JOINT DETERMINATION OF TRADE, PRODUCTION, AND FINANCIAL FLOWS IN THE MULTINATIONAL FIRM ASSUMING RISKY CURRENCY EXCHANGE RATES

A DOCTORAL THESIS IN MANAGEMENT

HELSINKI 1975
THE HELSINKI SCHOOL OF ECONOMICS
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A TWO-STAGE LINEAR PROGRAMMING MODEL BUILDING APPROACH

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FOREWORD

Abstract

Problems relating to big multinational business enterprises have been subject to much research effort and debate for about two decades. This results from the considerable role played by such enterprises in the world economy. Recently one emerging research trend in this context has been towards building operations research models for global decision making in the multinational firm. This is natural since research on the application of operations research techniques to the planning problems of national firms has been expanding at a fast pace. In the case of the multinational firm short-term (∼ tactical) management of finance has been subject to most research of this kind. The separation of the treatment of physical and financial flows is a simplification, however, which can lead to nonoptimal total plans.

Several authors have presented mathematical programming models for the (tactical) planning of finance in the multinational firm under the assumption that the physical activities (trade and production) have first been fixed at a preceding planning stage. The most advanced of these models are constructed to include the probabilistic nature of currency exchange rates.

Linear programming models have also been presented for the planning of trade and production in the multinational firm. Furthermore, linear programming models have been constructed for joint planning of physical and financial flows in the multinational firm under the assumption of deterministic currency exchange rates. In addition to the determinism of the currency exchange rates it should be noted that these models treat certain decision variables as fixed or omit them. These simplifications can lead to suboptimal total planning, too.
It is my purpose to propound how certain tactical decisions, which earlier O.R. modelling research has treated only separately, can be simultaneously treated in a manageable way in mathematical programming models for the multinational firm:

In this dissertation I show how the probabilistic nature of currency exchange rates (= "currency risk") can be taken into account in linear programming models for joint planning of trade, production, and financial flows in the multinational firm. This will mean that only the (discrete) probability estimates of future exchange rates have to be assumed to be available to the decision maker. The inclusion of currency risk is achieved by applying two-stage linear programming.

I also show how forward contracts can be included in linear programming models for joint determination of trade, production, and financial flows in the multinational firm facing risky currency exchange rates.

Furthermore, I demonstrate how the exposure of assets and liabilities to currency risk can be adjusted to fit into predetermined bounds.

I demonstrate the inclusion of interaffiliate transfer prices, which are declared for duty and tax assessment, as well as the inclusion of interaffiliate interest rates as decision variables in the kind of models discussed. I suggest a concept of "shadow transfer prices" for the analysis of interaffiliate transfer price bounds.

The construction of separate equations for the formulation of the objective function and the treatment of taxation is found to be advantageous.

I also briefly discuss the possibility of using the propounded normative model as a behavioral model of the multinational firm.

Particular attention is given to retaining readability, even at the cost of scientific rigor. This dissertation is thus meant to be applicable also as an advanced textbook in operations research and multinational management.
Acknowledgements

Above all I wish to acknowledge with gratitude the essential contributions made by Professor Veikko Jääskeläinen and Roy Dahlstedt, M.Sc. in preparing the ideas of this dissertation. Professor Jääskeläinen has given me invaluable guidance and much of his time as my thesis instructor. He actually provided me with most of the basic ideas needed in the research work to be presented. Roy Dahlstedt has helped me considerably by giving much of his time to discussions and to reading my text, and by his personal friendship.

I also wish to thank Professor Fedi Vaivio, Professor Aarni Nyberg, and Jyrki Malmio, M.Sc. for their help and time. Professor Jouko Manninen first suggested linearization to me for the handling of interaffiliate transfer prices. This suggestion is gratefully acknowledged.

The Helsinki School of Economics has offered a pleasant environment for research work, and very many good colleagues. Furthermore, I had the opportunity of being for almost three years a research fellow at the Institute of Basic Research at the Helsinki School of Economics at an earlier stage of my research work, which in spring 1973 led to the publication of my licentiate thesis in the same problem area. For this opportunity I sincerely thank Professor Jaakko Honko, Rector of the Helsinki School of Economics.

For checking the English language my thanks are primarily due to Mr. Philip Binham, M.A.

I also wish to take this opportunity to thank Professor Sakari Mattila for my mathematical education at the Helsinki School of Economics.
I am especially indebted to my dear wife Elina Salmi for her trust and patience.

I also acknowledge with gratitude financial support from the following sources: Liikesivistysrahasto (Outokummun rahasto), Vuorineuvos tekn. ja kauppat. tri h.c. Marcus Wallenbergin liiketaloudellinen tutkimus-säätiö, Jenny ja Antti Wihurin rahasto, Instrumentariumin Tiedesäätiö, Kauppatieteellinen yhdistys r.y., and Helsingin kauppakorkeakoulu.

Helsinki, 

March 1975

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1 PRELIMINARIES

An outline of prior research on the topic and of the research to be presented was already given in the Abstract of the Foreword. The primary purpose in this dissertation is to develop a manageable operations research formulation for simultaneous optimal assessment of decisions on trade, production, and finance in the multinational firm facing risky currency exchange rates. The secondary purpose of this dissertation is to suggest a tentative way of providing predictions of the behavior of the big multinational business enterprises in some particular situations.

1.1 INTRODUCTION TO THE MULTINATIONAL FIRM

What is meant by the multinational firm? There is a wide variety of definitions of the multinational firm in literature. It is characteristic of these

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1By a manageable model I mean a model that requires only obtainable input, is solvable numerically, and gives output on which decisions can be based in an actual decision-making situation.


definitions that the degree of multinationalism ranges from a national firm with straightforward export or import operations to a completely internationally owned and controlled world-wide conglomerate. The definition is naturally dependent on the particular problem under consideration. For our purposes the following definition is perhaps the most suitable. The multinational firm is an enterprise which controls producing facilities in more than one country.4

The concept of the multinational firm is elucidated with the help of Figures 1-1 and 1-2. The (uninational) firm is sometimes delineated by the chart given in Figure 1-1. The firm acquires a set of inputs (the trade flow directed towards the firm), transforms the inputs into outputs in the production processes (the production flow), and sells the resultant outputs (the trade flow directed outwards from the firm). This physical (trade and production) process is reflected in the financial process of the firm. The financial process is depicted by financial flows in Figure 1-1. The purchases of the inputs and the sales of the outputs cause certain financial flows. So does the transformation of inputs into outputs in the production processes. In addition, financial capital from (and to) the outside and its concomitant costs (e.g. borrowing, interest, equity issues, dividends, and so on) also cause a set of financial flows, which have not been separately drawn in Figure 1-1.5

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5This view has been propounded e.g. by Martti Saario.
FIGURE 1-1 The Structure of the Firm

FIGURE 1-2 The Structure of the Multinational Firm
(A two-country case)
Figure 1-2 delineates the multinational firm. For simplicity only the case of two countries is drawn in Figure 1-2. There is one producing affiliate drawn in each country. Each affiliate purchases inputs in its host country, transforms the inputs in the production processes, and sells the outputs in the host country. This is delineated by the trade flows and the production flows within each country. The financial flows within each country correspondingly delineate the payments for domestic purchases and the receipts from domestic sales as well as the financial incidents arising from the production processes. They also include other financial incidents, like borrowing, amortization, interest, dividends, etc. The multinational firm can import and export in the same way as an ordinary importing or exporting firm. In addition, the multinational firm is able to carry on trade between its affiliates in different countries. This is depicted by the trade flows between the affiliates. The multinational firm can also carry on financial transactions between its affiliates in different countries. This includes payments for interaffiliate trade, and interaffiliate loans, interest, dividends etc. This is reflected in Figure 1-2 in the interaffiliate financial flows. The interaffiliate trade and financial activities differ, however, in a fundamental way from conventional international trade and finance. The trade and finance activities take place between the units of the SAME firm. In other words, the relevant parties are no longer independent in the case of the multinational firm, as, for instance, the traditional theory of international trade presupposes. This fact can be utilized by the big multinational business enterprises, especially if their planning process is centralized enough. (In Figure 1-2 this prerequisite is depicted by the words "centralized planning".)
In addition it is important to note that there are different currencies in different countries. The exchange rates are not fixed. The relative values of the currencies can change via devaluations, revaluations, floating, and inflation.

When we discussed the multinational firm above, we noted that it can utilize such activities as are not open to the uninational firm, which plans and acts in a one-nation environment only. The essential features of the multinational firm are consequently contained in these plans and activities. It is instructive to assume that a reasonable set of the most noteworthy of these features can be presented in the framework of a categorization, which I give shortly. This categorization is by no means complete or unique. E.g. such important factors as know-how, organizational aspects, and size are not accounted for.\(^6\)

In general, when we consider the decision making process of the firm it is often customary to make a distinction between strategic and tactical planning. The main emphasis in the categorization to be given is on the tactical level, simply because the problems of this study relate to the tactical level of decision making in the multinational firm. Nevertheless, I commence on the strategic level.

Location adjustment. The multinational firm can within certain limits choose the countries where it locates its affiliates. It can invest its funds in options in different countries. It can construct and withdraw affiliates looking at the matter from a global viewpoint. Naturally

\(^6\)It seems plausible, however, to contend that a number of the decision variables which arise in the multinational firm, but not in the national firm, are contained in the items of the categorization. In a way the categorization is inconsequential, because it is intended primarily for the benefit of the reader.
in the case of capital in/(di)vestments these are matters of major strategic importance, and are influenced by a great number of different factors for consideration.

*Repatriation adjustment.* We have now moved down to the tactical level. The multinational firm can choose between different means in bringing home the revenues and its capital investments. The means vary from open repatriation to e.g. arranging unnecessary interaffiliate trade with appropriate transfer prices.

*Production adjustment.* The multinational firm can buy and sell goods and services between the affiliates located in different countries across national borders (cf. Figure 1-2). It can allocate its production globally between the affiliates in different countries. If necessary it can move its production over to other affiliates.

*Book profit adjustment.* The multinational firm can adjust its book profits in different countries and thus manipulate the taxes it pays. In addition to conventional book-keeping methods, this can take place by a suitable arrangement of trade, production, and finance.

*Currency risk adjustment.* The multinational firm can hedge and speculate against currency exchange rate changes by financial operations and/or by globally manipulating its trade and production pattern. The most important of the financial operations are raising loans and acquiring assets in suitable currencies, and making forward contracts.\(^7\)

What I have stated above is consistent with the following arguments and observations concerning

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\(^7\)These options are naturally not completely closed to a national firm, especially if it is engaged in imports or exports, but the perspective is much more limited in that case.
the multinational firm. National governments are worried about the ability of big multinational enterprises to move their production from one country to another, affecting national products, employment, imports and exports, and balances of payments of the host countries. National governments have tried to regulate the operations of the multinationals, e.g. to prevent them from evading local taxes. Labor unions in countries with high wage rates accuse multinationals of exporting jobs. Central banks are concerned about the balance of payments problems created by shifts in international financing. They fear that the multinational firms undermine the international currency system by hedging and speculating in "hot currencies". There are other similar points, but these may suffice as examples.

1.2 Research Problem

The problems of the research work to be presented are given below. They are given as they were stated before the research was completed.

1) How can we build a model for decisions to cover simultaneously the assessment of trade, production,
and financial plans in order to achieve optimal production, book profit, and currency risk adjustment in the multinational firm? What are the technical and conceptual difficulties and how can they be overcome? Tackle the following research task in order to find the answers: Construct for an idealized multinational firm a normative model for optimal production, book profit, and currency risk adjustment by means of simultaneous manipulation of suitable trade, production, and financial flows which are deemed to be decision variables. Call this the normative problem.

2) How can we predict the behavior of the multinational firm? What structure does the multinational firm give its various trade, production, and financial flows in different situations? How does the multinational firm react to economic environmental changes and measures of the host or the home country governments? Call this the behavioral problem.

The emphasis in the research project was very strongly on the normative problem. Most of my results on the behavioral problem can be attributed to my earlier research efforts. Concentrating on the normative problem turned out to be laborious enough.

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1.3 Review of Prior Research on Topic

The literature search on the topic indicates that much non-rigorous verbal text has been written on a variety of problems relating to the multinational firm, but that mathematical modelling is just beginning to develop.\(^{11}\) This view is corroborated by comments of other researchers in the field.\(^{12}\)

The perspective in prior (and this present) research on the topic can be visualized with the help of Figures 1-1 and 1-2. In the uninational case, elucidated by Figure 1-1, both basic and applied research on operations research models for planning have been developing at a fast pace for a relatively long time. Above all, ordinary linear programming has turned out to be an effective means particularly in the case of tactical planning problems. Now we are concerned with the multinational case. As can be seen in Figure 1-2, planning now becomes more complex. Constructing operations research models for multinational planning is a quite recent development. This can be seen in the fact that a major part of theoretical research results involving mathematical planning (and behavioral) models of the multinational firm have been achieved since the mid-1960's, and that a significant part of the relevant research results have been published not earlier than in the 1970's. In the same manner as


\(^{12}\) Many of the researchers whose works are reviewed in this section explicitly comment on this state of affairs.
in the early stages of uninational mathematical modelling, not much is known about real-life applications in multinational firms.

It is instructive (but not necessary) in this context to classify the prior relevant research on mathematical modelling for the multinational firm into five groups. This, of course, is a simplification. The groups will be 1) strategic planning models, 2) models for tactical production planning, 3) models for tactical planning of finance, 4) models for joint tactical planning of production and finance, and 5) descriptive models. This classification is based on distinctions between strategic/tactical planning, partial/integrated models, and planning/descriptive models, which are convenient here. For instance in the taxonomy strategic/tactical planning the location adjustment, discussed in Section 1.1, falls into the strategic planning range, while the rest of the adjustments are more tactical in nature. The decisions to be covered in this study are clearly tactical in nature. In prior research there is a rather clear-cut distinction, on the tactical level, between partial and integrated models. By partial models I mean models which cover either only physical or only financial activities of the multinational firm. By integrated models I mean models which simultaneously cover both physical and financial activities of the multinational firm. The concepts of planning and descriptive models mean the same as the concepts of normative and behavioral models.

\[\text{For a discussion and further references on strategic and tactical planning in the multinational firm see e.g. William A. Dymsza, Multinational Business Strategy, op.cit., Ch. 3.}\]
Strategic Planning

Few of the existing relevant applications of quantitative techniques deal with strategic planning problems of the multinational firm. David Rutenberg (1967) considered in his pioneering dissertation the possibility of viewing the operations of a multinational firm as a two-stage process, the first stage being the decision concerning which countries to operate in.¹⁴

Larry Merville (1971) constructed an investment decision model to solve the problem of rationing capital in the multinational firm.¹⁵ Using the E-model of chance-constrained programming he set the objective of maximizing the expected cash flow at the end of the planning horizon. The resultant model is a mixed integer linear programming model.

Niranjan Arya (1972) applied linear programming to formulation and analysis of capital budgeting models for multinational firms.¹⁶ All models developed in the study are based on the assumption that the parent company controls the operation of its subsidiaries. Arya's most general capital budgeting model maximizes the dividend payments to the parent company over the planning horizon.

Tactical Production Planning

From now on we will be discussing relevant research

on the tactical level. Much more research has been carried out on this level of planning problems of the multinational firm.

Veikko Jääskeläinen (1963) indicated how linear programming models can be applied to production adjustment in the multinational firm.\(^\text{17}\) He considered the case of a multinational firm with two factories located in different countries (cf. Figure 1-2). His model minimizes the total costs to the multinational firm by finding an optimal pattern of production and inter-factory trade in the face of relevant production and transfer (duty, transportation and similar) costs. The model in his unpublished paper is preliminary, however, omitting e.g. the effect of taxation.\(^\text{18}\)

Actually, most production planning models make no explicit distinction between multinational and unina- tional environments. E.g. Norman Driebeek (1969), in a textbook, gives an example of a linear programming model for production planning in the multinational firm, but he makes no factual distinction between a multinational and a unina- tional case.\(^\text{19}\) - These models are discussed no further here.

**Tactical Financial Planning**

David Rutenberg (1967) explored in his dissertation the possibility of using operations research approaches


\(^{18}\) In a multinational case, unlike a normal unina- tional case, taxation becomes a significant factor in assessing an optimal solution, since tax rates differ in different countries and this fact can be utilized by the multi- national firm (cf. book profit adjustment).

to planning in the multinational firm.\textsuperscript{20} In addition to generic planning prerequisites and concepts in the multinational firm he analytically tackled the following problem areas in his dissertation and later papers:\textsuperscript{21} the optimal use of tax havens and bilateral tax agreements; maneuvering liquid assets in the multinational firm, which is an extension of the former area; the optimal pricing and marketing effort for a product line marketed in several countries; modularization for multiple markets (this is again tactical production planning). The mathematical technique he adopted was stochastic programming with recourse.

John Petty II (1971) built a deterministic linear programming model for transfer pricing in the multinational firm.\textsuperscript{22} In his multiproduct model, production and trade between the affiliates (cf. Figure 1-2) in different countries are taken as predetermined and the transfer prices of the products are the decision variables. The model assesses these transfer prices in a way which minimizes the global sum of the costs included. He also modified his model to encounter multiple objectives by transforming some restrictions into goals using linear

\textsuperscript{20} David P. Rutenberg, \textit{Stochastic Programming with Recourse for Planning Optimal Flexibility in Multi-national Companies}, op.cit.


goal programming techniques.

Bernard Lietaer (1970) developed an operations research model to deal with foreign exchange risk in international business. He considered each asset and liability which is exposed to devaluation (or revaluation) a portfolio. He built a parametric quadratic programming model for portfolio selection (under risk) along the lines developed by Harry M. Markowitz and later by William Sharpe in their well-known publications. The decision variables in his multitemporal model relate to loan raising, acquiring assets, and making forward contracts in different currencies. The objective is to maximize global expected utility over the planning horizon. Lietaer's model can be criticized because it seems to require a prohibitive amount of data. It is also to be feared that if one tried to incorporate production and book profit adjustments into this currency risk adjustment model, the quantitative solution techniques for the envisaged model would be rather cumbersome. In other words, it is to be feared that this model would probably not be manageable enough.

Alan Shapiro (1971) developed in his dissertation models for optimal timing of various hedging operations against devaluation subject to risk. (The operations

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include forward contracts, discounting accounts receivable, advance purchase of inventory and other assets, and increasing prices.) He formulated his model versions as the particular class of dynamic programming problems called stopping rule problems. Shapiro and Rutenberg (1974) revisited the optimal timing of forward contracts in a joint paper.26 In his dissertation Shapiro built "a central model" for optimal short-term financing and hedging decisions in a multinational corporation with a special cash center for the routing of interaffiliate financial transactions. In order to minimize the global expected after-tax cost for the multinational corporation, the central model assesses the optimal quantities for the relevant financial flows. This central model utilizes as input the optimal timing of hedging decisions given by the proper models. In the central model there are surprisingly many technical errors, which are not, however, fatal.

Joint Tactical Planning of Production and Finance

Dileep Mehta and Isik Inselbag (1973) presented a linear programming model under certainty (= the model is deterministic) for simultaneous treatment of physical and financial flows in the multinational firm.27 The model maximizes the discounted global profit of a multi-product multinational firm over a multiperiod planning horizon. Interaffiliate transfer prices and interest


on interaffiliate loans are taken as predetermined and not treated as decision variables. Forward contracting is excluded. Also in this model there is a surprisingly large number of technical errors of varying importance. I have discussed Mehta's and Inselbag's model and the errors in detail elsewhere. Mehta and Inselbag suggest that the implementation of a factually centralized planning model for the multinational firm can best be achieved by means of "coordinated decentralization" with the help of the decomposition principle.

In my own licentiate thesis (1973) I built a one-product multi-period linear programming model for "production and profit adjustment" in the multinational firm under certainty. This model treats production, trade between the affiliates in different countries, transfer prices declared to the authorities, and interest on interaffiliate loans as decision variables. The objective is global profit maximization. I also showed how this model can be used in creating hypotheses of the behavior of the multinational firm. Presenting the relevant lines of thought was the primary aim of this thesis, model building and hypothesis creation being secondary, then.

The research results to be presented in this dissertation have been recorded previously in a preliminary way: Veikko Jääskeläinen and Timo Salmi (1974) demonstrated in a joint paper how currency risk can be taken into account in linear programming models for simultaneous

28 Timo Salmi, Monikansallinen yritys ja lineaarinen optimointi, op.cit., Ch. 4.
planning of trade, production, and financial flows in the multinational firm.\textsuperscript{30} In August 1974 I popularized the research results in a textbook written in Finnish.\textsuperscript{31} Furthermore, a small part of the results was given in my licentiate thesis.\textsuperscript{32}

\textbf{Descriptive Models}

Only two relevant mathematical programming or corresponding modelling approaches are known to me. There might be some more among econometric studies (although I do not think so), since in this respect the literature search may not be complete enough. This results from the fact that the behavioral problem received relatively little attention in my research work, as was noted earlier. Nevertheless, the trend is quite evident: the theory of the multinational firm remains to be written, while the theory of the (uninational) firm has been much considered. A large number of approaches have been used in the latter case.\textsuperscript{33} For instance models based on marginalistic and linear programming approaches are well established.

Thomas Horst (1970) extended the classical margina-


\textsuperscript{31} Timo Salmi, Monikansallinen yritys ja lineaarinen optimointi, op.cit.


\textsuperscript{33} See e.g. Thomas Naylor & John Vernon, Microeconomics and decision models of the firm (1969), and Ira Horowitz, Decision Making and the Theory of the Firm (1970).
listic model of the firm to a two-country case.  
He studied the model behavior under different tariff and tax rates. It seems to me that his dissertation can be deemed pioneering in this direction.

In my licentiate thesis (1973), as was already mentioned, I tentatively suggested the use of a linear programming model of the multinational firm for creating hypotheses of the behavior of the multinational firm. The selection of variables of a descriptive model of the multinational firm has been commented on by Roy Dahlstedt in a short paper.

1.4 RESEARCH APPROACH AND METHODOLOGY

We return to the research work in hand. We discuss the normative problem first. As stressed in Section 1.2 ("Research Problem") we concentrate on the normative problem. Figure 1-2 delineated the physical (trade and production) and financial flows in the multinational firm. As discussed earlier, the primary aim is to develop a manageable operations research modelling formulation for simultaneous optimal assessment of trade, production, and financial flows in the multi-

34 Thomas Horst, A Theoretical and Empirical Analysis of American Exports and Direct Investments (doctoral dissertation, 1970), and


national firm to achieve production, book profit, and currency risk adjustment; decisions which earlier O.R. modelling studies have treated only separately. In accordance with the normative problem statement, this is attempted via constructing a particular stochastic linear programming\textsuperscript{37} model to determine how an idealized multinational firm can maximize expected global profit in a multiple currency world when certain trade, production, and financial activities are defined as decision variables. (The intention is not to provide directly the potential links between the existing separate models, but to develop linear programming formulations to cover simultaneous optimal assessment of trade, production, and financial flows. Naturally the results from earlier research are utilized when applicable. Constructing direct links between existing models would be impractical, since for the most part they are not compatible as such.)

In detail: This model is to give, for the idealized multinational firm, on a global scale, an optimal pattern of production, sales, and purchases, as well as interaffiliate external transfer prices, loan raising from external and internal sources, interaffiliate interest rates, and forward contracting with outside parties. The model is to establish this pattern in the face of potential probabilistic changes in foreign exchange rates.

internal requirements on the currency exposure in each appropriate currency held by the multinational firm as a whole, relevant production costs, taxes in different countries, transportation costs between the affiliates, duties, costs of external loans, and forward contract rates.

Next we discuss an important point. The model is constructed for the multinational firm assuming an idealized situation. Consequently I use the expression "the idealized multinational firm". Basically the idealization means that the number of superfluous model details will be kept at a minimum, and that the model does not directly simulate any single existing multinational business enterprise. Thus, in the face of the normative problem stated, model details which have earlier been developed and considered by other researchers, can be kept simple. This is advisable, since in this way we can concentrate on the essential new points, without the burden of too many details. Nevertheless, the model is constructed in a manner which allows the incorporation of the omitted details, when required.  

Model details are elaborated next. The idealized multinational firm for which the model is built is

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38 For example, the model is a one-product model. Since the number of products in the model is not essential because of the nature of the problem statements, the model is thus kept simple in this respect. This suffices, because when needed an extension into a multi-product case could easily be made in principle. Many examples can be found of parallel extensions into multi-product models in uninaional as well as in multinational cases.

Actually, the model constructed by Mehta and Inselbag in particular (see footnote 27 for the reference) conveniently includes a great many such details as should be taken into account in a real-life planning situation, but which are simplified here, since they are not new theoretically!
assumed to be a one-product firm operating in several countries through its affiliates. Central planning is assumed in constructing the model. The objective is the maximization of expected global profit over the planning horizon stated in the currency of the home country. The currencies of the host and home countries are subject to fluctuations and potential devaluations as well as potential revaluations. Only the relevant discrete probability distributions of the future exchange rates need be assumed to be available to the decision maker. Two-stage linear programming techniques are applied. The planning horizon is divided into two subperiods (pre- and post- the expected change in the currency exchange rates). In the model the trade flows are decision variables which consist simply of external purchases of a raw material, external sales of a finished product, and interaffiliate trade in an intermediate product. The financial flows to be included as decision variables arise from external and interaffiliate sales and purchases, production costs, raising and repaying of both external and interaffiliate loans and paying interest on them, transportation costs, duties, and taxes. The incorporation of forward contracting in the model evokes decision variables for forward contracts made by each affiliate in each foreign currency with third parties (banks). Currency exposure of current assets and current liabilities of the multinational firm stated separately for each currency is defined as a set of decision variables which can be constrained to fit into predetermined bounds.

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Deterministic simulations are run with the model to check it as well as to demonstrate it and the decision rules it gives.

Next we discuss the behavioral problem. In the behavioral problem the aim is not to say how the multinational firm should act, but to try to find out how the multinational firm is likely (ex ante) to act in different situations.

To tackle this question of making predictions of the behavior of the multinational firm we proceed as follows. It is assumed tentatively that planning in the big multinational business enterprises is or will be rational enough for us to try to predict ("generate hypotheses" of the pattern of) their behavior by means of the normative linear programming model which is to be built for the normative problem.41

40 In connection with the multinational firm we are perhaps still more interested in the future than in the present.

41 The assumption of relative rationality made here is prompted by two circumstances. First, in the uninal national case there are examples of a similar approach. Second, this assumption is also prompted by the opinion that the big multinational business enterprises can and are beginning to use advanced quantitative techniques also in planning their global operations. This view is propounded e.g. by Veikko Jääskeläinen in discussions on the matter, by Norman Driebeek in Applied Linear Programming, op.cit., for example on page 42 where he comments on multipant, multitime period models. Also the findings by Guy Stevens are noteworthy here, although they concern another aspect of the multinational firm. He states in an abstract on his doctoral dissertation Fixed Investment Expenditure of Foreign Manufacturing Affiliates of U.S. Firms: Theoretical Models and Empirical Evidence (1967): "there is considerable evidence that the behavior of plant and equipment expenditures of foreign subsidiaries is consistent with a theory of worldwide profit maximization subject to financial constraints;" (continued)
Suggestions for action which the linear programming model gives are in an implicit form. We wish to discover and interpret verbally general features of action which the model suggests for the idealized multinational firm in various situations. This is done by deducing directly from the model and when this is too difficult, numerical experiments are made (i.e. deterministic simulations are run, cf. the normative problem) with suitably reduced versions (e.g. in numerical experiments only the two-country case is run). Situations with interesting interpretations are sought. The results are condensed and interpreted appropriately, and called hypotheses of the behavior of the multinational firm. The aim of the idealization is slightly different in connection with the behavioral problem from what it was in connection with the normative problem. The purpose is to simplify reality to facilitate a synthesis in order to tackle the behavioral problem.

Some of the lately published texts on operations research models for increasingly complex situations claim actual application in unnamed real-life multinational firms. Examples of such texts are William F. Hamilton & Michael A. Moses, "An Optimization Model for Corporate Financial Planning," Operations Research, May-June 1973, 677-692, and Bernard Lietaer, Financial Management of Foreign Exchange: An Operational Technique to Reduce Risk, op.cit. It should be stressed that these are selected instances of positive evidence or statements. Also negative evidence and statements exist. What has been said in this footnote is not conclusive in either direction.

42 The problem naturally arises of what is meant by "interesting".

43 It should be noted that actually these "hypotheses" are already fixed by the basic assumptions which are made for the normative linear programming model to assess the physical and financial flows in the idealized multinational firm.
1.5 IMPORTANCE OF TOPIC

In this section we discuss points in favor of the selection of the research problems of this study. In the next section we debate the related question of the contribution to knowledge attempted in the research work presented.

First, points are given which relate primarily to the normative problem.

One reason for the need of the proposed kind of research is the fact that so far simultaneous optimal assessment of both financial and physical activities to achieve production, book profit, and currency risk adjustment in the multinational firm facing risky currency exchange rates, has not been incorporated in a single operations research model. For example the few known normative mathematical model building studies made to tackle foreign exchange risks in the multinational firm define only financial operations as decision variables. They take production and trade flows and other factual decision variables as predetermined. The adjustments mentioned above have recently been the subject of research (as seen in Section 1.3 "Review of Prior Research on Topic"), though more or less separately. Now one evident major direction of further development is in incorporated models.

The theory of the multinational firm still remains largely unwritten. No significant progress is claimed towards that objective in the research work done. Nevertheless I believe that the development of such a theory

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44 In general it can be said for the importance of any topic involving the multinational firm that scholars stress the point that the influence exerted on world economy by the big multinational business enterprises is steadily increasing.
will finally be through results concerning operations research model building for managerial decision making in the multinational firm. I have pondered this question elsewhere.\textsuperscript{45} It is interesting to note in this connection that it seems intuitively evident that the theory of international trade is losing whatever relevance it may have had, if the multinational firm is not incorporated. Although the research project falls into the category of basic research and not applied research, one pragmatic point can be made for the need for this kind of research: Especially in the face of the current European integration, the impending resources crisis, and the developments in foreign exchange markets the role of consistent planning is becoming increasingly important for Finnish firms, which have to function in an international environment in competition with foreign-based multinational firms. And it seems highly probable that particularly U.S. based, and also other big multinational business enterprises use advanced operations research techniques for planning. Pertinent knowledge of planning tools, which can be applied in production, book profit, and currency risk adjustment in these multinational enterprises, is thus needed.

Second, points are given which relate primarily to the behavioral problem.

Big multinational business enterprises are an important factor in the world economy and their significance to the states in whose jurisdiction they act is notable.\textsuperscript{46}

\textsuperscript{45} Timo Salmi, The Multinational Firm: a Mathematical Programming Model Building Approach (licentiate thesis), op.cit., pp. 20-23 (Section 2.2.2: "Steps for Studying the Implications of the Multinational Firm").

\textsuperscript{46} For further discussion see e.g. Raymond Vernon, Sovereignty at Bay: the Multinational Spread of U.S. Enterprises (London, 1971).
Governments and other interested parties have expressed their concern over existing and potential possibilities of adjustment, which are open to the multinational firm. To be able to predict the behavior and reactions of the big multinational business enterprises is one prerequisite of sound economic planning. All the more so if the country is small or if the position of foreign firms is dominant within the economy. So far there is too little knowledge of this kind and research is needed.

1.6 Contribution to Knowledge Arising from This Research Work

The main contribution intended in this study is to provide an operations research modelling formulation to treat production, book profit, and currency risk adjustment in the multinational firm facing risky currency exchange rates, by means of simultaneous manipulation of suitable trade, production, and financial plans using special linear programming techniques. Depending on the point of view we hold this can be said to entail:

1) An extension of probabilistic operations research modelling formulation to simultaneously cover decisions which earlier have been treated separately.

47 For example the Nordic Council have recently debated upon the activities of big multinational business enterprises and the need for passing proper legislation on them.

48 The forms of contribution arising from the research project in hand were already touched upon in the previous section where we discussed the importance of the topic.

A doctoral dissertation must be based on "independent research work". This self-evident statement is naturally a necessary basis for any contribution to be made.
2) New research in the emerging research area of global decision making in the multinational firm by operations research techniques.

3) An attempt at a computationally feasible formulation which involves obtainable input and applicable output, to encounter, under risky currency exchange rates, decisions involving manipulation of trade, production, and financial plans in the multinational firm.\textsuperscript{49}

4) A new application of two-stage linear programming techniques.

When we pursue some aspects of the question of the attempted contribution to knowledge on a more detailed level, the following notions are noteworthy.

I show in this dissertation how the probabilistic nature of currency exchange rates can be taken into account in linear programming models for multinational planning also in the case of simultaneous assessment of physical and financial plans. This means that only (discrete) probability estimates need be assumed to be available to the decision maker.

I also show how forward contracts, interaffiliate transfer prices to be declared to the authorities, and interest rates on interaffiliate loans can be incorporated as decision variables. Previously these factors have been treated as predetermined or detached from joint planning of trade, production, and financial flows in the multinational firm. When these factors

\textsuperscript{49} In other words the model is kept manageable, cf. footnote 1. Manageability will be attested by 1) including in the model only constants for which proper estimates are readily available to the management of an enterprise, 2) retaining the linearity of the model, and 3) demonstrating that planning budgets can be readily derived from optimal solutions to the model.
are included, the relevant normative model does not lead to suboptimal total planning in this respect. Also in connection with the behavioral problem better results are to be expected when these decision variables are not treated as predetermined.50

I demonstrate restricting the currency exposure of assets and liabilities of the multinational firm to fit into predetermined bounds in models of the kind discussed.

In prior congenial models the handling of currency conversion has either been unnecessarily laborious or has been performed incorrectly. An improvement in this respect is attempted in this study. Since I have made experiments with the model to be presented in this dissertation, albeit using only fictitious data, it is to be hoped that the technical errors infesting a considerable portion of prior congenial research are not repeated.

1.7 PLAN OF THE TEXT

In this section I discuss the goals, mode, and plan of the text of this dissertation.

The text is designed for a reader who has some prior knowledge of linear programming, its application in managerial problems, and decision theory, but who need not have specialized in this field. The presentation of the model and its details is supported by parallel fictitious numerical examples. I also occasionally linger on mathematical trivia for the benefit of a non-mathematical reader. Thus the attempt to retain readability may even be made at the cost of scientific rigor. The reason for this manner of proceeding is that otherwise the circle of potential readers becomes

50 If empirical evidence supported this omission, this, of course, would be another matter.
all too small.

In Chapter 2 limitations and key assumptions of the study are discussed. First, general limitations of the topic are discussed together with the limitations and simplifications arising from the problem statements as well as from the research approach and methodology. Second, the outlines of the model frames, in the idealized situation for which the model is built, are taken up as an introduction to the further discussion on some limitations and key assumptions on a more detailed level.

In Chapter 3 the model is given under certainty (= the currency exchange rates remain fixed over the planning horizon). The model is given in a symbolic form. A numerical example is presented simultaneously, for the reasons already discussed. As noted earlier, superfluous model details are kept simple here, as well as throughout the whole dissertation, in order to demonstrate the essence of the ideas developed. This chapter presents the inclusion of interaffiliate transfer prices as decision variables in a linear programming model for joint planning of trade, production, and financial flows in the multinational firm. These are the transfer prices which are declared to the authorities for tax and duty assessment. The most convenient way to handle the objective function in a multiple currency planning situation is demonstrated, too. - In the numerical example appropriate budgets, which would facilitate an implementation, are derived from the optimal solution. - After a discussion on mathematical preliminaries, the concept of "shadow transfer prices" is developed. The opportunity cost of capital in the model is commented on, in connection with presenting the "conventional" shadow prices of the numerical example. - Last a sensitivity analysis is carried out in the numerical example, and the handling
of the maximization of profit after repatriation to the home country is briefly considered.

In Chapter 4 currency risk in the form of probabilistic future currency exchange rates is incorporated into the linear programming model of Chapter 3. The model can be kept linear by transforming the model into a two-stage linear programming model. Again a numerical example is given in conjunction with the presentation of the model. The numerical example is slightly altered from the numerical example of Chapter 3 in order to better demonstrate the model. Again appropriate budgets are derived from the optimal solution to the fictitious numerical example. At the beginning of the chapter the two-stage linear programming approach is reviewed.

In Chapter 5 the model under currency risk is augmented with further relevant decision variables. First, inter-affiliate loans and the interests on them are included as decision variables. (In Chapter 3 these variables are omitted for simplicity.) Then, decisions on forward contracting with third parties (banks) are incorporated in the model. Last, it is demonstrated how the currency exposure of current assets and liabilities in each pertinent currency held by the multinational firm can be fitted into predetermined bounds. The model can still be kept linear. The numerical example of Chapter 4 is enlarged appropriately and presented simultaneously with the main text. Deriving appropriate budgets, from the optimal solution to the fictitious numerical example, is demonstrated once more.

In Chapter 6 the behavioral problem is considered. The hypotheses generated as a result of tackling the behavioral problem are given. Details of the deduction
process and the deterministic computer simulation runs are not lingered on, since they are not essential in this dissertation.

Chapter 7 concludes the dissertation. The results deliberated in the main body of the text are summarized and evaluated. Some extension prospects of the model are taken up. Finally directions for relevant further research are suggested.
This chapter discusses the limitations and key assumptions of the study. Their background and the reasons for making them are considered. First, we discuss general limitations, simplifications, and key assumptions arising from the problem statements, the research approach and the goals of the text of this dissertation. This is done in order to give the reader a proper image of the limitations and key assumptions viewed on different levels of generality. Second, the selection of a production structure in the idealized multinational firm is discussed. The discussion on the limitations and key assumptions may appear unusually long, but in this way they are not deliberately hidden in the text. When relevant, they are naturally discussed also in later chapters.

2.1 LIMITATIONS AND KEY ASSUMPTIONS OF THE STUDY

In the previous discussion in Chapter 1 a host of limitations have already been stated more or less implicitly. Obviously, an explicit statement of every single limitation of a study is an impossible task. Here we try to form a general view of them: This section first comments on the limitations of mathematical model building approach in general. We then tackle limitations and key assumptions which relate primarily to the normative problem, and after that those which relate primarily to the behavioral problem. Last, the section comments

1There are the unstated silent assumptions as in every study.
on the nature of the numerical examples presented for the
benefit of a non-specialized reader.

First of all a general difficulty afflicting the
kind of research approaches discussed must be noted.
It is the inherent complexity of the international
environment and operations. This is the major reason
why constructing mathematical models for global decision
making in the multinational firm, as well as for theoretical
frameworks of the multinational firm, is as new a research
trend as it is. As in other similar research the negative
impact caused by this fact must be recognized.3

The statement of the normative problem entails at
least the following significant limitations and simplifi-
cations. We are concerned with the manufacturing multi-
national firm, since it is the most prevalent case.

2 I reviewed some relevant properties of a mathematical
programming model building approach (especially in
relation to the behavioral problem) in Timo Salmi,
The Multinational Firm: A Mathematical Programming
Model Building Approach (licentiate thesis), op.cit.,
p. 26-27 (Section 2.4.1: "Of Mathematical Programming
Model Building Approach"). The main points were that
mathematical models can include the influence of several
factors at the same time; that the possibilities of
sensitivity analysis are improved; and that mathematics
in itself provides a powerful tool for deduction. On
the other hand it was noted that the quantitative nature
of mathematical programming models affects the factors
which can be included; and that numerical solvability
(read: manageability) must be retained.

3 It should be noted that divergent views are held of
the applicability of O.R. in the multinational firm.
E.g. David Rutenberg, who has been referred to earlier,
seems firmly to hold the view, on the basis of his
research, that it is both possible and sound to use
O.R. techniques in the multinational problems of the
multinational firm. On the other hand, e.g. James S.
Shulman, Transfer Pricing in the Multinational Business
(doctoral dissertation, August 1966), seems to hold
the opposing view, which he substantiates in his doubts
concerning transfer pricing in the multinational firms
"by means of a mathematical approach to optimum solutions".
The results thus do not directly apply to e.g. multinational banking. As noted earlier such important factors as know-how, organization, and size are not accounted for, because this would be difficult considering the methodology applied and the present state of research in this field.

In the classification tactical/strategic planning we are concerned with problems of tactical planning level. Location adjustment is omitted because its inclusion would lead to both mathematical and conceptual difficulties. For convenience repatriation is only briefly touched upon in this study. These simplifications are fully compatible with precedents.

In accordance with the research approach and methodology the model is constructed for an idealized situation, which, as discussed, means that the model is kept simple and does not simulate any particular multinational business enterprise. This idealization is used in order to concentrate on the essential points (cf. Section 1.4: "Research Approach and Methodology"). Because the emphasis is on the relevant principles, such factors as the mechanics of international payments, details of

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5 Later we shall see that limits set by strategic considerations on tactical planning can be included in the model by applying suitable requirements at the end of the planning horizon. This is the conventional procedure for tactical planning models. In addition, in one instance also the objective function of the model reflects the relation between strategic and tactical planning levels (see Section 3.5.4).
tax-laws in different countries, and other similar factors, which are very relevant in actual practice, are left out. In particular individual decision-making applications many more (from our point of view, this time, trivial) model details would have to be included (cf. also footnote 38 in Chapter 1). Since decisions concerning trade, production, and financial flows in the multinational firm are covered simultaneously, fewer details can be included in this integrated model than in less comprehensive partial models.  

Next we discuss some limitations and assumptions which are more specific in nature.

In constructing the model, centralized planning without any organizational problems or friction is assumed for the idealized multinational firm. This important assumption, which is here only briefly commented on, could be discussed at great length. The motivation for making this assumption is simply the following. Every relevant prior research work, in the field we are considering, makes a similar assumption. A parallel assumption is prevalent also in operations research models for the uninational case (and in microeconomic theory!). Considering the precedents, I feel that making a contrary assumption would be too demanding. - Since making an assumption of centralized planning in the multinational case has been defended by other researchers in the field, I will only mention one new point in its favor. This point is the recent developments in global computer on-line systems via communication satellites, which warrant the possibility of arranging information networks on a global scale. Nevertheless, once more the weight of refuting points must be fully

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6 For definitions see page 10.
The objective of the multinational firm will be global nonrepatriated profit maximization over the planning horizon, stated in the currency of the parent country. Also this choice is prompted by a good number of precedents. This item warrants a comment. Howard Perlmutter propounded in 1965 a theoretical classification of multinational firms. This classification is now well-known and generally accepted. He classified the multinational firms into three categories: 1) home-country-oriented (ethnocentric), 2) host-country-oriented (polycentric), and 3) world-oriented (geocentric). At first sight it may seem that we are concerned with the ethnocentric case, but this is not so. Measuring the profits in the currency of the parent country actually only means that, in the model, the yields in the different currencies have to be computed in a common unit. The model is in accordance with a geocentric operation philosophy.

Two-stage linear programming techniques are applied. This fact gives rise to some particular limitations and simplifications. First, there are the general limitations

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8With one exception, which is made when sensitivity analysis in the objective function of the numerical example of Chapter 3 is carried out.

and key assumptions of linear programming. Thus, for example, the relevant environment of the idealized multinational firm, and the potential reactions of the environment, are taken into account in the model via the constraints, not via the objective function. No nonlinear relationships occur, and so on. Second, there are the limitations and key assumptions relating particularly to the two-stage formulation. From our point of view the most noteworthy limitation is the fact that in two-stage linear programming applications the model size tends to increase, endangering the manageability of the approach. Thus, only the future currency exchange rates are made probabilistic in the model. All the other factors are kept deterministic. Another kind of limitation of a two-stage linear programming approach is that the probabilities of the future states of the world have to be assumed to be independent of the decisions made. Thus, for example, it is assumed that the future currency exchange rates are independent of the actions of the idealized multinational firm under observation. This assumption, however, would have been made anyway,

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10 For an elementary discussion on the general limitations of linear programming see e.g. Frederick Hillier & Gerald Lieberman, Introduction to Operations Research (1967), pp. 135-138. The properties of linear programming, which are taken up in above, are proportionality, additivity, divisibility, and determinism (which is not relevant here). Furthermore, e.g. the "extreme" nature of linear programming solutions is relevant, too. (The optimum solution always lies at an extreme point of the region of feasible solutions.) For an elementary discussion on this aspect see e.g. Timo Salmi, The Multinational Firm: A Mathematical Programming Model Building Approach (licentiate thesis), op.cit., pp. 103-105.

11 For further references to the subject, see references given in footnote 37 of Chapter 1, and in footnote 4 of this chapter.

12 This results from the fact that each considered future state of the world must be accounted for in the model, as will be seen later.
since, just as in prior congenial studies, a contrary assumption would have been too difficult to handle. - The limitations and key assumptions of linear programming and two-stage linear programming could be discussed at great length, but this is not necessary, because relevant discussions can be found elsewhere.

When an operations research model is built, a large number of specific, but less strict, assumptions have to be made. The simplifications and assumptions for the model to be presented could be classified into two categories. First, there are the true simplifications and limiting assumptions. These concern model details, which cannot be included without further research. The essential simplifications are the following. 1) The payments for interaffiliate trade (cf. Figure 1-2) are effected in the currency of the host country of the selling affiliate (the trading currency is not a decision variable). 2) The affiliates always convert any foreign cash not needed for foreign payments into the currency of the relevant host country. In other words, the affiliates hold in cash only the currency of the relevant host country. 3) The currency exposure of assets and liabilities of the multinational firm is accounted for only via appropriate constraints. These simplifications are made, since neither my own research work nor that of the other researchers in the field have yet covered these aspects. (As will be seen in Chapter 7, these simplifications give rise to suggestions for further research.13) Second, there are the ostensible simplifications and limiting assumptions. These concern model details which can easily be included,

13 The inclusion of the trading currency seems rather straightforward on the basis of my preliminary analysis. This is as yet inconclusive.
if required. For example, losses cannot be carried forward in taxation in different countries in the model. The reason for this is that the assumption has to be made one way or the other. A contrary assumption could be easily reflected in the model. Another example of an ostensible simplification is the following. In some countries, as in Finland, book profits may be adjusted also by manipulating depreciation and by undervaluing inventories. These features are omitted, since a compatible way to handle this assumption can be found elsewhere, when necessary. Thus the non-essential model details can be kept simple, as was required for readability. Some similar details are discussed in the next section, and later when necessary, but I hope that the general outlines have been satisfactorily presented.

In the behavioral problem I ask: "how does the multinational firm react to economic environmental changes and measures of the host or the home country governments?" This sentence could be continued by adding "and what are the consequences to the host or the home economy?" This was omitted, because it is to be feared that the

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14 E.g. in this particular case in Veikko Jääskeläinen, Optimal Financing and Tax Policy of the Corporation, op.cit., pp. 75-77. (For a numerical example see Juha Nissilä, "Tavoitetulosajatteluun perustuvan yhdistetyn tulos- ja rahoitussuunnitelman laatiminen lineaarisen optimoinnin avulla," The Finnish Journal of Business Economics, No 3, 1974.)

15 I have considered one fictitious numerical example of the consequences on the balances of payments and gross national products of two host countries of the multinational firm, resulting from prohibiting capital movements between the two countries in question, in Timo Salmi, The Multinational Firm: A Mathematical Programming Model Building Approach (licentiate thesis), op.cit., pp. 68-71, and Timo Salmi, "Monikansallinen yritys tarkasteltuna matemaattisen optimointimallin avulla," op.cit., pp. 119-124. The numerical example also attempts to demonstrate that "international trade between [affiliates in] different countries is conceivable [at least in a taxless world economy] also in the absence of a comparative advantage." It also demonstrates that "the effects of government actions become more involved and might even be rather unexpected in the existence of the multinational firm."
study would grow all too large. Potential counter measures which could be taken by the governments against the adjustment activities of the multinational firm are not taken into account in tackling the behavioral problem. This would be too demanding at this stage.

No attempt is made to test the normative model in actual practice. This fact is admittedly an apparent flaw, but it can be justified by a good number of precedents and the difficulty of any other arrangement without access to multinational companies. No empirical verification of either the behavioral assertions made or the potential findings discovered is attempted in the strict sense of the word. Naturally general knowledge of the multinational firm and prior research were utilized during the research work to subjectively evaluate the points above. As a methodological detail it should also be mentioned that the simulation runs for Chapter 6 ("Selected Examples of Hypotheses of the Behavior of the Multinational Firm Generated via the Model") were based on fictitious data.

Last, we should note the nature of the numerical examples, which are given in introducing to the reader the model for the idealized multinational firm. These numerical examples are fictitious, too, as has been mentioned before. The input data used is not (nor is it meant to be) always in strict accordance with reality. The reason for this is that the primary purpose of the numerical examples is to demonstrate various model features to the reader.

\[16\text{ For instance, in the first numerical example the tax rate on corporate profits in a fictitious Finland will be } 60\%, \text{ and } 45\% \text{ in a fictitious England.}\]
2.2 Notes on the Structure of Production Assumed in the Model for the Multinational Firm

The selection of the production structure in the model to be presented for the idealized multinational firm is discussed in this section.

Linear programming models for production planning have been much considered in the uninational case and to some extent also in the multinational case.¹⁷ Hence production in our multinational firm can be made very simple in the light of the normative problem without a loss of generality. The idealized multinational firm is a one-product firm.¹⁸ Inventories are not included in the model (or rather it can be said that the inventory levels are kept constant). Including inventories would mean only including non-essential details.¹⁹ Figure 2-1 gives the structure of production in the idealized multinational firm in flow form. For simplicity only a two-country case is depicted in the figure. The product is constructed as follows. From a certain number of units of the raw material the firm can manufacture one unit of the intermediate product in a production phase called phase I. From another given number of units of the intermediate product the firm can then manufacture one unit of the final product in a production phase

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¹⁷ See especially Dileep Mehta & Isik Inselbag, "Working Capital Management of a Multinational Firm," op.cit. (In spite of its title the paper also tackles production planning in the multinational firm.) See also the review of tactical production planning in Section 1.3 ("Review of Prior Research on Topic").

¹⁸ See footnote 38 in Chapter 1.

¹⁹ In a model with a planning horizon of one period only, inventory costs are not necessarily a decisive factor.
FIGURE 2-1 Trade and Production Flows in the Idealized Multinational Firm

output market

| border |

output market

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final product

production phase II

intermediate product

production phase I

raw material

input market country i

---

a trade flow

---

a production flow

---

final product

production phase II

intermediate product

production phase I

raw material

input market country j
called phase II. We assume for simplicity that only the intermediate product can be transferred. Because of the simple structure of production, drawn in Figure 2-1, we can assume without a loss of generality that producing one final product requires exactly one intermediate product, and that producing one intermediate product requires exactly one unit of the raw material.\(^{20}\)

Anyhow, the point to be made is that in tackling the normative problem the selection of a production structure for the multinational firm can be considered inconsequential, since it can be easily altered, when required.

The rest of this chapter can be skipped without a loss of continuity. The selection of the particular production structure adopted originates actually from preliminary considerations on the behavioral problem.

If the production structure given in Figure 2-1 is used, it is possible to obtain some interesting variations of the basic structure. Figure 2-2 gives some examples of different possibilities. Figure 2-2 (a) gives a basic structure allowing also raw material and final product transfers. If a rather flexible and general view is taken, we can say that (b) represents a multinational firm horizontally integrated across national borders, (c) represents a multinational firm vertically integrated across national borders, and (d) represents a diversified multinational firm. It seems plausible to suggest that the existing big multinational business enterprises often fall into one of the above categories. For example in the automotive industry

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\(^{20}\) This has been demonstrated in an appendix of Veikko Jääskeläinen & Timo Salmi, "Joint Determination of Production and Financial Budgets of a Multinational Firm Facing Risky Currency Exchange Rates," op.cit., pp. 32-35.
FIGURE 2-2  Examples of Different Production Structures
horizontal integration is common, and oil companies are often examples of vertically integrated multinational firms. When hypotheses are generated concerning the pattern of behavior of the multinational firms, these cases could be treated separately, in order to analyze potential differences in their model behavior. This was not done in my research work, because the behavioral problem received little attention in my later research efforts, and consequently only the case presented in Figure 2-1 was touched upon. Nevertheless, the basic structure was constructed with the ideas of the above discussion in mind. The structure presented is the minimum structure covering a vertically integrated case. As can be seen in 2-2 (c), we must apply the trilogy final product/intermediate product/raw material, if we wish to have distinct production phases in two different countries.21

A further point can be noted. When transfers are made from one country to another, the transferred items must be given a transfer price, which is declared to the authorities for duty and tax assessment. The freedom of action in transfer pricing often depends on the ability of the authorities to assess at-arm's-length-prices22 for the transferred product. The possibilities in assessing this price depend on the existence of equivalent products on the market.23 The trilogy

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21 For further discussion of and references to horizontal and vertical integration in the multinational firm see John H. Dunning, "The Multinational Enterprise: the Background," op.cit., pp. 21-26.

22 Prices quoted between independent buyers and sellers.

23 If equivalent products exist on the market and "if customs and tax authorities on both sides were reasonably diligent, a firm would probably not try to declare the value of its exports to be less than their marginal cost of production or greater than their market price in the (continued)
final product/intermediate product/raw material is convenient in covering the different cases.\textsuperscript{24}


\textsuperscript{24} For further discussion see Timo Salmi, \textit{The Multinational Firm: A Mathematical Programming Model Building Approach} (licentiate thesis), op.cit., pp. 91-92.
This chapter presents a linear programming formulation under certainty for joint planning of trade, production, and financial activities including decisions on inter-affiliate transfer pricing. A numerical example involving production in two countries is given at the same time. An analysis of selected model features is carried out: The concept of "shadow transfer prices" is suggested, and opportunity cost of capital in the model is commented on. Operating budgets, and predicted income statements for tax purposes, and for internal usage, are derived for both affiliates of the numerical example. A numerical analysis of shadow prices, sensitivity, and the objective function is performed.

3.1 Flow Model, and the Concepts of Internal and External Transfer Pricing

Figure 2-1 in the previous chapter and Figure 3-1

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1 Basic mathematical knowledge of linear programming can be acquired e.g. from Jouko Manninen & Timo Salmi, Matemaattinen optimointi: teoriaa ja tekniikkaa (Helsinki, 1974).

2 Basic knowledge of the application of linear programming techniques in budgetary planning in the firm can be acquired e.g. from Veikko Jääskeläinen, Linear Programming and Budgeting (Malmö, 1975), or Veikko Jääskeläinen, Lineaarinen ohjelmointi ja budjetointi (Tapiola, 1972), and Juha Nissilä, "Tavoitetulosajatteluun perustuvan yhdistetyn tulos- ja rahoitussuunnitelman laatiminen lineaarisen ohjelmoinnin avulla," op.cit.

3 The following terminology is used in this dissertation. The idealized multinational firm has a headquarters in the home country, and one or several subsidiaries abroad. Both the headquarters and the subsidiaries are called affiliates.
give the model in a flow form. For simplicity, only a two-country case is depicted in the figures.

The intermediate product can be transferred between the affiliates of our idealized multinational firm. Because the affiliates are located in different countries, the transfer prices of the transferred items must be declared to the relevant customs and tax authorities for duty and tax assessment. Furthermore, the transfer prices are a basis of interaffiliate payments. The model to be presented is to assess these transfer prices simultaneously with the other decision variables.

These transfer prices declared to the authorities will be called the external transfer prices. The external transfer price is not necessarily the same transfer price as is credited and debited in the firm's accounts for internal profit control. This latter transfer price, which is used for monitoring the performance of divisions (profit centers), can be called the internal transfer price. Because a centrally planned decision-making situation is assumed, the internal transfer prices are not relevant. In centrally planned decision making the internal transfer prices must either be omitted or they must be adjusted properly to allow for the centralized overall solution. If this is not done, conflicts are bound to arise. Internal transfer pricing is omitted.

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4We assume ad valorem import duties in the model.
5The currency needed can be bought on this basis. The interaffiliate payments must be effected on the basis of these transfer prices, since they have to be contained in the proper commercial documents.
6For a further discussion on "internal" transfer pricing see any relevant accounting textbook, e.g. Charles T. Horngren, Accounting for Management Control: An Introduction (Englewood Cliffs, New Jersey, 1970), Ch. 12.
in the present model, because its inclusion would require considerable further research efforts, since there are no quantitative precedents treating simultaneously internal and external transfer pricing.\textsuperscript{7}

3.2 Mathematical Model,\textsuperscript{8} and a Fictitious Numerical Example

3.2.1 PREFACE

The symbols are divided into five groups:

1) The decision variables. They are identified by the letter X.

2) The constants which appear on the left-hand side of the constraints. They are identified by the letter A.

3) The constants which appear on the right-hand side of the constraints. They are identified by the letter B.

4) The constants which appear in the objective function. They are identified by the letter C.

5) Miscellaneous symbols.

\textsuperscript{7}As will be seen in Chapter 7 ("Conclusion"), this omission gives rise to a suggestion for further research.

\textsuperscript{8}The first published version of my model was given in Timo Salmi, "The Multinational Firm, a Mathematical Programming Model Building Approach: A Proposition for Research Work; Part II," The Finnish Journal of Business Economics, No. 2, 1972, pp. 134-147. The model in above is more extensive than the present model version. The former is a one-product multi-period model. (The reason for introducing a one-period model, here is better explained in the next chapter while including the currency risk.) In addition, the former model includes decisions on acquiring additional capacity, depreciation, dividends to headquarters, and inter-affiliate fees and royalties. Each component of the product can be transferred in the model presented in the above reference. [cf. Figure 2-2 (a)]. As is recalled, these features can be omitted here because of the idealization.
The symbols (1-4) do not follow the conventional mathematical notation. For convenience a FORTRAN programming language notation is adopted for the symbols. In other respects mathematical conventions are followed in notation. The miscellaneous symbols are given below:

\[
\begin{align*}
nc &= \text{number of affiliates involved} \\
nc &= \text{number of countries involved} \\
i,j &= \text{indexes indicating countries} \\
i=1 &= \text{the home country of the multinational firm.}
\end{align*}
\]

Figure 3-2 gives the trade and production flows together with the relevant decision variables in the numerical example to be presented parallel with the model. To simplify the discussion on the numerical example let us say that the affiliates of the multinational firm of the example are located in a fictitious Finland \((i = 1)\) and a fictitious England \((i = 2)\). It is to be stressed that this terminology is adopted for convenience only. It must not be taken literally. Thus the deviations from reality in the figures of the example are to be considered inconsequential.

Let the currency of the fictitious Finland be the Finnish mark\(^9\) (Fmk) and that of the fictitious England the pound (\(\pounds\)). The total profit to be maximized is computed in marks, since Finland will be the home country of the multinational firm of the numerical example. (The exchange rates will be 10 Fmk/\(\text{\pounds}\), i.e. 0.1 \(\text{\pounds}/\text{Fmk}\).)

The object in the numerical example is the derivation of the optimal operating budgets and thus demonstrate that the model gives sound output.

---

\(^9\)Hereafter the Finnish mark is called the mark for short.
FIGURE 3-2  Trade and Production Flows in the Numerical Example

- Final product
- Production phase II
- Intermediate product
- Production phase I
- Raw material

Finland

England
3.2.2 SALES/CAPACITY CONSTRAINTS

Denote:

\[ X_1(i) = \text{the number of units of the final product produced and sold in country } i. \]

\[ B_1(i) = \text{the minimum of the sales potential and the production capacity of production phase II in country } i. \]

We have:

\[ X_1(i) \leq B_1(i) \quad i=1,\ldots,nc. \]

The budgeted sales may not exceed the predicted demand (sales potential) for the final product. The production of the final product cannot exceed the production capacity of phase II. Since inventory levels are kept unchanged in the model, for simplicity, production and sales in each country are embodied in the same decision variables \( X_1(i) \).\(^{11}\) An affiliate can sell the final product domestically only, but this is not an actual simplification, since foreign sales can be routed through the relevant affiliates. The sales price of the final product has to be assumed to be predetermined, since otherwise linearity is endangered.\(^{12}\) In this tactical planning model, the above is not necessarily a severe simplification, since parametric routines can be applied, and, on the other hand, it has been posed for convenience. (See Figure 3-2.)

---

\(^{10}\) The expression "country i" is used for short to indicate "affiliate in country i" or "by affiliate in country i", or some other analogous expression, on many occasions in depicting the model formulations.

\(^{11}\) In the numerical example the parentheses will be omitted for convenience. (See Figure 3-2.)

\(^{12}\) This is an extremely common simplification both in the multinational and the national case.
in management literature that marketing decisions are often decentralized in multinational business enterprises.

In the numerical example it is assumed that in Finland the sales potential of the final product is 10,000 units at the predetermined price of 100 Fmk/unit. In England the corresponding figures are 10,000 units and £ 100/unit. X11 delineates the number of the items of the intermediate product designated to production phase II in Finland and X12 is the corresponding English figure, as seen in Figure 3-2. As will be recalled, exactly one unit of the intermediate product is required in producing one unit of the final product.\(^\text{13}\) Hence, in Finland, the sales constraint is

\[
\begin{align*}
X_{11} & \leq 10000, \\
\end{align*}
\]

and in England it is

\[
\begin{align*}
X_{12} & \leq 10000. \\
\end{align*}
\]

It is assumed that the capacity of production phase II is 17,500 units in both countries of the example. Hence, the following constraints are constructed:

\[
\begin{align*}
X_{11} & \leq 17500 \\
X_{12} & \leq 17500.
\end{align*}
\]

\(^\text{13}\) This idealization can be made here without a further loss of generality, as stated in footnote 20 in Chapter 2.
Because of the simplified structure of production assumed, these latter two constraints can be omitted, since they are dominated by the sales constraints. In a more general case involving both several products and the corresponding inventories, we would not proceed as simply as this. In that case production and sales of each product would be assigned separate decision variables. The capacity per unit (e.g. man hours/unit, etc.) required in production, and the total capacity (e.g. man hours, etc.) would be needed. Separate sales and capacity constraints, accounting for the inventories, would be constructed.¹⁴

The above figures already indicate that the affiliates of the numerical example are quite similar. This is arbitrary, but convenient, since it spares us a few tiresome details.

3.2.3 SUPPLY/CAPACITY CONSTRAINTS

Denote:

\[ X_2(i,i) = \text{the number of units of the intermediate product produced in country } i. \]

\[ B_2(i) = \text{the minimum of the supply potential and the production capacity of production phase I in country } i. \]

We have:

\[ X_2(i,i) \leq B_2(i) \quad i=1,\ldots,nc. \]

The production of the intermediate product cannot

¹⁴ See e.g. Dileep Mehta & Isik Inselbag, "Working Capital Management of a Multinational Firm," op.cit., or Timo Salmi, Monikansallinen yritys ja lineaarinen optimointi, op.cit., Ch. 4 and pp. 94-95.
exceed the production capacity of phase I. The budgeted purchases of the raw material may not exceed the supply limit. Owing to the idealization the production of the intermediate product and the purchases of the raw material in each country are embodied in the same decision variables $X_2(i,i)$.

In the numerical example it is assumed that the production capacity of production phase I is 25,000 units in both affiliates, during the planning period under observation. The corresponding constraints are

\[
\begin{align*}
X_{211} &\leq 25,000 \\
X_{222} &\leq 25,000,
\end{align*}
\]

where $X_{211}$ is the number of units of the intermediate product produced in the Finnish affiliate and $X_{222}$ is the corresponding number for the English affiliate.

It is assumed that the supply potential of the raw material is 40,000 units in each country of the example. Consequently we have the following constraints, which are dominated by the former constraints:

\[
\begin{align*}
X_{211} &\leq 40,000 \\
X_{222} &\leq 40,000.
\end{align*}
\]

In the case of a more general production structure, the comments of the previous section apply.

### 3.2.4 TECHNICAL AVAILABILITY CONSTRAINTS

Denote:
\[ X_2(i,j) = \text{the number of units of the intermediate product transferred from affiliate } i \text{ to affiliate } j. \]

We have

\[ X_1(i) + \sum_{j=1}^{nc} X_2(i,j) - \sum_{j=1}^{nc} X_2(j,i) \leq 0 \quad i=1,\ldots,nc. \]

The technical availability constraints arise from the simple fact that the final product cannot be produced unless enough units of the intermediate product are available.

As is seen in Figure 3-2, in the numerical example the number of the intermediate products transferred from Finland to England is denoted by the symbol \( X_{212} \), and the number of the units transferred in the opposite direction is denoted by \( X_{221} \). Consider the Finnish affiliate and the intermediate product. The initial inventory (0 units) plus the production in Finland (\( X_{211} \)) plus the imports from the English affiliate (\( X_{221} \)) must be equal to, or greater than, what is needed in the production phase II (1\( \cdot X_{11} \)) plus the exports to the English affiliate (\( X_{212} \)) plus the minimum ending inventory requirement (0 units). On the basis of this "in \( \geq \) out"-restriction we have the constraint

\[
\begin{align*}
\text{init.inv. from F prod.} & + \text{imports to F prod.} \\
0 & + X_{211} + X_{221} \geq X_{11} + X_{212} + 0.
\end{align*}
\]

This is equivalent to

\[
X_{11} - X_{211} + X_{212} - X_{221} \leq 0.
\]
For the English affiliate we have:

\[ X_{12} - X_{212} + X_{221} - X_{222} \leq 0. \]

3.2.5 DECISIONS ON EXTERNAL TRANSFER PRICING: TRANSFER PRICE CONSTRAINTS

Denote:

- \( X_{3(i,j)} \) if \( j \neq j \) the transfer value of all those units of the intermediate product which are transferred from affiliate \( i \) to affiliate \( j \) stated in the currency of country \( i \). (It is paid in cash.)

- \( Y_1(i,j) \) if \( j \neq j \) the transfer price of one unit of the intermediate product transferred from affiliate \( i \) to affiliate \( j \) stated in the currency of country \( i \). \( Y_1(i,j) = X_{3(i,j)}/X_{2(i,j)} \) if \( X_{2(i,j)} > 0 \). We say that \( Y_1(i,j) \) does not exist if \( X_{2(i,j)} = 0 \).

- \( A_1(i,j) \) the lower bound on the transfer price of one unit of the intermediate product transferred from affiliate \( i \) to affiliate \( j \). It is stated in the currency of country \( i \).

- \( A_2(i,j) \) the upper bound on the transfer price of one unit of the intermediate product transferred from affiliate \( i \) to affiliate \( j \). It is stated in the currency of country \( i \).

We have:

**Lower bounds**

\[ A_1(i,j)X_{2(i,j)} - X_{3(i,j)} \leq 0 \quad i=1,\ldots,nc \]

\[ -X_{2(i,j)} \leq A_2(i,j)X_{2(i,j)} + X_{3(i,j)} \leq 0 \quad i\neq j. \]

As discussed, the items transferred between the affiliates

---

\(^{16}\) See page 59.
must be given external transfer prices to be declared
to customs and tax officials for duty and tax assessment.
The external transfer prices are also a basis of inter-
affiliate remittances. The central management must
stipulate upper and lower limits on the external transfer
prices, because if they are made too high or too low
the authorities are bound to step in. These predetermined
limits are the constants $A_1(i,j)$ and $A_2(i,j)$. The external
transfer prices affect both the cash flows in different
affiliates and the global profit or loss$^{15}$ of the multi-
national firm. The repercussions are taken into account
in the current model. The model assigns optimal values
to the transfer prices$^{16}$ within the predetermined limits
set by the central management. Thus the transfer prices
are decision variables in the present model. In previous
models for joint tactical production and financial
planning the transfer prices have been treated as pre-
determined. Such a manner of proceeding can lead to
nonoptimal total plans in this respect.

The transferred items are always invoiced in the
currency of the host country of the exporting affiliate.
Thus the trading currency is not a decision variable.
The other congenial studies, which were referred to
in the above, implicitly make this same assumption in
addition to the assumption of predetermined transfer
prices.

If the limits on a transfer price are so narrow that
it can be considered as fixed or if it is predetermined,
it is possible to proceed conventionally by including

---

$^{15}$ The expression "yield" or "net income" will be on
occasions used as a synonym of "profit and loss".

$^{16}$ From now on "transfer pricing" will mean "external
transfer pricing", if not stated to the contrary.
the transfer price as a constant in the model and by omitting the transfer price constraints under observation. It should be noted, however, that if this is done, the information content of the model is reduced, since then the economic consequences of raising or lowering the transfer price bound are not explicitly reflected in the model output! (This will be elaborated in Section 3.3.2: "The Concept of Shadow Transfer Prices: An Extension of the Conventional Analysis of Shadow Prices".) Parametric procedures and re-solving could, naturally, be resorted to.

Consider the numerical example and a transfer from the English affiliate to the Finnish affiliate. Figure 3-3 depicts the transfer.

![FIGURE 3-3 Interaffiliate Transfer from England to Finland](image)

The number of units of the intermediate product to be transferred from England to Finland is the decision
variable X22l. The Finnish affiliate pays X32l pounds to the English affiliate for the transferred items. The resultant transfer price of one unit is X32l/X22l pounds. (The Finnish affiliate buys the pounds needed by paying for them in marks, as will be seen later in a discussion of further constraints. Also the duties and transportation costs arise in later constraints.)

Assume that the central management stipulates that for an item transferred from England to Finland the lowest transfer price still acceptable is £30, and the highest is £100. Since the transfer price of one transferred unit of the intermediate product is X32l/X22l pounds, the concomitant nonlinear transfer price constraints are as follows.

\[ \frac{X32l}{X22l} \geq 30 \quad \text{and} \quad \frac{X32l}{X22l} \leq 100. \]

When the first restriction giving the lower bound is multiplied by X22l, we get

\[ X32l \geq 30X22l. \]

From this expression follows the linear lower-bound-transfer-price-constraint for the Finnish affiliate, concerning the intermediate products imported from the English affiliate:

\[ 30X22l - X32l \leq 0. \]

In the above, the case X22l = 0 does not incur difficulties,

\[ \text{In this example the limits incidentally range between the marginal cost of the intermediate product and the market price of the final product (cf. footnote 23 in Chapter 2).} \]
since then $X_{321}$ must be equal to 0, and we define that the transfer price $Y_{121}$ does not exist.\textsuperscript{18}

By proceeding in the same way as in the above, we get the corresponding linear upper-bound-transfer-price-constraint, which is

$$-100X_{221} + X_{321} \leq 0.$$  

The potential transfer from Finland to England is treated analogously. The number of the intermediate products to be transferred is now $X_{212}$ and the concomitant transfer value is $X_{312}$ marks. The transfer price is thus $X_{312}/X_{212}$ marks. The limits on the transfer price are now given in marks, not in pounds, since the model is easier to handle, if, for the model, the relevant transfer prices are stated in the currency of the host country of the exporting affiliate. Assume that the central management has stipulated that for an item transferred from Finland to England the transfer prices must be between 400 Fmk and 900 Fmk. This requirement leads to the linear constraints

$$400X_{212} - X_{312} \leq 0 \quad \text{and} \quad -900X_{212} + X_{312} \leq 0.$$  

\textsuperscript{18} What is crucial here is that the definition is a legitimate one in connection with the Simplex-algorithm of linear programming.
3.2.6 LOCAL LOAN RAISING CONSTRAINTS

Denote:

\[ X_{4(i,i)} = \text{the loans raised from the local money market in country } i. \]

\[ B_3(i) = \text{the maximum amount of loans that can be raised from the local money market in country } i. \]

We have:

\[ X_{4(i,i)} \leq B_3(i) \quad i=1,...,nc. \]

The maximum amount of loans that can be raised must not be exceeded in the model solution. The number of money markets could be very easily increased simply by adding an index. Also multiple investments in short-term financial options could be included. Decisions on interaffiliate loans and their interest rates are included in Chapter 5 ("Extension of the Model under Currency Risk").\(^{19}\)

In the numerical example it is assumed that the maximum amount of loans that can be raised is 3 000 000 Fmk for the Finnish affiliate, and £ 300 000 for the English affiliate. \(X_{411}\) denotes the loan raising in Finland, and \(X_{422}\) denotes the respective English figure. The following constraints must be applied:

\[
\begin{align*}
X_{411} & \leq 3 000 000 \quad \text{(Fmk)} \quad \text{and} \\
X_{422} & \leq 300 000 \quad \text{(£)}. 
\end{align*}
\]

\(^{19}\) Since interaffiliate loans and their interest rates are not included in the numerical example of the current chapter, their general presentation is postponed, too, to keep the exposition simple. There is no compelling reason for the postponement.
3.2.7 CASH FLOW CONSTRAINTS

Denote:

\[ X_6(i) \] = the profit before taxes shown in books by affiliate i for tax assessment in country i.

\[ A_3(i) \] = the portion of the sales price of one unit of the final product sold in country i, which is collected in cash.

\[ A_4(i) \] = the portion of the sales price of one unit of the final product sold in country i, which is accrued as accounts receivable.

\[ A_5(i) \] = the cash portion of the variable cost of one unit of the final product incurred in phase II (cf. Figure 3-2) in country i.

\[ A_6(i) \] = the accounts payable portion of the variable cost of one unit of the final product incurred in phase II in country i.

\[ A_7(i) \] = the cash portion of the raw material and phase I variable costs of one unit of the intermediate product in country i.

\[ A_8(i) \] = the accounts payable portion of the raw material and phase I variable costs of one unit of the intermediate product in country i.

\[ A_9(i,j) \] = the transportation cost of one unit of the intermediate product transferred from country i to country j. The transportation cost is paid in cash by the exporting affiliate in the currency of country i.

\[ A_{10}(j,i) \] = the exchange rate of currency j in country i. (Note the order of the indices.)

\[ ^{20} \] These variables are not needed until the next set of constraints is presented, but I feel that they are best introduced here.
A11(i,j) = the duty coefficient on items which are transferred from country i to country j. It is the absolute rate of the duty imposed on the value of the intermediate products transferred from country i to country j multiplied by the appropriate currency exchange rate which is A10(i,j). The duty is paid in cash by the importing affiliate in the currency of country j.

A12(i) = the absolute rate of interest\(^{21}\) paid on the loans from the local sources in country i. Interest is paid in cash.

A13(i) = the absolute tax rate (on profit) in country i. Taxes are paid in cash.

B4(i) = the initial cash in bank less the cash portion of the fixed cost less the required minimum closing cash plus other assets which incur predetermined cash inflows (e.g. accounts receivable to be collected) less other liabilities which incur predetermined cash outflows during the planning period in country i.

We have:

\[
\text{cash sales} - \text{cash variable prod. costs} - \text{transportation} - \text{loans interest} - \text{tax} - \text{initial cash} \leq B4(i) 
\]

\[
[-A3(i)+A5(i)]X1(i) + A7(i)X2(i,i) + \sum_{j=1, j \neq i}^{n_c} A9(i,j)X2(i,j) - \sum_{j=1}^{n_c} X3(i,j) - \sum_{j=1}^{n_c} [A10(j,i)+A11(j,i)]X3(j,i) - [1-A12(i)]X4(i,i) + A13(i)X6(i) \leq B4(i) 
\]

It must be required for each affiliate that

\[
\text{initial cash} + \text{cash inflows} \geq \text{cash outflows} + \text{minimum closing cash}. 
\]

\(^{21}\) If an interest rate is 5%, the corresponding absolute interest rate is 0.05.
In corresponding linear programming models for unination planning situations, only one cash flow constraint is needed in a one-period model, but in the multinational case each affiliate must be treated separately.

The cash flow constraints, and the other financial constraints and equalities to be presented shortly, are denominated in the currency of the host country of the relevant affiliate. This manner of proceeding makes it easier to handle the multiple-currency situation, and formulation errors are hopefully avoided. In the face of prior research work done in this field, the above assumption involves no new simplifications.

The predetermined minimum closing cash balance affects the region of the feasible solutions of this tactical planning model. Since the firm is a going concern, and not liquidated at the end of the planning horizon, the continuation of the activities of the firm after the planning horizon must be accounted for in this manner in the model. Thus the conventional method of setting appropriate closing balance requirements is followed. (We discuss this question more fully later in Section 3.5.4.)

For simplicity, no differences are assumed between the purchase and sales rates of the currencies. Export duties are omitted in the model (although they are often relevant in actual practice). The import duties are levied on an ad valorem basis. Interests and taxes are paid in cash during the planning period. In all these cases contrary assumptions could be easily reflected in the model.

Figure 3-4 delineates the cash flows in the numerical example. Consider the components of the cash flow in the Finnish affiliate. The cash flow constraint for the Finnish affiliate is denominated in marks, while
that of the English affiliate is denominated in pounds.

Let us first consider the cash inflows. It is assumed in the numerical example that the sales of the final product incur no cash inflows. The sales revenue of the final product (1,000 Fmk/unit) is accrued in accounts receivable and the cash is received outside the planning period. (Consequently Figure 3-4 depicts the relevant sales revenue by a dashed arrow.)

The intermediate products exported to the English affiliate bring in a cash payment of X312 marks.

The loans raised from the local money market cause a cash inflow of X411 marks.

The initial balance sheets for both affiliates are seen below. They have been made as simple as possible for convenience.\(^ {22} \)

**FINNISH AFFILIATE**

**ASSETS**
- Cash in bank 6,000,000 Fmk
- External receivables 0 Fmk
- Inventories 0 Fmk
- Receivables from other affiliates 0 Fmk
- Fixed assets 15,000,000 Fmk

**LIABILITIES**
- External payables (including external short-term loans) 0 Fmk
- Long-term liabilities 0 Fmk
- Payables to other affiliates 0 Fmk
- Stockholders' equity 21,000,000 Fmk

**ENGLISH AFFILIATE**

**ASSETS**
- Cash in bank 600,000 £
- External receivables 0 £
- Inventories 0 £
- Receivables from other affiliates 0 £
- Fixed assets 1,500,000 £

**LIABILITIES**
- External payables (including external short-term loans) 0 £
- Long-term liabilities 0 £
- Payables to other affiliates 0 £
- Stockholders' equity 2,100,000 £

\(^ {22} \) The items are presented in a slightly unconventional ordering. The reason is better explained when currency exposure is discussed in Chapter 5. The noteworthy point here is that the data described should be readily available to the multinational firm in actual practice.
Let us next consider the cash outflows. Producing one unit of the final product in Finland incurs a variable cost of 250 Fmk in production phase II. We assume for simplicity that it is paid entirely in cash. This item causes a cash outflow of 250X11 marks. Call this cost the *variable cost II* in Finland.

Producing one domestic unit of the intermediate product in Finland incurs a variable cost of 300 Fmk in production phase I and for raw material. It is paid out entirely in cash. A cash outflow of 300X211 marks results. Call it the *variable cost I* in Finland.

The affiliates deal with each other on a c.i.f. basis. The transportation cost of one unit of the intermediate product is 100 Fmk/unit for a transfer from Finland to England. Thus the Finnish affiliate has to pay 100X212 marks in cash for its exports.

The payments for items imported from the English affiliate cause a cash outflow of 10X321 Fmk. X321 has been multiplied by the relevant exchange rate A10(2,1), which is 10 Fmk/£, since the transferred items are invoiced in pounds, and the invoice has to be converted into Finnish marks. Effecting the payment can be visualized as follows. The Finnish affiliate buys from the bank X321 pounds paying 10X321 marks for the pounds. Then these pounds are sent to the English affiliate as the payment for the discussed imports.\(^{23}\)

An ad valorem import duty of 10% is levied on the value of the items under observation. The duty is calculated on the basis of the value of the transferred items, which, in turn, depends on the transfer price

\(^{23}\) This is, of course, an oversimplification, but as discussed we are not concerned with the mechanics of international payments. A convenient proxy suffices.
of the items. The value of the items is $10X32l Fmk$, and thus the cash outflow to duty is $0.1 \cdot 10X32l = 1X32l$ marks. (The import duty in England is also 10%.)

The cash outflow for the interest on the loans raised is $0.05X41l$ marks, since it is assumed that the relevant interest rate is 5%.

The book profit shown in the books by the Finnish affiliate is denoted by the decision variable $X6l$. (For the English affiliate the corresponding variable is $X62$.) Since the tax rate on corporate profits is 60% in the fictitious Finland, a cash outflow of $0.60X6l$ marks results.

The fixed cost is 1 000 000 Fmk. It is paid entirely in cash. (The fixed cost in England is £ 100 000.)

The minimum closing cash balance required at the end of the planning period is 1 000 000 Fmk for the Finnish affiliate (£ 100 000 in England).

On the basis of the above discussion we see that the inequality below must hold for the Finnish affiliate.

\[
\begin{align*}
\text{initial cash} & \quad \text{exports} \quad \text{loan} \quad \text{v.c. II} \quad \text{v.c. I} \\
6 000 000 & \quad + \quad X3l2 & \quad + \quad X411 & \quad \geq \quad 250X1l & \quad + \quad 300X211 \\
\text{transportation} & \quad + \quad 100X212 & \quad + \quad 10X321 & \quad + \quad 1X321 & \quad + \quad 0.05X411 \\
\text{tax} & \quad + \quad 0.60X6l & \quad + \quad 1 000 000 & \quad + \quad 1 000 000. \\
\end{align*}
\]

This is tantamount to

\[
\begin{align*}
\begin{vmatrix}
250X1l & + & 300X211 & + & 100X212 & - & X3l2 & + & 11X321 \\
-0.95X411 & + & 0.60X6l & \leq & 4 000 000 & (Fmk).
\end{vmatrix}
\end{align*}
\]

\[24\] This is brought up since it cannot be seen in Figure 3-4.
The data for the English affiliate is seen in Figure 3-4. The only factual difference is that the tax rate on corporate profits is 45% in the fictitious England, as compared to the 60% of the fictitious Finland. The cash flow constraint for the English affiliate is

$$25X_{12} + 10X_{221} + 30X_{222} + 0.11X_{312} - X_{321}$$
$$- 0.95X_{422} + 0.45X_{62} \leq 400\,000 \, (£).$$

### 3.2.8 OPERATIONAL TREATMENT OF TAXATION: EQUATIONS FOR BOOK PROFIT AND LOSS

Denote:

- $X_7(i)$ = the loss shown in books by affiliate $i$ for tax assessment in country $i$.
- $A_{14}(i) = A_3(i) + A_4(i) - A_5(i) - A_6(i)$.
- $B_5(i)$ = the *deductible* fixed cost *plus* the depreciation for external purposes in country $i$.

We have:

$$sales - v.c. \, II \quad v.c. \, I$$
$$A_{14}(i)X_1(i) - [A_7(i)+A_8(i)]X_2(i,i) - \sum_{j=1}^{nc} A_9(i,j)X_2(i,j)$$
$$j \neq i$$
$$\sum_{j=1}^{nc} X_3(i,j) - \sum_{j \neq i} [A_{10}(j,i)+A_{11}(j,i)]X_3(j,i)$$
$$\sum_{j=1}^{nc} \text{exports} \quad \sum_{j \neq i} \text{imports + duty}$$
$$\text{interest} \quad \text{profit loss + depreciation}$$
$$- A_{12}(i)X_4(i,i) - X_6(i) + X_7(i) = B_5(i) \quad i=1,\ldots,nc.$$
The given equations define the profit and loss for tax assessment for each affiliate separately. Taxation is sometimes treated incorrectly or awkwardly in literature. First, book profits and losses should not be handled exactly analogously, because this is an oversimplification. When this is done, it actually indicates the silent assumption that the relevant government, in the model, subsidizes the affiliate in accordance with the tax rate in the case of a loss. If, on top of this, tax payments are included in cash flow constraints, the government is supposed to pay the subsidy in cash, in the model! Second, in at least one study only book profits are feasible. Like the former one, this is not a sound simplification, if it is not motivated. Third, most congenial models treat taxation directly in the objective function. This manner of proceeding makes a proper distinction between tax profits and losses impossible in linear programming models, and the consequences are as above. In addition, particularly in the multinational case, the handling of such objective functions is too laborious from the pragmatic point of view, because the handling requires onerous substitutions especially since the currency differences must be accounted for. The difficulties discussed are avoided when distinct auxiliary equations are constructed and book profits and losses are assigned separate variables.

The structure of the tax-laws assumed in the present model can be inferred from the equations and the discussion on the numerical example. The structure of the tax-laws could easily be changed. In the present version it may be noted that losses cannot be carried forward in taxations, and that manipulation of depreciation and inventory valuation is excluded.

Let us consider the numerical example. The model
is augmented with the equations for book profit and loss. The relevant decision variables are given below.

\[
X_{61} = \text{the book profit shown by the Finnish affiliate in marks.} \\
X_{71} = \text{the book loss shown by the Finnish affiliate in marks.} \\
X_{62} = \text{the book profit shown by the English affiliate in pounds.} \\
X_{72} = \text{the book loss shown by the English affiliate in pounds.}
\]

For the Finnish affiliate we construct the following equation:

\[
X_{61} - X_{71} = \text{the revenues subject to corporate income tax in Finland less} \\
\text{the deductible costs in Finland.}
\]

The Simplex-algorithm of linear programming always sees to it that if the above net income is positive then \(X_{71} = 0\), and \(X_{61}\) gives the book profits before taxes in Finland. If the above net income is negative, then \(X_{61} = 0\), and \(X_{71}\) gives the book loss in Finland.\(^{27}\)

Because the above equation resembles the Finnish cash flow constraint, a detailed demonstration is not necessary. Note the following differences in the components of the equation. It is assumed that the depreciation rate of fixed assets is 10% for each affiliate. Thus the depreciation amounts to 1 500 000 Fmk in Finland.

\(^{27}\) For the mathematical background see G. Hadley, *Linear Programming* (London/etc. 1962), pp. 168-169.
The deductible fixed cost is 1,000,000 Fmk. The accounts receivable from sales of the final product (1,000 Fmk/unit) must be included. As will be remembered, no accounts payable occur in the numerical example, but if they occurred they would have to be included now. Loan raising is not a revenue. Tax payments are not deductible. For the Finnish affiliate we have the following auxiliary equation:

\[
\begin{align*}
\text{sales-v.c.II} & - \text{v.c. I} + \text{transportation} + \text{exports} + \text{imports+duty} \\
750X_{11} & - 300X_{211} - 100X_{212} + X_{312} - 11X_{321} \\
\text{interest} & + \text{fixed cost+depr.} \\
- 0.05X_{411} & - X_{61} + X_{71} = 2,500,000 \text{ (Fmk).}
\end{align*}
\]

For the English affiliate we have

\[
\begin{align*}
75X_{12} & - 10X_{221} - 30X_{222} - 0.11X_{312} + X_{321} \\
- 0.05X_{422} & - X_{62} + X_{72} = 250,000 \text{ (£).}
\end{align*}
\]

3.2.9 OPERATIONAL TREATMENT OF THE OBJECTIVE: EQUATIONS FOR INTERNAL PROFIT AND LOSS

Denote:

- \(X_8(i)\) = the profit after taxes in country \(i\) in the currency of country \(i\) from the firm's internal point of view.

- \(X_9(i)\) = the loss after taxes in country \(i\) in the currency of country \(i\) from the firm's internal point of view.

- \(B_6(i)\) = the fixed costs in country \(i\) from the firm's internal point of view, in the currency of country \(i\).
We have:

\[
\text{sales} = \text{v.c. II} - \text{v.c. I} = A14(i)X1(i) - [A7(i)+A8(i)]X2(i,i) - \sum_{j=1, j\neq i}^{nc} A9(i,j)X2(i,j)
\]

\[
\text{transfortation} = \sum_{j=1}^{nc} \text{exports} - \sum_{j=1, j\neq i}^{nc} [A10(j,i)+A11(j,i)]X3(j,i) - A12(i)X4(i,i)
\]

\[
\text{interest} = \sum_{j=1}^{nc} \text{imports + duty} - \sum_{j=1, j\neq i}^{nc} [A10(j,i)+A11(j,i)]X3(j,i) - A12(i)X4(i,i)
\]

\[
\text{tax} = \text{profit} - \text{loss} = \text{int. fixed cost} = A13(i)X6(i) - X8(i) + X9(i) = B6(i) \quad i=1,\ldots,nc.
\]

The definition of the internal yield for each affiliate stated in the currency of the relevant host country can be read from the above equations. The definition could be easily changed, but this definition seems natural enough.

Note that the equations for internal profit and loss define the yield in each affiliate separately and that the yield is stated in the currency of the relevant host country. The handling of the objective function is more flexible when the equations for internal profit and loss are not substituted in the objective function. In addition, the possibilities of sensitivity analysis are enhanced, as will be seen in Sections 3.5.3 and 3.5.4. The substitution is a legitimate one, but from the pragmatic point of view it is burdensome. On the other hand the separate presentation of these auxiliary equations increases the size of the model.

Let us again consider the numerical example. The new variables are given below.

\[
X81 = \text{the internal profit in Finland in marks.}
\]

\[
X91 = \text{the internal loss in Finland in marks.}
\]

\[
X82 = \text{the internal profit in England in pounds.}
\]

\[
X92 = \text{the internal loss in England in pounds.}
\]

For the Finnish internal yield we have to construct the
following equation

\[
X_{81} - X_{91} = \text{revenues in Finland less costs in Finland}
\]

which is tantamount to

\[
750X_{11} - 300X_{211} - 100X_{212} + X_{312} - 11X_{321} + 0.05X_{411} - 0.60X_{61} - X_{81} + X_{91} = 1\,000\,000 \quad (\text{Fmk}).
\]

For the English affiliate we write

\[
75X_{12} - 10X_{221} - 30X_{222} - 0.11X_{312} + X_{321} - 0.05X_{422} - 0.45X_{62} - X_{82} + X_{92} = 100\,000 \quad (£).
\]

These equations are constraints in the present linear programming model, even if they relate to the objective function of the model.

3.2.10 NON-NEGATIVITY CONSTRAINTS

It is required that all the variables (the symbols identified by the letter X) must be non-negative in the linear programming model. The Simplex-algorithm automatically sees to this.

All the constraints have now been presented.
3.2.11 THE OBJECTIVE FUNCTION

Define:

\[ C_1(i) = \text{the evaluation coefficient on internal profits realized in affiliate } i, \text{ set by the central management. It is } \geq 0. \]

\[ C_2(i) = \text{the evaluation coefficient on internal losses realized in affiliate } i, \text{ set by the central management. It is } \geq 0. \]

We have:

\[
\text{maximize } P = \sum_{i=1}^{n_c} C_1(i)X_8(i) - \sum_{i=1}^{n_c} C_2(i)X_9(i). 
\]

The objective is to maximize the sum of the internal profits\(^{26}\) of the affiliates over the planning horizon stated in a common unit. For each affiliate the net internal income is calculated in the relevant host country currency. If a "geocentric operation philosophy" and a linear utility function are assumed, we can set \[ C_1(i) = C_2(i) = A_{10}(i,1). \] (\[ A_{10}(i,1) \] is the exchange rate of currency \[ i \] in the home country.) Thus the objective function will be denominated in the currency of the home country of the idealized multinational firm.

Consider the numerical example once more. The objective is to maximize the total profit of the two-affiliate multinational firm stated in Finnish currency. The yield in England must, therefore, be converted \textit{in calculations} into marks. This is done only for computations. It does not mean that the potential English profits must

\(^{26}\) or negative loss.
actually be repatriated in Finland.\textsuperscript{29}

The evaluation coefficients applied to the English profits and losses are simply made equal to the pertinent exchange rate (10 Fmk/£), since, in the numerical example, there is no reason to proceed differently. Thus the objective function of the numerical example is

\[
\text{max } P = X_{81} + 10X_{82} - X_{91} - 10X_{92},
\]

where \(P\) stands for the global yield to be maximized.

The model derived in the numerical example is presented in Exhibit 3-1.

\textsuperscript{29} If the profits were actually repatriated, this would incur repatriation costs, which would have to be included in the model. On the other hand, if repatriation is consistent with the long-term objectives of the multinational firm under observation, then the profits abroad would be valued less than the domestic profits, and this would be reflected through the evaluation coefficients. This theme is not pursued much further, since delving into questions of repatriation adjustment would require further research efforts. Section 3.5.4 comments, however, on repatriation of foreign profits in the model.
\[
\text{max } P
\]
\[
\begin{align*}
X11 & + 10X12 - 2X1 - 10X92 \\
X11 & -X211 + X212 - X221 \\
X12 & -X12 + X211 - X222 \\
& 400X212 - X312 \\
& -900X212 + X312 \\
& 30X221 - X321 \\
& -100X221 + X321 \\
& X411 \\
& X422 \\
& 250X11 + 300X211 + 10X212 - X312 + 11X321 - 0.95X411 + 0.60X61 \\
& 25X12 + 10X221 + 30X222 + 0.11X312 - X321 - 0.95X422 + 0.45X62 \\
& 750X11 - 300X211 - 100X212 + X312 - 11X321 - 0.55X411 - X61 + X71 = 2500000 \\
& 75X12 - 10X221 - 30X222 - 0.11X312 + X321 - 0.05X422 - X62 + X72 = 2500000 \\
& 750X11 - 300X211 - 100X212 + X312 - 11X321 - 0.55X411 - 0.60X61 - X81 + X91 = 1000000 \\
& 75X12 - 10X221 - 30X222 - 0.11X312 + X321 - 0.05X422 - 0.45X62 - X82 + X92 = 1000000 \\
\end{align*}
\]
\text{all variables } \geq 0
3.3 Analysis of Selected Model Features

In this section we discuss two model features. The shadow prices of the transfer price constraints do not directly give information about the economic effects, in the model, of changing lower or upper bounds on the transfer prices. This results from the fact that basically the transfer price constraints are nonlinear. The information would be useful, and a simple extension of the conventional manner of analysing the shadow prices of the relevant constraints is suggested in order to gain the information without having to re-solve the model.

The opportunity cost of capital in the model is discussed, too. It is suggested that no single opportunity cost of capital is meaningful in the multinational case.

3.3.1 MATHEMATICAL PRELIMINARIES: REVIEW OF CERTAIN FEATURES OF LINEAR AND MATHEMATICAL PROGRAMMING

The primal problem in linear programming is

\[
\text{maximize } f = c_1 x_1 + \ldots + c_n x_n \\
\text{subject to } a_{11} x_1 + \ldots + a_{1n} x_n \leq b_1 \\
\vdots \\
a_{m1} x_1 + \ldots + a_{mn} x_n \leq b_m \\
x_1, \ldots, x_n \geq 0
\]

or

\[
\text{maximize } f = c_1 x_1 + \ldots + c_n x_n \\
\text{subject to } a_{11} x_1 + \ldots + a_{1n} x_n + s_1 = b_1 \\
\vdots \\
a_{m1} x_1 + \ldots + a_{mn} x_n + s_m = b_m \\
x_1, \ldots, x_n, s_1, \ldots, s_m \geq 0.
\]
The corresponding dual problem is

\[
\begin{align*}
\text{minimize } g &= b_1 r_1 + \ldots + b_m r_m \\
\text{subject to } a_{11} r_1 + \ldots + a_{1m} r_m &\geq c_1 \\
&\vdots \\
& a_{1n} r_1 + \ldots + a_{mn} r_m \geq c_n \\
\end{align*}
\]

\[r_1, \ldots, r_m \geq 0.\]

Denote by \(x_j^0\) (\(j=1, \ldots, n\)) and \(s_i^0\) (\(i=1, \ldots, m\)) the optimal solution of the primal problem, and by \(f^0\) the respective value of the objective function. Assume that the optimal solution is finite. Denote by \(r_i^0\) (\(i=1, \ldots, m\)) the optimal solution of the dual problem.

**Theorem 1** The value of \(r_i^0\) tells us how much \(f^0\) increases if \(b_i\) is increased by one unit, providing that the optimal basis is not changed by the increase in \(b_i\).\(^{30}\) In economic applications \(r_i^0\) is called the shadow price of the \(i\):th constraint or resource.

**Theorem 2** If \(s_i^0 > 0\), or more generally if \(s_i\) is in the final basis, that is the \(i\):th constraint is not effectively binding, then \(r_i^0 = 0.\)^{31}

**Theorem 3** If \(x_j^0 > 0\), or more generally if \(x_j\) is in the final basis, then \(a_{1j} r_1^0 + a_{2j} r_2^0 + \ldots + a_{mj} r_m^0 = c_j.\)^{31}

---

\(^{30}\) Theorem 1 is an obvious corollary of the so-called Duality Theorem \((f^0 = \sum_{j=1}^n c_j x_j^0 = \sum_{i=1}^m b_i r_i^0)\). The proof of the Duality Theorem can be found e.g. in Saul I Gass, *Linear Programming: Methods and Applications* (London/ etc., 1964), pp. 84-87.

\(^{31}\) The proof can be found in G. Hadley, *Linear Programming*, op.cit., pp. 239-240.

The property of linear programming given by Theorems 2 and 3 is called complementary slackness.
The general mathematical programming problem can be formulated as follows.\(^{32}\)

Find a vector \(\hat{x}^0 = [x_1^0, \ldots, x_n^0]\), \(\hat{x}^0 \in X \subseteq \mathbb{R}^n\)\(^{33}\) such that

\[
f(\hat{x}^0) = \max_{\hat{x} \in X} f(\hat{x}), \quad X = \{\hat{x} | g_i(\hat{x}) \leq b_i \quad (i=1, \ldots, m), \quad x_j \in S_j \quad (j=1, \ldots, n)\},
\]

where \(f\) and \(g_i\) \((i=1, \ldots, m)\) are real valued functions of the vector \(\hat{x}\), \(b_i\) \((i=1, \ldots, m)\) are real numbers, and each \(S_j\) \((j=1, \ldots, n)\) is a subset of real numbers.

The following three concepts are used in association with the definition above. The variables: \([x_1, \ldots, x_n]\), the constraints: \(\hat{x} \in X\), and the objective function: \(f(\hat{x})\).

Mathematical programming can be of various special types depending on the refined definition of \(S_j\) \((j=1, \ldots, n)\), \(f\) and \(g_i\) \((i=1, \ldots, m)\). Linear programming is one of them.

### 3.3.2 THE CONCEPT OF SHADOW TRANSFER PRICES: AN EXTENSION OF THE CONVENTIONAL ANALYSIS OF SHADOW PRICES

Consider the transfer price constraint

\[
A1(i,j)X2(i,j) - X3(i,j) \leq 0.
\]

---


\(^{33}\) \(\hat{x} \in X\) means that the vector \(\hat{x}\) belongs to a set \(X\) as an element.

\(X \subseteq \mathbb{R}^n\) means that \(X\) is a subset of \(\mathbb{R}^n\), and can be equal to it. \(\mathbb{R}^n\) is the set of \(n\)-dimensional real vectors.
AI(i,j) is the lower bound on the transfer price of intermediate products, which are transferred from affiliate i to affiliate j. The constraint sees to it that the relevant transfer price, i.e. Y1(i,j), cannot be less than AI(i,j) when pertinent transfers are made, i.e. when X2(i,j) > 0. Suppose that the pertinent transfers occur in the optimal program. We wish to know how much the optimal value of the objective function (P) changes if AI(i,j) is changed by a definite figure. The answer to this question could be sought by using standard sensitivity analysis procedures for a variation in an aij when xj appears in the final basis. Using this method would be a bit unwieldy, however. Another line can be followed. The analysis is made decisively easier by the fact that AI(i,j) appears only in the constraint given above, and nowhere else in the linear programming problem in question.

Denote for short:

A = AI(i,j). (lower bound on the transfer price)
X2 = X2(i,j). (units transferred from affiliate i to j)
X2o = the value of X2 in the old optimum solution.
X2n = the value of X2 in the new optimum solution.
X3 = X3(i,j). (value of the transferred items)
X3n = the value of X3 in the new optimum solution.
r = the optimal value of the dual variable related to the constraint considered.
VP = the change in the optimal value of the objective function.
ε = a change in the right-hand side of the constraint considered.
a = a change in constant A.

For these procedures see Saul Gass, Linear Programming: Methods and Applications, op. cit., pp. 134-138, or Jouko Manninen & Timo Salmi, Matemaattinen optimointi: teoriaa ja tekniikkaa, op. cit., Ch. 4 ("Herkkyysanalyysi ja parametraalinen lineaarinen optimointi").
Assume that $X2^o > 0$, $X2^n > 0$, and that the same final basis is preserved throughout the analysis. If the constraint

$$AX2 - X3 \leq 0$$

is changed to

$$AX2 - X3 \leq \epsilon,$$

then

$$\nabla P = \epsilon,$$

according to Theorem 1. In the new optimum solution

$$AX2^n - X3^n \leq \epsilon$$

holds.

This can be written as

$$(A-\epsilon/X2^n)X2^n - X3^n \leq 0,$$

since we assumed that $X2^n > 0$. It is easy to see that

$$\nabla P = -aX2^n$$

by substituting

$$a = -\epsilon/X2^n,$$

i.e. $\epsilon = -aX2^n$.

Set $a = -1$. Then

$$\nabla P = -X2^n.$$

This figure is defined as the shadow transfer price in the model, since it tells us how much reducing the lower bound on the relevant transfer price by one unit increases the optimal value of the objective function. $\nabla P$ must be approximated, since $X2^n$ is unknown. One possible approximation is $\nabla P \approx X2^o$. A similar concept can be defined for raising the upper bound on a transfer price by one unit.

A numerical example will be considered later (in Section 3.5.2) of the upper bound.
3.3.3 OPPORTUNITY COST OF CAPITAL IN THE MODEL

This section presents a speculative and abstract sideline. It can be omitted without loss of continuity.

In management literature the concept of opportunity cost of capital to the firm is often referred to. It seems, however, that defining a single general opportunity cost of capital to the firm is meaningless.

Suggesting one single opportunity cost of capital to the firm implicitly contains the following way of thinking. If a firm and all possibilities open to it are embodied in a general mathematical programming model, then the opportunity cost of capital is given by a generalized shadow price relating to the financial constraint for the firm. (The general mathematical programming problem above is defined as in Section 3.3.1. By generalized shadow price we mean a figure that gives for the relevant mathematical programming model the change in the optimal value of the objective function $V_f(x)$ caused by a change of one infinitesimal unit $V_{b_1}$ in the pertinent resource.) The problem naturally arises: what is the "financial constraint for the firm"? Is it, for example, a loan raising constraint or a cash flow constraint? Defining one single opportunity cost of capital to the firm means that this question can be settled and "the financial constraint for the firm" exists.

In the multinational case there is a separate "financial constraint" for each affiliate. They indicate one single opportunity cost of capital to the multinational firm only if the respective shadow prices are all equal.

---

This model always exists in principle, but it can never be written out in actual practice, or in theory without simplifications.
Consider next the model for the multinational firm, which was presented. Define that the cash flow constraints can be deemed as the financial constraints to the idealized multinational firm. It is easy to demonstrate that no single positive ($> 0$) opportunity cost of capital exists in the model, except in very special cases. For simplicity, consider only two affiliates of the idealized multinational firm. The affiliates are denoted by indices $h$ and $k$, this time. Denote by $R_i$ the optimal value of the dual variable relating to the $i$:th constraint of the model. Then, according to the above, $R_{13}$ is the opportunity cost of capital to affiliate $h$, and $R_{14}$ is the opportunity cost of capital to affiliate $k$. The task is to verify that, when $R_{13} > 0$ and $R_{14} > 0$, then $R_{13} = R_{14}$ need not hold in the model.

If a financial constraint is not effective, then, according to Theorem 2, the corresponding shadow price is equal to zero. Consequently $R_{13}$ and $R_{14}$ can be positive only if the financial constraints are both effective. If no transfers occur between the affiliates, then the affiliates are independent units in the model, each having its own opportunity cost of capital defined by the relevant financial constraints. For the case where transfers occur it is sufficient to consider transfers in one direction only. Suppose that transfers are made from affiliate $k$ to affiliate $h$. Then $X_2(k,h) > 0$, and $X_3(k,h) > 0$ or at least it appears in the final basis. According to Theorem 3 we can write as follows.

\[36\] Incidentally, an example of a relation between the shadow price of the cash flow constraint for an affiliate and the shadow price of the loan raising constraint of the same affiliate will be considered in the numerical example in Section 3.5.2.
It is easy to see that $R_{13} = R_{14} > 0$ need not hold.
(It will be seen that in the solution of the numerical example $R_{13} = 0.060$ and $R_{14} = 0.282$.)

3.4 Solution to the Numerical Example

The rest of this chapter discusses the model in the framework of the numerical example, which was presented parallel with the model in Section 3.2.

3.4.1 COMPUTER OUTPUT

When the model given in Exhibit 3-1 is run on a computer, we get the following output.\(^{37}\) The variables identified by the letter $S$ are the slack variables of the inequality-constraints of the model.

---

\(^{37}\) It should be noted that the model is always degenerate. In principle, the Simplex-algorithm is not affected by this fact (except that the number of iterations increases; cycling is too rare to matter). Nevertheless, some standard computer codes, which have not been properly tested, may be affected by the degeneracy. The computer code programmed for the current research work has been documented in Timo Salmi, *The Multinational Firm: A Mathematical Programming Model Building Approach* (licentiate thesis) op.cit., and Jouko Manninen & Timo Salmi, *Matemaattinen optimointi: teoriaa ja tekniikkaa*, op.cit.
X61 = 1 250 000  (book profit in Finland, Fmk)
X422 = 221 336  (loan raising in England, £)
S3 = 15 750  (slack in Finnish production phase I, units)
S4 = 14 250  (slack in English production phase I, units)
X11 = 10 000  (final product sales in Finland, units)
X12 = 10 000  (final product sales in England, units)
X212 = 0  (exports from Finland to England, units)
S8 = 0  (slack of the second transfer price constr.)
S9 = 0  (slack of the third transfer price constr.)
X321 = 75 000  (value of Finnish imports from England, £)
X411 = 3 000 000  (loan raising in Finland, Fmk)
S12 = 78 664  (unused English loan raising potential, £)
X81 = 2 000 000  (internal profit in Finland, Fmk)
X82 = 278 664  (internal profit in England, £)
X221 = 750  (imports to Finland from England, units)
X62 = 233 933  (book profit in England, £)
X211 = 9 250  (intermediate product production in Finland, units)
X222 = 10 750  (intermediate product production in England, units)
REST ARE ZEROS

P = 4 786 640  (global profit, Fmk)

Also a list of the shadow prices of the constraints results from the computer run:

R1 = 130.46  (Finnish sales constraint)
R2 = 226.22  (English sales constraint)
R5 = 127.24  (Finnish technical availability constraint)
R6 = 169.66  (English technical availability constraint)
R10 = 0.989  (upper bound constraint of transfer price E→F)
R11 = 0.039  (Finnish loan raising constraint)
R13 = 0.060  (Finnish cash flow constraint)
R14 = 0.282  (English cash flow constraint)
REST ARE ZEROS
The shadow prices are often interpreted as the contribution to profit per unit of the limiting factor. The dimension of all the shadow prices in the current model are marks per unit of the relevant restricting resource, because the dimension of the objective function is marks.

The numerical solution is elucidated in Figure 3-5. 750 units of the intermediate product are transferred from the English affiliate to the Finnish one. The external transfer price per unit will be \( X_{321}/X_{221} \), that is \( 75000/750 \) units = \( 100/\) unit. The transfer price is thus made as high as possible. Note that the transfer prices are assessed only for the units to be transferred. It was defined, for mathematical reasons, that if no transfer occurs no respective transfer price exists.

3.4.2 DERIVING OPTIMAL OPERATING BUDGETS FOR THE MULTINATIONAL FIRM IN THE NUMERICAL EXAMPLE

Operating budgets are the prime tools of managing an enterprise. Conventionally the budgets are prepared by selecting one of the budgets as the basis (usually this is the sales, cash, or production budget of the firm) and adjusting the other budgets in the frames stated by the basic budget. If the resulting set of budgets is not satisfactory or feasible the basic budget is reconsidered, and the iteration process is repeated until a satisfactory set of budgets has been constructed as the result of the budgetary process.

The budgetary process can be carried out also by
FIGURE 3-5  Trade and Production Flows in the Optimum Solution of the Numerical Example

- Final product: 10,000 units
- Production phase II:
  - Intermediate product: 750 units
  - Production phase I: 9,250 units
- Raw material: 10,750 units

Transfer price: £100/unit
utilizing linear programming. In this case optimal budgets can be derived in one iteration after the necessary data have been collected. First, an appropriate linear programming model is constructed for the activities of the firm over the planning horizon. Then, after solving the program, the budgets are derived from the optimal solution. These budgets will automatically be compatible, if the linear programming model has been correctly constructed. - The same process is applicable for the multinational firm as well, as will be seen shortly in the framework of the numerical example. The difference is that in the multinational case a set of budgets has to be constructed for each affiliate, on the basis of the relevant linear programming model.

The sales budgets for the affiliates in the numerical example can be derived from the optimal values of the variables $X_{11}$, $X_{12}$, $X_{212}$, $X_{221}$, $X_{312}$, and $X_{321}$:

<table>
<thead>
<tr>
<th>FINNISH AFFILIATE</th>
<th>TOTAL</th>
<th>PRICE</th>
<th>AMOUNT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic sales</td>
<td>10 000 000 Fmk</td>
<td>1 000 Fmk</td>
<td>10 000 units</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ENGLISH AFFILIATE</th>
<th>TOTAL</th>
<th>PRICE</th>
<th>AMOUNT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic sales</td>
<td>1 000 000 £</td>
<td>100 £</td>
<td>10 000 units</td>
</tr>
<tr>
<td>Exports</td>
<td>75 000 £</td>
<td>100 £</td>
<td>750 units</td>
</tr>
</tbody>
</table>

\[1 075 000 £\]

38 For an introduction to a budgetary process based on linear programming, see the references in footnote 2 in this chapter.

39 Optimal considering the data collected. It is obvious that the result cannot be any better than the quality of the input data indicates.
The production, inventory, and purchase budgets could be derived in an analogous manner, but this is omitted, since these budgets would not be instructive, because of the simple production structure.

The cash budgets for each affiliate are given below. They are derived on the basis of the values of the variables which were included in the cash flow constraints.

**FINNISH AFFILIATE**

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cash</td>
<td>6 000 000 Fmk</td>
</tr>
<tr>
<td>CASH INFLOWS</td>
<td></td>
</tr>
<tr>
<td>loans</td>
<td>3 000 000 Fmk</td>
</tr>
<tr>
<td>Total cash available</td>
<td>9 000 000 Fmk</td>
</tr>
<tr>
<td>CASH OUTFLOWS</td>
<td></td>
</tr>
<tr>
<td>variable cost II 10 000·250</td>
<td>2 500 000 Fmk</td>
</tr>
<tr>
<td>variable cost I 9 250·300</td>
<td>2 775 000 Fmk</td>
</tr>
<tr>
<td>intermediate product imports 750·1 000</td>
<td>750 000 Fmk</td>
</tr>
<tr>
<td>duty 10% 750 000</td>
<td>75 000 Fmk</td>
</tr>
<tr>
<td>fixed cost</td>
<td>1 000 000 Fmk</td>
</tr>
<tr>
<td>interest 5% 3 000 000</td>
<td>150 000 Fmk</td>
</tr>
<tr>
<td>tax 60% 1 250 000</td>
<td>750 000 Fmk</td>
</tr>
<tr>
<td>TOTAL</td>
<td>8 000 000 Fmk</td>
</tr>
<tr>
<td>Closing cash</td>
<td>1 000 000 Fmk</td>
</tr>
</tbody>
</table>

**ENGLISH AFFILIATE**

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cash</td>
<td>600 000 £</td>
</tr>
<tr>
<td>CASH INFLOWS</td>
<td></td>
</tr>
<tr>
<td>intermediate product exports 750·100</td>
<td>75 000 £</td>
</tr>
<tr>
<td>loans</td>
<td>221 336 £</td>
</tr>
<tr>
<td>Total cash available</td>
<td>896 336 £</td>
</tr>
<tr>
<td>CASH OUTFLOWS</td>
<td></td>
</tr>
<tr>
<td>variable cost II 10 000·25</td>
<td>250 000 £</td>
</tr>
<tr>
<td>variable cost I 10 750·30</td>
<td>322 500 £</td>
</tr>
<tr>
<td>transportation 750·10</td>
<td>7 500 £</td>
</tr>
<tr>
<td>fixed cost</td>
<td>100 000 £</td>
</tr>
<tr>
<td>interest 5% 221 336</td>
<td>11 066 £</td>
</tr>
<tr>
<td>tax 45% 233 933</td>
<td>105 270 £</td>
</tr>
<tr>
<td>TOTAL</td>
<td>796 336 £</td>
</tr>
<tr>
<td>Closing cash</td>
<td>100 000 £</td>
</tr>
</tbody>
</table>

Figure 3-6 depicts the budgeted cash flows.
In addition to the above budgets it is easy to derive projected income statements both for book profits and for internal profits in each affiliate. Note that the projected income statements for book profits were not needed in order to derive the cash budgets, since $X_{61}$ and $X_{62}$ give the predicted book profits for each affiliate respectively. The projected income statements for book profit can be derived on the basis of the values of the variables which were included in the equations for book profit and loss. In actual practice these statements should naturally be drafted in accordance with the practice of the appropriate countries. In the numerical example, we have the following statements:

**FINNISH AFFILIATE**

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic sales</td>
<td>10,000,000 Fmk</td>
</tr>
<tr>
<td>Variable cost II</td>
<td>2,500,000 Fmk</td>
</tr>
<tr>
<td>Variable cost I</td>
<td>2,775,000 Fmk</td>
</tr>
<tr>
<td>Intermediate product imports</td>
<td>750,000 Fmk</td>
</tr>
<tr>
<td>Duty</td>
<td>75,000 Fmk</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>1,000,000 Fmk</td>
</tr>
<tr>
<td>Interest</td>
<td>150,000 Fmk</td>
</tr>
<tr>
<td>Depreciation</td>
<td>1,500,000 Fmk</td>
</tr>
<tr>
<td>Net income subject to taxation</td>
<td>1,250,000 Fmk</td>
</tr>
</tbody>
</table>

**ENGLISH AFFILIATE**

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic sales</td>
<td>1,000,000 £</td>
</tr>
<tr>
<td>Intermediate product exports</td>
<td>75,000 £</td>
</tr>
<tr>
<td>Variable cost II</td>
<td>250,000 £</td>
</tr>
<tr>
<td>Variable cost I</td>
<td>322,500 £</td>
</tr>
<tr>
<td>Transportation</td>
<td>7,500 £</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>100,000 £</td>
</tr>
<tr>
<td>Interest</td>
<td>11,067 £</td>
</tr>
<tr>
<td>Depreciation</td>
<td>150,000 £</td>
</tr>
<tr>
<td>Net income subject to taxation</td>
<td>233,933 £</td>
</tr>
</tbody>
</table>

Finally we can derive the income statements from the firm's internal point of view. The statements are denominated in the currencies of the host countries of
the affiliates, although the final evaluation is performed in the currency of the home country of the multinational firm. This results from the fact that the statements are derived from the equations for internal profit and loss.

### FINNISH AFFILIATE

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount (Fmk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic sales</td>
<td>10 000 000</td>
</tr>
<tr>
<td>Costs of goods sold</td>
<td></td>
</tr>
<tr>
<td>variable cost II</td>
<td>2 500 000</td>
</tr>
<tr>
<td>variable cost I</td>
<td>2 775 000</td>
</tr>
<tr>
<td>intermediate product imports</td>
<td>750 000</td>
</tr>
<tr>
<td>duty</td>
<td>75 000</td>
</tr>
<tr>
<td>Contribution margin</td>
<td>3 900 000</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>1 000 000</td>
</tr>
<tr>
<td>Interest</td>
<td>150 000</td>
</tr>
<tr>
<td>Tax</td>
<td>750 000</td>
</tr>
<tr>
<td><strong>NET INCOME</strong></td>
<td><strong>2 000 000</strong></td>
</tr>
</tbody>
</table>

### ENGLISH AFFILIATE

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td></td>
</tr>
<tr>
<td>domestic sales</td>
<td>1 000 000</td>
</tr>
<tr>
<td>intermediate product exports</td>
<td>75 000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1 075 000</strong></td>
</tr>
<tr>
<td>Costs of goods sold</td>
<td></td>
</tr>
<tr>
<td>variable cost II</td>
<td>250 000</td>
</tr>
<tr>
<td>variable cost I</td>
<td>322 500</td>
</tr>
<tr>
<td>transportation</td>
<td>7 500</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>580 000</strong></td>
</tr>
<tr>
<td>Contribution margin</td>
<td>495 000</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>100 000</td>
</tr>
<tr>
<td>Interest</td>
<td>11 066</td>
</tr>
<tr>
<td>Tax</td>
<td>105 270</td>
</tr>
<tr>
<td><strong>NET INCOME</strong></td>
<td><strong>278 664</strong></td>
</tr>
</tbody>
</table>

The value of the objective function is

\[
P = 2 000 000 + 10 \cdot 278 664 = 4 786 640 \text{ (Fmk)}.
\]
3.5 Analysis of the Solution of the Numerical Example

3.5.1 Characteristics of the Solution

In the numerical example the fictitious Finland is a country with higher corporate income taxes (60%) and the fictitious England is a country with lower corporate income taxes (45%). In the optimum solution the multinational firm contrives to reduce the impact of Finnish taxation by importing intermediate products from the English affiliate. The transfer price of these items is made as high as possible (£100/unit). Consequently, less taxes have to be paid, because of the increase in the ostensible costs of the final products sold in Finland. This is clearly an instance of book profit adjustment in the multinational firm by manipulating interaffiliate trade and transfer prices. In our example the direct costs of tax avoidance are composed of duty and transportation, but the total reduction in taxation turns out to be more significant.

There are several effectively limiting factors in the optimal program. First, the demand for the final product is a limiting factor in both countries. The current resources of the example firm would allow it to produce more.

The upper bound on the transfer price of the intermediate products transferred from the English affiliate to the Finnish affiliate is a limiting factor, too. This results from the fact that if the discussed transfer price could be made higher than the current upper bound,

40 See Section 1.1: "Introduction to the Multinational Firm".
it would be possible to decrease the impact of Finnish taxation still more effectively than before.

Funds are a limiting factor in the Finnish affiliate, but not in the English one. The Finnish affiliate would use the additional funds to import more tax-reducing intermediate products from the English affiliate. One interpretation of the situation is that the Finnish production is planned in a manner which satisfies the demand of 10,000 units for the final product in Finland. The units of the intermediate product needed in the production in Finland are preferably imported from the English affiliate because of tax considerations. As a result of the high transfer price applied, this procedure strains finance in the Finnish affiliate. Consequently domestic production dominates, since it strains the scarce financial resource less.

For the English affiliate finance is not an effectively limiting factor. This is reflected in the fact that the English loan raising capability of £300,000 is not completely used up. The cash flow constraint for the English affiliate is not effectively binding, even if the respective shadow price is positive (0.282 Fmk/£). The English affiliate only raises such an amount of loans as is necessary to balance the cash flows, because there is no incentive to hold idle cash. (The minimum cash balance requirement at the end of the planning horizon cannot be deemed idle cash, since the requirement results from the tactical nature of the planning situation. The continuity of action and general management policies must be accounted for.) Hence, the cash flow constraint for the English affiliate naturally becomes an equation in the optimal solution. The reason for the occurrence of a positive value for the respective shadow price in this numerical example is explained in the next section,
where shadow prices are discussed.

The technical availability constraints cannot be unambiguously considered as "limiting factors", because their function is to link the physical flows of the model. The technical availability constraints become equations for both affiliates in the example solution. This indicates that the initial inventories of the intermediate product (0 units in both countries) are not sufficient to cover the needs of the planning period without producing or importing.

3.5.2 ANALYSIS OF THE SHADOW PRICES

The aim of an analysis of shadow prices is to evaluate the economic significance of the limiting factors. The analysis is often (and on some occasions incorrectly) interpreted as an answer to the question: "what should the management be willing to pay for an additional unit of the limiting resource?"

Consider the shadow price of the English sales constraint: R2 = 226.22. It indicates that if the English demand for the final product could be increased by one final product, the optimum value of the objective function, i.e. the global profit to the firm, would increase by 226.22 marks. The dimension of the shadow price under observation is 226.22 marks/unit of the final product, even if the sales constraint in question concerns English sales. As will be recalled, the section of the objective function relating to the English affiliate was converted, in calculations, into the currency of the home country, i.e. Finnish marks, by applying the relevant evaluation coefficient (10 Fmk/$), selected by the central management. As is known the Simplex-algorithm of linear programming
automatically gives the shadow prices of all the inequality constraints in the model. Assessing a shadow price correctly via conventional management accounting methods is often a burdensome task (if there is more than one limiting factor). Consider as an example an analysis of the shadow price \( R_2 = 226.22 \) marks/unit of the final product.

In order to assess the marginal contribution of an English final product we have to subtract from the sales price the marginal costs which can be attributed to the product. These marginal costs consist of variable production costs, interest, and tax costs attributable to the relevant product. Let us first consider the pertinent tax cost:

<table>
<thead>
<tr>
<th>Description</th>
<th>£</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales price</td>
<td>100</td>
</tr>
<tr>
<td>Variable cost II</td>
<td>25</td>
</tr>
<tr>
<td>Variable cost I</td>
<td>30</td>
</tr>
<tr>
<td>Contribution subject to taxation</td>
<td>45</td>
</tr>
</tbody>
</table>

The tax rate on corporate profits was assumed to be 45%. Hence the tax-component of the costs is given by \( 0.45 \times 45 = 20.25 \).

Next consider the interest cost. Let \( \gamma \) be the marginal loan needed for producing one final product in England. The interest rate is 5% on the loan when considered on a pre-tax basis. The interest is deductible in taxation, however, and thus the effective after-tax interest rate is \((1-0.45)5\% = 2.75\%\). The marginal loan \( \gamma \) can be solved from the relationship below

\[
\gamma = \frac{v.c.II}{25} + \frac{v.c.I}{30} + \frac{\text{tax}}{20.25} + \frac{\text{interest}}{0.0275\gamma}.
\]

The above gives \( \gamma \approx £ 77.38 \). Hence, the loan-cost-
component is $0.0275 \cdot 77.38 \approx 2.13$.

When the marginal costs are deducted from the sales price of one final product, we see the marginal contribution of one unit of the English final product:

$$100 - 30 - 25 - 20.25 - 2.13 = £ 22.62 = 226.2 \text{ Fmk}$$

as should be the case.

Next consider the shadow price of the Finnish technical availability constraint: $R_5 = 127.24$. It indicates the value of one unit of the intermediate product in the Finnish affiliate. This is clearly seen from the relevant Finnish constraint:

$$\text{initial inventory less minimum closing inventory}$$

$$X_{11} - X_{211} + X_{212} - X_{221} \leq 0.$$

Increasing the right-hand side of the constraint by one unit can be interpreted as acquiring one additional unit of the intermediate product. Thus, $127.24$ Fmk is the highest price for one externally acquired unit of the intermediate product which would still not be unprofitable.

Next we consider the shadow prices relating to finance in the present model. Also some general features of shadow price analysis are discussed. The shadow price of the Finnish loan raising constraint $R_{11} = 0.039$ indicates that the highest interest rate the firm should be willing to pay is $3.9\% + 5\% = 8.9\%$ for each additional mark borrowed in Finland. On the other hand the shadow price of the Finnish cash flow constraint is $R_{13} = 0.060$, indicating that the management should be willing to pay an interest of $6\%$ for each additional mark borrowed in Finland. What is the reason for the difference between $8.9\%$ and $6\%$? A factual difference between the two
interest rates results only if the shadow prices are not properly interpreted. In the present numerical example the difference arises from the fact that the interest of the former additional mark is deductible in taxation, while the latter is not! An additional mark brought in through the Finnish cash flow constraint is not deductible, since the model includes no mechanism to that effect. The model, however, is not inadequate in this respect. The phenomenon is not a new one, but it is sometimes forgotten. Mathematically the shadow prices give only the increase in the optimal value of the objective function caused by an increase of one unit in the right-hand side of the relevant constraints, and nothing else, as can be seen in Theorem 1 of Section 3.3.1 ("Mathematical Preliminaries:..."'). Any economic interpretation of the shadow prices must be made bearing this fact in mind.\footnote{Another common mistake made in real-life applications is in forgetting that the feasible range of the shadow prices is limited by the next change in the optimal basis.} For example, several possibilities of defining an opportunity cost of capital arise. There is a definite relationship between the "interest rates" of 8.9% and 6% in the numerical example. In the current numerical example the relationship is rather complicated, and, therefore, we will consider the corresponding English shadow transfer prices instead.

The shadow price of the English loan raising constraint is $R_{12} = 0$. Raising additional loans in England is thus not advantageous. Nevertheless, the shadow price of the English cash flow constraint is positive: $R_{14} = 0.282$ Fmk/£. This figure indicates that for an additional non-deductible pound the management should be willing to pay up to 0.282 Fmk, i.e. 0.028 pounds,
assuming that the interest need not be paid in cash. This is tantamount to an interest rate of 2.82%. The additional pound would be used to substitute the original loan, for which the pre-tax interest rate is 5%, and the after-tax interest rate is \((1-0.45)5\% = 2.75\%\).

It is noted that 2.82% ≠ 2.75%. This difference results from the fact that the new additional pound substitutes \(1/[1-0.05(1-0.45)] = 1/0.9725\) old pounds, because for the latter we must consider the after-tax effect of paying the interest on it. (It was assumed that the interest on the local loans was to be paid in cash during the planning horizon. Furthermore, cash outlays for taxes are affected by the fact that the interest on the local loans is deductible, but the interest on the new additional pound is not.) Consequently, we have \(2.75\%/0.9725 \approx 2.82\%\), as should be the case.

So far we have been able to proceed within the framework of the conventional analysis of shadow prices. The transfer price constraints cannot be treated in the same way, as was seen in Section 3.3.2 ("The Concept of Shadow Transfer Prices: An Extension of the Conventional Analysis of Shadow Prices). The difficulty of assigning shadow prices to the upper and lower bounds of the transfer prices resulted from the fact that basically the transfer price constraints are nonlinear (e.g. \(X_{321}/X_{221} \leq 100\)). Consider the following question in connection with the numerical example. What is the increase in the optimal value of the objective function if the upper bound of the intermediate products imported by the Finnish affiliate from the English affiliate is raised from £100/unit to £101/unit? We know that the transfer price relating to the relevant constraint is \(R_{10} = 0.989 > 0\), which indicates that the upper bound under discussion is an effectively limiting factor in the optimum solution.
We wish to find out what happens to the optimal value of the objective function, when the coefficient of the decision variable $X_{221}$ is changed from -100 to -101. The shadow price $R_{10}$ indicates the effect of a change in the right-hand side of the constraint. The change is now in the left-hand side of the constraint. In accordance with the extended results of Section 3.3.2 the relevant change in the optimal value of the objective function, called the shadow transfer price, is

$$ VP = R_{10} \cdot X_{221}^n. $$

$X_{221}^n$ indicates the new optimal value of $X_{221}$ after increasing the discussed upper bound from £100/unit to £101/unit. If $X_{221}^n$ is approximated by using the old value, the relevant shadow transfer price is approximated by

$$ VP \approx 0.989 \cdot 750 \text{ Fmk/£} \approx 742 \text{ Fmk/£}. $$

If we re-solve the numerical example with the pertinent upper bound increased, we note that the true change in the value of the objective function is 732 Fmk. The approximation was fairly good in this case.

3.5.3 SENSITIVITY ANALYSIS

In this section we discuss the effect of changes

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42 In Section 3.3.2 a formula was derived for the shadow transfer price in the case of decreasing a lower limit on a transfer price. The non-mathematical reader is referred to Timo Salmi, *Monikansallinen yritys ja matemaattinen optimointi*, op.cit., pp. 76-79, for the new derivation.
in selected parameters on the nature of the optimal solution of the numerical example. The factors which will be analysed are the upper bound on the English-Finnish transfer price, and the evaluation coefficients on English (internal) profits and losses. The difficulty of analysing the effect of currency exchange rate changes by parametric linear programming\(^43\) and re-solving, which are the prime tools of sensitivity analysis, is noted.

When the upper bound on the transfer price of the items transferred from the English affiliate to the Finnish one is analysed, it is noted that "a critical value" exists between £90/unit and £89/unit. By the critical value I mean here a value past which the nature of the optimum solution is clearly changed. Figures 3-7 and 3-8 delineate the respective optimal solutions. When the Finnish affiliate can no longer import intermediate products at a transfer price high enough for tax avoidance, producing domestically becomes relatively more profitable, and there is no further need to continue the interaffiliate trade in the numerical example. The output of the relevant computer runs are documented in Exhibits 3-2 and 3-3.

A sensitivity analysis of a transfer price bound is particularly easy to perform for the model, since only one constant of the model has to be parametrized.

As will be recalled, the planning situation under observation is assumed to be centrally managed, and thus the objective function which is used, consists

\(^{43}\) For an introduction into parametric linear programming see e.g. Jouko Manninen & Timo Salmi, Matemaattinen optimointi: teoriaa ja tekniikkaa, op.cit., Ch. 4 ("Herkkyysnalyysi ja parametraalinen lineaarinen optimointi"), and Timo Salmi, Lineaarisen suunnittelun eräitä laajennuksia (unpublished master's thesis, 1970), Appendix IA ("Suomessa saatavilla olevista kirjastohoijelmista ja niiden toimintatavasta: parametraalinen lineaarinen suunnittelul"").
An Upper Bound of £90 on the Transfer Price of the Items Imported from the English Affiliate

**FIGURE 3-7**

An Upper Bound of £90 on the Transfer Price of the Items Imported from the English Affiliate

<table>
<thead>
<tr>
<th>Production Phase I</th>
<th>Production Phase II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Material</td>
<td>Intermediate Product</td>
</tr>
<tr>
<td>9,130 units</td>
<td>870 units</td>
</tr>
<tr>
<td>Final Product</td>
<td>Final Product</td>
</tr>
<tr>
<td>10,870 units</td>
<td>10,000 units</td>
</tr>
</tbody>
</table>

Transfer Price: £90/unit
FIGURE 3-8 An Upper Bound of £ 89 on the Transfer Price of the Items Imported from the English Affiliate
EXHIBIT 3-2  \( UB = \£ 90 \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X61</td>
<td>1 250 000</td>
<td>(book profit in Finland, Fmk)</td>
</tr>
<tr>
<td>X422</td>
<td>222 197</td>
<td>(loan raising in England, £)</td>
</tr>
<tr>
<td>S3</td>
<td>15 870</td>
<td>(slack in Finnish production phase I, units)</td>
</tr>
<tr>
<td>S4</td>
<td>14 130</td>
<td>(slack in English production phase I, units)</td>
</tr>
<tr>
<td>X11</td>
<td>10 000</td>
<td>(final product sales in Finland, units)</td>
</tr>
<tr>
<td>X12</td>
<td>10 000</td>
<td>(final product sales in England, units)</td>
</tr>
<tr>
<td>X212</td>
<td>0</td>
<td>(exports from Finland to England, units)</td>
</tr>
<tr>
<td>S8</td>
<td>0</td>
<td>(slack of the second transfer price constr.)</td>
</tr>
<tr>
<td>X221</td>
<td>870</td>
<td>(imports to Finland from England, units)</td>
</tr>
<tr>
<td>X211</td>
<td>9 130</td>
<td>(intermediate product production in Finland, units)</td>
</tr>
<tr>
<td>X411</td>
<td>3 000 000</td>
<td>(loan raising in Finland, Fmk)</td>
</tr>
<tr>
<td>S12</td>
<td>77 803</td>
<td>(unused English loan raising potential, £)</td>
</tr>
<tr>
<td>S9</td>
<td>52 174</td>
<td>(slack of the third transfer price constr.)</td>
</tr>
<tr>
<td>X62</td>
<td>232 368</td>
<td>(book profit in England, £)</td>
</tr>
<tr>
<td>X81</td>
<td>2 000 000</td>
<td>(internal profit in Finland, Fmk)</td>
</tr>
<tr>
<td>X82</td>
<td>266 802</td>
<td>(internal profit in England, £)</td>
</tr>
<tr>
<td>X321</td>
<td>78 261</td>
<td>(value of Finnish imports from England, £)</td>
</tr>
<tr>
<td>X222</td>
<td>10 870</td>
<td>(intermediate product production in England, units)</td>
</tr>
</tbody>
</table>

REST ARE ZEROS

\( P = 4 778 024 \) (global profit, Fmk)

R1 = 159.86 (Finnish sales constraint)
R2 = 226.22 (English sales constraint)
R5 = 122.95 (Finnish technical availability constraint)
R6 = 169.66 (English technical availability constraint)
R7 = 0.014 (lower bound constraint of transfer price F→E)
R10 = 1.147 (upper bound constraint of transfer price E→F)
R13 = 0.025 (Finnish cash flow constraint)
R14 = 0.282 (English cash flow constraint)

REST ARE ZEROS

EXHIBIT 3-3  \( UB = \£ 89 \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X61</td>
<td>1 862 242</td>
<td>(book profit in Finland, Fmk)</td>
</tr>
<tr>
<td>X422</td>
<td>246 786</td>
<td>(loan raising in England, £)</td>
</tr>
<tr>
<td>S3</td>
<td>15 000</td>
<td>(slack in Finnish production phase I, units)</td>
</tr>
<tr>
<td>S4</td>
<td>15 000</td>
<td>(slack in English production phase I, units)</td>
</tr>
<tr>
<td>X11</td>
<td>10 000</td>
<td>(final product sales in Finland, units)</td>
</tr>
<tr>
<td>X12</td>
<td>10 000</td>
<td>(final product sales in England, units)</td>
</tr>
<tr>
<td>X212</td>
<td>0</td>
<td>(exports from Finland to England, units)</td>
</tr>
<tr>
<td>S8</td>
<td>0</td>
<td>(slack of the second transfer price constr.)</td>
</tr>
<tr>
<td>S11</td>
<td>244 897</td>
<td>(unused Finnish loan raising potential, Fmk)</td>
</tr>
<tr>
<td>X211</td>
<td>10 000</td>
<td>(intermediate product production in Finland, units)</td>
</tr>
<tr>
<td>X411</td>
<td>2 755 103</td>
<td>(loan raising in Finland, Fmk)</td>
</tr>
<tr>
<td>S12</td>
<td>53 214</td>
<td>(unused English loan raising potential, £)</td>
</tr>
<tr>
<td>S9</td>
<td>0</td>
<td>(slack of the third transfer price constr.)</td>
</tr>
<tr>
<td>X62</td>
<td>187 661</td>
<td>(book profit in England, £)</td>
</tr>
<tr>
<td>X81</td>
<td>2 244 897</td>
<td>(internal profit in Finland, Fmk)</td>
</tr>
<tr>
<td>X82</td>
<td>253 213</td>
<td>(internal profit in England, £)</td>
</tr>
<tr>
<td>X321</td>
<td>0</td>
<td>(value of Finnish imports from England, £)</td>
</tr>
<tr>
<td>X222</td>
<td>10 000</td>
<td>(intermediate product production in England, units)</td>
</tr>
</tbody>
</table>

REST ARE ZEROS

\( P = 4 777 030 \) (global profit, Fmk)
of global profits to be maximized. Since the activities of the Finnish and the English affiliates are reflected in different currencies, the net internal income for the English affiliate was calculated in marks by applying the evaluation coefficients stipulated by the central management. In our numerical example, the evaluation coefficient was simply equated with the currency exchange rate of the pound in Finland, i.e. 10 Fmk/£. We wish to establish the model behavior in a case where the English yield is less valued than the Finnish yield.

A lower English evaluation coefficient may arise, for example, from considering the fictitious pound a weak currency. Be this as it may, an analysis of the evaluation coefficient of the net internal income denominated in pounds reveals that its critical value is between 9.63 Fmk/£ and 9.62 Fmk/£. The nature of the optimum solution is changed between these two values.

When the evaluation coefficient (for both profit and loss in England) is 9.63 Fmk/£, the optimum solution remains the same as was depicted in Figures 3-5 and 3-6. When the evaluation coefficient is 9.62 Fmk/£, the nature of the optimum solution is altered as seen in Figures 3-9 and 3-10. The net internal income in Finland is relatively valued so much more that in spite of

---

44 See also footnote 29 in this chapter.
FIGURE 3-9 Trade and Production Flows when the Evaluation Coefficient on the English Yield is 9.62 Fmk/£
the high tax rate in the fictitious Finland, the book profit is no longer pressed down by arranging interaffiliate trade with properly adjusted transfer prices. At first sight the preceding sentence may seem contradictory, but if more internal profits are realized in Finland, this leads also to showing more book profits in Finland, since the two are not negatively correlated.

Also currency exchange rates would be an important subject of sensitivity analysis. A change in the currency exchange rates is, however, simultaneously reflected in several constants of the linear programming models of the discussed kind! Hence a sensitivity analysis for the currency exchange rates is an awkward suggestion. Carrying out the sensitivity analysis by re-solving the model several times would be expensive in actual practice, since many changes would be needed in each step. Neither are the parametric routines of the standard computer codes effective, since they normally allow handling of parameters in one section of the model at a time only. Most often parametric routines are confined to the objective function or the right-hand sides of the constraints. The potential changes of currency exchange rates during the planning horizon must be accounted for in some other manner. This question will be discussed thoroughly in the next chapter where the handling of the currency exchange risk is included.

3.5.4 ANALYSIS OF A NON-GLOBAL OBJECTIVE FUNCTION: A FICTITIOUS CASE OF HOME-COUNTRY-ORIENTED OPERATING PHILOSOPHY

Finally, we conduct an experiment with the model in order to see the nature of the solution, if the
The objective function is no longer consistent with global profit maximization.

What happens if the net internal income in Finland is maximized? If we set the English evaluation coefficients both for profit and loss as equal to 0 Fmk/£, the objective function becomes:

$$\text{max } P = \text{X81} - \text{X91}.$$  

When the numerical example is re-solved applying the new objective function, the optimum model solution is as elucidated in Figures 3-11 and 3-12. The "optimum" solution is an unrealistically extreme case of repatriation adjustment. The losses in the English affiliate are extremely high. The continuation of the activities of the English affiliate would soon become impossible, and the authorities would soon step in, too. In this solution the same units of the intermediate products are first exported to the English affiliate and then imported back. The difference between the transfer prices is made as wide as possible. This is done in order to drain the resources of the English affiliate back to Finland. The English affiliate raises a local loan of £ 3 000 000 (which is the maximum potential) in order to pay for the transferred items. English production is terminated.

The basic reason for such a drastic suggestion as above lies in the fact that the model is a tactical planning model. In the above, the planning situation was deliberately detached from the more general planning framework. This point warrants further discussion:
FIGURE 3-11  \( P = X_{81} - X_{91} \): Trade and Production Flows

- **Final Product**: 10,000 units
- **Production Phase II**: 8,671 units, 900 Fmk/unit
- **Intermediate Product**: 8,671 units, £30/unit
- **Production Phase I**: 10,000 units
- **Raw Material**

Finland

England
local loans
1 091 707

Finland Fmk
payments for transfers
2 601 265 Fmk £ 260 127

currency conversion
payments for transfers
7 803 797 Fmk £ 780 380

England £
local loans
£ 300 000

fixed costs
£ 100 000

interest
£ 15 000

variable cost II
2 500 000 Fmk

fixed costs
1 000 000 Fmk

variable cost I
5 000 000 Fmk

interest
545 854 Fmk

tax
3 614 438 Fmk

867 100 Fmk transport. £ 86 710

X61 = 6 020 730 Fmk (book profit) P = 3 908 239
X81 = 3 908 239 Fmk (internal profit)

X72 = £ 950 000 (book loss)
X92 = £ 800 000 (internal loss)
In operations research applications of mathematical programming the pertinent constraints can often be classified into three categories. First, there are the *structural constraints*. The purpose of these constraints is to delineate the technical features of the planning situation. Sales constraints, capacity constraints, and the cash flow constraints are a few examples of such constraints. Second, there are the *statutory constraints*, which, in some cases, cannot be distinctly separated from the former (cf. the equations for book profit and loss). Nevertheless, for example, the liquidity requirements in banking models are a clearcut example of statutory constraints. Third, there are the *management policy constraints*. Operations research models naturally can cover only a part of the total activities of the firm in scope and time. (The same goes naturally for all models of the firm.) The factors not included in the model should nevertheless be somehow reflected in it: 1) The general management policies, applied in the particular firm, set certain outlines to the activities of the firm. The model solutions can, in some cases, be restricted accordingly by using proper management policy constraints. 2) The factors which have to be left outside the model decisions are often reflected in the mathematical programming applications by using proper ending balance requirements, as discussed earlier. 3) In some cases also the objective function can be made to reflect the general management policies.

---

45 For example the Finnish bank law requires that the cash balance has to be at least 20% of the short-term deposits plus 5% of certain longer-term liabilities.

46 Goal programming could be deemed as one instance.
Let us attempt a reformulation of the objective function of the present model. Set

\[
\max P = \frac{\text{Fprofit}}{X_{81}} - \frac{\text{Floss}}{X_{91}} - 10\frac{\text{Eloss}}{X_{92}}.
\]

As is seen in Figures 3-13 and 3-14 the optimal solution is less extreme. Repatriation adjustment is effected by exporting intermediate products to the English affiliate at a transfer price, which is made as high as possible (900 Fmk/unit).

### 3.6 Summary of Chapter 3

In this chapter I presented deterministic linear programming formulation techniques for joint planning of trade, production, and financial flows in the multinational firm, together with decisions on interaffiliate transfer pricing for external purposes. Deterministic future currency exchange rates were assumed throughout the chapter. I presented the formulation of interaffiliate transfer price constraints, and suggested the concept of shadow transfer prices, and derived a formula for calculating them. An operational manner was observed for handling taxations and constructing the objective function in linear programming models of the kind discussed. I also introduced a parallel fictitious numerical example of the idealized multinational firm, derived appropriate operating budgets, and analysed the optimal solution of the numerical example. This was done in order to demonstrate the model, and the fact that operating budgets can readily be derived from model output. The
FIGURE 3-13 $P = x_{81} - x_{91} - 10x_{92}$: Trade and Production Flows

$$
\begin{align*}
\text{production phase II} \\
\text{intermediate product} \\
\text{production phase I} \\
\text{raw material}
\end{align*}
$$

$10,000$ units

$$
\begin{align*}
\text{production phase II} \\
\text{intermediate product} \\
\text{production phase I} \\
\text{raw material}
\end{align*}
$$

$8,000$ units

$3,551$ units

$900$ Fmk/unit

$4,449$ units

Finland

England
variable cost II 2 500 000 Fmk
fixed cost 1 000 000 Fmk
variable cost I 4 065 300 Fmk
interest 609 139 Fmk
tax 2 204 303 Fmk

payments for transfers
3 195 650 Fmk

conversion

local loans
2 030 464

Finland Fmk

355 100 Fmk transportation

local loans
£ 300 000

England £

variable cost II £ 200 000
fixed cost £ 100 000
variable cost I £ 133 470
interest £ 15 000

duty

£ 31 957

X61 = £ 3 673 838 Fmk (book profit)
X81 = £ 2 969 535 Fmk (internal profit)
P = £ 2 969 535
X72 = £ 150 000 (book loss)
X82 = £ 0 (internal loss)
numerical example was discussed at length, partly because it will later (in Chapter 6) serve as an illustration of creating a hypothesis of the behavior of the multi-national firm.
4 INCORPORATION OF CURRENCY RISK, A TWO-STAGE LINEAR PROGRAMMING FORMULATION

4.1 Preface

The possibility of currency exchange rate changes as a result of floating, devaluations, or revaluations is a significant economic factor for multinational business enterprises as well as for domestic exporting and importing firms. The purpose of this chapter is to demonstrate how the economic impact of a probabilistic future change in the set of the currency exchange rates can be accounted for in linear programming models for joint planning of trade, production, and financial flows in the multinational firm, assuming that the objective is to maximize the firm's global expected net internal income.¹ This is done by incorporating currency risk in the model for the idealized multinational firm, which was presented in the previous chapter.

The expression "currency risk" is used here to indicate a planning situation where only the discrete probability estimates of the set of the future exchange rates are assumed to be available to the decision maker, i.e. the central management of the idealized multinational firm. The assumption that managements of multinational business enterprises are able to construct sound subjective probability distributions of the future exchange rates seems intuitively a realistic assertion. In the model of the previous chapter it was assumed that the future exchange rates are deterministic. This means that the

¹As has been pointed out, global profit maximization is the usual objective in the models of the kind discussed.
future currency exchange rates are known with certainty, or rather that the most likely, or some other, estimate is used as if it were certain. This is conventionally called planning under certainty. In this chapter only the relevant probability distribution is assumed to be known. This kind of a planning situation is conventionally called planning under risk. Hence the expression "currency risk".

A parallel numerical example is presented again. The exposition of the numerical example has been slightly altered in order to better demonstrate the model. The essential difference, both in the general model and in the numerical example, is that now a probabilistic change in the set of the exchange rates can take place during the planning horizon. The planning horizon is divided into two subperiods: pre- and post- the potential change in the set of the exchange rates.

The incorporation of the currency risk is achieved by applying a two-stage linear programming (often also called linear programming under uncertainty, for historical reasons) formulation. The next section reviews relevant aspects of two-stage linear programming. A reader who is very familiar with this method may omit the section without a loss of continuity.²

²For a relevant elementary introduction in the framework of a simple numerical example a non-mathematical reader is referred to Timo Salmi, Monikansallinen yritys ja lineaarinen optimointi, op.cit., pp. 154-164.
4.2 PRELIMINARIES: REVIEW OF RELEVANT TWO-STAGE LINEAR PROGRAMMING FEATURES

Consider a decision-making situation delineated by the following two-stage linear programming model

\[
\max E(z) = \sum_{j=1}^{k} E(c_j)x_j + \sum_{q=1}^{Q} \sum_{j=k+1}^{n} p_q d_{qj}y_{qj}
\]

subject to

\[
\sum_{j=1}^{k} a_{ij}x_j = b_i \quad i=1,\ldots,g \quad \text{(first stage constraints)}
\]

\[
\sum_{j=1}^{k} a_{qij}x_j + \sum_{j=k+1}^{n} h_{qij}y_{qj} = b_{qi} \quad i=g+1,\ldots,m \quad q=1,\ldots,Q
\]

all \( x_j \geq 0 \) and all \( y_{qj} \geq 0 \).

In matrix-notation this can be written as

\[
\max E(z) = E(\tilde{c}'\tilde{x} + \tilde{d}'\tilde{y})
\]

subject to

\[
A\tilde{x} + H\tilde{y} = \tilde{b}
\]

\[
\tilde{x} \geq \tilde{0}
\]

\[
\tilde{y} \geq \tilde{0}
\]

Figure 4-1 depicts the planning situation.

---

3Because the basic two-stage linear programming techniques can be considered well-established knowledge, I have omitted the original references. They can, however, be easily traced back from the references given in footnote 37 in Chapter 1 and footnote 4 in Chapter 2.
At the beginning of the first stage the values of the first stage variables \( \tilde{x} \) must be decided. This decision is made in the light of the subsequent knowledge. The constants \( a_{ij} \) (\( i=1,\ldots,g; j=1,\ldots,k \)) and \( b_i \) (\( i=1,\ldots,g \)) are known with certainty. It is assumed that there are \( Q \) possible sets \( \{d_{qj}, a_{qij}, b_{qi}\} \) with a known discrete probability distribution \( p_q = P(d_{qj}, a_{qij}, b_{qi}) \) (\( q=1,\ldots,Q \)).

The existence of these \( Q \) potential states of the world is the source of risk in the model. After the first stage variables \( \tilde{x} \) have been fixed, the true state of the world is revealed (revelation). After this the values of the proper elements of the second stage variable vector \( \tilde{y} \) are the basis of further action. (These elements are \( y_{qj} \) (\( j=k+1,\ldots,n \)), if the \( q \):th state of the world occurs.) Since the true state of the world is not known until the revelation, the economic consequences of all potential \( \tilde{y} \) values have to be reflected in the objective function. (The potential values of the second stage variables are based on the first stage decisions.) The economic consequences of the recourse actions (contingency plans) \( y_{qj} \) (\( j=k+1,\ldots,n \)) are given by

\[
(4-1) \quad \sum_{j=k+1}^{n} d_{qj} y_{qj}
\]
in the case of the occurrence of the q:th state of the world. This expression has to be weighed by the concomitant probability of occurrence, i.e. by $p_q$, because the expected value of the objective functional is to be maximized. Thus the economic consequences of the current decisions are appropriately reflected in the model.

Expression (4-1) reflects the economic consequences of the risk related to recourse actions in the model. It was assumed that only $Q$ different states of the world are accounted for in the planning situation. The second stage variables can behave in accordance only. The question naturally arises of what happens if none of the predicted states of the world occur. This is not essential, however, because in two-stage linear programming the decision maker is actually interested in the current decision, i.e. in assessing the $\hat{x}$ values. The $\hat{y}$ values are not strictly designated as decisions on true future actions. Their basic function is to reflect the economic impact of management's current decision on the objective functional in accordance with the decision makers' subjective estimates of the future.

The two-stage linear programming formulation which was presented above is called the *slack formulation of two-stage linear programming*. Another formulation, called the *fat formulation of two-stage linear programming*, is defined as follows:

$$\max E(z) = E(\hat{c}'\hat{x} + \hat{d}'\hat{y})$$
subject to

$$A\hat{x} + H\hat{y} = \hat{b}$$
$$\hat{x} \geq \hat{0}$$
$$\hat{y} \geq \hat{0}$$

where

$$A\hat{x} \leq \hat{b} \text{ holds.}$$

The fat formulation may also be restricted to concern only a selected set of the $A\hat{x} \leq \hat{b}$ constraints.
4.3 MATHEMATICAL MODEL, AND A FICTITIOUS NUMERICAL EXAMPLE

The following new miscellaneous symbols (cf. Section 3.2.1) are defined:

- \( q \) = index for the state of the world that occurs
- \( Q \) = number of possible states of the world that are accounted for in the model
- \( p(q) \) = probability of occurrence of the \( q \)th state of the world.

The decision variables are identified with the letters X and W.

4.3.1 SUBDIVIDING THE PLANNING HORIZON INTO TWO SUBPERIODS (PRE- AND POST- THE POTENTIAL CHANGE IN EXCHANGE RATES)

The planning horizon is divided into two subperiods as delineated by Figure 4-2.

FIGURE 4-2 Division of the Planning Horizon

<table>
<thead>
<tr>
<th>change in the set of the exchange rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>A10(j,i)</td>
</tr>
<tr>
<td>first subperiod</td>
</tr>
</tbody>
</table>

It is assumed that the set of the exchange rates can

*The expression "subperiod" is used instead of "stage" in order to avoid confusion.*
change only once during the planning horizon, lest the model becomes too complicated. The first subperiod corresponds to the period prior to the potential change in the exchange rates, while the second subperiod corresponds to the period after it.

Denote:

\[ A_{10}(j,i) \] = the exchange rate of currency \( j \) in country \( i \) during the first subperiod.

\[ A_{15}(j,i,q) \] = the exchange rate of currency \( j \) in country \( i \) during the second subperiod if the \( q \):th state of the world occurs.

The subdivision of the one-period planning horizon is based on the expected date of change of the currency exchange rates. The goodness of this estimate is not crucial, since only the current decision accounts. The predictions of the future events are needed only in reflecting the later economic impact of the first stage decisions. The later action can be planned anew when it is seen what really happens, but this is not of interest here. In the model we have to select the best current action on the basis of the central management's subjective estimates of the future, given in the \( A_{15}(j,i,q) \) constants and the relevant probabilities of occurrence, i.e. \( p(q) \).

The planning horizon consists of only one period subdivided into two subperiods. This selection results from the nature of the envisaged planning situation. The essence is in incorporating the impact of currency risk, and consequently a planning horizon consisting of a pre-change and a post-change segment is sufficient.

\footnote{In Section 3.2.7 we had \( A_{10}(j,i) \) = the exchange rate of currency \( j \) in country \( i \) (for the entire planning period).}
for our purposes. Furthermore, the size of the model would otherwise grow drastically.

The second subperiod currency exchange rates $A_{15}(j,i,q)$ with the relevant probabilities $p(q)$ of occurrence are the basis of the potential future states of the world (cf. Figure 4-2). Also other constants besides the currency exchange rates could be made (but are not) to differ for the different states of the world. For example, interest rates on loans typically vary in connection with devaluations and revaluations. This omission can be made without an actual loss of generality, since the principle for an inclusion is straightforward. The exclusion spares us tiresome details with no actual novelty value.

Let us consider the model for the idealized multinational firm on a more detailed level. The financial flows during the two subperiods are treated separately, since the second subperiod financial flows can be affected by the potential change in the set of the exchange rates. No difference is made between the trade and the production flows during the two subperiods. This seems intuitively quite a realistic assumption, since the physical plans of the firm often appear to be much more inflexible in the short run than the financial plans. Thus it is assumed here that trade and production plans are not immediately altered after the change in the exchange rates. (A contrary assumption could be reflected in the model.) It is important to understand that the model, nevertheless, particularly reflects the economic influence of the potential exchange rate changes, not only on the financial plans, but also on the trade and production plans to be made for the

---

6With the natural exception of the evaluation coefficients as will be seen in Section 4.3.6 ("Equations for Internal Profit and Loss, and the Objective Function").
whole planning period! - All the second stage variables (which must not be confused with the second subperiod financial flows!) in the model are related to the finance of the idealized multinational firm. The second stage variables in the model reflect future courses of action or the states of certain factors, for each potential state of the world. These factors are 1) the slacks of the cash flow constraints for each of the affiliates, 2) the book profits and losses declared for tax purposes in the different countries, and 3) the profit or loss from the firm's internal point of view in each affiliate.

The model is a tactical planning model. Again the factors which have to be left outside the model are accounted for by applying proper ending balance requirements, such as the minimum cash balance requirements. Now that the planning period has been subdivided it is possible to set requirements both at the end of the first subperiod and at the end of the second subperiod. In the model we set only minimum closing cash requirements at the end of the second subperiod. (In the next chapter currency exposure requirements are set at the end of the first and the second subperiod.)

In the numerical example it is assumed that the central management expects a devaluation of the pound (or a revaluation of the mark). Figures 4-3 and 4-4 illustrate the fictitious decision-making situation.

FIGURE 4-3 Central Management's Subjective Probability Distribution of the Future Exchange Rates

[Diagram showing probability distribution for the exchange rate of the pound in Finland Fmk/£]
During the first subperiod the exchange rate of the pound is 10 Fmk/£, i.e. 0.1 £/Fmk. The subjective estimate of the central management is that for the second subperiod the exchange rate of the pound will remain at 10 Fmk/£ (0.1 £/Fmk) with probability 0.7, and will change to 8 Fmk/£ (0.125 £/Fmk) with probability 0.3. The future exchange rate of the pound is in the numerical example the future state of the world not yet known at the beginning of the planning horizon, when the optimal plans are sought.

4.3.2 CONSTRAINTS REMAINING UNALTERED IN PRINCIPLE  
(but not necessarily in the numerical example)

Sales/capacity constraints (Section 3.2.2), supply/capacity constraints (3.2.3), technical availability constraints (3.2.4), and local loan raising constraints (3.2.6) remain unchanged. All these constraints are first stage constraints.

In the numerical example the following alterations

As before, no difference is assumed between sales and purchase rates of the currencies, for simplicity.
are made. In Finland the sales potential of the final product is 25,000 units and in England it is 5,000 units (in the prior numerical example the respective predicted demands were 10,000 units and 10,000 units). The tax rate in Finland is 45% and the respective corporate tax rate in England is 45% (in the prior example 60% and 45%). Except for the changes in the currency exchange rates and the evaluation coefficients no other factual alterations are made in the data of the numerical example. The new sales/capacity constraints are

\[
\begin{align*}
X_{11} &\leq 25,000 \\
X_{12} &\leq 5,000.
\end{align*}
\]

4.3.3 DECISIONS ON EXTERNAL TRANSFER PRICING WITH THE POSSIBILITY OF DELAYING OR NOT DELAYING THE INTER-AFFILIATE PAYMENTS: TRANSFER PRICE CONSTRAINTS

Denote:

\[ X_3(i,j) = \text{the transfer value of all those units of the intermediate product which are transferred from affiliate } i \text{ to affiliate } j \text{ for which the payment is effected during the first subperiod. It is denominated in the currency of country } i. \text{ Call this the cash-based exports of affiliate } i \text{ to affiliate } j. \]

\[ W_3(i,j) = \text{the transfer value of all those units of the intermediate product which are transferred from affiliate } i \text{ to affiliate } j \text{ for which the payment is effected during the second subperiod. It is denominated in the currency of country } i. \text{ Call this the credit-based exports of affiliate } i \text{ to affiliate } j. \]
Y_l(i,j) = the respective transfer price.
\[ Y_l(i,j) = \frac{X_2(i,j) + W_3(i,j)}{X_3(i,j)} \text{ if } X_2(i,j) > 0. \]
We say that Y_l(i,j) does not exist if X_2(i,j) = 0.

The rest of the symbols remain unchanged (cf. Section 3.2.5).

We have:

Lower bounds
\[ A_1(i,j)X_2(i,j) - X_3(i,j) - W_3(i,j) \leq 0 \quad i=1,...,nc \]

Upper bounds
\[ -A_2(i,j)X_2(i,j) + X_3(i,j) + W_3(i,j) \leq 0 \quad i\neq j. \]

The transfer prices are thus kept between the stated bounds A_1(i,j) and A_2(i,j).

Consider the numerical example. Figure 4-5 elucidates a potential transfer of intermediate products from the English affiliate to the Finnish one.

**FIGURE 4-5** The Mechanism of Interaffiliate Payments for a Transfer

<table>
<thead>
<tr>
<th>Finnish affiliate</th>
<th>transferred items</th>
<th>English affiliate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fmk</td>
<td>X_21</td>
<td>£</td>
</tr>
<tr>
<td></td>
<td>X_221</td>
<td>£X_321</td>
</tr>
<tr>
<td></td>
<td>first subp. payments</td>
<td>Fmk10X321</td>
</tr>
<tr>
<td></td>
<td></td>
<td>£X321</td>
</tr>
<tr>
<td></td>
<td>second subp. payments</td>
<td>Fmk10W321 q=1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>£W321</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fmk 8W321 q=2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>£W321</td>
</tr>
</tbody>
</table>

The payments for the intermediate product transfers can be effected either during the first subperiod or the second subperiod, or partially during both. Thus the decision on delaying the interaffiliate payments,
or not, is included in the model. In two-stage linear programming formulation the number of units to be transferred is a first stage variable $X_{221}$. The payment effected during the first subperiod by the Finnish affiliate for the imported intermediate products is $X_{321}$. It is denominated in pounds and it is a first stage variable. (The Finnish affiliate purchases the pounds needed with marks during the first subperiod.)

The payment during the second subperiod, $W_{321}$, is also denominated in pounds, and it is a first stage variable, too, even if it relates to the second subperiod. This results from the fact that the value of $W_{321}$ must be decided before the future currency exchange rate of the pound becomes known, since the transfer value of the transferred items, i.e. $X_{321} + W_{321}$ must be documented, and declared to the customs officials when the items are transferred.

The total value of the intermediate products transferred from England to Finland is $X_{321} + W_{321}$ pounds. Thus the transfer price of one pertinent unit is $(X_{321} + W_{321})/X_{221}$ pounds. As before, the predetermined transfer price bounds for the relevant items are £ 30 and £ 100. Thus it is required that

$$30 \leq (X_{321} + W_{321})/X_{221} \leq 100.$$  

These nonlinear constraints can be converted into linear transfer price constraints. Thus the transfer price constraints for the items transferred from England to Finland are

$$30X_{221} - X_{321} - W_{321} \leq 0 \quad \text{and} \quad -100X_{221} + X_{321} + W_{321} \leq 0.$$  

(lower bound)  

(upper bound)
The transfer price of the intermediate products transferred in the opposite direction, i.e. from the Finnish affiliate to the English one, is again limited between 400 Fmk/unit and 900 Fmk/unit:

\[
\begin{align*}
400X_{12} - X_{312} - W_{312} & \leq 0 \quad \text{and} \\
-900X_{12} + X_{312} + W_{312} & \leq 0.
\end{align*}
\]

(lower bound)

(upper bound)

The transfer price constraints are first stage constraints, since their content makes no difference between the different states of the world.

4.3.4 CASH FLOW CONSTRAINTS

Denote:

\[ X_{6(i,q)} = \text{the profit before taxes shown in books by affiliate } i \text{ for tax assessment in country } i, \text{ if the } q:th \text{ state of the world occurs.} \]

We have:

\[
\begin{align*}
\text{cash sales} & \text{ cash variable prod. costs} & \text{nc transportation} \\
& [-A3(i)+A5(i)]X_{1(i)} + A7(i)X_{2(i,i)} + \sum_{j=1}^{\text{nc}} A9(i,j)X_{2(i,j)} \\
& \text{nc exports} \quad \text{nc exports} \quad \text{nc imports + duty} \\
- \sum_{j=1}^{\text{first subp.}} X_{3(i,j)} - \sum_{j=1}^{\text{s. subp.}} W_{3(i,j)} + \sum_{j=1}^{\text{f. subp.}} [A10(j,i)+A11(j,i)]X_{3(j,i)} \\
& \text{nc imports + duty}
\end{align*}
\]

\[
\begin{align*}
& - \sum_{j=1}^{\text{payment}} \text{f. subp.} + \sum_{j=1}^{\text{payment}} \text{s. subp.} + \text{f. exchange rate payment} \\
+ \sum_{j=1}^{\text{second subperiod}} [A15(j,i,q)+A11(j,i)]W_{3(j,i)} - [1-A12(i)]X_{4(i,i)} \\
& \text{tax} \quad \text{initial cash...} \\
+ A13(i)X_{6(i,q)} & \leq B_{4(i)} \quad i=1,...,\text{nc} \quad q=1,...,Q.
\end{align*}
\]

\footnote{In Section 3.2.7 we had \( X_{6(i)} = \text{the profit before taxes shown in books by affiliate } i \text{ for tax assessment in country } i. \) The rest of the symbols remain as in Chapter 3 and are not repeated.}
Just as for the deterministic model presented in Chapter 3, it must be required that

\[
\text{initial cash } + \text{ cash inflows} \geq \text{ cash outflows} + \text{ minimum closing cash}.
\]

In the deterministic version we noted that each affiliate requires a separate cash flow constraint for the planning period, compared with the single cash flow constraint needed in a one-period uninational case. In the current stochastic model we must, in addition, construct a separate constraint for each potential state of the world included into consideration. Thus \(nc \cdot Q\) separate cash flow constraints have to be constructed now. This results from the fact that cash flows are different for each affiliate and each different state of the world. The interaffiliate payments during the second subperiod for imported items are affected. The constants \(A15(j,i,q)\) are, in fact, distinct for the different states of the world. Furthermore, tax payments depend on the occurring state of the world, and thus the resultant cash outlays are distinct. In the two-stage linear programming formulation the variables \(X6(i,q)\) are second stage variables.

Note that the formulation does not allow a violation of cash flow constraints. The cash flows have been constructed in accordance with the fat formulation of two-stage linear programming. The first stage variables must be chosen in a way that preserves feasibility in every state of the world.\(^9\) It would be very easy to make a contrary assumption, and define "distress loans",

\(^9\)\(X6(i,q)\) is a function of the first stage variables, and thus the statement is legitimate, although at first sight it may appear incomplete.
which would be utilized if cash becomes too short. This would be in accordance with the slack formulation of two-stage linear programming. Neither of the formulations is preferred here; the selection made is arbitrary. The minimum closing cash requirements are kept the same for each state of the world. If deemed fit, we could define $B_4(i,q)$ instead of $B_4(i)$.

Consider the numerical example. We discuss only the components of cash flows, which differ from the numerical example presented in connection with the deterministic model version. Two cash flow constraints are needed for both affiliates in the numerical example. Figure 4-5 elucidated the interaffiliate payments for items transferred from the English affiliate to the Finnish one. The future probabilistic second subperiod exchange rate of the pound affects the amount of marks which the Finnish affiliate has to pay to its bank during the second subperiod in order to buy the pounds it needs for paying the credit-based imports from the English affiliate (W321 pounds). In the Finnish cash flow constraint, which is constructed for the first state of the world accounted for (10 Fmk/$), a cash outflow of 10W321 results from the credit-based imports. In the Finnish cash flow constraint for the second state of the world (8 Fmk/$) a cash outflow of 8W321 Fmk results from the credit-based imports. A cash inflow of W321 pounds in both the English cash flow constraints results from the credit-based exports of the English affiliate. The transfers in the opposite direction cause second subperiod cash inflows and outflows in Finland and England respectively. These components can be seen in the cash flow constraints to be given shortly.

The cash outlays to duties are unaffected by the potential devaluation of the pound, since it is assumed
that the duty is paid during the first subperiod on
the basis of the invoiced total value of the items
to be transferred \((X312 + W312 \text{ or } X321 + W321)\).
Again, contrary assumptions would incur no difficulties.

The book profit shown by the Finnish affiliate, if
the first state of the world occurs, is denoted by \(X611\).
\(X612\) is the respective figure in the case of the occurrence
of the second state of the world. For the English affiliate
the symbols \(X621\) and \(X622\) are used respectively. All
these variables are second stage variables, since they
indicate the recourse action or the state of a factor
after the revelation of the true state of the world.

The minimum closing cash balance in the Finnish
affiliate at the end of the planning horizon is
1 000 000 Fmk for both potential states of the world.
No deficit in cash is allowed in this model version.
As mentioned, we could easily use some other formulation.
For the English affiliate the minimum closing cash balance
is £100 000 for both potential states of the world.

The cash flow constraint for the Finnish affiliate,
concerning the first state of the world, is

\[
\begin{align*}
\text{initial cash} & \quad \text{exports} \quad \text{loan} \quad \text{v.c. II} \\
6 \, 000 \, 000 & \quad + \quad X312 \quad + \quad W312 \quad + \quad X411 \quad \geq \quad 250X11 \\
+ \quad 300X211 & \quad + \quad 100X212 \quad + \quad 10X321 \quad + \quad 10W321 \\
\text{duty} & \quad \text{interest} \quad \text{tax} \\
+ \quad 0.1(10X321 \quad + \quad 10W321) & \quad + \quad 0.05X411 \quad + \quad 0.45X611 \\
\text{fixed cost} & \quad \text{min. cash balance} \\
+ \quad 1 \, 000 \, 000 & \quad + \quad 1 \, 000 \, 000. \\
\end{align*}
\]

The cash flow constraint for the Finnish affiliate,
concerning the second state of the world, is
\[ 6000000 + X312 + W312 + X411 \geq 250X11 \]
\[ + 300X211 + 100X212 + 10X321 + 8W321 \]
\[ + 0.1(10X321+10W321) + 0.05X411 + 0.45X612 \]
\[ + 1000000 + 1000000. \]

After a reformulation the two cash flow constraints for the Finnish affiliate become

\[ 250X11 + 300X211 + 100X212 - X312 - W312 + 11X321 \]
\[ + 11W321 - 0.95X411 + 0.45X611 \leq 4000000 \text{ (Fmk)}, \quad q = 1 \]

and

\[ 250X11 + 300X211 + 100X212 - X312 - W312 + 11X321 \]
\[ + 9W321 - 0.95X411 + 0.45X612 \leq 4000000 \text{ (Fmk)}, \quad q = 2 \]

The cash flow constraints for the English affiliate can be derived in a similar manner. They are

\[ 25X12 + 10X221 + 30X222 + 0.11X312 + 0.11W312 \]
\[ - X321 - W321 - 0.95X422 + 0.45X621 \leq 400000 \text{ (£)}, \quad q = 1 \]

and

\[ 25X12 + 10X221 + 30X222 + 0.11X312 + 0.135W312 \]
\[ - X321 - W321 - 0.95X422 + 0.45X622 \leq 400000 \text{ (£)}, \quad q = 2 \]

The cash flow constraints are second stage constraints.
4.3.5 EQUATIONS FOR BOOK PROFIT AND LOSS

Denote:

\[ X_7(i,q) = \text{the loss shown in the books by affiliate} \]
\[ \text{i for tax assessment in country i, if} \]
\[ \text{the q:th state of the world occurs.} \]

We have:

\[
\begin{align*}
\text{sales} & - \text{v.c. II} & \text{v.c. I} & \text{nc transportation} \\
A_14(i)X_1(i) & - \left[ A_7(i)+A_8(i) \right]X_2(i,i) & & \sum_{j=1}^{j\neq i} A_9(i,j)X_2(i,j) \\
& & & \text{nc exports} \\
& & \sum_{j=1}^{j\neq i} X_3(i,j) & \sum_{j=1}^{j\neq i} W_3(i,j) & \sum_{j=1}^{j\neq i} A_{10}(j,i)X_3(j,i) \\
& & & \text{nc exports} \\
& & & \text{nc imports} \\
& & & \text{payment} \\
& & & \text{f. subp.} \\
- & \sum_{j=1}^{j\neq i} A_{15}(j,i,q)W_3(j,i) & \sum_{j=1}^{j\neq i} A_{11}(j,i)[X_3(j,i)+W_3(j,i)] \\
& & & \text{interest} \\
& & & \text{book profit} \\
& & & \text{book loss} \\
& & & \text{+ depreciation} \\
A_12(i)X_4(i,i) & - X_6(i,q) & + X_7(i,q) = B_5(i) \\
i=1,\ldots,\text{nc} & q=1,\ldots,Q.
\end{align*}
\]

The equations for book profit and loss define for each affiliate and state of the world the profit or loss to be shown in the books for tax authorities in country i. The statements of the tax-laws are as before. In addition, profits and losses due to exchange rate changes are subject to corporate income taxes/deductible in taxations.

Consider the numerical example. Denote

\[ X_{611} = \text{the book profit shown in Finland if the first state of the world occurs.} \]
\[ X_{612} = \text{the book profit shown in Finland if the second state of the world occurs.} \]
\[ \vdots \]
\[ X_{722} = \text{the book loss shown in England if the second state of the world occurs.} \]
The above variables are second stage variables. For the Finnish affiliate we construct the following equations.

\[ X_{611} - X_{711} = \text{the revenues subject to corporate income tax in Finland when the first state of the world occurs} \]
\[ \text{less} \]
\[ \text{the deductible costs in Finland when the first state of the world occurs} \]

and

\[ X_{612} - X_{712} = \text{the respective factors in Finland when the second state of the world occurs} \]

For the English affiliate similar equations can be formed. The equations for book profit and loss are not explicitly given here for the numerical example, since they closely resemble the cash flow constraints. The equations for book profit and loss can be considered second stage constraints.

The usefulness of applying auxiliary equations, which was discussed in the previous chapter, is emphasized in the two-stage formulation, because one is spared a prohibitive amount of calculation. On the other hand the adverse impact on the size of the model must be noted.

4.3.6 EQUATIONS FOR INTERNAL PROFIT AND LOSS, AND THE OBJECTIVE FUNCTION

Denote:

---

Chapter 5 presents the model of the numerical example completely after the inclusion of interaffiliate loans and their interest rates, forward contracting decisions, (and currency position requirements).
\( X8(i,q) = \) the profit after taxes in country \( i \) in the currency of country \( i \) from the firm's internal point of view, if the \( q:th \) state of the world occurs.

\( X9(i,q) = \) the loss after taxes in country \( i \) in the currency of country \( i \) from the firm's internal point of view, if the \( q:th \) state of the world occurs.

\( C1(i,q) = \) the evaluation coefficient on internal profits realized in affiliate \( i \), set by the central management for the \( q:th \) state of the world.

\( C2(i,q) = \) the evaluation coefficient on internal losses realized in affiliate \( i \), set by the central management for the \( q:th \) state of the world.

We have:

\[
\begin{align*}
\text{sales} - \text{v.c. II} &= \text{v.c. I} - \sum_{j=1}^{\text{nc}} (A9(i,j)X2(i,j) + A14(i)X1(i) - [A7(i) + A8(i)]X2(i,i)) \\
&+ \sum_{j=1}^{\text{nc}} A9(i,j)X2(i,j)) + \sum_{j=1}^{\text{nc}} (A10(j,i)X3(j,i)) - \sum_{j=1}^{\text{nc}} A9(i,j)X2(i,j)) - \sum_{j=1}^{\text{nc}} A10(j,i)X3(j,i)
\end{align*}
\]

\[
\begin{align*}
\text{nc exports} &= \sum_{j=1}^{\text{nc}} X3(i,j) + \sum_{j=1}^{\text{nc}} W3(i,j) - \sum_{j=1}^{\text{nc}} A10(j,i)X3(j,i) \\
\text{nc imports} &= \sum_{j=1}^{\text{nc}} A15(j,i,q)W3(j,i) - \sum_{j=1}^{\text{nc}} A11(j,i)[X3(j,i) + W3(j,i)] \\
\text{interest} &= A12(i)X4(i,i) - A13(i)X6(i,q) - X8(i,q) - X9(i,q) \\
\text{payment} &= B6(i)
\end{align*}
\]

and

\[
\max E(P) = \sum_{q=1}^{Q} \sum_{i=1}^{\text{nc}} p(q) [C1(i,q)X8(i,q) - C2(i,q)X9(i,q)].
\]
The objective of the model is to maximize the *global expected profit* (net internal income) for the idealized multinational firm over the planning horizon. This objective is consistent with the precedents. The use of this kind of objective has been defended for example by the following arguments. 1) "In recent years, scholars of decision-theory have devised several ways of justifying the use of the expected value as the sole criterion for an optimal choice." 2) Linearity is otherwise endangered, and more is lost because of this fact than gained by an allegedly more realistic objective function. 3) Geocentric operating philosophy is an operational basis for quantitative planning in multinational business enterprises. The question of the objective of the firm could naturally be, once more, discussed at great length also in this dissertation. This is omitted, since nothing new would be gained here by a further discussion.

The profits and losses for the different affiliates are given in the relevant host country currencies. Consequently they are converted into a common unit in the calculations. When a geocentric operating philosophy and a linear utility function are assumed, we can consider setting \( C_1(i,q) = C_2(i,q) = A_1S(i,1,q) \), because \( A_1S(i,1,q) \) gives the relative values of the profits and losses denominated in the different currencies. (This is naturally not the only possibility.) Thus we have defined the global net internal income for each state of the world in a common unit (host country currency). Finally, these incomes must be weighed by the relevant probabilities of occurrence.

---

Consider the numerical example again. The auxiliary equations to define internal profit and loss in the host country currency must be constructed separately for the Finnish and the English affiliates, distinctly for the first and the second state of the world. Denote

- \( X_{811} = \) the internal profit in the Finnish affiliate in marks if the first state of the world occurs.
- \( X_{812} = \) the internal profit in the Finnish affiliate in marks if the second state of the world occurs.
- \( \vdots \)
- \( X_{922} = \) the internal loss in the English affiliate in pounds if the second state of the world occurs.

The above variables are second stage variables. For the Finnish affiliate we construct the following equations.

\[
X_{811} - X_{911} = \text{internal revenues in Finland when the first state of the world occurs less internal costs in Finland when the first state of the world occurs}
\]

and

\[
X_{812} - X_{912} = \text{the respective factors in Finland when the second state of the world occurs.}
\]

For the English affiliate similar auxiliary equations can be written. The equations for internal profit and loss are not presented explicitly in the numerical example here, since the principle should be clear by now. The equations for internal profit and loss can be considered second stage constraints.

The net internal income for the English affiliate is denominated in pounds for both states of the world. It is
converted into marks in calculation by applying the proper evaluation coefficients set by the central management. Finally, the net internal incomes, thus denominated in marks for both affiliates, are weighed by the appropriate probabilities of occurrence:

\[ q = 1 \implies p(q) = 0.7, \quad \text{and} \quad q = 2 \implies p(q) = 0.3. \]

If 10 Fmk/£ is used as the evaluation coefficient for \( q = 1 \) and 8 Fmk/£ for \( q = 2 \), the objective function is

\[
\max E(P) = 0.7 \times 811 + 0.3 \times 812 + 0.7 \times 10 \times 821 + 0.3 \times 8 \times 822 \\
- 0.7 \times 911 - 0.3 \times 912 - 0.7 \times 10 \times 921 - 0.3 \times 8 \times 922,
\]

i.e.

\[
\begin{align*}
\max E(P) &= 0.7 \times 811 + 0.3 \times 812 + 7 \times 821 + 2.4 \times 822 \\
&- 0.7 \times 911 - 0.3 \times 912 - 7 \times 921 - 2.4 \times 922.
\end{align*}
\]

4.4 Solution and Optimal Operating Budgets in the Numerical Example

After running the two-stage linear programming problem of the numerical example on a computer, the optimal solution can be elucidated with the help of Figures 4-6 and 4-7. Recall that \( q = 1 \) denotes the revelation of the first state of the world, meaning that the second subperiod exchange rate of the pound is 10 Fmk/£. \( q = 2 \) denotes the revelation of the second state of the world, meaning that the second subperiod exchange rate of the pound is 8 Fmk/£.

On the basis of the optimal solution of the model it is possible to write, for example, the cash budgets, the predicted income statements for tax assessment,
FIGURE 4-6  Trade and Production Flows in the Numerical Example Including Currency Risk

Finland

England
FIGURE 4.7
Financial Flows in the Numerical Example
Including Currency Risk

local loans
3 000 000 Fmk

sales revenue
(not in cash)
10 444 000 Fmk

Finland Fmk

variable cost II
2 610 000 Fmk

fixed cost
1 000 000 Fmk

variable cost I
3 008 100 Fmk

interest
150 000 Fmk

tax
q = 1 772 740 Fmk
q = 2 810 270 Fmk

duty

payments for transfers £ 41 700
q = 1 417 000 Fmk
q = 2 333 600 Fmk

England £

variable cost II
£ 125 000

fixed cost
£ 100 000

variable cost I
£ 162 510

transportation
£ 4170
and the income statements from the firm's internal point of view. In addition to indicating a separate set of budgets for each affiliate, the model indicates a separate set of budgets for each state of the world accounted for. In actual practice the best procedure would possibly be the application of a budgetary process based on flexible budgets.

Cash budgets

FINNISH AFFILIATE

\[ q = 1 \quad (10 \text{Fmk/£}) \quad q = 2 \quad (8 \text{Fmk/£}) \]

<table>
<thead>
<tr>
<th>Description</th>
<th>Initial cash</th>
<th>Loans</th>
<th>Variable cost II</th>
<th>Variable cost I</th>
<th>Imports</th>
<th>Duty</th>
<th>Fixed cost</th>
<th>Transportation cost</th>
<th>Fixed cost</th>
<th>Closing cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+ 6 000 000 Fmk</td>
<td>+ 3 000 000 Fmk</td>
<td>- 2 610 000 Fmk</td>
<td>- 3 008 100 Fmk</td>
<td>- 417 000 Fmk</td>
<td>- 1 000 000 Fmk</td>
<td>- 150 000 Fmk</td>
<td>- 772 740 Fmk</td>
<td>- 150 000 Fmk</td>
<td>1 000 460 Fmk</td>
</tr>
<tr>
<td></td>
<td>6 000 000 Fmk</td>
<td>3 000 000 Fmk</td>
<td>2 610 000 Fmk</td>
<td>3 008 100 Fmk</td>
<td>333 600 Fmk</td>
<td>1 000 000 Fmk</td>
<td>150 000 Fmk</td>
<td>810 270 Fmk</td>
<td>1 046 330 Fmk</td>
<td></td>
</tr>
</tbody>
</table>

ENGLISH AFFILIATE

<table>
<thead>
<tr>
<th>Description</th>
<th>Initial cash</th>
<th>Exports</th>
<th>Variable cost II</th>
<th>Variable cost I</th>
<th>Transportation cost</th>
<th>Fixed cost</th>
<th>Closing cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+ 600 000 £</td>
<td>+ 41 700 £</td>
<td>- 125 000 £</td>
<td>- 162 510 £</td>
<td>- 4 170 £</td>
<td>- 100 000 £</td>
<td>250 020 £</td>
</tr>
<tr>
<td></td>
<td>600 000 £</td>
<td>41 700 £</td>
<td>125 000 £</td>
<td>162 510 £</td>
<td>4 170 £</td>
<td>100 000 £</td>
<td>250 020 £</td>
</tr>
</tbody>
</table>

Predicted income statements for taxations

FINNISH AFFILIATE

<table>
<thead>
<tr>
<th>Description</th>
<th>q = 1</th>
<th>q = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic sales</td>
<td>+10 444 000 Fmk</td>
<td>10 444 000 Fmk</td>
</tr>
<tr>
<td>Variable cost II</td>
<td>- 2 610 000 Fmk</td>
<td>2 610 000 Fmk</td>
</tr>
<tr>
<td>Variable cost I</td>
<td>- 3 008 100 Fmk</td>
<td>3 008 100 Fmk</td>
</tr>
<tr>
<td>Imports</td>
<td>- 417 000 Fmk</td>
<td>333 600 Fmk</td>
</tr>
<tr>
<td>Duty</td>
<td>- 41 700 Fmk</td>
<td>41 700 Fmk</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>- 1 000 000 Fmk</td>
<td>1 000 000 Fmk</td>
</tr>
<tr>
<td>Interest</td>
<td>- 150 000 Fmk</td>
<td>150 000 Fmk</td>
</tr>
<tr>
<td>Depreciation</td>
<td>- 1 500 000 Fmk</td>
<td>1 500 000 Fmk</td>
</tr>
<tr>
<td>Net income subject to taxation</td>
<td>1 717 200 Fmk</td>
<td>1 800 600 Fmk</td>
</tr>
</tbody>
</table>
ENGLISH AFFILIATE

<table>
<thead>
<tr>
<th>Item</th>
<th>Domestic sales</th>
<th>Exports</th>
<th>Variable cost II</th>
<th>Variable cost I</th>
<th>Transportation cost</th>
<th>Fixed cost</th>
<th>Depreciation</th>
<th>Net income subject to taxation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+ 500 000 £</td>
<td>+ 41 700 £</td>
<td>- 125 000 £</td>
<td>- 162 510 £</td>
<td>- 4 170 £</td>
<td>- 100 000 £</td>
<td>- 150 000 £</td>
<td>(20 £)</td>
</tr>
</tbody>
</table>

FINNISH AFFILIATE

<table>
<thead>
<tr>
<th>Item</th>
<th>Domestic sales</th>
<th>Exports</th>
<th>Variable cost II</th>
<th>Variable cost I</th>
<th>Imports</th>
<th>Duty</th>
<th>Fixed cost</th>
<th>Interest</th>
<th>Tax</th>
<th>Net internal income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+10 444 000 Fmk</td>
<td></td>
<td>- 2 610 000 Fmk</td>
<td>- 3 008 100 Fmk</td>
<td>- 4 170 Fmk</td>
<td>- 1 000 000 Fmk</td>
<td>- 150 000 Fmk</td>
<td>- 772 740 Fmk</td>
<td>2 444 460 Fmk</td>
<td></td>
</tr>
</tbody>
</table>

ENGLISH AFFILIATE

<table>
<thead>
<tr>
<th>Item</th>
<th>Domestic sales</th>
<th>Exports</th>
<th>Variable cost II</th>
<th>Variable cost I</th>
<th>Transportation cost</th>
<th>Fixed cost</th>
<th>Depreciation</th>
<th>Net internal income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+ 500 000 £</td>
<td>+ 41 700 £</td>
<td>- 125 000 £</td>
<td>- 162 510 £</td>
<td>- 4 170 £</td>
<td>- 100 000 £</td>
<td>- 150 000 £</td>
<td>150 020 £</td>
</tr>
</tbody>
</table>

**Expected global profit**

\[
E(P) = 0.7 \times 2 444 460 + 0.3 \times 2 490 330 \\
+ 0.7 \times 150 020 + 0.3 \times 150 020 = 3 868 409 \text{ (Fmk)}.
\]

4.5 Summary of Chapter 4

In this chapter I presented incorporation of currency risk in linear programming models for joint planning of trade, production, and financial flows. The planning
period was first divided into two subperiods. For the second subperiod only the discrete probability distribution of the currency exchange rates was assumed to be known. This probabilistic set of currency exchange rates constituted the unknown future state of the world. Two-stage linear programming techniques were applied for a maximization of the global expected profit. I introduced a parallel fictitious numerical example of the idealized multinational firm, and derived appropriate operating budgets for each affiliate and potential state of the world. The discussion in general was made rather brief in comparison with Chapter 3, because the points which could already be presented there were not lingered over any more.
5 EXTENSION OF THE MODEL UNDER CURRENCY RISK

5.1 Preface

Hedging against (and speculating with) currency exchange rate changes is a significant problem in multinational business. The question of selecting proper means of hedging and speculating arises in multinational business enterprises as well as in national firms with export and/or import operations. In this dissertation we are interested in the multinational firm. If a change in the currency exchange rates seems likely in future, a plan of action should be made. The most usual means of hedging and speculation are 1) forward contracting, and 2) raising loans and investing in short-term financial options denominated in relevant currencies.

A forward contract is a binding agreement, between the customer and a bank, on a purchase or a sale of an agreed amount of foreign currency on a future date\(^1\) at an exchange rate which is already agreed on when the contract is written ("now"). If the contract is made, the future payment for the foreign currency is independent of the true exchange rate (spot rate) on "the future date", when the payment is effected. Forward contracting is most commonly used to determine in advance the currency exchange rate for a specific future date on which a financial transaction involving a foreign currency will take place (a payment for imports, etc.).\(^2\)

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\(^1\)Most usually the future date is after 30, 60, 90 or 180 days, but sometimes (though rarely) even after two years.

\(^2\)For the Finnish usage see e.g. Timo Salmi, *Monikansallinen yritys ja lineaarinen optimointi*, op.cit., pp. 147-149.
Forward contracting can in some countries on occasions also be used for pure speculation without any relation to a payment pending in a foreign currency. Forward contracting naturally incurs costs which must be weighed against the estimates of the future spot rates.  

Raising loans and investing in short-term financial options in different currencies is most commonly used in order to manage the net currency exposure of a suspect currency. Net currency exposure means the difference between current assets and current liabilities denominated in the relevant currency. For example, the economic consequences of a devaluation to a firm depend on the firm's current assets and current liabilities denominated in the currency being devalued.

Conventionally the hedging and speculation operations have been treated per se in literature, or in connection with only partial models for the multinational firm. However, if optimal total plans are sought this is inadequate. The hedging and speculation operations discussed should be considered in conjunction with both physical and financial operations of the multinational firm (and the merely exporting and/or importing national firm, but as stated we discuss the multinational firm in this dissertation).

The purpose of this chapter is to demonstrate how forward contracting can be planned in conjunction with trade, production, and financial planning in the multinational firm. This chapter also discusses how currency

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4This question has been discussed especially by Bernard Lietaer (cf. footnote 23 in Chapter 1).

5See Section 1.3 for the definition.
exposure of current assets and liabilities of the multinational firm can be adjusted to fit into predetermined bounds in conjunction with assessing the physical and financial plans of the multinational firm. These features are introduced by incorporating the appropriate decisions in the model for the idealized multinational firm. The currency exposure is accounted for only via proper constraints, because not enough is known to proceed differently in a manageable manner. To begin with, however, decisions on interaffiliate loans and their interest rates are incorporated in the model for the idealized multinational firm.

Note that Chapter 4 introduced the impact of currency risk, i.e. the probabilistic future exchange rates, on planning trade, production, and financial flows in the multinational firm. The main theme of this chapter is the further introducing of hedging (and speculation) against the probabilistic future currency exchange rates. This is done in the model, which already accounts for the currency risk in planning the trade, production, and financial flows.

A parallel numerical example is presented again. The numerical example is the same as in Chapter 4, with relevant extensions in it. In the numerical example only the new input data are presented while introducing the new model features. The detailed derivation of the model in the numerical example is omitted. It can be found elsewhere.\(^6\) The model in the numerical example is given in its final form towards the end of the chapter.

\(^6\)Timo Salmi, *Monikansallinen yritys ja lineaarinen optimointi*, op.cit., Ch. 6.
5.2 Inclusion of Interaffiliate Loans and Their Interest Rates as Decision Variables

Denote:

\[ X_4(i,j) = \text{the loan granted by affiliate } i \text{ to affiliate } j \text{ in the currency of country } i. \]  
(The loan is raised during the first subperiod, and repaid after the planning horizon.)

\[ X_5(i,j) = \text{the interest on the loan } X_4(i,j), \text{ which is paid by the affiliate } j \text{ to affiliate } i \text{ during the first subperiod in the currency of country } i. \]

\[ W_5(i,j) = \text{the interest on the loan } X_4(i,j), \text{ which is paid by the affiliate } j \text{ to affiliate } i \text{ during the second subperiod in the currency of country } i. \]

\[ A_{16}(i,j) = \text{the lower bound on the absolute interest rate on the loan } X_4(i,j). \]
\[ A_{16}(1,2) = 0.03 \text{ (3\%)} \]
\[ A_{16}(2,1) = 0.03 \text{ (3\%)} \]

\[ A_{17}(i,j) = \text{the upper bound on the absolute interest rate on the loan } X_4(i,j). \]
\[ A_{17}(1,2) = 0.08 \text{ (8\%)} \]
\[ A_{17}(2,1) = 0.08 \text{ (8\%)} \]

\[ B_7(i) = \text{the maximum allowed amount of interaffiliate loans that can be granted by affiliate } i. \]

It is stated in the currency of country \( i \).
\[ B_7(1) = 1000000 \text{ Fmk} \]
\[ B_7(2) = \£ 100000 \]

\[ B_8(i) = \text{the maximum amount of interaffiliate loans that affiliate } i \text{ is allowed to receive. It is stated in the currency of country } i. \]

The interest rates on interaffiliate loans are thus decision variables in the model. The payment of the

\footnote{The numerical example is given piece by piece after each relevant constant.}
interest can be effected during the first subperiod or the second subperiod, or partially during both. Thus the decision on delaying the payment of the inter-affiliate interest, or not, is included in the model. In the two-stage linear programming formulation the variables $X_4(i,j)$, $X_5(i,j)$, and $W_5(i,j)$ are all first stage variables, since their value must be assessed prior to the revelation of the true future state of the world.

**Interaffiliate Interest Rate Constraints**

The absolute interest rate on an interaffiliate loan must be kept between the stated limits $A_{16}(i,j)$ and $A_{17}(i,j)$. This kind of limitation can arise either from statutory reasons or may result from the management policies, as was discussed in Section 3.5.4, where the constraints of mathematical programming models for managerial applications were classified into structural, statutory, and management policy constraints. The absolute rate of interest on the loan granted by affiliate $i$ to affiliate $j$ is $[X_5(i,j) + W_5(i,j)]/X_4(i,j)$ when $X_4(i,j) > 0$. Otherwise we say that it does not exist. The following inequalities must hold:

$$A_{16}(i,j) \leq [X_5(i,j) + W_5(i,j)]/X_4(i,j) \leq A_{17}(i,j).$$

They are equivalent to

\[
\begin{align*}
\text{Lower bounds} & \quad A_{16}(i,j)X_4(i,j) - X_5(i,j) - W_5(i,j) & \leq 0 & \quad i=1,\ldots,nc \\
\text{Upper bounds} & \quad -A_{17}(i,j)X_4(i,j) + X_5(i,j) + W_5(i,j) & \leq 0 & \quad i\neq j.
\end{align*}
\]

**Loan Granting Constraints**

The total loan granting by an affiliate is limited
by the initial funds available, by general management policies, and by statutory regulations, like restrictions on financial capital movements out of the relevant country. The minimum of these factors is embodied in the constant $B7(i)$. We have

$$\sum_{j=1, \ldots, nc, j \neq i}^{nc} X_{4(i,j)} \leq B7(i) \quad i=1, \ldots, nc.$$  

**Interaffiliate Loan Raising Constraints**

The total loan raising by an affiliate is also limited by several factors. Although the loans are repaid after the planning horizon, no more loans should be taken than can be repaid in the future. There may be restrictions on importing foreign financial capital, and also other similar reasons can restrict the total loan raising. Only the sum of the loans raised from abroad is restricted below, but in some cases also the individual interaffiliate loans would have to be restricted, depending on the planning situation. We have

$$\sum_{j=1, \ldots, nc, j \neq i}^{nc} A_{10(j,i)} X_{4(j,i)} \leq B8(i) \quad i=1, \ldots, nc.$$  

In the numerical example no interaffiliate loan raising constraints are written, since in a two-country case they are equivalent to the loan granting constraints in format.

The constraints presented so far in this section
are first stage constraints.

**Cash Flow Constraints**

As has been observed it must be required that (stated in the relevant currency)

\[
\text{initial cash} + \text{cash inflows} \geq \text{cash outflows} + \text{minimum closing cash}
\]

for each affiliate \(i=1,\ldots,nc\); and for each state of the world \(q=1,\ldots,Q\).

The inclusion of the interaffiliate loans and their interests in the model give rise to additional terms in the cash flow constraints, since new cash inflows and cash outflows result.

The new terms in cash inflows are

\[
\text{interest received for each affiliate}
\]

\[
\sum_{j=1}^{nc} \sum_{j \neq i} A10(j,i)X4(j,i) + \sum_{j=1}^{nc} X5(i,j) + \sum_{j=1}^{nc} W5(i,j) \quad i=1,\ldots,nc
\]

\[
j \neq i \quad \text{exchange rate} \quad j \neq i
\]

---

The new terms in cash outflows are

\[
\text{loans granted interest paid for each affiliate}
\]

\[
\sum_{j=1}^{nc} X4(i,j) + \sum_{j=1}^{nc} A10(j,i)X5(j,i) + \sum_{j=1}^{nc} A15(j,i,q)W5(j,i) \quad i=1,\ldots,nc
\]

\[
j \neq i \quad j \neq i
\]

\[
j = 1 \quad \text{second subperiod} \quad j \neq i \quad \text{exchange rate}
\]

\[
j = 1,\ldots,nc \quad q = 1,\ldots,Q.
\]

**Equations for Book Profit and Loss**

The exchange gains or losses on interaffiliate loans are realized beyond the horizon and therefore they are excluded from the equation for book profit and loss for the period. On the other hand, the interest on
these loans is accounted for in the book profit and loss equations. Consequently the equations for book profit and loss must be augmented with the following terms.

\[
\sum_{j=1}^{nc} X5(i,j) + \sum_{j \neq i}^{nc} W5(i,j) - \sum_{j=1}^{nc} A10(j,i)X5(j,i)
\]
\[
- \sum_{j=1}^{nc} A15(j,i,q)W5(j,i) \quad i=1,...,nc \quad q=1,...,Q.
\]

Equations for Internal Profit and Loss

These equations are augmented with the same terms as the equations for book profit and loss. In addition, the exchange gains/losses on loans can be included. This inclusion gives rise to the following additional terms in the equations for internal profit and loss. (These terms are not included in the equations for internal profit and loss in our numerical example.)

\[
+ \sum_{j=1}^{nc} [A10(j,i)-A15(j,i,q)]X4(j,i) \quad i=1,...,nc
\]
\[
\text{first} \quad \text{second} \quad q=1,...,Q.
\]

To conclude, we discuss a feature of the loans in the model. An affiliate can raise loans from the local money market and from other affiliates. Loans from foreign money markets have not been introduced as separate decision variables. These loans can, however, be routed through the appropriate affiliate, and thus no actual simplification is made. Figure 5-1 illustrates this.
The freedom of action for the multinational firm is enhanced if the loan from the foreign money market is actually conveyed through another affiliate, as in Figure 5-1, since then the interest rates can be manipulated in between. If this is not the case the proper interaffiliate interest rate can be fixed in the model.

5.3 Inclusion of Forward Contracting Decisions

Denote:

\[ Z_{1\left(i,j\right)} = \text{the amount of currency } j \text{ to be purchased from bank by affiliate } i \text{ during the second subperiod on the basis of the forward contract made in the beginning of the first subperiod.} \]

\[ A_{18\left(j,i\right)} = \text{the exchange rate applied on the contracted amount of currency } j \text{ in country } i, \text{ during the second subperiod.} \]

\[ A_{18\left(2,1\right)} = 9.50 \text{ Fmk/£} \]
\[ A_{18\left(1,2\right)} = 0.11 \text{ £/Fmk} \]

Consider a forward contract made by affiliate \( i \) in the beginning of the first subperiod in order to buy \( Z_{1\left(i,j\right)} \) units of currency \( j \) from a bank on a future date,
at an exchange rate which is already agreed upon at the beginning of the first subperiod. On the basis of this contract the bank sells the agreed $Zl(i,j)$ units of currency $j$ during the second subperiod at the predetermined exchange rate, i.e. $A18(j,i)$, regardless of the true second subperiod exchange (spot) rate, i.e. $A15(j,i,q)$. Affiliate $i$ on its part agrees to purchase the $Zl(i,j)$ units of currency $j$ during the second subperiod at the exchange rate of $A18(j,i)$, regardless of the true exchange rate $A15(j,i,q)$.

$Zl(i,j)$ is a first stage variable, because its value must be decided before the true state of the world, that is the true value of the second subperiod exchange rate $A15(j,i,q)$, is revealed. Note that we have adopted an additional letter $Z$ to identify a decision variable.

It was assumed for simplicity that only liabilities are accrued in foreign currencies for any affiliate (e.g. resulting from imports, and interaffiliate loans). The assets of an affiliate are always accrued in the currency of the host country of the affiliate. This results from the fact that in the idealized multinational firm items transferred between the affiliates are invoiced in the currency of the exporting affiliate (e.g. in the numerical example the transfers from the English affiliate to the Finnish one in pounds), and the interest payments on interaffiliate loans are effected in the currency of the host country of the affiliate receiving the interest. On the other hand, it is assumed that a loan from a foreign affiliate is immediately converted into the host country currency. Nevertheless, the pertinent liability is factually denominated in the currency relevant to the loan granting affiliate. The foreign currency needed for any foreign payment is purchased by the importing/owing affiliate at the instant of the payment. In other
words, what has been said above means that the affiliates of the idealized multinational firm hold in cash only the currency of the relevant host country.

In connection with forward contracting decisions in the model the same simplifying assumption still holds. Only the forward contracts in order to purchase foreign currencies from the bank are included. It would naturally be possible to include also forward contracted sales of foreign currencies to the bank. This is omitted because of the simplification discussed above.

**Cash Flow Constraints**

We wish to establish the impact of making the forward contract $Z_1(i,j)$ on the cash flow constraints of affiliate $i$. It is assumed that if more of the currency $j$ is acquired on the basis of the contract than is needed by affiliate $i$ for the second subperiod payments in currency $j$, the surplus is immediately converted into currency $i$, at the exchange rate $A_{15}(j,i,q)$. If the amount acquired on the basis of the contract is less than is needed by affiliate $i$ for the second subperiod payments in currency $j$, the deficit is immediately bought using currency $i$ to buy the lacking amount of the foreign currency at the exchange rate $A_{15}(j,i,q)$.

It is demonstrated below that the net impact of a forward contract on the cash flows is independent of the amount of trade transactions covered with it.

Assume first that the contract $Z_1(i,j)$ can be made without covering any transaction at all with it. Then, during the second subperiod affiliate $i$ must purchase $Z_1(i,j)$ units of currency $j$ at an exchange rate of $A_{18}(j,i)$. A cash outflow of $A_{18}(j,i)Z_1(i,j)$ results in currency $i$. Because no transactions were covered,
affiliate i consequently possesses $Z_l(i,j)$ units of currency $j$, which it sells at the true spot rate, i.e. $A_{15}(j,i,q)$. A cash inflow of $A_{15}(j,i,q)Z_l(i,j)$ results in currency $i$. Thus the net cash flow caused by the forward contract discussed is

$$[A_{15}(j,i,q) - A_{18}(j,i)]Z_l(i,j)$$

stated in currency $i$.

Assume next that a payment of $W(j,i)^8$ has to be effected in currency $j$ during the second subperiod, and that the contract $Z_l(i,j)$ is used to cover the payment. Note that, in principle, the cash flow constraints already contain a term $A_{15}(j,i,q)W(j,i)$ to indicate a cash outflow. - Consider, first, the case where $Z_l(i,j) \geq W(j,i)$. The payment $W(j,i)$ causes a cash outflow of $A_{18}(j,i)W(j,i)$ in currency $i$, because the forward contract covers the payment completely. On the other hand the cash outflow $A_{15}(j,i,q)W(j,i)$ must be cancelled, lest the payment is accounted for twice. The remainder $Z_l(i,j) - W(j,i)$ of the forward contract causes a cash outflow of $A_{18}(j,i)[Z_l(i,j) - W(j,i)]$ in currency $i$. Now affiliate $i$ possesses $Z_l(i,j) - W(j,i)$ units of currency $j$, which it converts into currency $i$. This gives rise to a cash inflow of $A_{15}(j,i,q)[Z_l(i,j) - W(j,i)]$ in currency $i$. When the relevant cash flows are combined, the net cash flow in currency $i$ caused by the forward contract under observation is

$$
\begin{array}{c}
- A_{18}(j,i)W(j,i) + A_{15}(j,i,q)W(j,i) \\
- A_{18}(j,i)[Z_l(i,j) - W(j,i)] \\
+ A_{15}(j,i,q)[Z_l(i,j) - W(j,i)] \\
= [A_{15}(j,i,q) - A_{18}(j,i)]Z_l(i,j).
\end{array}
$$

\hspace{1cm}^8 W(j,i) is actually used for short to indicate $W_3(j,i)$ and $W_5(j,i)$. 
Consider, then, the case where $Z_l(i,j) \leq W(j,i)$. A cash outflow of $A_18(j,i)Z_l(i,j)$ results in currency $i$ for the portion of the payment $W(j,i)$ which is covered. The rest of the payment, i.e. $W(j,i)-Z_l(i,j)$, gives rise to a cash outflow of $A_15(j,i,q)[W(j,i)-Z_l(i,j)]$ in currency $i$. On the other hand the cash outflow $A_15(j,i,q)W(j,i)$ must be cancelled, lest the payment is accounted for twice. When the relevant terms are combined, the net cash flow in currency $i$ caused by the forward contract discussed is arrived at:

\[- A_18(j,i)Z_l(i,j) - A_15(j,i,q)[W(j,i)-Z_l(i,j)] + A_15(j,i,q)W(j,i) = [A_15(j,i,q)-A_18(j,i)]Z_l(i,j)\].

Once more the same result, which is independent of $W(j,i)$, was obtained, and the proof is complete.

The new terms in the cash flow constraints are

\[
\sum_{j=1}^{nc} [A_15(j,i,q)-A_18(j,i)]Z_l(i,j) \quad \text{for second subp. forward contract} \quad q=1,\ldots,Q.
\]

---

Equations for Book Profit and Loss, and for Internal Profit and Loss

Because profits and losses due to currency exchange rate changes are included in tax considerations, the same terms are augmented (with changed signs) as included in cash flow constraints. The principle "the net impact of a forward contract is independent of the foreign payments covered with it" applies also for these auxiliary
Statutory restrictions on forward contracting are quite common. For example the appropriate Finnish law requires that a forward contract must be based on a trading or a debt relationship. In the model it is required that the forward contract made by affiliate $i$ for currency $j$ may not exceed the relevant second subperiod payments for imports and interaffiliate interest. Thus we set

$$Z_{1(i,j)} - W_{3(j,i)} - W_{5(j,i)} \leq 0 \quad i=1,\ldots,nc \quad j=1,\ldots,nc \quad i\neq j.$$

Note that contrary to the other financial constraints the forward contract constraints are not denominated in the currency of the affiliate under observation. These constraints are first stage constraints.

The interaffiliate loans $X_{4(j,i)}$ were excluded from the above, since it is arbitrarily assumed that a forward contract cannot be made for a long enough period. Furthermore the relevant repayments are effected outside the planning horizon. First subperiod payments $X_{3(j,i)}$ and $X_{5(j,i)}$ are excluded, since they would not be accepted by the bank, because the proper commercial documents show that these payments are effected in cash.

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9Consider, for example, what happens if you make a binding agreement with a bank to buy £ 100 three months from now at a rate of 9.50 Fmk/£ and after the three months you note that the spot rate for the pound is 8.00 Fmk/£.
5.4 Model, Solution, and Optimal Operating Budgets in the Numerical Example

The numerical example is the same one as presented in the previous chapter, with the additional input data, which were given after the presentation of the relevant constants. The numerical example now includes decisions on interaffiliate loans, their interest rates, and delaying or not delaying the interest payments. The essential extension of the numerical example is the inclusion of forward contracting. In order to solve the numerical example we must solve the linear programming problem given on the next five pages.
Sales/capacity constraints

\[ X_{11} \leq 25\,000 \text{ (units)} \]
\[ X_{12} \leq 5\,000 \text{ (units)} \]

Supply/capacity constraints

\[ X_{211} \leq 25\,000 \text{ (units)} \]
\[ X_{222} \leq 25\,000 \text{ (units)} \]

Technical availability constraints

\[
\begin{align*}
\text{to } F \text{ prod. from } F \text{ prod.} & \quad \text{exports} & \quad \text{imports} & \quad \text{init.inv. - min.cl.inv.} \\
X_{11} - X_{211} + X_{212} - X_{221} & \leq 0 \text{ (units)} \\
\text{to } E \text{ prod.} & \quad \text{imports} & \quad \text{exports} & \quad \text{from } E \text{ prod.} \\
X_{12} - X_{212} + X_{221} - X_{222} & \leq 0 \text{ (units)}
\end{align*}
\]

Transfer price constraints

\[
\begin{align*}
\text{bound exports} & \quad \text{first subp. p.} & \quad \text{second subp. p.} \\
400X_{212} - X_{312} - W_{312} & \leq 0 \text{ (lower bound } F \to E \text{ Fmk)} \\
-900X_{212} + X_{312} + W_{312} & \leq 0 \text{ (upper bound } F \to E \text{ Fmk)} \\
30X_{221} - X_{321} - W_{321} & \leq 0 \text{ (lower bound } E \to F \text{ £)} \\
-100X_{221} + X_{321} + W_{321} & \leq 0 \text{ (lower bound } E \to F \text{ £)}
\end{align*}
\]

Local loan raising constraints

\[ X_{411} \leq 3\,000\,000 \text{ (Fmk)} \]
\[ X_{422} \leq 300\,000 \text{ (£)} \]
Interaffiliate loan granting/raising constraints

\[ \text{loan } F \rightarrow E \]
\[ X_{412} \leq 1000000 \quad (\text{Fmk}) \]

\[ \text{loan } E \rightarrow F \]
\[ X_{421} \leq 100000 \quad (\text{£}) \]

Interaffiliate interest rate constraints

\begin{align*}
\text{interaffiliate interest } p. \\
\text{first subp.} & \quad \text{second subp.} \\
0.03X_{412} - X_{512} - W_{512} & \leq 0 \quad (\text{lower bound}) \\
-0.08X_{412} + X_{512} + W_{512} & \leq 0 \quad (\text{upper bound}) \\
0.03X_{421} - X_{521} - W_{521} & \leq 0 \quad (\text{lower bound}) \\
-0.08X_{421} + X_{521} + W_{521} & \leq 0 \quad (\text{upper bound})
\end{align*}

Forward contract constraints

\begin{align*}
\text{£ contract} \\
- W_{321} - W_{521} + Z_{12} & \leq 0 \quad (\text{Finland})
\end{align*}

\begin{align*}
\text{Fmk contract} \\
- W_{312} - W_{512} + Z_{21} & \leq 0 \quad (\text{England})
\end{align*}

Cash flow constraints

\begin{align*}
\text{v.c. II} & \quad \text{v.c. I} & \quad \text{transport.} & \quad \text{exports} & \quad \text{import} \\
250X_{11} + 300X_{211} + 100X_{212} - X_{312} - W_{312} + 11X_{321} \\
\text{ports + duty} & \quad \text{local loan} & \quad \text{loan } F \rightarrow E & \quad \text{loan } E \rightarrow F & \quad \text{interest received} \\
+ 11W_{321} - 0.95X_{411} + X_{412} - 10X_{421} - X_{512} - W_{512}
\end{align*}
interest paid £ contract tax init.-min. cash. bal. + f. cost
\[ + 10X521 + 10W521 - 0.5Z12 + 0.45X611 \leq 4000000 \] (Fmk, \( q=1 \))

\[ 250X11 + 300X211 + 100X212 - X312 - W312 + 11X321 \]

imports
\[ + 9W321 - 0.95X411 + X412 - 10X421 - X512 - W512 \]

interest £ contract tax
\[ + 10X521 + 8W521 + 1.5Z12 + 0.45X612 \leq 4000000 \] (Fmk, \( q=2 \))

(the potential devaluation of the pound affects the underlined terms of the cash flow constraints)

\[ 25X12 + 10X221 + 30X222 + 0.11X312 + 0.11W312 - X321 \]
\[ - W321 - 0.1X412 + X421 - 0.95X422 + 0.1X512 + 0.1W512 \]
\[ - X521 - W521 + 0.01Z21 + 0.45X621 \leq 4000000 \] (£, \( q=1 \))

\[ 25X12 + 10X221 + 30X222 + 0.11X312 + 0.135W312 - X321 \]
\[ - W321 - 0.1X412 + X421 - 0.95X422 + 0.1X512 + 0.125W512 \]
\[ - X521 - W521 + 0.025Z21 + 0.45X622 \leq 4000000 \] (£, \( q=2 \))

**Equations for book profit and loss**

s.p.-v.c.II
\[ 750X11 - 300X211 - 100X212 + X312 + W312 - 11X321 \]
\[ - 11W321 - 0.05X411 + X512 + W512 - 10X521 - 10W521 \]
book profit book loss fixed cost + depreciation
\[ + 0.5Z12 - X611 + X711 = 2500000 \] (Fmk, \( q=1 \))

\[ 750X11 - 300X211 - 100X212 + X312 + W312 - 11X321 \]
\[ - 9W321 - 0.05X411 + X512 + W512 - 10X521 - 8W521 \]
\[ - 1.5Z12 - X612 + X712 = 2500000 \] (Fmk, \( q=2 \))
Equations for internal profit and loss

\[ 75X_{12} - 10X_{221} - 30X_{222} - 0.11X_{312} - 0.11W_{312} + X_{321} + W_{321} - 0.05X_{422} - 0.1X_{512} - 0.1W_{512} + X_{521} + W_{521} - 0.01Z_{21} - X_{621} + X_{721} = 250000 \quad (\text{\pounds}, q=1) \]

\[ 75X_{12} - 10X_{221} - 30X_{222} - 0.11X_{312} - 0.135W_{312} + X_{321} + W_{321} - 0.05X_{422} - 0.1X_{512} - 0.125W_{512} + X_{521} + W_{521} + 0.025Z_{21} - X_{622} + X_{722} = 250000 \quad (\text{\pounds}, q=2) \]
Non-negativity constraints

It is required that all the variables (the symbols identified by the letters X, W, and Z) must be non-negative.

Objective function

\[
\text{max } E(P) = 0.7X_{811} + 0.3X_{812} + 7X_{821} + 2.4X_{822} \\
- 0.7X_{911} - 0.3X_{912} - 7X_{921} - 2.4X_{922}.
\]

When the model is run on a computer the output which is presented on the next page is obtained. The optimal solution can be elucidated with the help of Figures 5-2 and 5-3. In the optimum solution the transfer price of the intermediate products transferred from the English affiliate to the Finnish one is \( f 100 \) (28 334 / 283.34). The interest rate on the interaffiliate loan granted by the English affiliate to the Finnish one is 8% \([(8000/10000) \cdot 100]\). The Finnish affiliate must buy 28 334 + 8 000 = 36 334 pounds from a bank in order to pay for its imports and interaffiliate interest. Both these payments are effected during the second subperiod according to the optimal model solution. The Finnish affiliate makes a forward contract for 36 334 pounds at the forward rate of 9.50 Fmk/£ offered by the bank. Consequently the second subperiod cash outflow in marks resulting from the future foreign payment is known in advance with certainty in spite of the currency risk. For the contracted 36 334 pounds the currency risk is eliminated.

This time, only the predicted income statement for
VARIABLES

S1 = 13 194.25 (slack in Finnish sales constraint, units)
X822 = 150 000.00 (q=2, internal profit in England, £)
S3 = 13 477.59 (slack in Finnish production phase I, units)
S4 = 19 716.64 (slack in English production phase I, units)
X11 = 11 805.75 (final product sales in Finland, units)
X12 = 5 000.00 (final product sales in England, units)
S7 = 0.00 (slack in F→E transfer price l.b. constr.)
S8 = 0.00 (slack in F→E transfer price u.b. constr.)
S9 = 19 833.00 (slack in E→F transfer price l.b. constr.)
X421 = 100 000.00 (loan from England to Finland, £)
X411 = 3 000 000.00 (local loan raising in Finland, FmK)
S12 = 300 000.00 (unused English loan raising potential, £)
S13 = 1 000 000.00 (slack in Finnish loan granting constr. FmK)
Z12 = 36 334.00 (%-forward contract in Finland, £)
S15 = 0.00 (slack in F→E interest rate l.b. constr.)
S16 = 0.00 (slack in F→E interest rate u.b. constr.)
S17 = 5 000.00 (slack in E→F interest rate l.b. constr.)
W521 = 8 000.00 (second subperiod interest on E→F loan, £)
S19 = 0.00 (slack in Finnish forward contr. constr., £)
S20 = 0.00 (slack in English forward contr. constr., FmK)
X612 = 2 374 085.00 (q=2, book profit in Finland, FmK)
X811 = 2 805 746.00 (q=1, internal profit in Finland, FmK)
S23 = 50 000.00 (q=1, slack in English cash flow constr., £)
S24 = 50 000.00 (q=2, slack in English cash flow constr., £)
X812 = 2 805 746.00 (q=2, internal profit in Finland, FmK)
X611 = 2 374 085.00 (q=1, book profit in Finland, FmK)
W321 = 28 334.00 (second subp. payment for Finnish imports, £)
X722 = 0.00 (q=2, book loss in England, £)
X211 = 11 522.41 (intermediate prod. production in Finland, units)
X221 = 283.34 (Finnish imports from England, units)
X222 = 5 283.34 (intermediate prod. production in England, units)
X821 = 150 000.00 (q=1, internal profit in England, £)
REST ARE ZEROS
P = 4 215 745.00 (global expected profit)

DUAL VARIABLES

R2 = 411.100 (English sales constraint)
R5 = 219.200 (Finnish technical availability constraint)
R6 = 274.000 (English technical availability constraint)
R10 = 1.461 (E→F transfer price u.b. constraint)
R11 = 0.292 (local Finnish loan raising constraint)
R14 = 3.464 (English loan granting constraint)
R18 = 2.192 (E→F interest rate u.b. constraint)
R21 = 0.296 (Finnish cash flow constraint for q = 1)
R22 = 0.032 (Finnish cash flow constraint for q = 2)
Finnish taxation is presented, because it suffices to demonstrate the derivation of optimal operating budgets. As noted earlier, the model including currency risk indicates a separate set of statements and budgets for each state of the world accounted for. In the numerical example in hand the same statement is relevant for both potential states of the world, since the risk in the future currency exchange rate of the pound (which is the only source of risk in the numerical example) is eliminated by making the forward contract, which covers exactly the pounds needed for second subperiod foreign payments in Finland. Note that this state of affairs results from the figures assumed. If, for example, the forward rate of the pound were 9.90 Fmk/f, the contract would not be made, since then it would no longer be deemed advantageous. The budgets would then be distinct for the different states of the world.

Predicted income statement for Finnish taxation

\[
q = 1 \quad q = 2
\]

<table>
<thead>
<tr>
<th>Description</th>
<th>Value (Fmk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic sales</td>
<td>+11 805 750</td>
</tr>
<tr>
<td>Variable cost II</td>
<td>-2 951 438</td>
</tr>
<tr>
<td>Variable cost I</td>
<td>-3 456 723</td>
</tr>
<tr>
<td>Imports</td>
<td>-269 173</td>
</tr>
<tr>
<td>Duty</td>
<td>-28 334</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>-1 000 000</td>
</tr>
<tr>
<td>Domestic interest</td>
<td>-150 000</td>
</tr>
<tr>
<td>Interaffiliate interest</td>
<td>-76 000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>-1 500 000</td>
</tr>
<tr>
<td>Net income subject to taxation</td>
<td>2 374 082</td>
</tr>
</tbody>
</table>
FIGURE 5-2  Trade and Production Flows

Finland

 production phase I

 raw material

11,522.41

 production phase II

 intermediate product

11,805.75

 final product

5,283.34

 £100/unit

England

 production phase I

 raw material

5,000.00

 production phase II

 intermediate product

283.34

 final product

15,000.00

 production phase II

 intermediate product

11,522.41

 final product

5,283.34

 £100/unit
5.5 Inclusion of Currency Position Requirements

If the real value of an asset of the multinational firm (or any asset) denominated, say, in currency \( j \) is affected by a change in the pertinent exchange rates, the relevant asset is called an exposed asset. The same goes respectively for a liability. In that case we speak of an exposed liability. It is customary to define:

\[
\text{net currency exposure in currency } j = \text{exposed assets in currency } j - \text{exposed liabilities in currency } j.
\]

Consider for example a devaluation of currency \( j \). (The net currency exposure has conventionally been considered in the face of expected devaluations, but the concept is relevant in the cases of revaluations and floating as well.) If the net currency exposure in currency \( j \) is positive the firm suffers a loss as a result of the devaluation.\(^{10}\) If it is negative the firm gains as a result of the devaluation. For a revaluation the relation is reversed.

The exposed assets and exposed liabilities in different currencies are reflected in the balance sheets for the affiliates. The balance sheet of an affiliate of the idealized multinational firm can be written in the following format.

\(^{10}\) Consider what happens if you hold in cash £100 and the pound is devalued by 20%. Consider what happens if the pound is revalued instead. What if you owe, say, £60?
<table>
<thead>
<tr>
<th>ASSETS</th>
<th>LIABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash in bank</td>
<td>External payables</td>
</tr>
<tr>
<td></td>
<td>(including external</td>
</tr>
<tr>
<td></td>
<td>short-term loans)</td>
</tr>
<tr>
<td>External receivables</td>
<td></td>
</tr>
<tr>
<td>Inventories</td>
<td>Long-term liabilities</td>
</tr>
<tr>
<td>Receivables from</td>
<td>Payables to other</td>
</tr>
<tr>
<td>other affiliates</td>
<td>affiliates</td>
</tr>
<tr>
<td>Fixed assets</td>
<td>Stockholders' equity</td>
</tr>
</tbody>
</table>

The assets and liabilities which are assumed to be exposed are separated by the dashed lines in the balance sheet formula above. Note that this time we are not interested in the balance sheets of the affiliates per se. They are used as a convenient basis for deriving the exposed assets and liabilities denominated in different currencies, because the items indicated are denominated in the respective host country currencies in our idealized case.

When, in general, a geocentric operating philosophy is assumed it is sound to derive only the total exposed assets in each currency, and the respective total liabilities, of the multinational as a whole. It is not necessary then to calculate the net currency exposures in the different currencies for each affiliate separately, as conventional management literature suggests.

The choice of which assets and liabilities are assumed to be exposed, and which not, depends on several factors. Consider the items in the balance sheet formula which was presented. Cash and external short-term loans are clearcut cases. External receivables and payables can be considered as exposed, except when they are protected by special clauses passing the risk on to the external party. (Naturally also a partial exposure of an asset or a liability is possible.)
Inventories are often an ambiguous case. Here they have been considered as not exposed, but we could, again, proceed also on the basis of a contrary assumption. For example, if the cost caused by a devaluation can be shifted on to customers by increasing the relevant prices, the inventories can be considered as not exposed, or only partially exposed. Fixed assets are not included in exposed assets, since their real value is preserved. Long-term liabilities and stockholders' equity are excluded because of their long-term nature, but also this point can be argued. Inclusion or exclusion of receivables from other affiliates and payables to other affiliates depends on the currency which they are denominated in. This point is inconsequential, however, since in the aggregate balance sheet of the multinational firm, which is the balance sheet we are interested in, these terms cancel one another out.

In the previous model versions under currency risk we implicitly assumed that the decision maker is neutral to the risk arising from the exposure of assets and liabilities denominated in a suspect currency. This assumption is not in accordance with the observed behavior of managers. Their behavior indicates aversion to risk. (In other words the expected value criterion is not sufficient.) Thus, in order to prevent the possibility of considerable losses due to changes in the real values of assets and liabilities as a result of a potential change in the currency exchange rates, the net currency exposure, or some other figure deemed fit to indicate currency exposure position, in each currency can be restricted to fit into predetermined bounds in the model. The currency exposure position in each currency must first be calculated in the model. The positions can be separately calculated at the end of the first sub-
The new currency exposure positions, (1) and (2), are calculated on the basis of the initial positions in each currency and the relevant changes in the positions. The initial net currency exposures and the predetermined changes are accounted for in the right-hand constants of the equations for the net currency exposure positions, which will be presented shortly. In the model the decisions on activities which, in the balance sheet, simultaneously affect an exposed and a non-exposed item, alter the net currency exposure. On the other hand, decisions which
are reflected, in the balance sheet, in two items which are both exposed, or both non-exposed, do not alter the net currency exposure positions. As discussed, a geocentric operating philosophy is assumed. Hence the new net currency exposures in the different currencies are calculated on the basis of the aggregate balance sheet of the idealized multinational firm.

Equations for Currency Exposure Positions at the End of the First Subperiod

Denote:

\[ X_{10}(i) = \text{the positive net currency exposure in currency } i \text{ at the end of the first subperiod.} \]

\[ X_{11}(i) = \text{the negative net currency exposure in currency } i \text{ at the end of the first subperiod.} \]

\[ B_9(i) = \text{the initial net currency exposure in currency } i \text{ adjusted for predetermined changes during the first subperiod.} \]

We have:

\[
\begin{align*}
\text{domestic} & \quad \text{domestic} & \quad \text{variable cost II} \\
\text{cash sales} & \quad \text{credit sales} & \quad \text{cash accounts payable} \\
-A_3(i)X_1(i) & \quad -A_4(i)X_1(i) & \quad [A_5(i) + A_6(i)]X_1(i) \\
\end{align*}
\]

\[
\begin{align*}
\text{variable cost I} & \quad \text{transportation} \\
\text{from exports} & \quad \text{for imports} \\
\sum_{j=1}^{nc} & \quad \sum_{j=1}^{nc} \\
-j=1 & \quad j=1 \\
\end{align*}
\]

\[
\begin{align*}
\sum_{j=1}^{nc} X_3(i,j) & \quad \sum_{j=1}^{nc} A_{10}(j,i)X_3(j,i) \\
j\neq i & \quad j\neq i \\
\end{align*}
\]

\[ A_3(i) \text{ indicates this time the first subperiod cash sales price. Earlier no distinction was necessary. We proceed as if the necessary alterations had been made in all the relevant constants in the equation under observation.} \]
\[\text{local interest} \quad \text{duty} \quad \text{local interest}\]
\[\sum_{j=1, j \neq i}^{nc} A11(j,i)[X3(j,i)+W3(j,i)] + A12(i)X4(i,i)\]
\[\text{interaffiliate loans granted} \quad \text{interaffiliate loans raised} \quad \text{cash interest on } X4(i,j)\]
\[\sum_{j=1}^{nc} X4(i,j) - \sum_{j=1, j \neq i}^{nc} A10(j,i)X4(j,i) - \sum_{j=1}^{nc} X5(i,j)\]
\[\text{cash interest on } X4(j,i) \quad \text{forward contracts covering currency } i\]
\[\sum_{j=1}^{nc} A10(j,i)X5(j,i) + \sum_{j=1, j \neq i}^{nc} Z1(j,i)\]

\[\text{positive net exposure} \quad \text{negative net exposure} \quad \text{adjusted initial exposure}\]
\[X10(i) - X11(i) = B9(i) \quad i=1, \ldots, nc,\]

(\(\text{where } i \text{ now indicates currencies, not affiliates}).

Let us discuss selected items in the above set of equations.

Domestic cash sales in country \(i\) cause a cash inflow in currency \(i\). Sales made on cash terms is debited to "cash in bank" and credited to "stockholders' equity". The net currency exposure is increased, since the former is considered as exposed, and the latter as not exposed.

Domestic credit sales are debited to "external receivables" (exposed), and credited to "stockholders' equity" (not exposed). The net currency exposure is increased.

Similarly, variable cost \(II\) is debited to "stockholders' equity" (not exposed) and credited to "cash in bank" (exposed), and thus the net currency exposure is reduced.

Consider next the interaffiliate credit sales. They

---

\(^{12}\) Profit and loss are included in "stockholders' equity", since it consists of retained earnings and capital stock.
are excluded. Inventories were defined as not exposed. Interaffiliate credit sales do not affect the net currency exposure. This can be confirmed by aggregating the relevant entries in the accounts of the affiliates involved. If the inventories were defined as exposed then the net currency exposure in currency $i$ would be increased by $\sum_{j \neq i} W_3(i,j)$ and decreased by $\sum_{j \neq i} A_{10}(j,i)W_3(j,i)$ as a result of the interaffiliate credit-based sales.

Local loans are excluded, since raising a local loan is debited to "cash in bank" (exposed), and credited to "external payables" (exposed).

Consider next the impact of an interaffiliate loan in the framework of an idealized numerical example. Figure 5-5 illustrates the accounts of a fictitious Finnish and a fictitious English affiliate, and their aggregate balance sheet. (1) Initially the Finnish affiliate holds 100 000 Fmk in cash. Hence the initial net currency exposure in marks is 100 000 Fmk. (2) The initial balance in the English affiliate is assumed to be zero for simplicity. Consequently, the initial net currency exposure in pounds is £0. The Finnish affiliate grants a loan of 100 000 Fmk to the English affiliate. (3) In the accounts of the Finnish affiliate the loan is debited to "receivables from other affiliates" and credited to "cash in bank". (4) In the accounts of the English affiliate "cash in bank" is credited with £10 000, since the fictitious English affiliate converts the marks immediately into pounds. "Payables to other affiliates" is credited with 100 000 Fmk, since the loan is denominated in marks. The new net currency exposures in marks and in pounds can be seen in the aggregate balance sheet of the multinational firm. Entries (c)
**FIGURE 5-5  Interaffiliate Loans and Net Currency Exposure**

**Finnish accounts**

<table>
<thead>
<tr>
<th>(a) cash in bank</th>
<th>(b) stockholders' equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 100 000 Fmk</td>
<td>100 000 Fmk (3)</td>
</tr>
<tr>
<td>100 000 Fmk</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) receivables from other affiliates</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3) 100 000 Fmk</td>
</tr>
</tbody>
</table>

**English accounts**

<table>
<thead>
<tr>
<th>(d) cash in bank</th>
<th>(e) stockholders' equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) £ 0</td>
<td></td>
</tr>
<tr>
<td>(4) £ 10 000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(f) payables to other affiliates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>100 000 Fmk (4)</td>
</tr>
</tbody>
</table>

**Multinational accounts**

<table>
<thead>
<tr>
<th>aggregate balance sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c) 100 000 Fmk</td>
</tr>
<tr>
<td>100 000 Fmk (b)</td>
</tr>
<tr>
<td>(d) £ 10 000</td>
</tr>
<tr>
<td>100 000 Fmk (f)</td>
</tr>
</tbody>
</table>

Note that these accounts are designated to the firm's internal usage.
and (f) are cancelled. (b) is defined as not exposed. (d) is defined as exposed. Consequently the new currency exposures are 0 Fmk and £10 000 respectively. This is tantamount to a reduction of 100 000 Fmk and an increase of £10 000 in the net currency exposures. This result is in accordance with the equations given for the net currency exposure positions.

It could be easily seen in a similar framework that interaffiliate cash-based sales and interaffiliate interest give rise to the changes indicated in the equations presented.

The forward contracts, which are made in order to purchase currency i from banks in the different countries at predetermined exchange rates during the second sub-period, reduce, with the contracted amount, the net currency exposure in currency i at the end of the first subperiod. The geocentric operation philosophy and the fact that the net currency exposure is consequently calculated by the currency only, is clearly reflected in the term involving forward contracts. It is the sum of all forward contracts in currency i regardless of the affiliates making the contracts!

In the numerical example of Section 5.4 the net currency exposures are 12 200 000 Fmk and £677 000 (when given with three significant digits).

Constraints for the Net Currency Exposure Positions at the End of the First Subperiod

Denote:

\[ B10(i) = \text{the maximum net currency exposure in currency } i \text{ tolerated by the central management at the end of the first subperiod.} \]

We have:
The net currency exposure may be either positive or negative (or zero). Hence the two different sets of variables are needed. The constraints presented above are first stage constraints in the two-stage linear programming formulation.

In the numerical example solved in Section 5.4 the pound is the suspect currency. If we required that the net currency exposure in pounds must not increase during the first subperiod, the basic nature of the optimal solution would remain unaltered. The English affiliate then sells more intermediate products to the Finnish affiliate (1250 units) than before (283 units), but at a lower transfer price (£ 54/unit) than before (£ 100/unit). (Recall that the relevant transfer price bounds are £ 30 and £ 100.) The forward contract made by the Finnish affiliate in order to purchase pounds at the agreed forward rate during the second subperiod is changed from £ 38 334 to £ 75 000. The value of the objective function is reduced approximately by 85 000 Fmk, which indicates the cost of adjusting the net exposure in pounds to fit into the stipulated range. The shadow price of the constraint under observation is \( 1.10 \) Fmk/£.

**Equations for Currency Exposure Positions at the End of the Planning Horizon**

Denote:

\[ X_{12}(i,q) = \text{the positive net currency exposure in currency } i \text{ at the end of the planning horizon if the } q:th \text{ state of the world occurs.} \]
X_{13}(i,q) = \text{the negative net currency exposure in currency } i \text{ at the end of the planning horizon if the } q:\text{th state of the world occurs.}

B_{11}(i,q) = \text{the initial net currency exposure in currency } i \text{ adjusted for predetermined changes during the planning horizon in the occurrence of the } q:\text{th state of the world.}

We have:

\text{domestic sales revenue and v.c. II} - A_{14}(i)X_{1}(i)\text{ } + \text{ variable cost I } [A_{7}(i)+A_{8}(i)]X_{2}(i,i)

\begin{align*}
\text{transportation} & + \sum_{j=1, j\neq i}^{n_c} A_{9}(i,j)X_{2}(i,j) - \sum_{j=1, j\neq i}^{n_c} X_{3}(i,j) - \sum_{j=1, j\neq i}^{n_c} W_{3}(i,j) \\
\text{exports} & \\
\text{imports} & + \sum_{j=1, j\neq i}^{n_c} A_{10}(j,i)X_{3}(j,i) + \sum_{j=1, j\neq i}^{n_c} A_{15}(j,i,q)W_{3}(j,i) \\
\text{first subperiod exchange rate} & \\
\text{duty} & + \sum_{j=1, j\neq i}^{n_c} A_{11}(j,i)[X_{3}(j,i)+W_{3}(j,i)] + A_{12}(i)X_{4}(i,i) \\
\text{interest from} & \\
\text{interaffiliate loans granted} & - \sum_{j=1, j\neq i}^{n_c} X_{4}(i,j) + \sum_{j=1, j\neq i}^{n_c} A_{10}(j,i)X_{4}(j,i) - \sum_{j=1, j\neq i}^{n_c} X_{5}(i,j) \\
\text{interest paid on interaffiliate loans raised} & \\
\text{loans granted} & - \sum_{j=1, j\neq i}^{n_c} W_{5}(i,j) + \sum_{j=1, j\neq i}^{n_c} A_{10}(j,i)X_{5}(j,i) + \sum_{j=1, j\neq i}^{n_c} A_{15}(j,i,q)W_{5}(j,i) \\
\text{interest paid on interaffiliate loans raised} & \\
\text{income} & + A_{13}(i)X_{6}(i,q) - \sum_{j=1, j\neq i}^{n_c} [A_{15}(j,i,q)-A_{18}(j,i)]Z_{1}(i,j) \\
\text{tax} & \\
\text{profit from forward contracts} & \\
\text{second subperiod forward exchange rate} & \\
\end{align*}
The net currency exposures for each currency, at the end of the planning horizon, are defined separately for each of the Q states of the world accounted for. The forward contracts will have been realized before the end of the planning horizon. The gains and losses realized because of the forward contracts made by affiliate i affect the net currency exposure in currency i, since the accounts "cash in bank" (exposed) and "stockholders' equity" (not exposed) are involved. The rest of the items in the above equations should be obvious on the basis of the earlier discussion.

Constraints for the Net Currency Exposure Position at the End of the Planning Horizon

Denote:

\[ B_{12}(i) = \text{the maximum net currency exposure in currency } i \text{ tolerated by the central management at the end of the planning horizon.} \]

We have:

\[ X_{12}(i,q) - X_{13}(i,q) \leq B_{12}(i) \quad i=1,\ldots,nc \]
\[ q=1,\ldots,Q. \]

These constraints are second stage constraints.

5.6 Summary of Chapter 5

In this chapter I presented the incorporation of forward contracting decisions and currency exposure
position requirements in linear programming models for joint planning of trade, production, and financial flows in the multinational firm facing risky currency exchange rates. I demonstrated that the net effects resulting from making a forward contract are independent of using it to cover foreign transactions. I suggested that it is sufficient to calculate the currency exposure positions for each currency when a geocentric operating philosophy is accepted for the multinational firm. The affiliates need not be considered separately. A parallel fictitious numerical example was presented once more.
6 SELECTED EXAMPLES OF HYPOTHESES OF THE BEHAVIOR OF THE MULTINATIONAL FIRM GENERATED VIA THE MODEL

6.1 PREFACE

We turn our attention to the behavioral problem of the study. The task contained in the statement of the behavioral problem (cf. Section 1.2), and the approach chosen (cf. Section 1.4), is to generate hypotheses of the pattern of the behavior of the multinational firm by utilizing the normative model for the idealized multinational firm, which has been presented in the previous chapters. It was assumed tentatively that planning in big multinational business enterprises is, or will be, rational enough for us to proceed by interpreting the suggestions for general lines of action, which the model gives, in terms of the multinational firm, and to call them hypotheses of the behavior of the multinational firm.

This chapter presents selected examples of the hypotheses arrived at. The hypotheses to be presented are not necessarily new in literature. They are quite obvious and trivial, since the purport is to control the plausibility of the approach chosen in the behavioral problem. The point is that, at least in the respects presented, the model leads to results to be expected in the light of reality. While validating a model and an approach naturally requires rigorous testing, getting expected results motivates the presentation of the results, the approach, and the model for a later, thorough testing.¹

¹As will be seen in the next chapter, this point gives rise to one suggestion for further research.
Note that it was not intended to make an overall hypothesis of the assessment of trade, production, and financial flows in multinational firms or of their reactions to economic measures of the host country governments. Our purpose was to create only a few hypotheses which are connected to the questions in the behavioral problem. The hypotheses have been derived by deducing directly from the model, and, when this was too difficult, experiments were made with it. First, a preliminary analysis was performed to unravel interesting partial problems. These partial problems were then studied more thoroughly. When experiments were resorted to, the maximum set used was three affiliates. The details of the derivation of the hypotheses, with the exception of the first one, are completely omitted in the text, since describing them would be too onerous. The reason for this is in the abundance of the details involved.

Two kinds of pitfalls occur when the problem is tackled with the help of the model for the idealized multinational firm:

1) The deductions made to obtain the results must be correct in connection with the model.
2) The results (and the model from which they are derived) should be plausible in the light of reality, either current or future.

6.2 The Hypotheses

The fictitious numerical example presented in Chapter 3, which discussed the deterministic model, gives one example of the model behavior and the interpretation of the characteristics of the optimal solution.² On

²This is the reason why especially that numerical example was discussed at length.
the basis of this numerical example, the analysis of its optimal solution, and computational experience with the model, the following wording of a hypothesis is arrived at.

**Hypothesis 1** Multinational firms arrange inter-affiliate trade with manipulated transfer prices in order to avoid excessive taxation in countries with high corporate income tax rates.

It is important to note that although the hypothesis could easily have been made on the basis of general knowledge of big multinational business enterprises, it has been derived in strict accordance with the process described.

With the exception of the above hypothesis the details of the derivation are omitted completely in this thesis, for the reasons stated.

After performing a more extensive analysis of the numerical example of Chapter 3 than was given in the text, Hypothesis 1 can be reformulated:

**Hypothesis 2** Multinational firms may arrange inter-affiliate trade with manipulated transfer prices in order to avoid taxation in countries with high tax rates, but only if the tax rates are different enough, and providing that the transfer prices can be adjusted freely enough. In that case, two different kinds of policies can arise, whose relative efficiency depends on other particulars: 1) The affiliate in the high tax rate country imports from other affiliates at high transfer prices. 2) The affiliate in the high tax rate country exports to other affiliates at low transfer prices.
"Different enough" and "freely enough" are defined by contrasting the tax avoided and revenue received per transferred item with costs incurred by the transfer, the latter including revenues forgone because of the transfer. Slight differences in taxations do not induce transferring of components for book profit adjustment purposes. Further experiments indicate that the hypothesis is relevant under currency risk also.

Hypothesis 3  Multinational firms may utilize inter-affiliate loans and manipulate their interest rates in order to avoid excessive taxation in countries with high corporate income tax rates.

The hypothesis states that loans may be made between affiliates of multinational firms also when funds are not actually needed by the affiliate raising the loans. These interaffiliate loans may be even granted mutually between two affiliates if the tax rates are different enough, and providing that the interest rates can be made to differ, and the affiliate which is required to grant the loan is short of funds. In this "back-and-forth"-system, the affiliate in a low tax rate country charges a high interest rate and pays a low interest rate.

Taxation is "excessive" when general management policies do not require the paying of as much taxes in a country as the corporate income tax rate indicates.³

Note that the above three hypotheses do not indicate

³Although excluded in the model presented, minimum tax payment constraints as management policy constraints can be easily assumed.
global tax minimization. The activities of the multinational firm are jointly planned in the model, tax payments being only one factor.

Hypothesis 4 Interaffiliate loans with manipulated interest rates are preferred in tax avoidance, and in shifting of funds, to arranging of interaffiliate trade with manipulated transfer prices.

In the model, when other reasons do not induce interaffiliate trade transactions, their use is inferior to loans, since they involve more costs in the form of duty and transportation. This is obvious, because in the model there is no transfer charge on money.

Hypothesis 5 If financial transfers are not permitted from a country where a multinational firm has an affiliate with unused financial potential, the utilization of the blocked funds is effected via interaffiliate trade with manipulated transfer prices, provided that the relevant transfer prices can be adjusted freely enough. When the transfer prices can be made low enough, components are exported from the affiliate with the blocked funds in order to sell them elsewhere. When the transfer prices can be made high enough, the affiliate with the blocked funds imports components in order to shift the unused financial potential elsewhere.

*Which would be needed elsewhere in the multinational firm.*
Hypothesis 6  If finance is a scarce resource in a multinational firm for one affiliate and there is idle financial resource for another, and financial capital movements are not permitted from the latter country, taxation in the two countries has a restrictive influence on the use of transfer of components as a means of shifting the idle financial resource. The ranges of the transfer prices which make interaffiliate transfers advantageous in shifting funds, narrow as tax rates increase.

The restrictive effect of taxation on transferring of components, if the purpose of arranging the transfer is shifting funds between affiliates, is explained as follows. When transfers are made, total taxation (in this case the taxation in both countries involved) increases with increasing tax rates when counted per unit of transferred item. When tax rates are high enough, the advantage gained in the face of the scarce resource by the transferring of components, is not enough to offset the disadvantage caused by the increased total taxation per transferred unit. Note carefully that Hypothesis 6 concerns transfers of components in order to shift funds. The hypothesis does not say anything about the transferring of components for any other purpose.

Hypothesis 7  Producing in a low cost country is preferred to producing in a high cost country. Sales in a high price country are preferred to sales in a low price country.

This hypothesis is really simple and obvious and it is given only in order to highlight the discussion in the previous section about obtaining expected results.
The hypotheses selected for presentation relate to book profit adjustment and production adjustment, but not to currency risk adjustment in multinational business enterprises. This results from the fact that the hypotheses presented are to a considerable extent attributable to my earlier research efforts, which did not yet include currency risk. Nevertheless, the hypotheses have now been checked with additional experiments using the model under currency risk also. Thus currency risk has been included in the behavioral problem, too, although no hypotheses are given concerning currency risk adjustment.

\footnote{For the references see footnote 10 in Chapter 1.}
7 CONCLUSION

This chapter concludes the discussion. After summarizing and briefly discussing the results of the study, we discuss some extension prospects of the model, and finally three suggestions for relevant further research are made.

7.1 REVIEW OF RESEARCH RESULTS

The main result of this dissertation is the development of a two-stage linear programming formulation for joint planning of physical and financial activities of a multinational firm in an environment with currency exchange rate changes to be expected. This was achieved by constructing a two-stage linear programming model for the multinational firm assuming a situation where all the non-essential factors can be omitted for simplicity (= the idealization). I showed how the probabilistic nature of currency exchange rates can be taken into account in linear programming models for joint planning of trade, production, and financial flows in the multinational firm. I also showed how forward contracting can be planned in conjunction with global tactical planning of physical and financial activities in the multinational firm. I demonstrated how the exposure of assets and liabilities of the multinational firm can be adjusted to fit into predetermined bounds in models of

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1The reader is advised to reread Section 1.2 ("Research Problem").

2Only the relevant discrete distribution of future currency exchange rates has to be available to the decision maker.
the kind presented. Furthermore, after introducing the concepts of internal and external transfer prices, I demonstrated the inclusion of interaffiliate external transfer prices and interaffiliate interest rates as decision variables in models of the kind discussed. The possibility of interaffiliate delayed payments was included. The suggested formulations make it possible to consider, within the framework of a single model, decisions which previously had to be treated separately.  

I also suggested the use of the concept of "shadow transfer prices" to supplement the conventional analysis of shadow prices in deterministic linear programming models for the multinational firm. The interpretation of the conventional shadow prices was discussed in the deterministic case. We saw the danger of misinterpreting the shadow prices of cash flow constraints as the maximum interest rate on external borrowing which is still not unprofitable. I observed some aspects of evaluation of foreign profits in the multinational firm, in a discussion on sensitivity analysis concerning especially the objective function of the model. It was suggested that in some cases it is advisable to account for the restrictions set by strategic considerations, also through the objective function of corresponding tactical level models, and not only via the constraints. In discussing

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3For example, the developments of the recent past in exchange markets suggest that both physical and financial plans should be jointly determined. At times, the possibility of currency re- or devaluations and floating currency exchange rates warrant only hedging operations. At other times, the expected changes in exchange rates make it advisable to change production and trade flows between the affiliates of a multinational firm. Ideally, one should therefore consider both of these strategies, or a combination of them, using a single model.
forward contracting (in the probabilistic model), I noted that in assessing the profitability of a forward contract, it is not necessary to assign the contract to any particular export or import transaction (except for statutory reasons). I pointed out that in a centrally planned multinational firm the currency exposure of assets and liabilities need be calculated only for each currency held by the multinational firm as a whole. It is not necessary to compute the currency exposures separately for each affiliate, only for each currency. Furthermore, I suggested that by always raising foreign loans through the relevant affiliates the flexibility of operations of the multinational firm is enhanced, because the interest rates can then be manipulated.

In constructing the model the requirement of manageability was appreciated, although this point received little attention earlier in the text. The components of "manageability" are as follows: 1) A Model requires only obtainable input, 2) is solvable numerically, and 3) gives output which facilitates an implementation in actual decision-making situations. It is quite easy to see without further discussion that the model presented only requires input which is available readily enough in actual decision making situations. The construction of the model was carried out applying two-stage linear programming techniques. Thus the model could be retained in linear form even after the inclusion of the currency risk and the other new factors. This is a very important point when numerical solvability is considered. In real-life operations research applications, which involve factors and interrelationships in great numbers, linear programming still seems to be by far the most useful mathematical programming technique. Nevertheless, model size is still a critical factor,
which I have no intention of belittling. 4 The presentation of the model for the idealized multinational firm was supported by parallel fictitious numerical examples. These numerical examples also serve to show that appropriate operating budgets can be derived from the optimal solutions. Such budgets are the prime tools of managing the plans in firms. Thus the model presented clearly gives applicable output. - It was seen that the construction of separate equations for the formulation of the objective function and for the treatment of taxation makes the handling of the model easier. Actually, from my computational experience with even simplified numerical experiments I know that constructing an objective function, which originally involves sections in different currencies, is a numerical exercise that is extremely hazardous, without using auxiliary equations. 5

In the early chapters a fairly extensive inquiry was made into the background of the research problem and the limitations of the approach chosen.

A secondary result of the research work presented in this dissertation is in tentatively suggesting and discussing a procedure for creating hypotheses of the behavior of multinational firms, and in demonstrating

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4 Even allowing for the "coordinated decentralization" discussed e.g. in Dileep Mehta & Isik Inselbag, "Working Capital Management of a Multinational Firm," op.cit., pp. 73-75. See also the second paragraph in Section 2.1 of the dissertation in hand ("Limitations and Key Assumptions of the Study")

5 It is my guess that the neglect of this obvious and mathematically really trivial point is the cause of the prevalent erroneous or awkward handling of currency conversions and taxations in the models of a few researchers in this field. It is noteworthy that a parallel development can be found in the uninational case.
seven simple hypotheses arrived at with this procedure.

The results presented on "the behavioral problem" are mostly due to my earlier research efforts, which I have discussed elsewhere. These earlier results have now been checked with more extensive numerical experiments (not recorded explicitly in the text) and, in addition, the currency risk has entered into the considerations. The hypotheses presented are not new in literature, but they give an intuitive check on the plausibility of the approach to the behavioral problem.

7.2 Additional Extension Prospects of the Model and Directions for Further Research

It is my purpose in this last section to make three suggestions for research work in the research area discussed.

In considering the formulations which were presented for an incorporation of new dimensions into linear programming models for normative planning of physical and financial activities in the multinational firm, the following "true" simplifications were made.

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6 For the references see footnote 10 in Chapter 1.

One reason for bringing up the results also here is that it is to be hoped that in this way the tentative approach presented is submitted to a wider circle of readers for criticism.

The references in footnote 10 in Chapter 1 also contain a speculative discussion on the repercussions of the existence of the multinational firm on the theory of international trade, and on the effects of government actions. These points were referred to in footnote 15 of Chapter 2, but otherwise they were omitted, since the results are not concise enough to be presented here.

7 See page 38.
1) The trading currency in interaffiliate trade was not a decision variable. 2) The affiliates held in cash only the currency of the relevant host country. 3) The future impact of the currency exposure on the value of the multinational firm's assets and liabilities was included only via constraints limiting the currency exposures to fit into predetermined bounds. It seems that the first two items could probably be included in a relatively straightforward manner after defining the proper decision variables and assessing the proper interrelationships. The third item is conceptually more difficult. First an inquiry into stock and flow concepts would be needed together with an inquiry into handling currency risk in this case. I think that the most difficult part is probably keeping the model still linear.

Two-stage linear programming techniques were applied. An attempt could be made also at a chance-constrained formulation. This would be motivated, first, by the curiosity concerning the applicability of this approach into the problem, and, second, by a need to keep the size of the model as small as possible.

According to the discussion in the preceding two paragraphs the following suggestion for research work is made.

**Suggestion 1.** Consider linear programming formulations for planning physical and financial activities in the multinational firm. Find out how to include the interaffiliate trading currencies as decision variables, how to include multiple-currency cash in the affiliates, and how to evaluate the impact of the currency exposure via the objective function of the proper models. Make an inquiry into the applicability of a chance-constrained linear programming approach to the research problem.
Personally, I prefer to follow this suggestion should I continue my research efforts in accordance with these particular suggestions.

At the beginning of Chapter 3 the concepts of external and internal transfer prices were discussed. The internal transfer prices, which are traditionally used for evaluating divisional performance of profit centers in large firms, were omitted in this study. It was propounded that if the internal transfer prices are used in spite of the assumption of centralization in the relevant decision-making situation, they should be corrected to account for the centralized solution. This gives rise to the second suggestion for further research.

**Suggestion 2.** Consider how the internal transfer prices should be assessed in the multinational firm, when their purpose is to motivate the affiliates into making independent decisions which lead to optimal total planning of physical and financial activities in the multinational firm. Establish a functional connection between internal and external transfer prices.

Above we discussed further research primarily on "the normative problem". In the case of "the behavioral problem" the following three-point scheme for further research work is suggested.

**Suggestion 3.** 1) Create further and more particular hypotheses of production, book profit, and currency risk adjustments in the multinational firm. Cover the different structures of the multinational firm.\(^8\) 2) Check whether feasible initial values of the constants

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\(^8\) See Figure 2-2.
give plausible numerical ranges for the values of the decision variables. Include or exclude model details if necessary. 3) Subject the results to rigorous empirical testing as behavioral hypotheses of the multinational firm.

I once more point out that to my knowledge not enough formal work has been done in the particular area relating to descriptive models of the multinational firm, and, in addition, I venture to claim that a fully-fledged theory of the multinational firm still remains to be written, although there has been some good work done in this direction. In my opinion here is a promising area with many untapped possibilities for a researcher interested in multinational firms and empirical research work.

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9If the idealization made (see Section 1.4: "Research Approach and Methodology", esp. the behavioral problem) is not a good one in connection with the behavioral problem, the model must be revised accordingly.
REFERENCES


