n_{0g} + \frac{A_g}{2} g(\omega t).$$

(2.33)

Here, $V_{0b}$ is a constant bias voltage, $n_{0g}$ a DC offset to the gate driving (remember that $n_g = C_g V_g/e$), $A_g$ the peak-to-peak (pp) amplitude and $g(\omega t)$ a periodic function of time with period $\tau = 1/f = 2\pi/\omega$. Different driving methods consisting of adding an extra modulation to $V_b$ are experimented in Pubs. IV and V.

There are four main control parameters of the device, the driving amplitude, the gate offset, the driving frequency and bias voltage. Ideally, a good metrological standard is highly insensitive to such parameters. However, and since life is is not ideal, the SINIS ST presents sensitivity to changes in these. The dependence on the driving amplitude is often the most explored one since the current forms clear wide plateaus against this parameter and by varying it one remains in the stability zone. As can be seen from Fig. 2.5(d) these plateaus fall close to the expected values and
can be very flat, which points to a weak sensitivity to amplitude in that range. Additionally, the shape of the plateau allows one to explore the different regimes that enable tunneling errors. These are always present as can be seen from the deviations from the red dashed line in the inset of Fig. 2.5(d). It is possible to identify errors due to leaking and Andreev reflection [21] at low amplitudes and back-tunneling at high amplitudes. These last ones will be further detailed in Chapter 6 where Pubs. IV and V are presented.

One parameter to which the current in this device is specially insensitive is the gate offset \( n_{0g} \). As discussed in Ref. 9 this factor only affects which current plateaus appear. This is easily explained from the stability diagram of Fig. 2.5(b) as follows. More than one electron can be transmitted by increasing the amplitude of the drive, this is one of the dependencies on \( A_g \), see Fig. 2.5(d), therefore the current is actually given by

\[
I = Nef, \tag{2.34}
\]

with \( N \) an integer number of electrons transferred. Increasing the amplitude enlarges the path of Fig. 2.5(b) so that it spans more Coulomb diamonds corresponding to different island charge states. So, when the driving spans diamonds from \( n = -1 \) to \( n = 1 \) two electrons are transferred per cycle and so on. Now, if we take into account that the path is symmetric with respect to \( n_{0g} \), we see that by setting \( n_{0g} = 0.5 \) the path can span from \( n = 0 \) to \( n = 1 \) giving one electron transmitted. At higher amplitudes, the path can span from \( n = -1 \) to \( n = 2 \) transferring three electrons but not from \( n = -1 \) to \( n = 1 \) or from \( n = 0 \) to \( n = 2 \) so that the transfer of two electrons only is impossible. One concludes that by setting the offset at gate degeneracy, only current with an odd number of electrons can be generated. For example, see Fig. 2.5(d) where data taken with \( n_{0g} \) at gate degeneracy are shown. For this precise device \( r \neq 1 \) and hence degeneracy is not at \( n_{0g} = 0.5 \) for all biases. However, by setting the offset in the gate closed state the conduction is only activated when the path already crosses states from \( n = -1 \) to \( n = 1 \), transferring two electrons and skipping the transmission of only one electron, allowing only conduction of an even number of electrons. For offsets at intermediate values, any \( N \) is allowed.

Perhaps the most relevant driving parameter is the frequency, since it sets the magnitude of the generated current. In fact, not only accuracy is important for a good standard but sufficient magnitude is relevant too. A current close to 100 pA is often mentioned as sufficient for realizations of the ampere in metrology [3, 87]. To achieve this in SINIS ST, parallel current pumping has been implemented [88]. Such implementation is possible in semiconducting pumps [89]. As a rule of thumb, the device loses synchronization to the driving signal as its frequency increases. Hence accuracy with respect to ideal current decreases as the current increases.

This is the case because the rates at which tunneling occurs are set by the
junction resistance and not by $\delta \epsilon^{\pm}_{L/R}$, since these events usually happen at $\delta \epsilon^{\pm}_{L/R} \approx \Delta$ the rates are constant. Hence at some point the driving rate becomes comparable to the tunneling one. As a consequence some events may be lost or tunneling in the wrong direction (back-tunneling) may occur, the latter errors are inspected in Publication IV and mitigation strategies are explored.

Finally, the dependence of current generated on bias voltage is less direct in so far as it also depends on other parameters. More specifically, the sensitivity of the current on bias voltage is higher when frequency is high so that missed and back-tunneling events are more likely. At low voltages the device has less time to respond to changes in the driving signal, and when the frequency is high enough, error events take place. Additionally, the minimum driving amplitude for tunneling activation depends on $V_b$, and as such at low amplitudes the current presents a large sensitivity to bias voltage. Such dependence is also manifested at high amplitudes and high frequency, when back-tunneling is the dominant error. A most important condition that enhances the dependence of current on bias is a high QP density. A high number of non-relaxed QPs allows for a higher leaking current which deteriorates the blockade inside the Coulomb diamonds and increases the spread of the current plateaus with respect to $V_b$, this characteristic is used in Pub. I and in Ref. 18 to quantify $n_{QP}$. It is also shown that this density increases with driving frequency.

In this presentation the influence of the shape of $g$ has been ignored since a sinusoidal waveform allows for an easier experimental study of error processes and new applications. This is mainly due to the harmonic distortion along the lines through which signals are applied in experiments. Because of this, sinusoidal signals are exclusively used here. We will simply mention that this dependence can be just reduced to how fast the driving passes the tunneling thresholds and how long it remains far from them. In Pubs. III and IV it was demonstrated that the former is the case. Additionally, it has been shown that for example back-tunneling is enhanced by fast-varying waveforms such as square signals [77, 90]. Furthermore, tailoring of waveforms has been tackled [91].

2.5.2 Markovian description

The turnstile operation can be modelled based on the Markovian description outlined generally in Subsection 2.3.2 with the important difference that Eq. (2.24) cannot be used. This is because such a stationary state cannot include time evolution. Instead, a periodical steady state has to be used where the state of the system has the same periodicity of the driving signal. Hence we have

$$\frac{d}{dt} \mathbf{p}(t) = A(t) \mathbf{p}(t).$$
The time dependence of matrix $A$ is contained in $n_g$ which is present in the chemical potential of the island (Eq. (2.8)).

To calculate the current, we propose an evolution equation for the state averaged charge in the island $\langle q \rangle_s$ [24]

$$
\frac{d}{dt} \left[ \begin{array}{c}
p (t) \\
\langle q \rangle_s (t)
\end{array} \right] = \left[ \begin{array}{cc}
A (t) & 0 \\
0 & b (t)^T
\end{array} \right] \left[ \begin{array}{c}
p (t) \\
\langle q \rangle_s (t)
\end{array} \right].
$$

(2.36)

In Eq. (2.36) the subscript $s$ indicates an average over charge states. Furthermore, $b$ is defined in Eq. (2.25). In fact, in order to calculate any transport quantity of the system, extended equations similar to Eq. (2.36) have to be proposed, this is exploited in Pub. III. Notice that Eq. (2.36) includes Eq. (2.35) in the first row.

Here, a solution to Eq. (2.36) by exponentiation is proposed, such that

$$
\left[ \begin{array}{c}
p (t) \\
\langle q \rangle (t)
\end{array} \right] = \exp \left( \int_0^t dt' \left[ \begin{array}{cc}
A (t') & 0 \\
b (t')^T & 0
\end{array} \right] \right) \left[ \begin{array}{c}
p (0) \\
\langle q \rangle (0)
\end{array} \right].
$$

(2.37)

Now, based on the assumption of a periodic steady state, we propose periodic boundary conditions for $p$ with the same period as $n_g$. This is reasonable since the tunneling rates, and hence $A$, have the same periodicity as this signal. Additionally, one can approximate the integral by discretizing the driving cycle in $m$ intervals of size $\delta \tau = \tau / m$, with $\tau$ the period of the signal. From the form of Eq. (2.37), the exponential can be regarded as a propagator. At the end of the period this propagator is given by

$$
\tilde{U} (\tau) = \prod_{k=1}^m \exp \left( \delta \tau \left[ \begin{array}{cc}
A (t_k) & 0 \\
b (t_k)^T & 0
\end{array} \right] \right),
$$

(2.38)

where $t_k = (k - 1) \delta \tau$. Evidently, Eq. (2.38) is exact for $m \to \infty$, however, in practice, the propagator is evaluated for a finite number of time intervals, usually three hundred is sufficiently large.

We decompose $\tilde{U} (\tau)$ as

$$
\tilde{U} (\tau) = \left[ \begin{array}{cc}
U (\tau) & 0 \\
U_b^T (\tau) & 0
\end{array} \right].
$$

(2.39)

Then, by imposing periodic conditions in Eq. (2.37) one gets

$$
p (\tau) = p (0) = U (\tau) p (0).
$$

(2.40)

Thus, the calculation of $p$ has been reduced to determining the eigenvector corresponding to the eigenvalue 1. Finally, the average charge can be calculated as $\langle q \rangle (\tau) = U_b (\tau) \cdot p (0)$ according to Eq. (2.37). As a result, the average current can be written as $I = \langle q \rangle (\tau) / \tau$. The inset of Fig. 2.5(d) shows results of this procedure as black solid lines.
 Crucial for the development of Pub. I is the calculation of the heat transferred to the island in the turnstile regime. This is readily done by replacing the vector $\mathbf{b}$ in Eq. (2.36) with the vector $\mathbf{q}$ described in Section 2.4, and by replacing the charge transferred $q$ with the energy injected to the island $E$. Thus, the power transmitted to the island is given by $\dot{Q} = \langle E \rangle / \tau$. For Pubs. II and III the power transfer to the leads was calculated by reformulating the vector $\mathbf{q}$ so that $q_{i}^{L/R} = \dot{Q}_{i \rightarrow i+1}^{S,L/R} + \dot{Q}_{i \rightarrow i-1}^{S,L/R}$ where $\dot{Q}_{i \rightarrow i+1}^{S,L/R} = \dot{Q}_{i \rightarrow i \pm 1}^{S,L/R} + \delta_{L/R}^{i \rightarrow i \pm 1}$. With this, the power transmitted to the left/right lead is calculated as $P_{L/R} = \langle E_{L/R} \rangle / \tau$. 
3. Experimental methods

In this chapter the methods followed in all experimental publications on which this thesis is founded are presented. First, fabrication processes common to Pubs. I, II, IV and V, such as sample patterning and deposition, are outlined in Section 3.1. Section 3.2 describes the refrigeration required for carrying out the experiments of this thesis. Next, in Section 3.3 the electrical wiring of the used refrigerators is described. Finally, in Section 3.4 the measurement setups external to the refrigerator and general steps for obtaining data are detailed.

3.1 Fabrication process

All devices are deposited on top of commercial 4-inch silicon wafers coated with a 300 nm thick thermally grown silicon oxide layer. Every nano and microfabrication process starts with patterning the shape of the wished sample, which has been previously computer-generated. In the procedures involved in this thesis, electron-beam lithography (EBL) on a polymeric resist is used for this purpose. EBL is done with a Vistec EBPG5000+ system operating at 100 kV at different electronic doses. For this, the substrate is coated with a uniform thin layer of resist, this is achieved by pouring the substance on the wafer and spinning, then it is soft-baked in order to evaporate solvent. Additionally, in all fabrications done in these works, electron-beam (e-beam) metal evaporation is used in order to metallize the structures. This physical vapor deposition (PVD) of metals happens in a vacuum chamber (around $4 \times 10^{-7}$ mbar) by heating a metal target, once the heat causes the atoms of this target to detach from it, they follow a route directly to the substrate. The target is placed in an open crucible and is hit by an electron beam generated from a filament with high current (typically over 20 A) and accelerated by high voltage (9 kV). The thickness and deposition rates are controlled by a gauge based on the resonance frequency of a quartz crystal exposed to the metallic vapor.

For Pubs. I and II the fabrication process included the making of ground
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planes which filter environmental photons that cause leakage [19]. Furthermore, bottom gate electrodes are also made so that the coupling between it and the transistor island is enhanced. The process carried out to do this is sketched in Fig. 3.1. Through a soft mask of Allresist AR-P 6200.13 defined by EBL, a 2 nm adhesion layer of titanium is deposited by e-beam evaporation, directly on top of the wafer. Next, 30 nm of gold is evaporated followed by an additional 2 nm titanium layer for protection. After deposition, excess resist and metal on top of that resist are removed by soaking the wafer in acetone. In these processes, 20 \( \mu m \times 20 \mu m \) alignment marks are also created so that further structures can be patterned with accurate alignment on top. The ground planes and gate electrodes are galvanically isolated from structures on top by depositing a 50 nm layer of aluminium oxide via atomic layer deposition (ALD) carried out in a commercial Beneq TFS-500 reactor. For the devices tested in Pubs. I and II a second deposition step is done. Two coarse electrodes and bonding pads are created on top of the ALD layer in a posterior EBL round. Through a similar mask 2 nm Ti plus a 30 nm thick AuPd layers are deposited and these structures are formed. These will be later directly contacted by transistor leads and function as normal-metal traps [18, 92–94]. As before, acetone is used to remove excess metal and resist. What follows is the creation of the smallest characteristics of the devices (such as the island and tunnel junctions). The necessary steps for this are shared by Publications I, II, IV and V. In the latter two, the samples are fabricated with only these steps.

In order to create the small features of the samples, a process based on a hard germanium mask is used [3, 95–97]. This process is summarized in Fig. 3.2. For preparing this mask, the wafer is coated with a 400 nm layer of poly(methyl methacrylate-methacrylic acid) (P(MMA-MAA)), then a thin 22 nm layer of germanium is deposited by e-beam evaporation, this

---

**Figure 3.1.** Process for fabricating ground planes. (a) Initial preparation of the substrate. (b) EBL (electron-beam lithography) exposure of the resist. (c) Development of the exposed pattern. (d) Deposition by e-beam evaporation of titanium and gold. (e) Removal of excess resist and metal. (f) Deposition of aluminium oxide by atomic layer deposition (ALD). Thicknesses are not at scale.
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Figure 3.2. Process for fabricating tunnel junctions by germanium mask. (a) Initial preparation of the germanium based mask. (b) EBL (electron-beam lithography) exposure of the top resist. (c) Development of the exposed pattern. (d) Etching of the germanium exposed in the previous step by reactive ion etching (RIE) with carbon tetrafluoride, exposing P(MMA-MAA) sectors. (e) Transfer of the pattern to the P(MMA-MAA) layer by RIE with anisotropic molecular oxygen. (f) Undercut aperture by RIE with isotropic molecular oxygen. (g) Electron-beam evaporation of metals, aluminium is first deposited and then oxidised in situ, then copper is deposited at an angle such that it contacts the oxide layer. (h) Removal of excess germanium, resist and metal. Thicknesses are not at scale.

is finally coated with a 50 nm layer of poly(methyl methacrylate) (PMMA), see Fig. 3.2(a). Then, the desired pattern is defined by EBL, usually for nanometric structures a dose of 1100 µC/cm² is used. Several arrays containing many devices are created and later separated by cleaving the wafer for easier handling. After cleavage, each small chip is soaked in a solution of methylisobutyl-ketone (MIBK) and isopropanol (1 to 3 by weight) to remove the exposed PMMA. Later, the pattern created in the PMMA layer is transferred to the germanium layer by etching it using reactive ion etching (RIE) with carbon tetrafluoride (CF₄). Inside the same chamber, the exposed P(MMA-MAA) sections are removed by RIE with molecular oxygen. This is also used to open an undercut pattern necessary for creating small contacts between different metal layers. These RIE steps are carried out in an Oxford Instruments Plasmalab 80Plus reactor. Following RIE, either the ALD layer (for Pubs. I and II) or the wafer (for Pubs. IV and V) are exposed. Then, the structures can be metallized following
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the multiple-angle shadow evaporation method [46]. For creating tunnel junctions, aluminium is first deposited at certain angle of the substrate with respect to the metal target. Then, static in situ oxidation of the layer is done with pressures between 1 and 2 mbar, creating an aluminium oxide layer. Next, copper is evaporated at an appropriate angle so that the previously created structure is contacted. Although Fig. 3.2(g) illustrates the procedure for fabricating tunnel junctions, direct contacts between metals are also created with this method by ignoring the oxidation step. In the final evaporation, also additional coarse electrodes and bonding pads are created. For the case of Pubs. I and II, also the bonding pad corresponding to the gate is capacitively coupled to the previously created electrode. In Pubs. III and IV the gate electrode and bonding pads are created at the same time as the other components of the sample, constituting what is known as a side gate.

3.2 Refrigeration

The effects observed in the works of this thesis can only be revealed in devices operated at low temperature. For example, the low QP density condition requires $T \ll 0.3 \, \text{K}$ and, for some of the studied samples, the dominance of charging energy gives $T \ll 1 \, \text{K}$. Additionally, superconductivity in aluminium is only achieved below 1.2 K. Furthermore, low temperatures provide a low error rate for SINIS ST and low leak current in the blockade of SINIS single-electron transistors. It is also at these temperatures that NIS thermometry is of great help [30, 31], such method is central for the development of Pub. II. Hence, temperatures of hundreds (or even tens) of milikelvin are mandatory.

These temperatures are nowadays routinely achieved with $^3\text{He}-^4\text{He}$ dilution refrigerators. The experiments described in this thesis were carried out using different custom-made dilution refrigerators designed and built in my research group and described in Ref. 98. The cooling power of the refrigerators comes from the heat of mixing two helium isotopes when these are in what is called a dilute phase [99]. Such heat is provided by the enthalpy of mixing two separate phases as explained in what follows. At temperatures below $870 \, \text{mK}$ a mixture of $^3\text{He}-^4\text{He}$ (with more than 6.6% of the lightest isotope) is separated into two phases, one rich in $^3\text{He}$ (concentrated phase) and the other rich in $^4\text{He}$ (diluted phase). At lower temperatures the former further dilutes so that it tends towards pure $^3\text{He}$, while the latter reaches an equilibrium concentration of $^3\text{He}$. To take advantage from the non-zero enthalpy of mixing, $^3\text{He}$ is removed from the diluted phase, which sits at the bottom of the concentrated phase due to its higher density. This causes a non-zero osmotic pressure (for a detailed description see Refs. 99 and 100) that drives $^3\text{He}$ atoms from the
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concentrated phase to the dilute one. In order to do this, mixing enthalpy is taken from the environment of the mixing chamber.

As mentioned, the refrigerator consists of a mixing chamber that contains the concentrated and diluted phases. However, to reach these states one needs to cool the $^{3}$He-$^{4}$He mixed gas from room temperature down to 870 mK where it becomes liquid. To achieve this, the dilution unit is enclosed in a stainless steel can which is sealed from the surroundings with an indium seal. This can is first pumped down to $< 10^{-3}$ mbar and pre-cooled in liquid nitrogen via a $^{4}$He exchange gas, after full thermalization the fridge is immersed in a dewar with liquid $^{4}$He so that the dilution unit and mixing chamber reaches 4.2 K. Then, the exchange gas is pumped out ($\sim 10^{-5}$ mbar) of the can so that good thermal isolation between the 4.2 K environment and the dilution unit is achieved. The fridge has a 4 K stage in which the mixture exchanges heat with the helium bath, then it is cooled down to $\sim$ 1 K in a subsequent step. This temperature is achieved by pumping evaporated $^{4}$He from a copper pot inside the can. Then, the mixture reaches the dilution unit which is made of either Stycast 1266 [101] or Torlon plastics and mounted in a stainless steel chassis. When in this unit, the mixture passes through heat exchangers of Teflon finally reaching the mixing chamber. Mixture is removed from the mixing chamber, as required for cooling, through the still line. This cold outflowing mixture exchanges heat with the incoming hotter one, providing additional pre-cooling for condensation. The chamber is thermally attached with a sintered silver rod to a stage to which the holder of the samples under test is connected.

The temperature of this sample holder is known by monitoring the resistance of a ruthenium oxide resistor (Scientific Instruments, Inc., model RO-600) thermally anchored to it, which is measured through lock-in techniques. This resistance is temperature calibrated in a dedicated cooldown against a Coulomb Blockade Thermometer (CBT) [102] operating in the primary mode. The temperature of the mixing chamber can be increased by applying current to a heating resistor installed in this stage.

3.3 Electrical wiring

In the works of this thesis, the way to access the properties of the devices is through measurements of currents and voltages. Also, the excitations required for carrying out experiments are electrical signals, both DC or AC. Consequently, the described refrigerators are also equipped with appropriate wiring so that such signals can be injected or measured from the sample.

The DC wiring is composed of two sections, the first one, from room temperatures to the 1 K stage of the refrigerator, is resistive twisted pair.
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These, connect a PCB inside the can to BNC terminals in a breakout box at room temperature so that connections to scientific instrumentation are done easily and with versatility. The other side of the PCB is connected to Thermocoax cable which goes down to the sample stage, where it is properly thermalized, and connects to a 20-pin commercial male connector fixed to this stage. The sample holder can then be easily attached to it via a corresponding female connector, which is connected to the leads of a PCB, which are terminated in bonding pads. In this holder, the fabricated chip is fixed with help of vacuum grease. The bonding pads patterned in the chip are connected to these leads with aluminium bonds.

The AC signals are similarly transported through two sectors of 50 Ω characteristic impedance coaxial lines. The first one is a stainless steel line which connects a SMA connector attached to a separate breakout box at room temperature, to a 20 dB attenuator which is followed by a feedthrough into the vacuum can at the 4.2 K stage. Inside the vacuum can, a superconducting niobium-titanium cable connects the 1 K stage with a SMA connector in the sample stage. The central line of this connector is electrically attached to a lead in the PCB. In Pubs. I and II this lead is directly bonded to the gate electrode of the sample. In Pubs. IV and V it is capacitively coupled to another lead through a surface mount capacitor of 22 nF. The latter lead is inductively coupled to a third one via a surface mount inductor of 33 µH leading to a DC connection. This configuration forms a bias tee, which is connected to the source and gate electrodes of a SINIS transistor.

A final comment about the protection of the sample from stray radiation produced by hotter stages of the refrigerator: Although the vacuum can itself offers some protection from the photons emitted in the 4.2 K stage, the sample holder counts with a brass lid which offers further protection from the radiation produced in the still, 1 K and 4 K stages [19]. Furthermore, the Thermocoax cables and attenuators play a vital role in reducing thermal noise, which causes similar effects.

3.4 Measurement setup and some measurement routines

The custom setup used in the experimental work is described in this section, this kind of setup refers to every instrumentation external to the employed refrigerators.

In order to generate the required DC signals, isolated voltage sources were used (Stanford Research Systems SIM928) placed inside the SIM900 frame of Stanford Research Systems. These sources provide low noise DC voltage required for biasing the source and (in the case of Pubs. IV and V) the gate electrodes. Since the minimum voltage that can be provided by these sources is 1 mV, which provides a single electron with an energy one
order of magnitude larger than the superconducting gap of bulk aluminium (\(\sim 180 \mu\text{eV}\)), it needs to be reduced. This is readily done by using custom made voltage dividers with the smallest resistance of around 100 \(\Omega\), giving zero output current, which is important since no external current must be injected into the devices, specially for the correct operation of SINIS ST. The reason for this absence of current is the high asymptotic resistance of the tested devices, from two to four orders of magnitude larger than the division resistance, for voltages close to the blockade and superconducting gap the ratio is even higher. For applying AC signals, arbitrary function generators are used, in Pubs. I and II the Tektronix AFG3252 is used, and in Pubs. IV and V the Keysight 335222B is used. Also in this case, the minimum generated peak-to-peak amplitudes are too large for the needs of the research. At the used frequencies, a custom-made divider does not work for reducing the amplitudes, instead commercial attenuators are used. Although 20 dB attenuation is already applied inside the fridge, additional attenuation needs to be applied at room temperature. In Pubs. I and II the DC offsets of the signals are applied from the function generators and hence divided by the attenuators and not by voltage dividers as in Pubs. IV and V.

The works presented here only involve DC measurements (with AC driv-
ings), hence the setup of the used refrigerators offers sufficient capabilities for acquiring the relevant data. The highest measured current in the turnstile operation is on the level of tens of picoamperes. On the other hand, the maximum measured voltage is around hundreds of microvolts. Therefore, additional amplification is mandatory, however at such low frequencies room temperature amplifiers are good enough. For current, diverse models of commercial (FEMTO Messtechnik GmbH) variable transimpedance amplifiers were used, typically amplifications ranging $10^9 - 10^{13}$ V/A are needed. For voltage measurements, room temperature low noise amplifiers typically at 80 dB gain are used. Naturally, the output of both amplifiers is a voltage proportional to the input. This voltage is then measured with a digital multimeter. The sources and multimeters are all computer controlled, and as such the procedures for measuring are easily carried out following commands in programming routines.

In what follows some general routines which are used in all the experimental works are briefly described. In all the experimental works the characterization of the transistors is important, hence IV curves need to be recorded. This is done simply by fixing the voltage bias and sweeping the gate voltage, ideally through three periods, provoking Coulomb oscillations in the device, see Fig. 3.3(a). Subsequently, the voltage bias is varied and the sweep repeated. To complete the characterization of the device, these measurements are compared to the calculations described in Section 2.4. The various parameters of the system $\Delta$, $E_c$, $R_T$, $r$, $\eta$ and the temperatures of the leads, are varied so that the calculated curves corresponding to maximum and minimum current reproduce the data. This procedure results in curves such as the solid red ones in Figs. 2.3(a) and 2.4(a). As a result, the parameters of the system are estimated and fixed.

Another important general procedure is the measurement of current plateaus, see Fig. 2.5(d). These are typically measured with the offset of the driving signal tuned to the gate open state. Consequently, first the signal has to be set to this point. To do this, a modulation of certain frequency that provides a large enough current (say $f = 20$ MHz) is applied, then the offset of the signal is swept over several gate periods. If the amplitude of the signal is small enough, sharp peaks around the gate offset corresponding to the open state appear, see Fig. 3.3(b). Once the voltage of the peak is located the signal is tuned around the gate open state. When this is done, the signal with the desired frequency is applied, the amplitude of which is then increased so that at least the first current plateau can be distinguished. The described procedure is repeated several times (from 5 to 15), the obtained curves are then averaged in order to reduce noise. Results may be altered due to random jumps in the background charge which might occur after the tuning process, see Fig. 3.3(c). Such iterations are left out of this averaging process. A similar procedure is followed in Pub. II for measuring the power injected to the transistors leads.
4. Detection of quasiparticle extraction with the SINIS ST

This chapter concentrates on Publication I, where QP extraction by biased $S_1S_2$ junctions was demonstrated. The QPs were extracted from one of the leads of a SINIS ST and the QP density was probed by the current emitted by the device. This demonstrates that these devices can be used as detectors for non-equilibrium QPs. Section 4.1 describes briefly why QP extraction is relevant in present quantum technologies. A description of the method for extracting QPs is presented later in Section 4.2. Then, the mechanism of QP poisoning in the SINIS ST and its consequences are shown in Section 4.3. Finally, evidence of QP extraction in a SINIS ST is presented in Section 4.4.

4.1 Why QP extraction?

This question is answered in two parts. The first one answers to the question: Why is it important to mitigate QPs? The second one answers to why opt for extracting instead of trapping QPs?

The first answer comes from the needs of quantum technologies [16, 17]. For these, superconductors are widely used and non-equilibrium QPs are ever present. These excitations negatively affect the correct operation of quantum bits based on Josephson junctions [14, 103–107]. This is clear in charge qubits where QP tunneling across the junction takes the system out of the desired computational basis [108, 109]. Furthermore, superconducting excitations also cause decoherence in other types of Josephson junction-based qubits [110–113], such as the transmon [114] and flux qubits [115, 116]. Additionally, superconducting QPs also lead to increased decoherence in Majorana qubits based on nanowires [117–121]. Here, excitations relax into the Majorana zero modes by exchange with other fermionic states. A similar situation happens in Andreev state qubits [122]. Other devices in which QP creation is detrimental for their proper functioning are superconducting microcoolers and kinetic inductance detectors. In the former kind, the cooling power is compromised because the cre-
Detection of quasiparticle extraction with the SINIS ST

vation of excitations causes a back-flow of hot electrons and a reduction in the superconducting gap [31, 123, 124]. While in the latter devices the sensitivity is reduced given that an excess of QPs “obfuscates” the ones arising from the signal to measure, increasing then the noise floor of the detector [125–128]. Also, in superconducting resonators excitations hinder their performance by shifting their resonance frequency and reducing their quality factor [126, 129, 130].

One could argue that QPs might not be such an issue if they can be thermally suppressed, that is, if $k_B T \ll \Delta$. However, these are ever present and even generated by the bare operation of the device, such as in the case of microcoolers [123]. Nowadays, different sources of a perennial density of QPs have been revealed. On one hand, experiments have unveiled the presence of excitations generated by cosmic rays and ionizing radiation [131–135]. However, hotter stages of the cryogenic setups have also been demonstrated to generate non-ionizing photons capable of breaking Cooper pairs [18, 72, 74, 103, 105, 136, 137]. Furthermore, it has been shown that phonons generated in the recombination of QPs might also contribute to their generation elsewhere in the system [138–141]. Opposed to these findings, a non-stationary source of QPs has been revealed [142]. Although its identity is still unknown, acoustic relaxation has been suggested as one of the culprits [143].

The second question is answered by looking at the several attempts recently done to reduce the density of QPs. A natural approach is trapping QPs far from the critical operational point in normal metals [92–94], where their energy is promptly dissipated, as shown in Pub. III and in Ref. 144. Similarly, QPs can be trapped in superconducting films with a lower gap [15, 145–147], which can be achieved by variations in the thickness of the films or by using different materials. Other methods consist on enhancing the diffusion of unpaired excitations by modifying the geometry of the superconductor or increasing its thickness [18, 148, 149], trapping QPs in magnetically generated vortices [129, 148, 150, 151], depositing normal-metal phonon traps [141, 152, 153] or even blocking them by modulating the gap of the active superconducting region [154, 155]. It is clear then that almost all these methods belong to what could be called passive QP suppression, meaning that these are either trapped, their generation rate reduced or simply avoided. These methods cannot be adjusted once the device has been fabricated. In contraposition, the method discussed in this chapter shows an extraction of QPs that can be tuned. Hence it is versatile and active in the sense that external work needs to be supplied for QP extraction.
4.2 The $S_1IS_2$ junction as a QP extractor

In this section, the potential of a biased Josephson junction for removing QPs from one of its leads into the other is explored. This was first theorized in 1961 [156] by estimating the enhancement of the superconducting gap $\Delta_1$ of a superconductor. This is the result of extracting QPs from this superconductor and adding them to another of gap $\Delta_2$ through a tunnel barrier, in what is called in Pub. I a $S_1IS_2$ junction. The conditions for this extraction are that $\Delta_1 < \Delta_2$ and that a voltage bias $e|V_{SIS}| < \Delta_1 + \Delta_2$ is applied. The condition for maximal efficiency is $e|V_{SIS}| \approx \Delta_2 - \Delta_1$, hence this extraction method can be referred to as “active”. This was experimentally verified in 1979 [157] by probing the enhancement of $\Delta_1$ via an additional Josephson junction. Further experiments were carried out confirming this observation [158, 159], also cooling of the smaller gap superconductor was observed through this effect [160] and the enhancement of the even parity life-time of a single-Cooper-pair transistor was achieved by using $S_1IS_2$ junctions to extract QPs from one of its leads [28].

These results can be understood from a simplified picture in the semiconductor model of the BCS DOS as in Figs. 4.1(a)–(c). First, suppose that the superconductor with the smaller gap has a non-zero density of excitations. In panel (a), the junction is unbiased and, as such, no QP current flows. In this situation there is no extraction. When the junction is biased so that $e|V_{SIS}| = \Delta_2 - \Delta_1$, the singularities of both DOSs align and QPs have a high tunneling rate towards the larger gap superconductor, since there is a low density of excitations in it due to the higher gap, see Fig. 4.1(b). This situation of QP tunneling is kept until $e|V_{SIS}| \geq \Delta_2 + \Delta_1$, when QPS are created on both leads in ohmic transport, see Fig. 4.1(c).

One can formalize this picture and show that there exists indeed a positive cooling power from the smaller gap superconductor by modelling the $S_1IS_2$ junction as a resistively shunted Josephson junction. The superconductor with gap $\Delta_1$ ($\Delta_2$) has a QP temperature of $T_1$ ($T_2$). Of interest is the cooling power of the junction rather than the current-voltage properties of the system. The junction is connected in series to a resistance $R_S$ and fed by a bias $V_{SIS}$. The system dynamics is described by

$$V_{SIS} = IR_S + \frac{\hbar \dot{\phi}}{2e},$$ (4.1)

$$I = I_c \sin \varphi.$$  

Where $I_c$ is the critical current of the junction, which can be approximated by [161]

$$I_c = \frac{1}{eR} \int_{\Delta_1}^{\Delta_2} dE \frac{\Delta_1 \Delta_2 [1 - 2f_i(E)]}{\sqrt{E^2 - \Delta_1^2} \sqrt{E^2 - \Delta_2^2}}.$$ (4.2)

Here, $f_i$ denotes the distribution function of the superconductor $i$ (often taken to be the Fermi-Dirac function) and $R$ is the junction normal-state
Figure 4.1. The S₁S₂ junction. (a) Semiconducting model of the DOS of the unbiased S₁S₂ junction. The superconductor S₁ has a non-zero QP (yellow circles) density that cannot diffuse efficiently. The gray zone represents the insulating barrier. (b) When the junction is biased so that $e|V_{SIS}| = \Delta_2 - \Delta_1$, QPs have more probability to transfer to S₂. (c) When the junction is biased so that $e|V_{SIS}| = \Delta_2 + \Delta_1$, QPs are created in both leads and the density in S₁ increases. (d) Normalized cooling power (left axis) of a S₁S₂ tunnel junction against the voltage bias. The legend indicates the Dynes parameter used for both leads. The right axis shows practical values of the power for $R = 5 \text{k}\Omega$ and $\Delta_2 = 236 \mu\text{eV}$. Furthermore, $T_1 = 300 \text{mK}$ and $T_2 = 50 \text{mK}$. Arrows indicate some relevant biasing points. The inset depicts the same curves for a wider bias range.

The power transferred from the lead $i = 1$ is $P^{(1)}(t) = P^{(1)}_{QP}(t) + P^{(1)}_{J}(t)$,
where $P_{\text{QP}}^{(1)}(t)$ is

$$P_{\text{QP}}^{(1)}(V(t)) = \frac{1}{e^2 R} \int dE \ n_2 (E - eV(t)) n_1 (E) (E - eV(t)) \times$$

$$\left[ f_2 (E - eV(t)) - f_1 (E) \right],$$

and expresses the power transmitted by QPs. Here $n_i$ are the density of QP states of each lead, subgap states are also considered if the DOS is given by Eq. (2.19). On the other hand, the Josephson contribution is written as

$$P_{\text{cos}}^{(1)}(t) = P_{\text{cos}}^{(1)} \cos \varphi + P_{\text{sin}}^{(1)} \sin \varphi [162],$$

where

$$P_{\text{cos}}^{(1)}(V(t)) = -\frac{1}{e^2 R} \int dE \ n_2 (E - eV(t)) n_1 (E) \frac{\Delta_1 \Delta_2}{E} \times$$

$$\left[ f_2 (E - eV(t)) - f_1 (E) \right].$$

The cooling power from the lead $i = 1$ is given by

$$P^{(1)} = \left\langle P_{\text{QP}}^{(1)} \left( \frac{\hbar \dot{\varphi}(t)}{2e} \right) + P_{\text{cos}}^{(1)} \left( \frac{\hbar \dot{\varphi}(t)}{2e} \right) \cos \varphi(t) \right\rangle_t.$$  

The sine component of the Josephson contribution vanishes upon averaging over a period $T = \hbar \pi / e (V_{\text{SIS}}^2 - I_c^2 R_S^2)^{-1/2}$ for $V_{\text{SIS}} > I_c R_S$.

When $\Delta_1 < \Delta_2$, this power is positive and the average is sharply peaked at $e |V_{\text{SIS}}| = \Delta_2 - \Delta_1$, as shown in Fig. 4.1(d). This gap difference was achieved in Pub. I by depositing aluminium layers of different thicknesses [163] but could be also realized by using different materials. For the specific case of these calculations, $T_1 = 300$ mK and $T_2 = 50$ mK. Observe that the lead $i = 1$ is cooled down for $e |V_{\text{SIS}}| < \Delta_1 + \Delta_2$, see the inset of Fig. 4.1(d), and hence QPs are extracted from this lead as long as the bias is kept in this range. However, this is strictly true only for $\eta \to 0$ as the figure shows. For a larger Dynes parameter on both terminals QPs are created in the lead with larger gap even though $\Delta_1 < e |V_{\text{SIS}}| < \Delta_1 + \Delta_2$ and leak to $S_1$. Hence, one expects heating in this lead for these bias values in junctions with higher $\eta$ and this model shows it, see Fig. 4.1(d). There is a sharp increase in cooling power at $e |V_{\text{SIS}}| = \Delta_2 - \Delta_2$ when the singularities in both DOSs align as in Fig. 4.1(b), although the more subgap states there are, the more smeared this maximum is.

### 4.3 Quasiparticle poisoning in the SINIS ST

Before describing the results of Publication I, evidence of the use of the SINIS ST as a probe for the QP density in one of its leads is provided. The operation of the SINIS ST has been already described from the picture described in Fig. 2.5(d). This description suggests that the bare operation of the device creates excitations in both superconducting leads. In fact,
Detection of quasiparticle extraction with the SINIS ST

**Figure 4.2.** SINIS ST as a QP probe. (a) False-color scanning electron micrograph; blue designates superconductor and red normal-metal. (b) Current plateaus measured at $f = 10$ MHz (filled circles) zoomed to $I \approx e f$, legend shows the used bias voltage for the measured data. Solid lines are simulated. (c) As in panel (b) for $f = 40$ MHz. (d) QP temperature in the narrow lead deduced from current simulations as those of panels (b) and (c) for different operation frequencies. (e) QP density extracted from data of panel (d) and Eq. (2.32) as a function of operation frequency. (f) Calculated current at the plateau as a function of the QP density for different operation frequencies.

This is directly demonstrated in Pub. II. It was also shown before through assignment of non-zero QP temperatures in Markovian models with which current plateaus were accurately reproduced [18]. Additionally, it has been concluded that the ultimate challenge for reaching metrological accuracy in SINIS STs is the QP poisoning generated by its operation and environment [3, 18]. A more concerning and restrictive feature is the fact that at higher operation frequencies QPs are injected faster and concentrate more around the junctions. Therefore, the concentration of QPs increases with the operation frequency. In Pub. I this has been revisited in a more controlled way.

It was demonstrated in Ref. 18 that diffusion of QPs is less effective in a narrow and long lead. Furthermore, in quickly opening, wedge-like leads these diffuse more efficiently. If such leads are then contacted to normal-metal traps, then the QPs are efficiently evacuated. Therefore, in order to isolate the poisoning, the devices of Pub. I are designed with one narrow and long lead while the other one is wedge-like, see Fig. 4.2(a). Two examples of pumping plateaus generated from this device are shown in Figs. 4.2(b) and (c) where the plateaus are generated at $f = 10$ MHz and $f = 40$ MHz. Each panel shows three curves of current emitted with different bias voltages (filled dots) and each curve has been simulated (solid lines).
with the Markovian model explained in Subsection 2.5.2. One modification was done to this model for Pub. I, the temperature of the normal-metal island was not a free parameter but solved for self-consistently, similarly as it is done for DC simulations. This temperature is considered to be periodically steady. QP temperature is also constant through the operation, given that the operation frequency is always faster than the effective excitation relaxation rate [74]. However, only the temperature of the narrow lead is considered as a free parameter and the one of the wide lead is considered to be the same as the bath temperature [18]. In conclusion, the accurate reproduction of the current plateaus through these simulations allows to determine the QP temperature of the narrow lead (Fig. 4.2(d)) and also the QP density \( n_{QP} \) in the vicinity of the junction (Fig. 4.2(e)) through Eq. (2.32). This reveals the possibility to use the SINIS ST current as sensitive QP probe and also that QP density increases with operation frequency. The current at the plateau shows enough sensitivity to QP density although it decreases with increasing operation frequency, see Fig. 4.2(f). Hence, the SINIS ST is a suitable platform to evaluate active QP extraction by \( S_1IS_2 \) junctions.

### 4.4 Active QP suppression in the SINIS ST

In the two previous sections it was shown that biased \( S_1IS_2 \) junctions can extract QPs from \( S_1 \) and that SINIS STs with the geometry shown in Fig. 4.2(a) can function as probes for \( n_{QP} \) in the narrow lead. In Pub. I these two facts come together to achieve an in situ control of QPs, both injection and extraction. The used geometry is similar to the one shown in Fig. 4.2(a). However, two additional superconducting leads are created and deposited at the bottom of the ST forming tunnel junctions with the long narrow lead in order to demonstrate and quantify QP extraction, see Fig 4.3(a). These two leads join far from the device forming a Superconducting QUantum Interference Device (SQUID) and have a higher superconducting gap. Such geometry allows suppression of the supercurrent flowing through the \( S_1IS_2 \) junctions by inducing a magnetic flux \( \Phi = \Phi_0/2 \), with \( \Phi_0 = h/(2e) \) the flux quantum. With this, the energy provided by the bias applied to the SQUID \( V_{SIS} \) is spent in QP transport. The experimental setup is briefly sketched in Fig. 4.3(a).

In broad terms, the system dynamics can be described from Fig. 4.2(b). The operation of the SINIS ST creates one QP per operation cycle in the narrow lead. For these, the relaxation channels are diffusion towards far away normal-metal traps, recombination and scattering. The first one of these has been proven to be inefficient for long and narrow leads [18] and the other two are exponentially suppressed at low QP temperature [74]. In spite of this, the QPs can tunnel through the \( S_1IS_2 \) junction when
Figure 4.3. Active QP extraction in a SINIS ST. (a) Scanning electron micrograph of a model device for QP extraction, red is normal-metal, light blue is superconductor with small gap and dark blue is superconductor with larger gap. (b) Sketch of the operation of QP extraction and probing. The ST operation injects a QP (yellow circle) from the island (light red) into the superconducting lead (light blue), these are extracted into S₂ (dark blue) but also scattered recombined and diffused. (c) Measured current plateaus for \( f = 5 \text{ MHz} \) at \( V_{\text{SIS}} = 0 \) (filled circles) and \( V_{\text{SIS}} = 50 \mu V \) (open diamonds) against driving amplitude, solid lines are simulations, the legend indicates the transistor bias. (d) As in panel (c) for \( f = 80 \text{ MHz} \). (e) Estimated temperature of the narrow ST lead as a function of the operation frequency, the legend indicates the applied \( V_{\text{SIS}} \), notice the reduction in temperature for non-zero \( V_{\text{SIS}} \). (f) Estimated \( n_{\text{QP}} \) in the narrow lead as function of \( f \) for different \( V_{\text{SIS}} \), see the legend in panel (e).

properly biased without flowing back, thus reducing the QP density in the superconductor. This reduction is manifested in the accuracy of the current emitted by the ST, a high \( n_{\text{QP}} \) drives the current away from the ideal value in the first plateau \( I = e f \) and makes it more sensitive to the bias voltage \( V_b \). Conversely, a low \( n_{\text{QP}} \) reverses these effects, Figs. 4.3(c) and (d) illustrate this by showing current plateaus at different biases for \( V_{\text{SIS}} = 0 \) (filled circles, cooler-off case) and \( eV_{\text{SIS}} \sim \Delta_2 - \Delta_1 \) (open diamonds, cooler-on case) for \( f = 5 \text{ MHz} \) and \( f = 80 \text{ MHz} \), respectively. Compared to the cooler-off case, the current is more accurate and less sensitive to bias in the cooler-on regime for both operation frequencies, confirming the reduction of \( n_{\text{QP}} \) when the \( S_1\overline{S_2} \) junction is properly biased. The results of the simulations that aid with the quantification of \( n_{\text{QP}} \) are shown as solid lines in Figs. 4.3(c) and (d).

The QP temperatures extracted from the simulations are shown in Fig. 4.3(e). As expected, these increase with operation frequency. Added to the case for unbiased \( S_1\overline{S_2} \) junction, two other cases of \( V_{\text{SIS}} \neq 0 \) are shown. In the first case, the results are comparable to those of Fig. 4.2(d), where no \( S_1\overline{S_2} \) junctions are present. The rate of QP injection is the same in the correct turnstile operation of any device of this type. One might think that
the energy of the injected QPs depends on the parameters of the device, specifically the charging energy and tunnel resistance. However, these energies are $\sim \Delta_1$ on average and the overall distribution of QP energies depends mainly on the value of the superconducting gap. Thus, one should expect similar QP temperature for devices with similar lead geometries and gaps, as is the case. This will be clear when Pubs. II and III are discussed in Chapter 5. Furthermore, the QP temperature is reduced for $eV_{\text{SIS}} < \Delta_1 + \Delta_2$, this reduction is more drastic for $eV_{\text{SIS}} \sim \Delta_2 - \Delta_1$, red dots in Fig 4.3(e). The corresponding densities can then be determined from Eq. (2.32), see Fig. 4.2(f). According to a model based on heat conduction $n_{\text{QP}}$ with no active extraction must hold [18]

$$\nabla^2 n_{\text{QP}} = \lambda^2 (n_{\text{QP}} - n_{\text{QP},0}). \quad (4.8)$$

Where, $\lambda^2 = (k_B/\Delta)^{1/2} \sqrt{2d} / (\sqrt{\pi} \rho_n \sigma_T)$ with $d$ the lead thickness, $\rho_n$ the resistivity of the film in normal-state and $\sigma_T$ the conductance of a tunnel-contacted normal-metal trap. The injection is taken into account by setting the boundary condition $-A \kappa_S \nabla T = P_{\text{inj}}$ where $A$ is the cross-sectional area of the lead, $\kappa_S$ is the heat conductivity of a BCS superconductor given by [164]

$$\kappa_S = \frac{2 \Delta^2}{e^2 \rho_n T} e^{-\Delta/k_BT}, \quad (4.9)$$

and $P_{\text{inj}} = \Delta_1 f$ which is a good approximation as will be seen in Chapter 5. One can ignore the normal-metal trap as for the case of the device in Fig. 4.3(a) and hence

$$n_{\text{QP}} = e^2 D (E_F) P_{\text{inj}} \rho_n l \left( \frac{\pi}{2k_BT \Delta_1} \right)^{1/2} + n_{\text{QP},0}. \quad (4.10)$$

In Eqs. (4.8) and (4.10), $n_{\text{QP},0}$ is a background QP density in the regime $f = 0$, also $l$ is the length of the lead and $w$ its width. The solid line in Fig. 4.3(f) shows results of Eq. (4.10) using the appropriate parameters of the device. These calculations are in qualitative agreement with data for $V_{\text{SIS}} = 0$, showing that in this system the resulting QP densities are product of an equilibration between generation and diffusion. However, in this particular calculation $\rho_n \sim 90 \Omega\text{nm}$ while a lower resistivity $\rho_n \sim 31 \Omega\text{nm}$ in the normal-state was measured.

More importantly, for $eV_{\text{SIS}} \neq 0$ $n_{\text{QP}}$ decreases and it is suppressed by an order of magnitude when $eV_{\text{SIS}} \sim \Delta_2 - \Delta_1$. The biased $S_1\text{IS}_2$ junction also achieves QP suppression in the regime $f \to 0$ in the same proportion. In spite of this, a non-zero density of the order of tens of QPs per cubic micrometer remains as a product of interactions with the environment. This is evident from an extrapolation of the data in Fig. 4.3(f) to $f = 0$. This extraction of quiescent QPs should be possible to detect as a reduction of the subgap DC current of the transistor as done in Ref. 18.

The reduction of QPs by an $S_1\text{IS}_2$ junction and how this extraction is manifested in the current of a SINIS ST have been shown. It is now possible
to establish the properties shown in Fig. 4.1(d), mainly that the maximum extraction power is achieved at $e |V_{SIS}| = \Delta_2 - \Delta_1$ and that the extraction is kept as long as $e |V_{SIS}| < \Delta_2 + \Delta_1$. This is done by tuning the amplitude of the gate signal to the middle of the plateau, sweep $V_{SIS}$ and verify the current decrease, see Fig 4.4(a) for measurements of this kind at different ST bias voltages. The already mentioned reduction in QP density is also observable in Fig. 4.4(b). Maximum reduction in QP density is achieved for $eV_{SIS} = \Delta_2 - \Delta_1$ as shown in Fig. 4.4(c) and pointed by the dashed line in the figure. This indicates a maximum in cooling power for this biasing condition. Furthermore, Fig. 4.4(a) is symmetric with respect to $V_{SIS} = 0$. Thus, the maximum cooling power of the $S_1S_2$ junction happens when $e |V_{SIS}| = \Delta_2 - \Delta_1$, as expected from the calculations in Section 4.2. However, the features of the extraction peak are not clear in this figure. In Pub. I this is ascribed to the high Dynes parameter of the $S_1S_2$ junction and to low-frequency noise in the application of $V_{SIS}$ [28, 160]. Additionally, the peak around $eV_{SIS} = \Delta_1$ is in stark contrast with the features of Fig. 4.1(d). The contradiction is twofold, in junctions with low presence of sub-gap states, no heating is expected until $eV_{SIS} = \Delta_1 + \Delta_2$; and in junctions with a high Dynes parameter, a sustained heating for $eV_{SIS} > \Delta_1$ is expected. Instead a sharp peak of QP density is seen and then further reduction in $n_{QP}$ for $\Delta_1 < eV_{SIS} < \Delta_2$. The fact that this peak is seen close to $\Delta_1$ might suggest that energy is mainly invested in QP creation in $S_1$ and not in QP transport from $S_1$ to $S_2$. When the provided energy is increased, it is preferably spent in QP transport. On the other hand, a
model including multiple Andreev reflections [165] shows this peak and the one of Fig. 4.4(d) near $\Delta_2$ in the conductance of $S_1 IS_2$ junctions, besides additional peaks. Further considerations about temperature distribution in both superconductors might improve the model presented in Section 4.2. Finally, the heating of $S_1$ due to ohmic conduction of QPs predicted in Fig. 4.1(d) for $eV_{SIS} = \Delta_2 + \Delta_1$ is clear in Fig. 4.4(a). It is evident then that the SINIS ST can be used not only as a probe for $n_{QP}$ in its leads but it can also be used as detector of non-coherent transport in $S_1 IS_2$ junctions.
5. The SINIS ST as a frequency to power converter

This chapter is dedicated to Publications II and III. A new application of the SINIS ST as a frequency to power converter is presented here and further assessment of its working properties is shown. First, a brief introduction to the recent redefinition of the SI is done in Section 5.1. Then, the mechanism used in Pub. II for producing precise amounts of power is introduced in Section 5.2. Later, the measurement setup used for detecting the emitted power in Pub. II is justified in Section 5.3 with the help of some of the results from Pub. III. Evidence and characterization of the metrological potential is presented in Section 5.4. Following this, the discussion is extended towards an evaluation of the ultimate power accuracy achievable in this realization in Section 5.5. Finally, future proposals are presented in Section 5.6.

5.1 Revision of the SI

The SI is the system of units chiefly used around the world allowing to harmonize the measurements taken in different times and places. As such it is one of the cornerstones of the economy and trade worldwide [12]. Recently, calls for redefinitions of its units have been made [166]. The fact that some base units were defined in terms of artefacts, properties and implausible measurement setups raised criticism. Some units like the ampere were either unrealisable [12], its definition useless at extreme scales [167] (as for the kelvin) or, in the case of the mass prototype, simply fluctuating with time [168]. To solve these issues, the SI was revised in 2019 and four base units redefined [10, 12]. The goal was to put aside any definition referred to specific devices, experiments, materials or standards and redefine the units in terms of constants of Nature. This had already been done for the metre, the second and the candela providing lasting and reproducible measurements. However, the conference in charge of the revision went beyond this and redefined the whole system by fixing the numerical values of seven constants of Nature. With this, SI units, base and derivates are
defined in terms of these constants. The numerical values of the fixed constants are identical to CODATA 2017 ones [169] so that the revision did not entail important practical changes. In consequence, the hierarchy of the units (definition of derivates in terms of base units) is not valid any more, although the base units retain such designation. The fixed constants of Nature are: the speed of light \(c\), Planck’s constant \(h\), electron’s charge \(e\), ground state hyperfine transition frequency of the caesium 133 atom \(\Delta\nu_{\text{Cs}}\), Boltzmann’s constant \(k_B\), Avogadro’s number \(N_A\) and luminous efficacy of monochromatic radiation of frequency 540 THz \(K_{\text{cd}}\). In the works of this thesis, of special importance are \(e, \Delta\nu_{\text{Cs}}\) and in the case of Pubs. II and III, \(h\). Because of this, examples of how the ampere and the watt are obtained in terms of these constants are shown.

In the revised SI, the ampere is defined as

\[
1 \text{ A} = \frac{e\Delta\nu_{\text{Cs}}}{(1.602176634 \times 10^{-19}) (9192631770)},
\]

notice the similarities with Eq. 2.34 which gives the current generated by the SINIS ST. Compare this definition with the previous definition of the unit of current in terms of the force exerted on each other by two parallel current-carrying conductors of infinite length and negligible cross-section, which was impractical. In turn, the watt is defined as

\[
1 \text{ W} = \frac{h (\Delta\nu_{\text{Cs}})^2}{(6.62607015 \times 10^{-34}) (9192631770)^2},
\]

notice that this definition can be also written as \(\propto (h\Delta\nu_{\text{Cs}}) \Delta\nu_{\text{Cs}}\), so that it is analogous to the definition of Eq. (5.1). Such analogy is crucial for the foundations of Pubs. II and III.

5.1.1 Do we really need a new power standard?

It has been already said in the Introduction and in Section 2.5 that the redefinition of the SI calls for realizations of the units. These realizations are the measurement of a quantity together with its uncertainty, of the same type of the unit consistent with its definition [11]. This leaves enough freedom for creating standards for the many units of the system. The watt is no exception and as such some attempts to realize it have been made, mainly in the electric domain. These consist on the use of programmable Josephson voltage standards [170] or Josephson waveform synthesizers [171], which are chains of Josephson junctions for AC voltage sampling. These are always referred to as a definition of the watt in terms of the volt, ampere and ohm. On the other hand, and perhaps more interestingly, realizations of the watt are also pursued in the domain of radiometric metrology. These are based on the fact that nowadays emission of single photons is feasible, if one is able to generate \(N\) monochromatic
photons of frequency $\nu$ at a rate $f$, then the power emitted is $P = Nh\nu f$. Notice the similarity with the definition of watt in Eq. (5.2). Thus, single-photon metrological power standards comprise beautiful realizations that are fully consistent with the watt definition but also with the basic definition of power: an energy emitted $(Nh\nu)$ per unit of time $(f)$ in what has been coined in Pub. II as frequency-to-power conversion (FPC). Many metrological proposals for radiative power standards have been done [172–175] based on single-photon emission [176–178] or detection [179, 180]. Even though these alternatives for power standards based on FPC are captivating, photon emission efficiencies are still under 60% [178] which makes further improvements mandatory in order to meet metrological requirements. This, added to the fact that no FPC realizations for the watt outside the optical domain exist, called for efforts to solve these issues. In summary, the answer to the title of this subsection is yes. In the remaining of the chapter it is shown that Pubs. II and III contribute to extend proposals for power standards beyond electrical and optical realizations.

5.2 Frequency-to-power conversion by quasiparticle injection

It is in the context of FPC that the highlighted analogy between the definitions (5.1) and (5.2) is relevant. It was shown that realizations of FPC comprise elegant realizations of the watt and also that single-electron transport are the most reliable ones for the ampere in what could be called frequency-to-current conversion based on Eq. (2.34). Both resort to the most basic definition of the dimension they realize and its unit according to the revision of the SI. Furthermore, they transduce a frequency into another quantity. With these parallelisms one might wonder if FPC could be achieved in hybrid solid-state systems such as the SINIS ST, given that this device implements frequency-to-current-conversion. To answer this, a sequence similar to the one shown in Fig. 2.5(c) is used to understand how a simple driven NIS junction can achieve FPC, see Fig. 5.1(a). In this system, the normal-metal island is capacitively coupled to a gate electrode and driven with a radio-frequency (RF) signal, similar to the operation of the SINIS ST. At some point in the cycle, the chemical potential of the normal-metal charged with a single electron aligns to the edge of the superconducting gap of the lead with a singularity in DOS and hence the electron can tunnel rapidly through the insulating barrier. This creates a QP in the superconductor with energy close to $\Delta$, which diffuses through the superconductor with the dynamics described in Chapter 4. In the meantime, the gate driving evolution aligns the chemical potential of the discharged normal-metal with the gap edge. This enables a tunneling event of an electron from the superconductor into the normal-metal, creating a QP in there [60] again with an energy close to the gap. Later in time, the
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Figure 5.1. SINIS ST for frequency-to-power conversion. (a) Sketch of the periodic driving sequence of a NIS junction. When the chemical potential of the normal-metal island (left) is aligned with the superconducting gap edge of the lead (right), an electron tunnels. At some point of the driving cycle, the island is charged with one electron which tunnels out and creates a QP with energy $\sim \Delta$ in the lead. Then, the chemical potential of the discharged island is lowered and one electron tunnels into the island creating another QP with energy $\sim \Delta$ in the lead. At the end of the cycle, a total energy $\sim 2\Delta$ has been injected into the lead. (b) Colored electron scanning micrograph of the device together with the experimental circuitry used for FPC in Pub. II. Red denotes normal-metal, blue superconductor. (c) Zoom to the normal-metal island of the used SINIS ST.

System resets and the cycle starts again. In total, this operation leaves a net energy transfer of $2\Delta$ to the superconducting lead, which is done $f$ times per second when this operation is repeated. The power transferred to this lead is

$$P = 2N\Delta f.$$ (5.3)

The factor $N$ appears because more than two QPs can be created in the S lead when the driving provides large enough energy as in the single-electron turnstile. Notice the resemblance between Eqs. (5.3) and (2.34). This shows how FPC can be achieved in a NIS junction. This also shows that FPC is possible in a SINIS ST. In fact, the descriptions depicted in Fig. 5.1(a) and in Fig. 2.5(c) are analogous.

In Pub. II, FPC was demonstrated by operating the device depicted in Fig. 5.1(b). Such device consists of two main parts, the turnstile and the detector. The former has the usual composition of a SINIS ST, although the leads have been designed so that injected QPs are not easily relaxed, see Fig. 5.1(c). The latter part of the system is composed of a normal-metal trap (long red structures in Fig. 5.1(b)) which is contacted through insulating barriers to four superconducting leads (vertical blue structures in Fig. 5.1(b)). These bolometers are directly contacted to the turnstile leads so that QPs can transfer to the trap with no energy loss. In the following section, the use of these bolometers for FPC detection embedded in such a geometry is justified by means of a thermal model. Furthermore, the working principle of this detector is presented so that the FPC results can be discussed separately later.
5.3 Bolometric detection for FPC

This section consists of three parts. First, the use of SINIS structures as thermometers is described. Second, the working principle of the bolometer is described and its calibration shown. Finally, a thermal model developed in Pub. III which justifies the use of the bolometer in this case is discussed.

5.3.1 Thermometry with SINIS devices

The bolometer used for FPC detection requires measurement of the electronic temperature of the normal-metal trap. The NIS contacts formed between the normal-metal traps and the vertical superconducting fingers in Fig. 5.1(b) serve for this purpose [30]. This is because of the sensitivity to temperature of the current-voltage characteristics of these structures. In Pub. II, two of these contacts in series, forming a SINIS structure, were used for measuring. The typical low-temperature IV features of these are shown in Fig. 5.2(a). As expected for a large normal-metal island, a current blockade for bias between $\pm 2\Delta/e$ appears. It is in this blockaded zone that conductance of these structures is specially sensitive to the electronic temperature of the normal-metal, see Fig. 5.2(b). This conductance increases with increasing temperature, although its variation notably saturates at low and high temperatures due to subgap leakage and QP poisoning, respectively [31, 181]. In a variation of this, if the junctions are biased with a constant current source, the voltage drop works as a linear sensor of temperature within an interval, see Fig. 5.2(c). Such interval depends on the current bias applied to the junctions and its normal-state tunnel resistance [31]. Typically, one fits a linear curve (red line in Fig. 5.2(c)) to the linear region of a control calibration (blue dots in Fig. 5.2(c)). In this control calibration, the temperature of the mixing chamber is increased and the sample is let to thermalize so that the substrate phonons, normal-metal phonons and normal-metal electrons equilibrate, hence acquiring a common temperature. In conclusion, it is possible to track the temperature of the electronic ensemble in the normal-metal trap by using current-biased SINIS junctions.

5.3.2 Bolometer calibration

The second component of the FPC detector is the bolometer itself, that is, a device whose temperature rise is monitored in order to measure the energy absorbed by it. A simple demonstration that the normal-metal traps used in Pub. II and shown in Fig. 5.1(a) work as sensitive bolometers can be done by using the setup shown in Fig. 5.2(d). If the normal-metal strip is nominally identical to the one used in the FPC device, the setup works also as a calibration method for the employed detector. This was
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precisely the case in Pub. II. In this setup, the temperature of the strip is monitored by current-biased SINIS junctions. Furthermore, the energy to heat up the strip is provided by the power dissipated due to a current flowing through the film. This current is provided by a source consisting of a voltage source in series with a large resistance compared to that of the strip. Hence the desired current $I$ is set by the former resistor. This current flows with no dissipation through the superconducting wires clean contacted to the bolometer. Other two superconducting probes serve for monitoring the voltage $V$ dropped in the normal-metal strip. Hence, the power dissipated there is readily calculated as $P = IV$. At the same time the electronic temperature $T_e$ of the trap is measured. Plotting these data as in Fig. 5.2(e) it is possible to distinguish that

$$ P = IV = C \left( T_e^5 - T_0^5 \right), \quad (5.4) $$

with $C$ a constant and $T_0$ the mixing chamber temperature. Eq. (5.4) resembles Eq. (2.28) so that it is possible to ascertain that the interaction

---

**Figure 5.2.** SINIS junction as bolometer. (a) Current $I$ through a SINIS structure as a function of the applied bias voltage $V_b$. Data taken from a similar device to the one measured in Pub. II. (b) Current-voltage characteristics of the same device for different environment temperatures zoomed to subgap biases. The colors of the curves correspond to the colorbar, which indicates the mixing chamber temperature $T_0$ at which data were taken. (c) Typical calibration curve of a current biased SINIS thermometer. The voltage dropped in the structure $V_{th}$ is measured while varying the temperature $T_0$ of the mixing chamber, see the blue dots. There is a region that can be well fitted by a linear function, see the solid red line. (d) Colored scanning electron micrograph of the system used for calibrating a bolometer and setup, blue is superconductor and red normal-metal. A current $I$ generated by a voltage source in series with a large resistance is passed through the normal-metal. The voltage $V$ dropped in it is measured while the voltage $V_{th}$ dropped in the current biased SINIS junctions is monitored. (e) Power injected to the normal-metal $P_{inj} = IV$ (green dots) as function of the difference between the fifth powers of the normal-metal electronic temperature $T_e$ and mixing chamber temperature. Eq. (5.4) is fitted to the data, see the solid black line. (f) Zoom-in to the zone of low injected power. Different lines correspond to different values of $C$ and fit well the data. This increases uncertainty in this parameters. The dashed line corresponds to the fit of panel (e).
resulting in an increase of electronic temperature is the electron-phonon coupling [31, 66] and that the phonon bath temperature corresponds in good approximation to the mixing chamber temperature [67]. Even though $C$ would correspond to $\Sigma V$, in Pub. II this quantity is calibrated as a whole instead of only estimating $\Sigma$. This rules out any uncertainty in the volume considered in the electron-phonon interaction. Such calibration is done by fitting a straight line to the data of Fig. 5.2(e), see the black solid line. In the range of powers relevant for FPC in a hybrid turnstile (from tens of attowatt to a few femtowatt) the sensitivity of the bolometer persists although more uncertainty in $C$ appears. This uncertainty is evident from Fig. 5.2(f) where it is seen that lines corresponding to different values of $C$ also fit well the calibration data. In Pub. II, an uncertainty of $\sim 10\%$ in this parameter is estimated. This uncertainty will have to be taken into account in the frame of FPC as a power standard. In summary, mixing the temperature sensitivity of SINIS junctions with the sensitive response to power injection of normal-metals through electron-phonon interaction helps to create a proper tool for FPC detection.

5.3.3 Pertinence of SINIS bolometric detection for FPC

It has been shown that the detector used is effective for measuring a power dissipated in the normal metal. However, given that a known power is injected to the superconducting lead ($\sim 2\Delta f$), how much of this power can be detected by the bolometer in the geometry shown in Fig. 5.1(b)? In other words, how much of the QP power is dissipated in the short superconducting lead? How much of the power that arrives to the normal-metal is dissipated there? And, how much leaks through the long superconductor?

These questions were answered through a model of heat transport in Pub. III. The system modelled is depicted in Fig. 5.3(a). It consists of three directly contacted main sectors, superconductor $S_1$ of length $\ell_1$, normal-metal $N$ of length $\ell_N$ and superconductor $S_2$ of length $\ell_2$. Notice that this geometry is similar to the composition of the leads in Fig. 5.1(b). Within each sector, the power in each infinitesimal section $Q(x)$ must satisfy conservation, that is, the continuity equation. In the model, it is assumed that the only energy dissipation channel is the electron-phonon interaction power $\dot{Q}_{e-ph}$ given by Eq. (2.28) for the normal-metal. This is reasonable since the QP density in the superconducting leads is low and the power is transported by QPs there and by electrons in the normal-metal. Hence

$$\frac{d}{dx} Q(x) = -\dot{q}_{e-ph}. \quad (5.5)$$

Where, $\dot{q}_{e-ph}$ is the power per unit length of the electron-phonon interaction. Here, the problem is already reduced to one-dimensional transport. In order to effectively add energy transport to the model, Fourier's law is...
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Figure 5.3. Efficiency of bolometric detection of FPC. (a) Scheme for the transport model of heat generated by FPC from a driven normal-metal island $N_1$. The power $P$ is injected via electron tunneling (yellow dot) and diffuses through the superconductor $S_1$ and is later dissipated as heat $Q_{e-ph}$ via electron-phonon interaction in the normal-metal $N$ at nearly constant temperature $T_N$. Some of the power leaks to the superconductor $S_2$ which is thermalized to bath temperature $T_0$ at the other end. (b) Trapping efficiency $\theta$ as function of bath temperature calculated by solving Eq. (5.7) numerically (dots) for different injected powers given by the driving frequencies in the legend. The trapping efficiency from Eq. (5.11) is also shown as solid black line. Inset: typical temperature difference profile along the structure.

Invoked, therefore

$$\frac{\dot{Q}(x)}{A_i} = -\kappa_i \frac{d}{dx} \theta(x). \quad (5.6)$$

Where, $\theta(x) = T(x) - T_0$, with $T(x)$ the temperature of the system at distance $x$ from the turnstile junction and $T_0$ is the bath temperature. Additionally, $\kappa_i$ is the heat conductivity of the sector $i$ and $A_i$ is the corresponding cross-sectional area. For the case of superconducting sectors, the conductivity is simply denoted by $\kappa_S$. Replacing Eq. (5.6) into Eq. (5.5) gives

$$\frac{d}{dx} \left[ -A\kappa_i \frac{d}{dx} \theta(x) \right] + \dot{q}_{e-ph} = 0. \quad (5.7)$$

Equation (5.7) is solved for each section separately. In order to obtain $\theta(x)$, the following conditions are imposed. First, the power injected to the system at $x = 0$ is $P$, which for the case of FPC is $\sim 2\Delta f$. Second, the energy current $\dot{Q}(x)$ is continuous in each interface between superconductor and normal-metal. Third, the temperature difference is similarly continuous in the interfaces. Finally, since $S_2$ is intentionally contacted to a large normal-metal trap at its right end, then $\theta(\ell_1 + \ell_N + \ell_2) = 0$.

Furthermore, material consideration have to be taken into account. This is done by introducing the appropriate expressions for the heat conductivity and the electron-phonon coupling interaction. In the normal-metal, $\dot{q}_{e-ph} = \Sigma_N A_N (T^5 - T_0^5)$ and $\kappa_N$ is given by the Wiedemann-Franz law. In the superconducting sectors, $\dot{q}_{e-ph} = a\Sigma S A_S T_0^4 e^{-\Delta/k_B T} \theta$ [74] with $a \approx 5$ and $\kappa_S$ is given in Eq. (4.9). With all these, plus the boundary and continuity conditions, the overheating of the system $\theta(x)$ can be uniquely determined for a given set of system parameters. However, since this model was designed in order to verify that the proposed geometry is appropriate for
detecting FPC, it is more important to calculate the energy current through the system. This is readily done using Eq. (5.6). Additionally, the power dissipated in the normal-metal sector is given by \( \dot{Q}_N = \dot{Q}(\ell_1 + \ell_N) - \dot{Q}(\ell_1) \).

How much energy is dissipated there in comparison to the injected quantity can be estimated with

\[
\vartheta = \frac{\dot{Q}_N}{\dot{P}},
\]

which is called trapping efficiency in Pub. III. If this quantity is equal to 1, then all the injected energy is dissipated in the normal-metal and not elsewhere in the system. Hence, it is available for measuring via bolometric detection as proposed for FPC. Otherwise, bolometric detection fails to account for the whole energy injected by the FPC device as it is either dissipated in \( S_1 \) or leaked to \( S_2 \). In Pub. III, Eq. (5.7) had to be solved numerically, since it is a non-linear equation, and the dependence of \( \vartheta \) on \( T_0 \) calculated. A typical resulting temperature profile is shown in the inset of Fig. 5.3(b). The strong variation of \( \theta \) in the whole \( S_1 \) sector (first blue curve) and its weak change in the entire \( S_2 \) suggests that almost all the energy is dissipated in \( N \). This is confirmed with the results shown in the main panel of Fig. 5.3(b) for two device operation frequencies (different injection powers), see the filled circles. Below \( T_0 \sim 120 \text{ mK} \), \( \vartheta \sim 1 \) and even though it decreases for higher temperatures, it is around 95\% for 190 mK, which is in line with results from Ref. 144. These numbers are calculated with material and geometric parameters corresponding to the design used in Pub. II and shown in Fig. 5.1, hence justifying its use for bolometric detection of FPC.

In addition to serving for justifying the design of FPC, the presented model may also be useful for optimizing it. In Publication III such an analysis was done by making two assumptions which helped simplifying Eq. (5.7). It was assumed that the power injected to the system is small or the bath temperature is high, then \( \theta \ll T_0 \). Furthermore, the small QP density approximation remains valid \( k_B T \ll \Delta \). Therefore, Eq. (5.7) in the superconducting sectors becomes

\[
\frac{d^2}{dx^2} \theta(x) - \lambda^2 \theta(x) = 0.
\]

Here, \( \lambda = \sqrt{\alpha \Sigma e^2 \rho_n / (2\Delta^2) T_0^{5/2}} \). Besides, a constant temperature was assumed for the normal-metal sector. This is justified by looking at the numerical solution of the full model, see the temperature profile of the N sector in the inset of Fig. 5.3(b). Equation (5.9) is solved in the superconducting sectors with the same boundary and continuity conditions as for Eq. (5.7). Thus, the trapping efficiency in this simplified approach is given by

\[
\vartheta = \frac{\text{sech}(\lambda \ell_1)}{1 + \varpi [A_1 \tanh (\lambda \ell_1) + A_2 \coth (\lambda \ell_2)] T_0^{-5} e^{-\Delta / k_B T_0}}.
\]
with \( \varpi = \frac{2 \lambda \Delta^2}{(5 e^2 \rho_n \Sigma_N \mathcal{V})} \), \( A_i \) the cross-sectional area of sector \( S_i \) and \( \mathcal{V} \) the volume of the normal-metal. For typical systems based on aluminium, such as the one depicted in Fig. 5.1(b), and at typical cryogenic temperatures, \( \lambda \ell_i \ll 1 \). Thus, Eq. (5.10) becomes

\[
\vartheta \approx \left( 1 + \frac{\varpi A_2 \lambda T_0^{-5} e^{-\Delta/k_B T_0}}{\lambda \ell_2 T} \right)^{-1}.
\]

Equation (5.11) reproduces accurately the characteristics of \( \vartheta \) for the full model, see the solid line in Fig. 5.3(b). Hence, it and Eq. (5.10) are a good starting point for optimizing the FPC detection scheme. This discussion will be expanded towards the end of the chapter, after experimental evidence of FPC in the device of Fig. 5.1(b) has been presented.

### 5.4 ST for frequency-to-power conversion

Having shown how bolometric detection and the proposed geometry are suitable for measuring FPC, it is now possible to focus on the experimental evidence of the phenomenon. As previously discussed, FPC should be feasible in driven NIS junctions. However, a SINIS ST operation also relies on the same QP injection principle making it a suitable platform for testing FPC. In fact, operation of a single driven NIS junction is nothing else but the zero bias operation of the SINIS ST and thus this latter device was tested in Pub. II. Because of this, the description is divided between the zero bias and non-zero-bias regime.

#### 5.4.1 Zero bias regime

The main result of Pub. II is that FPC is achievable in a SINIS ST. The data plotted in Fig. 5.4 allow to make this conclusion. To obtain these, the temperature of both normal-metal traps was monitored and then transduced to power through Eq. (5.4). Then, the powers injected to both leads (\( P_i \), with \( i = R, L \) for left and right, respectively) are added so that \( P_T = P_L + P_R \). The device was operated at zero bias and with \( f = 80 \text{ MHz} \) for several gate amplitudes \( A_g \) and gate offsets \( V_{0g} \) such that \( V_g = V_{0g} + A_g/2 \sin (2 \pi f t) \). Since \( V_b = 0 \), no current flows through the system so that energy current can be generated with no net particle flow within one driving cycle. The resulting operation is the same as expected for a NIS junction. It is remarkable that the main features of Fig. 5.4 are shared with measurements of the same type for current through the biased ST. On one hand, clear steps against \( A_g \) appear making this realization suitable for a watt realization. On the other hand, a diamond pattern appears when sweeping \( V_{0g} \). This results from the Coulomb interaction and reveals the nature of power injection through QP tunneling as described. Additionally, several plateaus may appear for higher \( A_g \). However, most
remarkable is that $P_T \approx 2N\Delta f$ at the plateaus. This confirms that FPC is possible in solid-state devices and hence extending the applications of the SINIS ST, in which two potential metrological standards coexist.

A closer inspection to measurements done with $C_g V_{0g}/e = 0.5$ allowed in Pub. II to assess the accuracy of FPC in a SINIS ST compared to Eq. (5.3). It is clear that FPC is possible for a wide range of frequencies allowing generation of a similarly wide range of power magnitudes, see Fig. 5.5(a).

This conversion is still robust against the driving amplitude although clear deviations from the expected value appear as shown in Fig. 5.5(b). Such deviation appears to increase with $f$ and with $A_g$, it will be shown and argued later that this is actually the case in general. It was mentioned in Subsection 2.5.2 that $P_T$ could also be modelled with the Markovian model described there. The solid lines in Fig. 5.5(b) are obtained within this model and by adding the resulting quantities $P_{L/R}$. The dependence of the data on $A_g$ is closely followed by the calculations and if uncertainty on the used bolometer calibration (see Fig. 5.2(f)) is taken into account, quantitative agreement is achieved.

The dynamics of QP injection in this regime is easily understood by looking at the behavior of the average instantaneous power and current, see Fig. 5.5(c). For this, Eq. (2.35) is solved numerically and the vectors $q^L/R(t)$ and $b(t)$ averaged with the help of the obtained $p(t)$. Then, the evolution in time of these averages can be plotted, however it is more useful to plot these against one of the $\delta\epsilon_{L/R}/\Delta$, notice that for $V_{b} = 0$, $\delta\epsilon_{L} = \delta\epsilon_{R}$. In the case of Figs. 5.5(c) and (e), the power and current
Figure 5.5. Frequency-to-power conversion at zero bias. (a) Power $P_T$ measured (dots) at $n_{0g} = 0.5$ against gate amplitude $A_g$. Plateaus form near $2\Delta f$. The legend indicates the driving frequency, solid lines correspond to simulations based on the Markovian model. Dashed red lines indicate the values given by Eq. (5.3). (b) Close up of panel (a) to the power plateaus. (c) Average total power (red line) and average current through the left junction (blue line) as function of the evolution of energy difference for filling the island. (d) Sketch of frequency-to-power conversion at zero bias. When the island (center) is charged and its chemical potential aligns with the gap edge of both leads (right and left), the electron can tunnel through any junction. Later in the cycle it can tunnel back also through any junction. (e) Average total power around $\delta\epsilon^+ = \Delta$ for different driving frequencies given in the legend, notice that the peaks become broader with larger operation frequency.

are plotted against $\delta\epsilon^+$. Notice that narrow peaks of power (red line) and current (blue line) appear around $|\Delta|$, this indicates that tunneling happens mainly around this energy. When $\delta\epsilon^+ = -\Delta$ then $\delta\epsilon^- = \Delta$ given that at $V_b = 0$, $\delta\epsilon^- = -\delta\epsilon^+$. Only the current through one junction is plotted, it is clear that the QP tunnels in and out of the island mainly through the same junction. This is sketched in Fig. 5.5(d) where it is clear that tunneling occurs stochastically through either junction, confirming the picture of Fig. 5.1(a) and the equivalence of SINIS ST and driven NIS junction. The increasing FPC error with increasing operation frequency can also be deducted from plotting the instantaneous power at different frequencies, see Fig. 5.5(e). It is clear that the peaks become less sharp for higher operation frequencies and broaden against $\delta\epsilon^+$, which indicates that tunneling happens for $|\delta\epsilon^+| \gtrsim \Delta$. Thus, the injected QPs might have higher energies and then, on average, the power transferred is higher than expected.

It remains yet unclear what determines which junction the QP tunnels through. The individual contribution of each lead to total injected power might hint towards an answer, see Figs. 5.6(a) and (b). It is evident that the
average power injected to each superconductor is not equal. In consequence, in the first plateau one of these is higher than $\Delta f$ and the other lower. In Pub. II a device with non-identical junctions was measured and as a result the inequality between $P_R$ and $P_L$ arises on average. Although it is possible that tunneling occurs for $\delta\epsilon_{L/R} \lesssim \Delta$ because of sub-gap states, $P_i < \Delta f$ does not necessarily means that a QP is injected to one of the leads with energy below $\Delta$. Rather, it means that for this lead tunneling is less frequent through the junction that connects it with the island. The device tested in Pub. II has the particularity that $R_R > R_L$ and $P_R < P_L$ in the first plateau. This points towards the idea that QP tunneling occurs more frequently through the less resistive junction, under the same energetic landscape single-electron tunneling is more likely through the more transparent junction. The following calculation strengthens this assertion.

In general, Eqs. (2.18) and (2.26) can be expressed as

$$\Gamma_{n \rightarrow n \pm 1}^{L/R} = \frac{1}{R_{L/R}} \chi_{n \rightarrow n \pm 1}^{L/R} \left( \delta\epsilon_{L/R}, T_N, T_S^{L/R} \right) \quad (5.12)$$

$$\dot{Q}_{n \rightarrow n \pm 1}^{N,L/R} = \frac{1}{R_{L/R}} q_{n \rightarrow n \pm 1}^{N,L/R} \left( \delta\epsilon_{L/R}, T_N, T_S^{L/R} \right). \quad (5.13)$$

The quantities $\chi$ and $\dot{q}$ are functions of the island charge state and temperatures as well as bias voltage. It has already been shown that at $V_b = 0$, $\delta\epsilon_L = \delta\epsilon_R \equiv \delta\epsilon$. Therefore, and making the important but reasonable
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assumption that $T_{LS}^{\text{S}} = T_{LR}^{\text{R}}$, it is evident that $\chi_{n \rightarrow n \pm 1}^{L} = \chi_{n \rightarrow n \pm 1}^{R}$ and $\dot{q}_{n \rightarrow n \pm 1}^{N,L} = \dot{q}_{n \rightarrow n \pm 1}^{N,R}$. As a consequence, the same reasoning extends to $\dot{Q}_{n \rightarrow n \pm 1}^{S,L/R}$. Therefore, the total power injected to the left/right superconductor is given by

$$P_{L/R} = \frac{1}{R_{L/R}} \sum_n p_n (\dot{q}_{n \rightarrow n + 1}^{S} + \dot{q}_{n \rightarrow n - 1}^{S}).$$  (5.14)

Here $\dot{q}_{n \rightarrow n \pm 1}^{S} = \dot{q}_{n \rightarrow n \pm 1}^{N} \mp \epsilon \pm \chi_{n \rightarrow n \pm 1}$. Now, it is clear that

$$\frac{P_{R}}{P_{L}} = \frac{1}{1/R_{R}} \sum_n p_n (\dot{q}_{n \rightarrow n + 1}^{S} + \dot{q}_{n \rightarrow n - 1}^{S}) = \frac{R_{L}}{R_{R}} = r.$$  (5.15)

This result should also hold on average in time, and can be easily testable experimentally. It is clear from Figs. 5.6(c) and (d) that this is the case and that the power ratio does not differ from $r$ by more than 10%. As suggested by Eq. (5.15) the ratio is highly independent of gate parameters, namely amplitude, frequency or offset. Such results confirm the understanding of how QPs are injected through either junction in the zero bias regime.

5.4.2 Non-zero bias regime

In Pub. II, the non-zero bias operation of a SINIS ST for FPC was also inspected. It was seen that this regime entails a different QP injection dynamics compared to the zero bias operation. When $V_{g} \neq 0$, frequency-to-current conversion and FPC converge in the SINIS ST, see Figs. 5.7(a) and (c). However, for such operation two different regimes appear, namely the high and low driving frequency ones. In the low frequency regime, accurate single-electron current appears so that the working principle of the turnstile is in line with what is depicted in Fig. 2.3(c). The QP injection dynamics in this regime is shown in Fig. 5.7(b). Notice that QPs have to tunnel through both junctions so that a non-zero current appears. This requires a power injection to both leads every driving cycle. Since both QPs are created at approximately the same energies, it is natural for the powers measured at each lead to be approximately equal. This is clearly shown in Fig. 5.7(a) (blue and red dots) together with calculations (solid lines). Also, $P_{R} \gtrsim P_{L}$ contrary to the zero bias operation. This is because the device tested in Pub. II holds $R_{R} > R_{L}$. In contrast to the zero bias case, in the non-zero operation tunneling through the most resistive junction is bound to happen. However, since the tunneling rate through that junction is smaller, then the energy of the tunneling electron has chance to increase due to the evolution of $V_{g}$ before tunneling occurs. Therefore, a QP with energy $\gtrsim \Delta$ will be created in the corresponding lead. On the other hand, the tunneling rate through the smaller resistance junction is larger and hence tunneling occurs for lower energies. Since the same amount of QPs is injected to both leads at the same rate, the power injected to the lead
contacted through the most resistive junction is slightly larger than the one injected through the other junction. In any case, for the non-zero bias regime the electron tunneling happens unidirectionally, allowing for an almost equal repartition of the total injected energy since tunneling occurs near the gap energy. Such direction is set by the polarity of the bias voltage and hence the direction of tunneling is not stochastic any more.

However, such unidirectionality can be broken in the non-zero bias operation by increasing the driving frequency. Curves in Fig. 5.7(c) were obtained with a higher frequency than in panel (a) and exhibit different features. One remarkable difference is that the current plateau bends down towards larger gate amplitudes. Such deviation from ideal single-electron current is easily explained from the stability diagram of an ST. In there, thresholds enabling electron tunneling against the bias direction are crossed before higher current plateaus are reached. This is further explored and explained in Chapter 6 when details of Pubs. IV and V are discussed. In brief, these thresholds might be crossed before the desired tunneling event takes place and the one against the bias occurs. As a
result, the current decreases. Fig. 5.7(d) illustrates this process, at high
enough amplitudes the chemical potential of the island can also align
with the gap edge of the lower QP branch of the lead in which a QP has
already been created. If driving is fast enough, then tunneling to the other
lead might not occur and the aforementioned alignment causes so-called
“back-tunneling”. This creates two QPs in the first lead and none in the
second one. Such process is similar to the dynamics of the zero bias regime.
However, there is still a preferred tunneling direction set by the bias, as
the non-zero current suggests. These back-tunneling events occur more
frequently through the less resistive junction as expected by extrapolating
the rationale of Eq. (5.15). Data and simulations shown in Fig. 5.7(c) back
this description by exhibiting a bend down also in $P_R$ at larger $A_g$. The
expression “fast enough driving” used in this description will be clarified
in Chapter 6 in the context of back-tunneling suppression.

The transition from the zero to the non-zero bias regimes is also studied
in Pub. II and some results shown in Fig. 5.8. In an ideal case the ST
current should be a step-like curve when measured at the plateau and
plotted against voltage bias. However, for non-zero temperature and for
increasing operation frequency these characteristics smear, see Fig. 5.8(a).
Other reason for such a smearing is that back-tunneling events are also
present at low biases even though driving frequency is low. Hence, for
a given operation frequency there is a minimum bias voltage at which
unidirectional electron transport is activated. For higher biases, $I \sim e f$
although not for every operation frequency, see inset of Fig. 5.8(a). This
behavior of the pumped current is also visible in the ratio $P_R/P_L$, see
Fig. 5.8(b). First, at zero bias all the curves converge to $r$, as required
by Eq. (5.15). As the bias increases and electron tunneling transits from
directionally stochastic to unidirectional the ratio approaches 1 as the
cartoons of Fig. 5.8(b) suggest. For low frequencies this transit happens at
lower bias voltages and the curve is more sharply peaked around $V_b = 0$.
On the contrary, at larger frequencies this transit happens at larger biases
because back-tunneling is more likely. Figs. 5.8(c)–(e) show this change of
transport regime for the current and power plateaus. The power injected
through the most transparent junction decreases with voltage bias while
the one injected through the most resistive one increases. This happens
as the current plateaus become flatter. Also, as the operation frequency
increases the statistical distribution of energies of the tunneling electron
broadens, as suggested by Fig. 5.5(e). This gives as a result a higher
average injected energy and hence higher total power also for non-zero
bias, see Fig. 5.8(f). Since $\delta \epsilon_{L/R}$ changes rapidly compared to tunneling
rates for high operation frequencies, the energy selectivity in tunneling is
very sensitive to bias voltage in such regime. Hence, the injected power is
less robust against $V_b$ for increasing operation frequency. The Markovian
model catches the features previously described, see solid lines in all panels.
Figure 5.8. Change of frequency-to-power conversion with voltage bias. (a) Pumped current $I$ as function of bias voltage $V_b$ measured (dots) at constant $A_g$ in the first plateau and at different driving frequencies given in the legend. Dashed red lines indicate the ideal pumped current. Inset, zoom in around $I = ef$. Solid lines are simulations. (b) Power ratio against $V_b$, measured (dots) and simulated (solid line). Solid red line indicates the value $r$ required for $V_b = 0$. Cartoons show how the charge transport regime transits from unidirectional to directionally stochastic with changing $V_b$ and how this relates to the power ratio. Colors correspond to legend of panel (a). (c) Current plateaus against $A_g$ for $f = 120$ MHz measured (dots) at different bias voltages corresponding to the legend of panel (d). Solid lines are calculated. Red dashed line indicates the ideal value $I = ef$. (d) Power injected to the left lead against $A_g$ for different biases corresponding to the legend. Notice how the measured data (dots) and calculations (solid lines) show that the power at the plateaus decreases as charge transport transits from directionally stochastic to unidirectional. Dashed red line indicates the ideal value. (e) As in panel (d) for the right lead. Notice how the power plateau increases with bias voltage as transport transits from directionally stochastic to unidirectional. (f) Total power generated $P_T$ against $V_b$. Measured data (dots) correspond to panel (b) and solid lines are calculations. Colors correspond to panel (a).

Some final remarks about the implementation of FPC first presented in Pub. II. There, the superconducting gap $\Delta$ was estimated \textit{in situ} in the DC characterization of the SINIS single-electron transistor. This allowed an accurate measurement of the constant relating frequency to power in Eq. (5.3). Having to measure $\Delta$ adds uncertainty to the overall measurement, which in Pub. II is estimated to be $< 1\%$. This ends the parallelism drawn between FPC and single-electron transport current standards based on Eq. (2.34). While in the latter the transducing constant ($e$) is fixed by definition in the former it needs to be measured and has an associated uncertainty. Finally, uncertainty arising from the calibration of the bolometer at relevant measured powers (see Fig. 5.2(f)) must also be taken into account.
5.5 Ultimate accuracy

It has been established that FPC can be realized in SINIS STs even at zero bias voltage, meaning that it should be possible to implement it in a driven single NIS junction. Also, it was shown that deviations from Eq. (5.3) occur and that operation parameters such as the driving frequency and amplitude affect how large these errors are. In Pub. III, attempts to derive expressions relating errors in energy injection and operation parameters were made. There, the case of a driven NIS junction is studied.

The first step to do this is to recognize that there is some probability that during a tunneling event a QP is created at an energy different than \( \delta \epsilon \), the change in normal-metal chemical potential. Therefore, the joint probability of creating a QP in the lead with energy \( E \) such that an energy \( \delta \epsilon \) is given to the electron tunneling out \((-\)) or in \((+\)) must be calculated. Such probability is given by

\[
P(E, \delta \epsilon) = -p'_\pm (\delta \epsilon) \Pi(E|\delta \epsilon). \tag{5.16}
\]

The probability in Eq. (5.16) consists of two parts. First one is the probability density of tunneling when the island chemical potential is \( \delta \epsilon \), \( -p'_\pm (\delta \epsilon) \). In Pub. III, the whole analysis was reduced to the tunneling event out of the island \((-\)). Hence, only half of the driving period is taken into account. It recognized that during the charging of the island in the second half of the period the injection dynamics are the same. Thus, \( p'_\pm (\delta \epsilon) \equiv p'(\delta \epsilon^-) \) and in terms of the probability of having one electron in the island at time \( t \), \( p(1, t) = p(t) \)

\[
p'(\delta \epsilon^-) = \frac{1}{\delta \epsilon^-} \frac{d}{dt} p(t). \tag{5.17}
\]

The second part is the probability of creating a QP of energy \( E \) in the lead through tunneling given that the island chemical potential is \( \delta \epsilon^- \). It is given by

\[
\Pi(E|\delta \epsilon^-) = \frac{n_S(E) f_N(E - \delta \epsilon^-) [1 - f_S(E)]}{\int dE' n_S(E') f_N(E' - \delta \epsilon^-) [1 - f_S(E')]}.
\tag{5.18}
\]

Therefore

\[
\langle E^n \rangle = \int dE E^n \int d(\delta \epsilon^-) P(E, \delta \epsilon^-). \tag{5.19}
\]

It is possible then to find the average energy added in half a driving cycle but also its standard deviation and more.

The system evolution is contained in \( p(\delta \epsilon^-) \). In order to calculate this, the master equation of the time evolution has to be solved, see Eq. (2.35). However, plain expressions for injection errors can only be achieved by doing some simplifications. Otherwise, the numerical processes used in this thesis for modelling experimental data should be used. In the first
place, only the part of the cycle in which the electron tunnels out of the island is studied. Hence, Equation (2.23) becomes
\[
\frac{d}{dt}p(t) = -\Gamma(t) p(t),
\]
where, \( \Gamma = \Gamma_{1 \rightarrow 0} \). Equation (5.20) has the solution
\[
p(t) = p(0) e^{-\int_{0}^{t} \Gamma(t') dt'},
\]
(5.21)
Furthermore, \( T = 0 \) and no subgap states (\( \eta = 0 \)) are assumed. Thus, \( \Gamma = 1/(e^2 R_T \sqrt{(\delta \epsilon^-)^2 - \Delta^2}) \). Finally, it is possible to reduce the whole injection dynamics to a small interval of time in which \( \delta \epsilon^- \sim \Delta \). Quite generally \( \delta \epsilon^-/\Delta = \dot{v} t \), with \( \dot{v} \propto E_c A_g f \) constant. Under these assumptions, Eq. (5.21) becomes
\[
p(v) = \exp \left[-1/(2\Omega) \left(v \sqrt{v^2 - 1} - \ln \left[v + \sqrt{v^2 - 1}\right]\right)\right].
\]
(5.22)
We set \( \Omega = e^2 R_T \dot{v}/\Delta \), coined in Pub. III as the dimensionless frequency. For low \( \Omega \)
\[
p(v) \approx \exp \left[-2\sqrt{2}/(3\Omega) (v - 1)^{3/2}\right].
\]
(5.23)
For this probability, the deviation of the average injected energy from the value required by ideal FPC (\( \Delta \)) is given by
\[
\frac{\langle E \rangle}{\Delta} - 1 \approx \frac{\Gamma (2/3)}{3^{3/4}} \Omega^{2/3} \approx 0.313 \Omega^{2/3}.
\]
(5.24)
Here \( \Gamma(x) \) is the gamma function. Hence, errors in energy injection increase with increasing operation frequency, driving amplitude, charging energy and tunnel resistance. One of the important results of Pub. III is that a compromise between \( R_T \) and \( f \) can be found in order to inject larger powers with low errors. For lower \( R_T \) and \( E_c \), larger operation frequencies can be used so that \( \Omega \) is kept constant with constant injection errors.

Additionally, higher moments of \( P(E, v) \) are calculated in Pub. III. As a result, the standard deviation of the injected energy \( \sqrt{\delta E^2} = \sqrt{\langle E^2 \rangle - \langle E \rangle^2} \) is readily calculated and given by
\[
\frac{\sqrt{\delta E^2}}{\Delta} = \left(3/2\sqrt{2}\right)^{2/3} \sqrt{\frac{\Gamma (7/3)}{5} - \frac{\Gamma (5/3)}{9} \Omega^{2/3}} \approx 0.400 \Omega^{2/3}.
\]
(5.25)
The zero-frequency noise in power generation arising from the stochastic nature of energy injection is proportional to \( \sqrt{\langle \delta E^2 \rangle} / n \). Typically, the number of averaged cycles in FPC is of the order of \( n = 10^6 \). Hence, errors in power arising from injection noise are much less than the systematic ones, which are given by Eq. (5.24), and can be neglected. The third central moment can also be calculated and hence the skewness of \( P(E, v) \)
\[
\frac{\langle (E - \langle E \rangle)^3 \rangle}{\langle (E - \langle E \rangle)^2 \rangle^{3/2}} = \frac{\langle \delta E^3 \rangle}{\langle \delta E^2 \rangle^{3/2}} \approx \frac{0.520^3 \Omega^2}{0.400^3 \Omega^2} = 2.197.
\]
(5.26)
Therefore, the distribution of injected energies is ideally leaning towards $E > \langle E \rangle$ for any operation and device parameter.

Equations (5.24)–(5.26) are plotted in Fig. 5.9 and compared to numerical calculations using the method described in Subsection 2.5.2. Notice that for low Dynes parameter the approximations provided by the simplified model follow the numerical calculations. Therefore, they can be regarded as a reliable estimation of the statistics of energy injection in FPC and its ultimate accuracy.

For calculating higher energy moments in the framework of the averaging method described in Subsection 2.5.2, it is necessary to find a vector $q^{(n)}$ such that

$$
\frac{d}{dt} \begin{bmatrix}
\langle E^n \rangle_s (t) \\
q^{(n)} (t)
\end{bmatrix} =
\begin{bmatrix}
A (t) & 0 \\
q^{(n)} (t)^T & \langle E^n \rangle_s (t)
\end{bmatrix}.$$

(5.27)

Where the subscript $s$ implies once again averaging over all the considered charge states, and $A$ and $p$ are as in Eq. (2.35). The components of the correct vector are $q_i^{(n)} = \dot{Q}_{i \rightarrow i+1}^{(n)} + \dot{Q}_{i \rightarrow i-1}^{(n)}$, where

$$
\dot{Q}_{i \rightarrow i\pm 1}^{(n)} (\delta \epsilon \pm) =
\frac{1}{RT e^2} \int dE \left( E - \delta \epsilon \pm \right)^n n_S \left( E - \delta \epsilon \pm \right) f_N (E) \left[ 1 - f_S (E - \delta \epsilon \pm) \right].
$$

(5.28)

These quantities are derived by extrapolating from Eq. (5.19).
The average injected energy drops below $\Delta$ at higher Dynes parameter and low $\Omega$, see the numerical calculations in Fig. 5.9(d). This is understood from the increasing existence of more subgap states with higher Dynes parameter. When $v \lesssim 1$ and subgap states are available, then tunneling rates are non-zero although small. If $v$ varies slowly compared to tunneling rates around this point, the electron tunnels. For faster variations of $v$, such that the tunneling rates below the gap are negligible in comparison, the probability of tunneling is also negligible. Such conditions of slowly varying $v$ around subgap states might be achieved at low driving frequencies and low driving amplitudes. Also, since tunneling rates are inversely proportional to tunnel resistance, having more transparent junctions increases the likelihood of having $\langle E \rangle < \Delta$. The fact that low $\Omega$ promotes subgap tunneling is also seen from the standard deviation and skewness, see Figs. 5.9(b) and (c). For $\Omega \to 0$, the probability density of injected energy becomes wider because now the electron can tunnel for $E \geq \Delta$ as well as for $E < \Delta$. Therefore, the standard deviation grows in this injection regime but naturally decreases for lower $\eta$. Furthermore, this function also has more weight towards energies below the gap in this regime and for certain $\Omega$ it becomes completely symmetric with respect to $\langle E \rangle$ giving a zero skewness. For lower $\Omega$ more subgap tunneling occurs and the function leans towards $E < \Delta$ and the skewness becomes negative.

To add robustness to the model based on injected energy probability $P(E, \nu)$ and to model errors arising from superconducting sub-gap states, the contributions of these to tunneling rates must be considered. For this, the tunneling rate for $E < \Delta$ is written as

$$\Gamma_{i \to i \pm 1}(\delta \epsilon \pm) \approx \frac{1}{e^2 R T} \int dE \frac{\eta}{(1 - E^2)^{3/2}} f_{\text{N}} \left( E - \delta \epsilon \pm \right) \left[ 1 - f_{\text{S}}(E) \right]. \quad (5.29)$$

Eq. (5.29) is derived by expanding the BCS DOS given in Eq. (2.19). This rate can be now used in the simplified model at zero temperature and replaced in Eq. (5.21) to calculate the probability of the state with the island filled. This is subsequently used to calculate the average injected energy by doing the same procedure as before, which gives

$$\langle E \rangle \approx \left( \frac{\pi}{2} - 1 \right) \frac{\eta}{\Omega} e^{-\eta/\Omega} + \left( 1 + 0.313 \Omega^2/3 \right) e^{-\eta/\Omega}. \quad (5.30)$$

For $\eta \to 0$, Eq. (5.30) tends to Eq. (5.24), as required. Equation (5.30) also forms an accurate expression for assessing injection errors in FPC as seen from the comparison with numerical calculations (dots) in Fig. 5.9(d).

To finalize the analysis of the ultimate possible accuracy of FPC in a driven NIS junction in the single-electron injection process, non-zero temperature was taken into account. In Figure 5.9(e), the numerical calculations (dots) seem to show that deviations from $\Delta$ increase by a constant term with respect to Eq. (5.24) and numerical calculations at zero temperature. At $T \neq 0$, the Fermi-Dirac distribution function shows a
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tail above the Fermi level with thermally populated states. Electrons that
occupy these have more probability of tunneling to states with \( E > \Delta \) for
any driving conditions. To take this into account in the analytical model,
\( \eta = 0 \) is again assumed. Because the increase in errors seems constant
throughout \( \Omega \), then calculations for \( \Omega \rightarrow 0 \) should suffice. As a result, for
\( \Omega \rightarrow 0 \)
\[
\frac{\langle E \rangle}{\Delta} - 1 \approx \frac{1}{2} \frac{k_B T}{\Delta}. \tag{5.31}
\]
Equation (5.31) suggests that deviations from ideal in energy injection are
given by equipartition of energy at zero driving frequency. Numerical calcu-
lations of Fig. 5.9(e) verify that this is the case. Such calculations were per-
formed for \( \Delta = 200 \mu eV \) and \( T = 100 \text{ mK} \), in consequence \( k_B T/(2\Delta) \approx 0.022 \).
When considering the full driving cycle that includes two tunneling events,
the thermally generated injection error is \( k_B T \) in line with energy equipar-
tition. Remarkably, these thermally generated deviations are dominant
over any other of the previously exposed errors except the systematic error
of Eq. (5.24). Hence, FPC in driven NIS junctions should be performed at
low temperatures.

In Pub. III, accuracy in FPC was also related with the detection method
employed in Pub. II. Errors in detection add to the errors analyzed in this
section. As a final remark, it should be mentioned that errors in detection
first relate to trapping efficiency in Eq. (5.11) as \( 1 - \vartheta \). From Fig. 5.3(b) it can
be deduced that these errors can be \( \ll 1\% \) for temperatures below 100 mK.
Additionally, detection errors are related to uncertainties due to noise
in measuring temperature. This is given by the fluctuation-dissipation
theorem \[ 182 \] \( S_Q = 10\Sigma_N \nu N k_B T^6 \), which intervenes in the signal-to-noise
ratio of a measurement with bandwidth \( \nu \) as
\[
2 \langle E \rangle f / \sqrt{S_Q \nu} \sim 2 \Delta f / \sqrt{S_Q \nu}.
\]
Typically, \( \nu \leq 1 \text{Hz} \) and the signal-to-noise ratio is \( \sim 10^4 \). Hence, it is also
not dominant over thermally generated injection errors and those caused
by increasing \( \Omega \).

5.6 Future implementations

This chapter finalizes with suggestions of more optimal FPC realizations
based on the evidence gathered in Pubs. II and III. The first suggestions
come from the increase of trapping efficiency so that detection errors are
avoided. From Eq. (5.10) it is clear that the main source of efficiency
reduction in the geometry depicted in Fig. 5.3(a) is heat leakage to the lead
\( S_2 \). This leak is suppressed by increasing the length \( \ell_2 \). However, the FPC
bolometric detection can be improved by implementing a device like the
one shown in Fig. 5.10(a) in which a single NIS junction is used. Here,
the normal-metal is periodically driven and the superconducting lead is
terminated by a directly contacted normal-metal trap whose temperature
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\[ \vartheta \approx \text{sech}(\lambda \ell), \quad (5.32) \]

where \( \lambda \) is as in Eq. (5.9). From Eq. (5.32) it is possible to see that the efficiency in this configuration drops just over 0.3% from unity at lengths as long as 100 \( \mu m \) and temperatures as high as 180 mK, see Fig. 5.10(b). This is a clear improvement compared to the setup originally used. Naturally, in shorter superconducting leads heat dissipation is further suppressed and more can be detected, which increases the efficiency as seen in the inset of Fig. 5.10(b). For \( \ell = 1 \mu m \), efficiencies practically do not depart from unity in a wide range of temperatures. Also, errors incurred in thermal measurements are reduced since only one bolometer needs to be monitored. However, such configuration has a drawback. Since no charge transport can be measured in this system, an in situ independent measurement of the superconducting gap via tunnel spectroscopy cannot be done. Thus, such a measurement needs to be done in a separate device.

Additionally, deviations from Eq. (5.3) can be reduced by improving the device parameters. As an example, the power injection of a device with \( \Delta = E_c = 200 \mu eV \), \( R_T = 200 k\Omega \) and \( \eta = 10^{-6} \) was simulated in Pub. II, see
Fig. 5.10(c). All these characteristics are achievable with standard fabrication. Furthermore, a temperature of $10 \text{ mK}$ is assumed for the island and $150 \text{ mK}$ for the superconducting lead. These are also routinely realized conditions. With these, the thermally generated deviation is around 0.4%. The low Dynes parameter reduces the errors due to injection of QPs below the gap. With this, injection at frequencies as low as $1 \text{ MHz}$ has maximum deviation below $2\Delta f$ of around 0.2%. The other parameters allow injection with frequencies as high as $80 \text{ MHz}$ with maximum deviation $\sim 3.8\%$ in the first plateau. The improvement of the power plateaus in Fig. 5.10(c) is notorious when compared to those in Fig. 5.5(b). Of course, even better accuracies can be achieved by choosing and implementing different parameters following the discussion on ultimate accuracy of FPC.

Finally, it is clear from the working principle of FPC in a SINIS ST and its ultimate accuracy analysis that the main contribution to accuracy is the energy selectivity of the tunneling event. This is, the ability to transmit electrons within a narrow energy band. Therefore, a natural alternative for FPC would be to contact the normal-metal island with a sharply peaked DOS such as the one devised in quantum dots [183]. This would also allow to tune at will its energy level and hence the transducing constant of FPC. In this setup, the synchronization of electron tunneling with the driving signal would be guaranteed by the Coulomb interactions in the island.
6. Single-electron current generation with RF biased SINIS STs

The objective of this chapter is to present the results of Pubs. IV and V. While in previously exposed publications the SINIS ST was driven by only applying a periodic electrical signal to the gate electrode, in Pubs. IV and V an extra modulation to the bias of the device was added. By adding this extra modulation, new driving paths in the stability diagram are realized. How this is possible is presented in Section 6.1, providing a common ground for the results contained in both publications. Then, the presentation focuses on the outcomes of Pub. IV in Section 6.2. These are the suppression of back-tunneling errors in single-electron currents. Finally, the results of Pub. V showing that single-electron currents can be generated in SINIS STs with zero-average bias are shown in Section 6.3.

6.1 New driving methods

In Pubs. IV and V current in a SINIS ST was generated by driving the device with modified modulation cycles. In the present section, it is explained how this can be done. First, recall from Section 2.5 that the working principle of the SINIS ST can be easily understood from its stability diagram, such as the one depicted in Fig. 6.1(a). In brief, single-electron currents can be generated by devising a trajectory that crosses tunneling thresholds cyclically in the appropriate fashion. So far, all the applications of the SINIS ST have made use of such a trajectory with constant bias voltage. This path, henceforth called flat driving, can be represented in the stability diagram by a straight line parallel to the \( n_g \) axis, see the magenta line in Fig. 6.1(b). Furthermore, it is easily implemented by injecting any periodic electrical signal to the gate electrode of the device. However, one could be more creative and think of different trajectories that could cross the tunneling thresholds in a way such that single-electron currents are generated. All the other curves shown in Fig. 6.1(b) also cross tunneling thresholds in the correct sequence. For example, starting from the red dot and going in the clockwise sense, the black ellipse crosses first the thresh-
Figure 6.1. Creation of new driving methods. (a) Stability diagram of a SINIS single-electron transistor, only two Coulomb diamonds corresponding to none (green) and one (orange) electron in the island are shown. The diamonds are bounded by tunneling thresholds, solid (dashed) lines correspond to tunneling processes into (out of) the island and blue (red) correspond to tunnelings though the right (left) junction. (b) New driving methods can be created by bias modulation, the red dot indicates a starting point for following the elliptical trajectory. \( A_b \) corresponds to the peak-to-peak (pp) amplitude of the bias modulation; \( A_g \), pp gate amplitude; \( V_{0b} \), DC bias level; \( n_{0g} \), normalized DC gate level. (c) Colored scanning electron micrograph of the sample measured in Pubs. IV and V along with the used setup. The modulations \( h \) and \( g \) are general periodical waveforms. Bias tee inductances are 33 \( \mu \)H and capacitances 22 nF.

old for an electron to tunnel out of the island through the right junction. Since the island is now empty, crossing other thresholds for a tunneling out of the island do not trigger such an event unless these correspond to other diamonds, this is, other charge states. The next relevant event is then the crossing of a threshold for an electron to tunnel into the island through the left junction. This leaves one electron transferred from the left lead to the right one just as when the flat driving is followed. Finally, the trajectory closes and the cycle can be repeated so that a net current is created. The other paths cross tunneling thresholds in the same sequence and can generate a single-electron current.

The geometric description of the paths hints toward ways of implementing them experimentally. For example, the ellipse of Fig. 6.1(b) is generated by

\[
\begin{align*}
V_b &= V_{0b} + \frac{A_b}{2} \cos(\omega t), \\
n_g &= n_{0g} + \frac{A_g}{2} \sin(\omega t), \tag{6.1}
\end{align*}
\]

which parametrizes the curve \((2/A_g)^2(n_g - n_{0g})^2 + (2/A_b)^2(V_b - V_{0b})^2 = 1\) that clearly describes the black ellipse of Fig. 6.1(b). In fact all the trajectories shown in that panel can be generated in a similar way. The
brown curve requires \( V_b \propto \sin (2\omega t + \phi) \) and the blue one can be generated by using triangular waveforms. These examples show that adding an extra periodic signal to the bias voltage is needed to create new methods for driving a SINIS ST. This was implemented in Pubs. IV and V by using bias tees as shown in Fig. 6.1(c) and described in Section 3.3. Furthermore, the single-electron transistor depicted in the figure was used. This has never been explored before in SINIS STs although modifications to driving schemes beyond the use of different waveforms have been realized in semiconducting quantum dot-based single-electron pumps [184, 185].

6.2 Suppression of back-tunneling by RF biasing

In this section, the rationale behind Pub. IV and its results are shown. The objective of that work is to show that a new driving protocol can be used to suppress back-tunneling errors while keeping the operation frequency of the turnstile. This is a radical change of mindset with respect to error suppression in SINIS STs. To show this, a brief description of typical tunneling errors and methods used to tackle these is made.

6.2.1 Common errors events

It is clear from the multiple figures of current generated with SINIS STs shown so far that these do not follow Eq. (2.34) exactly. It turns out that the non-idealities of the SINIS structure described in Chapter 2 among others are the cause of these deviations from ideal behavior. One of these non-idealities is the presence of a non-zero density of superconducting QPs in the leads of the transistor. Errors arising from it are one of the main topics of Chapter 4 and methods for their alleviation are already mentioned there. Therefore, they are not discussed further here.

In addition, the presence of sub-gap states also alters the correct dynamics of the SINIS ST as was shown in the context of FPC in Fig. 5.9(d). These tunneling events between normal-metal states and superconducting states below the gap cause loss of synchronization with the driving signal and therefore decrease accuracy in the context of frequency-to-current conversion. In Section 2.4 it was explained that such a leakage can be modeled by expressing the QP DOS as in Eq. (2.19) with the Dynes parameter. However, the Dynes DOS also has its origin in the interaction of tunneling electrons with the electromagnetic environment [19]. This interaction also favors tunneling events before the corresponding thresholds are crossed. Hence these two non-idealities affect the proper operation of SINIS STs by enhancing the leakage of the junctions [19, 72, 186, 187]. Typically, these errors can be tackled by engineering the electromagnetic environment of the device under test so that high frequencies are filtered out [19, 188, 189].
Such engineering consists on placing the SINIS structure for example in a high impedance environment [188–190] or in parallel with a large capacitance [19], as well as in tightly sealed sample carriers [191].

The next important non-ideal tunneling event is caused by Andreev reflection [63] which constitutes an error in single-electron tunneling since it changes the normal-metal island charge state by two electrons [20, 192]. Andreev reflection consists effectively on the tunneling of a Cooper pair while an electron or hole is reflected by the barrier. Because of this, the effect of having Andreev reflection in the operation of SINIS STs is to increase the current generated and therefore $I > I_{ef}$ [21]. These events have already been described when explaining the DC operation of the SINIS single-electron turnstile and its rates given in Eq. (2.20), which have been measured in NIS interfaces by electron counting [192, 193]. In Eq. (2.20) some characteristics are notorious. In particular notice that Andreev reflection rates decrease much quicker with tunnel resistance than single-electron tunneling ones. Additionally, since no QP tunneling into the superconductor is involved in this process the QP DOS (Eq. (2.5)) does not appear and neither does the QP distribution function. Very importantly, the energy of the intermediate state is taken into account through the parameter $\xi$. This parameter is given by the rate of single-electron tunneling out of this intermediate state which smears the logarithmic singularity of the transmission amplitudes in the perturbative treatment of Andreev reflection [3, 20]. However, since two-electron tunneling rates are sensitive to the energy of this intermediate state only for $\delta \epsilon^\pm > 2 (\Delta - E_c)$ [25] in this thesis the value $\xi / \Delta = 10^{-5}$ has been set for the cases where these error events are important. The last notable characteristic to which the Andreev reflection rates are sensitive is the number of conducting channels $N$ which can be written as $A / A_{ch}$ with $A$ the area of the junction and $A_{ch}$ the area of an individual conduction channel. This last value is often taken to be $30 \text{nm}^2$ [21] although for Pub. I $10 \text{nm}^2$ fitted better the data.

Given its dependence to tunneling resistance, Andreev reflection can be suppressed by increasing it even though this causes further problems that will be understood later. A better way to suppress these errors is to use a device for which $E_c > \Delta$ since in this condition Andreev reflection is energetically forbidden [21, 90].

Other important errors are, for example, photon-assisted Andreev reflections [194] that can be suppressed by proper engineering of the electromagnetic environment. It has also been proposed that inelastic co-tunneling can be suppressed by these means [195]. This additional non-ideality occurs in SINIS transistors and affect the ST accuracy by transmitting one electron from one lead to the other at once. This occurs as an Andreev reflection that takes place through one junction and a single-electron tunneling event through the other one. After this event, the island charge state changes by one while one electron has passed though the device [20].
Finally, tunneling events can be missed and hence average current might be reduced. These errors can be taken into account by counting electrons with capacitively coupled on-chip electrometers [196] which has been done in semiconducting platforms [197, 198] although exploratory measurements have been done in SINIS devices [199]. With this, errors can be statistically accounted and the transferred current corrected accordingly.

Another error event that might occur is tunneling through one junction against the bias direction, hence reducing the generated current below $e f$. These are the errors that have been suppressed by RF biasing the SINIS ST in the experiments described in Pub. IV. In the following subsection these events are described with more detail.

### 6.2.2 Back-tunneling errors

In this subsection a sinusoidal gate driving is assumed, however the results can be extended to a general periodic gate driving. Here, the conditions necessary for enabling back-tunneling errors are described. Considering the time evolution of the quantities $\delta\epsilon_{L/R}$ of Eq. (2.17) is a good starting point for this purpose. Typical time evolution of these in a driving cycle of period $\tau$ is shown in Figs. 6.2(a) for tunneling events out of the island and (b) for tunneling into the island. In ideal operation, the tunneling event corresponding to the first curve to cross the tunneling thresholds ($\delta\epsilon_{L/R} = \Delta$, magenta dashed lines) is the one that occurs and prevents such event from occurring through the other junction. Therefore, at the end of the cycle shown in Figs. 6.2(a) and (b) one electron is transmitted from the left to the right lead considered here as positive current. This sequence corresponds to the flat driving path of Fig. 6.1(b). However, notice that the curves corresponding to tunneling events going in the opposite direction also have an amplitude that allows them to cross the thresholds which occurs some time $\delta t$ after the first crossing.

The description above is too ideal and does not take into account the finite response times of the device encoded in the single-electron tunneling rates of Eq. (2.18). It might occur that the time $\delta t$ is too short for allowing the electron to tunnel through the expected junction due to high operation frequency. In this case a (undesired) tunneling event through the opposite junction is likely to occur. This is the origin of back-tunneling errors. However, a couple of simplifications might help to understand what constitutes a “too short” $\delta t$ and clarify the origin of these errors, but more importantly how to avoid them.

Suppose a device with perfect junctions ($\eta = 0$) and operated at zero temperature. Thus, the single-electron tunneling rates become $\Gamma \left( \delta\epsilon_{L/R} \right) = 1/\left( e^2 R_{L/R} \right) \sqrt{\left( \delta\epsilon_{L/R} \right)^2 - \Delta^2}$ for $\delta\epsilon_{L/R} \geq \Delta$ and zero otherwise. It is clear that for competing events the most resistive junction sets the lower bound
Figure 6.2. Back-tunneling errors. (a) Evolution of $\delta \epsilon^-$ for the left and right junction (see the legend) within a period $\tau$. Dashed magenta line indicates the tunneling threshold. The other terms are explained in the text. (b) As in panel (a) for $\delta \epsilon^+$. (c) $\varphi$ versus $\Omega$ for different DC biases $eV_b$ indicated in the legend. (d) Pumping plateaus generated by the device measured in Pub. IV against normalized gate $V$ amplitude with the flat driving at $f = 5\text{ MHz}$ and different $V_0$, see the legend. Dots are measured data and solid lines represent simulations. Inset, zoom in to the area enclosed by the red square.

of $\delta t$, since for this the tunneling rates are lower. In the analysis of this subsection the subscript $M$ refers to the most resistive junction and $\delta \epsilon^+_M$ indicates the energy difference corresponding to the desired forward tunneling event through such junction. This tunneling rate is expressed then as

$$\Gamma^+_M = \frac{1}{(e^2R_M)^{\frac{1}{2}}} \sqrt{(\delta \epsilon^+_M)^2 - \Delta^2}. \quad (6.2)$$

Furthermore, the probability of preserving the island charge state before the curve corresponding to back-tunneling event crosses the threshold (time $t_-\rightarrow t_-$ in Fig. 6.2(a)) can be approximated to $p(t_-\rightarrow t_-) \approx e^{-\varphi}$ where

$$\varphi = \int_{\delta t} dt \Gamma^+_M. \quad (6.3)$$

Notice that if $\varphi$ is large, it is likely is that the desired tunneling event has taken place before $t_-\rightarrow t_-$ since $p(t_-\rightarrow t_-) \rightarrow 0$. Thus, for larger $\varphi$ undesired back-tunneling events are less likely.

With this, it is possible to understand which operating conditions as well as device characteristics increase $\varphi$ by plotting this quantity against $\Omega$ which is defined as

$$\Omega = \frac{e^2R_M}{\Delta^2} \frac{d\delta \epsilon^+_M}{dt} \bigg|_{\Delta \epsilon^+_M = \Delta}. \quad (6.4)$$
Here, $\Omega$ has been defined for a general $\delta \epsilon$ while for Eq. (5.23) it is defined for a constant $\dot{v}$. These two coincide when the maxima (or minima) of $\delta \epsilon$ are far from $\Delta$. Therefore, $\Omega \propto R_M f$ and has an inverse proportionality to $V_b$ which depends on the specific driving waveform used. This gives an additional proportionality to $E_c$ and $A_g$. It is clear from Fig. 6.2(c) that $\varphi \propto \Omega^{-1}$ and proportional to $V_b$. This gives clear hints to find conditions that enhance back-tunneling. It is evident that a larger $R_M f$ product enables more errors against the bias. Additionally a larger driving amplitude or charging energy also contribute to these errors. On the contrary, by increasing $V_b$, back-tunneling can be suppressed. The occurrence of such events has the natural consequence of diminishing the current generated by the ST and has the typical footprint of bending down the current plateaus when plotted against $A_g$ as shown in Fig. 6.2(d). Notice from the figure how the increase of bias voltage allows to avoid the bending down of the plateau. The data (dots) shown in Fig. 6.2(d) are from the device measured in the experiments carried out for Pubs. IV and V for which $R_T = 4.53 \text{M}\Omega$ operated at $f = 5 \text{MHz}$. The solid lines are calculated based on the Markovian model from which the above analysis follows. The agreement between the calculations and data gives validity to this simplified picture. The fact that back-tunneling errors show some proportionality to $\Omega$ similar to the one defined for Eq. (5.23) is not mere coincidence. After all, back-tunneling errors and errors in FPC are related in so far as both are consequence of the retarded response of the SINIS ST to the driving signal, which is degraded by increasing $R_T$ or $f$.

It is clear then, that a large $\varphi$ indicates suppression of back-tunneling events. This suppression can be done in two ways, either by increasing $\Gamma_M \uparrow$ or by increasing $\delta t$. The former has been achieved by decreasing $R_T$ or $E_c$ in the device, although this might enable Andreev reflection and enhance two-electron tunnel events. The latter option can be realized by increasing the voltage bias since this lowers the energy difference curves for back-tunneling events and lifts the ones corresponding to forward-tunneling events. However, doing this might increase leakage current in leaky junctions or even two-electron errors in the case where Andreev reflection is allowed. Additionally, one might opt for decreasing the driving frequency but this will decrease the generated current which is detrimental for metrological purposes. In Pub. IV, $\delta t$ is increased by devising a new driving method using the ideas exposed in Section 6.1, avoiding then the necessity to decrease the driving frequency or that of improved device fabrication. This new driving method and the way it increases $\delta t$ are the topic of the next subsection.
Single-electron current generation with RF biased SINIS STs

Figure 6.3. Parabolic driving for back-tunneling suppression. (a) Parabolic driving generated by Eqs. (6.5), conventions are as in Fig. 6.1(a). (b) Time evolution of $\delta\varepsilon$ induced by the flat driving (solid lines) and the parabolic protocol (dashed lines) for tunneling through either junction, see the legend. These curves were generated with the conditions $A_g = 1.06$, $V_{0b} = 200 \mu V$ and $A_b = 0, 240 \mu V$. (c) As in panel (b) for $\delta\varepsilon$. (d) Parameter $\varphi$ as a function of the bias pp amplitude calculated for the same simulated conditions of panels (b) and (c) and $f = 5 MHz$.

6.2.3 New driving method and its effects on time evolution

The trajectory used in Pub. IV for suppressing back-tunneling errors is depicted in Fig. 6.3(a) where a zoom of the stability diagram is shown. This path is a parabola with negative concavity, which can be generated by setting

$$V_b = V_{0b} + \frac{A_b}{2} \cos(2\omega t),$$

$$n_g = n_{0g} + \frac{A_g}{2} \sin(\omega t).$$

(6.5)

Where $A_g$ and $A_b$ are peak-to-peak (pp) amplitudes as in Eq. (6.1). By substitution of Eqs. (6.5) it is possible to verify that

$$V_b = V_{0b} + \frac{A_b}{2} - 4\frac{A_b}{A_g^2} (n_g - n_{0g})^2,$$

(6.6)

which is the equation of a parabola that opens down. Therefore, this driving method is called the parabolic driving from now on. Notice that when $A_b = 0$ in Eq. (6.6), $V_b = V_{0b}$, and the parabolic driving reduces to the flat one. For the purposes of Pub. IV $n_{0g}$ was chosen to be the open gate position at the bottom of the parabola. Following Eq. (2.30) this position is such that

$$n_{0g} = \left( V_{0b} - \frac{A_b}{2} \right) \frac{r - 1}{4E_c (r + 1)} + 0.5.$$

(6.7)

This was chosen so that odd current plateaus appear at full width against $A_g$ and even plateaus do not appear. For the device measured in the
experiments reported in Pub. IV, $r \approx 0.15$ and the Coulomb diamonds in its stability diagram are tilted. It needs to be mentioned that the precise $n_{0g}$ value only affects how wide the plateaus that appear are and which ones do, and does not modify the resulting effect of back-tunneling suppression.

The extra bias modulation introduces a new degree of freedom to the evolution of the quantities $\delta \epsilon^{\pm}_{L/R}$. In the case in which $V_b = V_{0b}$ the shape of $\delta \epsilon^{\pm}_{L/R}$ inherits that of $V_g$ as shown in Figs 6.2(a) and (b). However, for the parabolic protocol of Eqs. (6.5) the shape of the evolution of these quantities is less trivial. Figs. 6.3(b) and (c) show a comparison between $\delta \epsilon^{\pm}_{L/R}$ for the flat (solid line) and the parabolic (dashed line) driving, for single-electron events out of (Fig. 6.3(b)) and into (Fig. 6.3(c)) the island. The most remarkable difference between the cases of the parabolic and flat driving is the size of $\delta t$ defined in Fig 6.2(a). It is evident that in the case of the parabolic protocol $\delta t$ increases and as a result so does $\varphi$. Figure 6.3(d) shows this quantity calculated with the characteristics of the device reported in Pub. IV for a gate amplitude that allows back-tunneling, $V_{0b} = 200 \mu V$ and $f = 5$ MHz and as a function of $A_b$. Notice that $\varphi$ increases with $A_b$ and that for the mentioned conditions the data and simulations shown in Fig. 6.2(d) exhibit clear back-tunneling using the flat driving. Thus, the parabolic driving reduces the likelihood of back-tunneling events by suppressing them. One would expect then an increase in current at the plateaus for the conditions in which back-tunneling is strong which is discussed in the following section.

### 6.2.4 Experimental evidence

Experiments of current pumping with appropriate operation frequency and DC bias with different $A_b$ verify whether the parabolic driving suppresses or not back-tunneling errors, see Fig. 6.4(a). Notice that for the case of $A_b = 0$, this is for the flat driving, strong back-tunneling is observed so the case presented is a good test for the new driving method. By increasing $A_b$ the paths shown in Fig. 6.4(b) are traced in the stability diagram of the sample measured, colors of the paths correspond to the data (dots) shown in Fig. 6.4(a). Together with these, simulations (solid lines) based on the Markovian model followed in the other parts of the thesis are shown. Notice that there is good agreement between such calculations and the data confirming that the paths drawn in Fig. 6.4(b) are the ones followed. It is evident that current at high $A_g$ progressively increases while $A_b$ is increased so that an actual plateau starts to form. This, added to the argument of the previous subsection based on Fig. 6.3(d) confirms that use of the parabolic driving suppresses back-tunneling errors and improves generation of single-electron currents in SINIS STs. This last assertion is backed by calculating the deviations from $ef$ as in Fig. 6.1(c) where $|I/(ef) − 1|$ is plotted in a logarithmic scale. When $A_b = 0$, the minimum
Figure 6.4. Suppression of back-tunneling errors by parabolic driving. (a) Current plateaus using the parabolic driving at \( f = 5 \text{MHz} \) and \( V_{0b} = 100 \mu\text{V} \), the legend designates the used pp bias amplitude. Dots represent measurements, solid lines simulations. (b) Depiction of the used protocols in the stability diagram of the measured device. Colors correspond to the curves of panel (a), (c) and (d). Conventions are described in Fig. 6.1(a). (c) Deviation from ideal current of data and calculations of panel (a) with the same legend. (d) Zoom in to the area enclosed by the red square in panel (a) which shows how the parabolic driving enhances back-tunneling-induced leakage at low \( A_g \). (e) Current calculated at the plateau \( (A_g = 1.2) \) as a function of the pp bias amplitude of the parabolic protocol, curves from right to left go from \( V_{0b} = 0 \) to \( 360 \mu\text{V} \) in steps of \( 20 \mu\text{V} \).

Deviation is just over \( 10^{-2} \) in the expected plateaus although it is clearly above \( 10^{-1} \) in most of the used \( A_g \) interval. For increasing \( A_b \) the interval for which accuracy is better than \( 10^{-1} \) is wider even though the minimum deviation increases, this will be explained later. For the highest shown \( A_b \) errors are below \( 10^{-2} \) with a clear plateau just above \( 10^{-3} \). Also the dashed line indicating \( I = e f \) is crossed which makes the deviations go below \( 10^{-3} \) for a short interval depending on the density of \( A_g \) points used. Further results on back-tunneling suppression exhibiting better accuracies and further improvements can be found in Pub. IV. Results of Fig. 6.4 are shown in this thesis since they highlight the important characteristics of back-tunneling suppression in a clearer way.

The resulting reduction in current at the start of the plateau shown
in Fig. 6.4(d), which is a zoom of panel (a) to that zone, is due to an enhancement by the parabolic driving of back-tunneling-induced leakage. This is not in contradiction with back-tunneling suppression since the former error event only takes place when energy differences for back-tunneling are \(\lesssim \Delta\). Notice that for these quantities maxima increase with \(A_b\), whereas for forward-tunneling maxima the opposite occurs, see Figs. 6.3(b) and (c). This fact covers importance in the case in which back-tunneling-induced leakage is relevant at low \(A_g\). There, \(\delta \epsilon\) for forward and back-tunneling are \(\sim \Delta\) and tunneling contrary to the bias might occur if there is presence of sub-gap states and the operation frequency makes the corresponding \(\delta t\) short enough. On the other hand, the discussed back-tunneling suppression is exhibited when \(A_g\) is large enough so that \(\delta \epsilon\) for back-tunneling events clearly crosses the thresholds. Here the maxima of \(\delta \epsilon\) do not play an important role and the dynamics is as described in the previous subsection. Therefore, by having less leaky junctions (with lower \(\eta\)) current increases towards \(e f\) for low \(A_g\). Finally, in Pub. IV the robustness of the generated current against \(A_b\) using the parabolic protocol was also studied by simulations with constant \(A_g\), see Fig. 6.4(e). It is evident and positive that for low \(V_{0b}\) a strong variation of current with \(A_b\) is seen and even at \(V_{0b} = 0\) a plateau with \(I \sim e f\) is recovered. However, such a strong variation is also seen at higher DC bias and accuracy is quickly lost already for \(V_{0b} = 260 \mu V\). Even though all the curves converge at some point around \(e f\), flatter plateaus against \(A_b\) are necessary for metrological purposes.

Further analyses were made in Pub. IV regarding better implementations of the parabolic driving. The main interest was in inspecting whether this could be used for suppressing back-tunneling errors in higher currents than the ones experimentally generated. The simulations described in Subsection 2.5.2 were used for this purpose. With these, the current plateaus generated by a device with \(R_T = 200\ k\Omega\) and \(E_C = 1.2\Delta\) were calculated when operated at \(f = 240\ MHz\), see Fig. 6.5(a). It is evident that such a device can achieve currents \(\sim e f\) at that operation frequency operated with the flat driving. Given the low simulated Dynes parameter \((10^{-6})\) it also achieves these currents with deviations from \(e f\) below \(10^{-3}\), see Fig. 6.5(b). However, some curves exhibit strong back-tunneling which is mostly suppressed when the parabolic driving is used. For example, at \(V_{0b} = 200\ \mu V, A_b = 0\) (brown curve in Fig. 6.5(a)) the current exhibits reduction below \(e f\). Once a bias amplitude of \(A_b = 250\ \mu V\) is applied, most of the bend down of the plateau disappears. Current is now \(I \gtrsim e f\) for most of its width showing that back-tunneling suppression is also achieved using the parabolic driving at high operation frequencies. In the case simulated in Pub. IV, the current calculated with the parabolic protocol outperformed any of the cases simulated with the flat driving even those in which back-tunneling does not lead to current reduction below \(e f\). This
Figure 6.5. Back-tunneling suppression at high operation frequencies. (a) Current plateaus simulated with the Markovian model at $f = 240$ MHz for a system with parameters given in the text. The legend shows the simulated biasing conditions. Solid lines were calculated with $A_b = 0$. (b) Deviation from ideal current of the curves in panel (a) with the same legend. (c) Calculated current at the plateau ($A_g = 2.0$) against the bias amplitude $A_b$. Legend indicates the used $V_{0b}$.

is evident from panel (b) of Fig. 6.5. There it is shown that the current simulated with the parabolic protocol has a wider $A_g$ region below $10^{-3}$ than any other curve and even a tiny kink below $10^{-4}$ appears. On the other hand, current generated with the parabolic driving at $f = 240$ MHz can also create plateaus against $A_b$ as required, see Fig. 6.5(c). Notice that even for low $V_{0b}$, some current of the order of $e f$ can be recovered. The main idea of Pub. IV was to demonstrate back-tunneling suppression even at high frequencies of SINIS ST operation. However, it is clear that the parabolic driving could have metrological potential as accuracies for large current are required for current standards.

6.3 Current generation at zero-average bias

It has been remarked already that under a pure DC bias and $V_b = 0$, $\delta e_R^+ = \delta e_L^+$ and as consequence the curves presented in Figs. 6.2(a) and (b) collapse on top of each other and oscillate around zero. As a result $\delta t = 0$ and back-tunneling might occur at any operation frequency no matter the device parameters. Remember from FPC at zero bias that in this regime the tunneling direction is completely stochastic and hence directionality is totally lost. This leads to no current generated by a SINIS ST on average. However, in Pub. V a driving approach inspired in the parabolic one was used to generate single-electron currents even though $\langle V_b \rangle = 0$, so that
bias is zero on average. Such driving method is

\[
V_b = \pm \frac{A_b}{2} \cos (2\omega t),
\]

\[
n_g = n_{0g} + \frac{A_b}{2} \sin (\omega t).
\]

Notice that for \( V_b \) the DC level is zero and thus it averaged to zero within a full driving period. Equations (6.8) parametrize the parabolas shown in the stability diagram of Fig. 6.6(a). If the positive sign in the bias modulation is used, the path labelled (1) is followed, otherwise trajectory (2) is followed. It was also shown that this flip of sign, which is only a phase shift of \( \pi \), allows for inversion of the current direction. Although this is the first demonstration of zero-average bias bidirectional single-electron current generation in SINIS ST, it is not the first time that a mesoscopic all-metallic device has been used for generating single-electron currents at zero-average bias. This was already achieved in the first all-metallic demonstration of single electron transport in single-electron pumps [32, 200]. These also allowed for current reversal by phase shifting of the driving as was also possible in silicon quantum dot single-electron pumps operated by three waveforms [185].

The possibility of generating single-electron current with the driving of Eq. (6.8) can be easily understood by inspecting the time evolution of the quantities \( \delta \epsilon_{L/R}^\pm \), see Figs. 6.6(b)–(e). Figures 6.6(b) and (c) depict the case of path (1) for tunneling events out of and into the island, respectively. It is notorious that now \( \delta \epsilon_R^\pm \neq \delta \epsilon_L^\pm \) and in consequence \( \delta t \neq 0 \), specifically \( \delta \epsilon_R^- = \Delta \) before \( \delta \epsilon_L^- \) matches this value. Thus, provided that the time

\[
\text{Figure 6.6. Parabolic driving for zero-average bias current generation. (a) Zoom to the stability diagrams of a SINIS single-electron transistor around the overlap of Coulomb diamonds. The parabolic drivings described by Eqs. (6.8) are shown and labelled by (1) and (2) for the + and - sign, respectively. The flat driving at } V_{0b} = 0 \text{ labelled (0) is also shown for reference. Conventions are as in Fig. 6.1(a). (b) Time evolution of } \delta \epsilon^- \text{ induced by the trajectory (1) for either junction, see the legend. Dashed line indicates the tunneling threshold. (c) As in panel (b) for } \delta \epsilon^+. \text{ (d) As in panel (b) for the path (2) of panel (a). (e) As in panel (d) for } \delta \epsilon^+.\]
Figure 6.7. Current plateaus generated at zero-average bias. (a) Current plateaus produced with drivings (1) and (2) at \( f = 1 \text{ MHz} \) against gate signal amplitude, the legend indicates the used pp bias amplitude. The curves were generated following the same color trajectory depicted at the center of the panel. (b) Close-up for the current flowing from left to right around \( I = e_f \), indicated by the dashed line. (c) Close-up for the current flowing from right to left around \( I = -e_f \).

elapsed between both threshold crossings is long enough, an electron will most certainly tunnel out of the island through the right junction. On the other hand, it is evident from panel (c) that \( \delta e_L^+ = \Delta \) before \( \delta e_R^+ = \Delta \) implying that an electron tunnels into the island most likely through the left junction. An electron is transferred from the left lead to the right lead with a probability higher than the reverse process. Thus, if this process is repeated, a current \( I = +e/\tau = +e_f \) is created by driving the device along path (1) of Fig. 6.6(a). Notice that here the positive direction for current has been defined from left to right. If, on the other hand, the SINIS ST is driven along trajectory (2) the evolution of \( \delta e_{L/R}^+ \) is as shown in Figs. 6.6(d) and (e) for tunneling events out of and into the island, respectively. Following the same rationale as before, it is possible to conclude that at the end of the period an electron is transferred from the right lead to the left one and therefore a current \( I = -e_f \) is generated. Notice that such direction inversion should be possible by shifting the bias modulation by \( \pi \).

The driving given in Eqs. (6.8) was implemented in the experiments reported in Pub. V with the same SINIS ST as employed in Pub. IV. With this, the description above was confirmed by measurements of current pumping at different \( A_g \) and \( A_b \), see Fig. 6.7(a). The curves in the figure have been generated by measuring with \( n_{0g} = 0.5 \) which is the gate open position for the device at \( V_b = 0 \). Furthermore, an operation frequency \( f = 1 \text{ MHz} \) was used. It is notable that a non-zero current has been generated. However, it is even more remarkable that current \( \sim \pm e_f \) has been produced confirming that the used driving frequency provides a large enough \( \delta t \) for the characteristics of the measured device, see Figs. 6.7(b) and (c). Besides, wide plateaus around \( e_f \) against \( A_g \) have also been achieved and these were generated at different \( A_b \) values, which suggests
that plateaus are also created against this parameter. The path along which the device is driven in order to generate each current curve is shown in Fig. 6.7(a) with corresponding color. It is then evident that the postulated current reversal by phase shift is possible and analogous to flipping the DC bias polarity of the ST when the flat driving is used. It is important to highlight that the driving described by Eqs. (6.8) is not the only option with which single-electron current at zero-average bias could be generated. In fact, any shape devised for creating a time evolution of \( \delta \epsilon_{L/R}^{\pm} \) similar to the one shown in Figs 6.6(b) and (c) should create a single-electron current provided it is driven at an appropriate frequency. The similarity pointed here is that one threshold for a tunneling event through one junction is crossed and later another threshold for the contrary tunneling event through the opposite junction is also crossed. For example, the driving method of Eqs. (6.1) generating an ellipse would not create single-electron currents when \( V_{0b} = 0 \) since it would cross tunneling thresholds for emptying the island through the right junction first and later for filling it through the same junction. This creates a permanent cycle of back-tunneling through the right junction and the average current at the end of the cycle would be zero. Finally, these new driving methods could help in increasing accuracy in FPC following the discussion on its ultimate accuracy. One could devise a path that slows down the threshold crossings while keeping the driving frequency so that \( \dot{v} \) decreases hence decreasing \( \Omega \).
7. Conclusions

This thesis was centered on applications of SINIS STs. It presented new concepts and experiments as well as exploited some already known ones. Exploiting these applications, I first described in Chapter 4 how in the experiments of Pub. I we achieved reduction of QPs in the leads of a SINIS ST. This was done by biasing a SQUID in which one of the leads was shared with a SINIS ST and the other had a larger energy gap. Such device presented a probe for the density of QPs in the lead shared with the SQUID. Because of the geometry of the lead, QPs did not relax efficiently. At the same time, the turnstile constituted a continuous injector of one QP per driving cycle so that poisoning was promoted by its operation. Hence, QP density was high around the insulating junction connecting the lead to the normal-metal island. A reduction of QP density in the ST lead was observed as long as the voltage bias applied to the SQUID was such that $e|V_{\text{SIS}}| < \Delta_1 + \Delta_2$. Furthermore, the most efficient reduction condition was observed at $e|V_{\text{SIS}}| = \Delta_2 - \Delta_1$ confirming theoretical calculations based on QP transport along a Josephson junction. However, some poisoning enhancement in the biasing interval below the sum of gaps was observed which was not caught by the model. Despite of this, a reduction in QP density of one order of magnitude was achieved around the most efficient biasing point. This was observed at all investigated injection rates and even extrapolated to DC operation. Hence we demonstrated an active, extra channel for suppression of QPs.

In Chapter 5, I presented an extension of the applications of the SINIS ST, which was the objective of the experiments reported in Pub. II and the central topic of discussion in Pub. III. A new realization of the watt, the SI unit of power, was presented by using the typical operation of the SINIS ST and measuring the temperature rise in a normal-metal trap contacted to each lead. It was shown that the total average power injected to the leads is $\sim 2\Delta f$ as a result of creation of two QPs per operation cycle close to the edge of the superconducting gap. Robustness of this result was seen against many operation parameters such as driving frequency, gate driving amplitude and bias voltage among others. Remarkably, power
generation was also observed at zero bias voltage when no net particle and charge current are produced. Hence two clearly distinct operation regimes were described, namely the zero and non-zero bias ones. In the former, power is distributed among the leads according to the transparency of the junctions joining them to the island. In the latter, power is distributed almost equally between both leads ignoring the asymmetry between tunnel resistances. The discussion was extended to the deviations of the injected power from $2\Delta f$ as reported in Pub. III. This was done by inspecting a simplified mathematical model of a driven NIS junction from which analytical expressions for the statistics of energy injection were derived. As a result, it was seen that average deviations increase as $(R_T f / \Delta)^{2/3}$ but also that presence of sub-gap states is detrimental for energy selectivity. Furthermore, increasing temperature also represents one of the main contributions to errors in power injection as electrons in the tail of the non-zero temperature Fermi-Dirac distribution give an excess of $\frac{1}{2}k_B T$ energy to the created QP. In addition, it was verified that for a measurement with large averaging, as typically done, noise in energy injection caused by the stochastic nature of single-electron tunneling can be largely suppressed compared to other error mechanisms. Equally important was the presented model of heat transport along the lead geometry employed in Pub. II. With this, it was possible to determine that errors stemming from the inability to dissipate all the power of the QP in the normal-metal trap were negligible below 120 mK compared to other deviations. This result was used in this thesis to justify the geometry used in experiments but it also constitutes an important result of Pub. III.

Finally, I showed the results and rationale of Pubs. IV and V in Chapter 6. It was shown that driving paths can be diversified by adding a modulation to the bias line of the SINIS ST which had not been done before in this device. It was also seen that with these new driving methods single-electron currents could be naturally generated. By inspecting the nature of back-tunneling errors it was possible to propose that a driving method created by bias and gate modulation could suppress such errors while keeping the driving frequency high. Such method consisted on applying a signal with twice the frequency to the source lead compared to that injected to gate. A clear suppression of back-tunneling errors was verified by observing an increase in the current at the pumping plateau so that accuracy with respect to $ef$ was increased by one order of magnitude as seen in the experiments reported in Pub. IV. Additionally, it was possible to generate single-electron currents with a bias averaging to zero in time by employing a similar bias driving. This was shown in Pub. V were it was clear that such method for generation of single-electron currents was robust against biasing conditions.

Envisions of future work were made in Chapters 4–6 for the results of each publication, respectively.
7.1 Personal reflection

Initially, the SINIS ST appeared to me as “only” a source of single-electron currents and candidate for a standard of the ampere. Even though the ability to emit single electrons at a known and very controllable rate is a great achievement, it soon became clear that the current generated by SINIS STs was an instrument that could give access to more information about the device itself and the environment to which it is connected. Already in the works of Pub. I the ability of single-electron currents to act as a probe for quantifying the presence of superconducting excitations in the device was clear. When preparing the experiments for Pubs. IV and analyzing its results it became clear to me what opportunities the turnstile offered for understanding and describing the dynamics of events that lead to reductions in current accuracy. These opportunities arise mainly from the fact that such events obviously leave a trace in the measured average current. Therefore, when these are properly understood, strategies to avoid them can be created and accuracy can be improved. However, it was the proposal of new driving methods as shown in Chapter 6 that led us to appreciate the dynamics of back-tunneling and not the other way around. This novel proposal also opened the window towards new ways in which single-electron currents could be generated in the SINIS ST and this was exploited in a very preliminary demonstration carried out in Pub. V. In spite of these results one question remained unresolved and its answer paved the way to the main finding of this thesis and to expand the applications of the device. This is, with how much energy is a QP created in the leads of a SINIS turnstile during its operation? It turned out that the answer was that these excitations were created close to the edge of the gap as was hinted previously and that such behavior was robust against the many operation parameters of the device. This allowed to propose that the SINIS ST could be used as a realization of the watt, the SI unit of power, since it can give with certain accuracy a power $\sim 2\Delta f$ which is the main conclusion of Pub. II. The deviations of the actual measured power from this value was the main motivation of Pub. III. The willingness to understand the underlying dynamics of QP injection also played an important role. Then simple expressions that related the operation parameters of the device to deviations from the expected power value were found and hence the understanding of the working principle of the device was simplified. Additionally, in Pub. III we saw that the temperature of the system had direct impact on the injected power by means of an equipartition-like relation. Thus, at the end of the work it was clear that the SINIS ST is not only a device that has the potential to be a current standard but also that its composition and internal dynamics offers new applications that can be exploited. Maybe new ones could be found in the future.
References


References


[33] H. K. Onnes, Further experiments with liquid helium. C. On the change of electric resistance of pure metals at very low temperatures etc. IV. the resistance of pure mercury at helium temperatures, in Through Measurement to Knowledge (Springer Netherlands, 1991) pp. 261–263.


References


References


